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The Effectiveness of Concrete-Representational-Abstract Integration Framework within Word-Problem Solving for Students Experiencing Mathematics Difficulty

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**The Effectiveness of Concrete-Representational-Abstract Integration
Framework within Word-Problem Solving for Students Experiencing
Mathematics Difficulty**

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The University of Texas at Austin, 2023

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With my dissertation project, I investigated the efficacy of concrete-representational-abstract integration (CRA-I) framework alongside word-problem solving (WPS) schema instruction (i.e., CRA-I-Schema instruction) to teach WPS to students with mathematics difficulty (MD). I investigated whether participation in the CRA-I-Schema instruction led to improved WPS performance as compared to participation in Schema-Alone control condition (i.e., students only received schema instruction in WPS). I randomly assigned, blocking by grade level, 10 students to the CRA-I-Schema instruction condition and 12 students to the Schema-Alone condition. The intervention occurred 25 min per session for three to four sessions per week for 10 sessions. Results indicated the CRA-I-Schema instruction had a greater impact on students' *Understanding of Word Problems* ($g = 0.74$), *Word-Problem Solving* performance ($g = 0.56$), and *Addition Facts* ($g = 0.58$) relative to Schema-Alone students. Further, the pretest of *Understanding of Word Problems* significantly moderated the impact of the CRA-I-Schema instruction. Findings of a social validity survey revealed that the CRA-I-Schema instruction was satisfying and effective for the students. These results indicated that 1) conceptual understanding of WPS could significantly impact students' WPS performance; 2) integrating a CRA-I approach into WPS instructional strategies could benefit students' WPS performance; 3) it is imperative to

account for the diverse range of students' needs when implementing the CRA-I-Schema instruction in classroom.

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Chapter I: Introduction

Overview

Mathematics difficulty (MD) impacts students across mathematics content areas (Nelson & Powell, 2018). Students with MD have more difficulty than peers without MD in early numeracy skills such as counting, fact retrieval, mathematics fluency, place value, and understanding mathematics operation signs (Andersson, 2010; Cirino et al., 2015; Nelson & Powell, 2018). Similarly, for word-problem solving (WPS), students with MD, compared to their peers without MD, experience higher rates of difficulty in mathematics language and word-problem tasks (Arsenault & Powell, 2022; Kingsdorf & Krawec, 2014; Pongsakdi et al., 2020).

Mathematical WPS is one of the most important skills students need to be proficient on so they can solve every day mathematics problems (Sepeng & Sigola, 2013). WPS is related to students' future mathematics pathways and later learning achievements, career aspirations, and even future job performance (Dowker et al., 2016). For this reason, WPS represents a major emphasis in almost all mathematics standards from kindergarten to high school (NCTM, 2014). Unfortunately, the National Assessment of Educational Progress (NAEP, 2022) reported that 64% of fourth graders struggled to meet proficient levels in mathematics. Furthermore, students with MD experience greater levels of difficulty with WPS than students without MD (Kingsdorf & Krawec, 2014).

Significance of My Study

Over last two decades, a sizeable literature has begun to accumulate with an emphasis on helping students to solve problems in concrete-representational-abstract (CRA) framework (e.g., Flores et al., 2016; Flores et al., 2019; Flores et al., 2020; Hinton & Flore, 2019; Miller & Kaffar, 2011; Witzel et al., 2008). The CRA framework has also been studied on a wide variety of

mathematics outcomes, including number sense, mathematics facts, and computation.

Furthermore, some researchers have studied the effect of CRA within WPS instruction (Flores et al., 2016; Kim et al., 2015).

In one study, Flores et al. (2016) investigated the effect of using CRA within schema-based WPS instruction for three students. The author team found functional relation for three students' WPS performance in that they demonstrated the effectiveness of the combination of two evidence-based interventions (i.e., CRA framework and schema instruction), for students who demonstrated lower WPS performance with persistent error patterns. Nevertheless, one limitation Flores et al. (2016) stated was that the multiple-probe across students design (one type of single case design) prevented their study from generalizing beyond the three students in their study.

Kim et al. (2015) conducted a multiple-probe design across participants to determine the effects of fraction WPS instruction involving explicit teaching of the CRA framework for 3 low-performing English learners. In their study, all participants reached grade-level mastery on WPS and maintained skills after the intervention ended. Similarly, Kim et al. also emphasized one limitation of their study was that all intervention sessions were implemented in one-to-one instructional settings. It is possible that the findings may not translate to other classroom settings, including general education classroom or resource classroom.

Regarding the two limitations in Flores et al. (2016) and Kim et al. (2015), my study will employ randomized controlled trial (RCT), which is one type of group designs, to test the effect of CRA-I-Schema framework on students' WPS in a setting of small group. By conducting group design study, my study result will be more practical for generalization to other settings, such as resource room or other small group setting. In addition, I conducted a group design

study, which involved 22 students in both intervention and control groups. Even though it was under powered, I used this sample size to tentatively explore other variables that may moderate the effect of the CRA-I-Schema framework on WPS.

Powell (2011) conducted a literature review on WPS by using schema instruction, and she concluded that students profited from organizing word problems via schemas, and having explicit method for conceptualizing their solutions for each schema. In my study, the whole lesson will be also taught in an explicit method. Based on Powell (2011), there are multiple ways of organizing information when using schema solving word problems such as schematic diagram and mathematical equation. In current study, CRA-I approach will also improve the organization of word-problem solving by using the CRA-I chart to solve the computation part of word problems. So due to the more organized format of CRA-I-Schema intervention, I propose CRA-I-Schema intervention will improve the WPS performance more than Schema-Alone instruction.

Definitions

Mathematics Difficulty

In my study, I refer to students with mathematics disability or difficulty as students experiencing *mathematics difficulty (MD)*. In the literature, *mathematics disability* refers to a student with a school-identified learning disability with Individualized Education Program goals related to mathematics (Gena et al., 2022; Powell et al., 2020). *Mathematics difficulty* is defined as lower mathematics performance compared to peer, and this is typically determined by a teacher or via scores on a researcher-implemented assessment (Bryant et al., 2016; Geary et al., 2012; Navarro et al., 2012; Swanson, 2012). In my study, students will be defined as having MD if their test score is lower than 25th percentile of the *Mathematics Problem Solving subtest of the Stanford Achievement Test* (SAT-10, Pearson Inc., 2004).

Word-Problem Solving

Word-problem solving (WPS) involves using reading comprehension and vocabulary knowledge to capture the meaning of the text and identify the problem type (Cook et al., 2020; Kintsch & Greeno, 1985). Then, students need to set up and solve the problem, often by using computation. In this study, *word problems* refer to mathematics problems in which students read a short story problem that ends with a computational question about the story. For example, students may read a story about Tom and his dogs with the story ending, “*How many pets does Tom have in total?*” After reading this word problem, teachers and tutors may consider the following questions: (1) How do I help students understand the question? (2) What is the missing information in this question? (3) Which schema (e.g., Total, Difference, or Change) represents the conceptual understanding of the word problem? (4) Which visuals and tools could make it easier to understand the schema of the problem and support solving the problem? (5) Which computational strategies could be used to solve the problem?

Schema

A schema is a framework, outline, or plan for solving a problem (Marshall, 1995). In mathematics, students can use schemas to organize information from a word problem in ways that represent the underlying structure (i.e., concepts) of a problem type. Pictures or diagrams, as well as number sentences or equations, can be used to represent schemas. With schema instruction, students learn to identify word problems by schema and apply a specific strategy to the schema. In the early elementary grades, students learn three different schemas: Total, Difference, and Change (Fuchs et al., 2008). In Total problems, students combine parts together for a total. In Difference problems, students compare an amount that is greater and an amount

that is less to find the difference. In Change problems, students start with an amount, and the amount increases or decreases to a new end amount (Powell et al., 2020).

Concrete-Representational-Abstract Integration (CRA-I) Framework

The CRA framework is an instructional technique that entails presenting students with different representations of mathematics concepts (Lafay et al., 2019). The three components of the CRA framework involve the following: Concrete (C) involves learning with physical manipulative objects. Concrete manipulatives include but not limited to the following, pattern blocks, base ten blocks, tiles, cubes, fraction circles, square tiles, and plastic/paper money, etc., (Bouck & Flanagan, 2010; Bouck & Park, 2018). Representational (R) involves use of pictorial representations. It is modeled after concrete manipulatives, but they are delivered through a computerized tool or application. It has been defined as an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge (Bouck et al., 2018; Moyer et al., 2002). And Abstract (A) involves doing mathematics with abstract notation (e.g., Arabic numbers or symbols).

The CRA-I framework represents an adapted version of the CRA graduated instructional sequence. It incorporates the simultaneous presentation of the mathematics content through concrete manipulatives, sketches of the concrete manipulatives, and abstract notation (Strikland, 2017). These multiple representations help students develop both conceptual and procedural knowledge (Hudson & Miller, 2006). CRA-I framework may offer teachers with an effective method for progressing through the curriculum in a timely fashion while still providing effective instruction. Furthermore, incorporating the abstract explanation during mathematics computation may assist in the transition from concrete manipulatives to abstract understanding (Pashler et al., 2007).

Statement of Purpose

Woodward et al. (2012) provide recommendations for effective WPS instruction that are especially relevant to students who struggle in mathematics. Teachers should model the non-hidden and hidden problem-solving process. Students need to be taught how to use visual representations as well as learn how to articulate their problem-solving processes. Thus, interventions include components of visual representations, and problem-solving processes are critical to help students learn WPS.

The purpose of the study is to implement concrete-representational-abstract integration (CRA-I) framework alongside WPS schema instruction (i.e., CRA-I-Schema instruction) to teach WPS to students with MD. In my study, I used randomized controlled trial (RCT), and investigated whether participation in instruction with the CRA-I-Schema instruction leads to improved WPS performance for students in intervention condition compared to students in a Schema-Alone control condition (i.e., students only received schema instruction in WPS).

Research Questions

I addressed the following research questions: First, what is the efficacy of the CRA-I-Schema instruction compared to Schema-Alone instruction on a measure of WPS for students with MD? Second, what is the efficacy of the CRA-I-Schema instruction compared to Schema-Alone instruction on a measure of Addition and Subtraction for students with MD? Third, do effects differ based on students' prior knowledge of WPS? Fourth, what is students' attitude towards CRA-I-Schema instruction and Schema-Alone instruction?

Chapter II: Review of Literature

In this chapter, I will introduce the challenge of WPS for students with MD. Then, I will discuss WPS intervention research for students with MD. More specifically, I will present the related intervention research in CRA and schema interventions on WPS. Finally, I will state why it may be important to use CRA within schema instruction.

Students with MD

Only 3% to 8% of school-age students receive a formal identification of a learning disability in mathematics (Devine et al., 2013; Geary, 2004). Recent data indicate the percentage of students with disabilities who are proficient in mathematics is as low as 9% and only as high as 16% in eighth and fourth grades, respectively (U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, 2018). However, many students experience persistent, low mathematics achievement without an official learning disability diagnosis (Nelson & Powell, 2018). In this study, we focus on students with MD (i.e., mathematics difficulty) of which a few students may have an identified disability but many more do not.

Students with MD often experience difficulty in early numeracy, calculation, mathematics language, and WPS, which continue to cause problems for students with MD at the secondary level (Calhoun et al., 2007; Fuchs et al., 2008; Nelson & Powell, 2018; Kingsdorf & Krawec, 2014; Prediger et al., 2019; Stock et al., 2010; Wei et al., 2013). Compared to the other areas of difficulty, WPS proficiency requires substantially more conceptual and procedural knowledge (Kingsdorf & Krawec, 2014). Besides the difficulty in different mathematics areas, students with MD also demonstrate difficulty in cognitive skills such as planning and organizing

tasks, filtering relevant from irrelevant information, and regulating their attention (Li & Geary, 2013; Swanson et al., 2014; Watson & Gable, 2010). WPS requires the seamless blend of these mathematics skills and cognitive skills, and also other critical skills, such as reading ability and mathematical knowledge (Fung et al., 2017).

Word-Problem Solving

Solving word problems presents a major challenge for many elementary and middle-school students (Skinner & Cuevas, 2023). WPS often proves difficult because of the numerous steps involved in solving a problem from start to finish (Fuchs et al., 2014; Powell, 2011; Sharpe et al., 2014). In terms of WPS strategies, the most popular strategy reported by 70 elementary teachers in a study by Bruun (2013) was circling, underlining, or highlighting key information in the problem. Similarly, the second most popular strategy taught by elementary teachers was identifying key words (Bruun, 2013). Key-word strategies can be problematic and misleading, which may encourage students to focus on specific key words, and remove the full meaning of the problem (Karp et al., 2019). Powell et al. (2022) conducted an analysis of key words within word problems determined key words are only effective in about 50% of single-step word problems and in less than 10% of multi-step word problems. Therefore, the key word strategy is an ineffective word problem practice.

Several research teams have contributed to a literature base of effective strategies on improving WPS outcomes for students with MD. Evidence-based WPS strategies for students with MD include use of meta-cognitive strategies and graphic organizers (Case et al., 1992; Krawec et al., 2013; Swanson et al., 2014; van Garderen, 2007). Woodward et al. (2012) provided recommendations for effective problem-solving instruction that are especially relevant for students with MD. These recommendations suggest that teachers should model problem-

solving processes in addition to using visual representations that align with the problem-solving process. Over the last two decades, WPS research has focused on using schemas to solve word problems (e.g., Cook et al., 2020; Fuchs et al., 2019; Griffin et al., 2018; Jitendra et al., 2013; Peltier et al., 2020; Powell et al., 2021). Schemas have been reported as more effective than other techniques for teaching WPS to students with MD (Jitendra et al., 2015; Zhang & Xin, 2012). With schema word-problem instruction, students learn to identify word problems by schema and apply a specific strategy to the schema. In the next section, I will provide an overview of the research related to schema instruction.

Schema Instruction for Word-Problem Solving

Developing schemas for categorizing word problems proves beneficial for helping students identify novel problems as belonging to familiar categories (Ng & Lee, 2009). Schema instruction aims to improve students' ability to analyze the underlying structure of story problems and identify potential solution paths. The underlying structure is the concept(s) represented within the word problem. Schema instruction has been identified as an evidence-based instruction for WPS (Jitendra et al., 2015; Peltier & Vannest, 2017; Powell, 2011; Zhang & Xin, 2012).

Over the past several decades, schema instruction has been used with students with MD across grade levels (Fuchs et al., 2014; Fuchs et al., 2021; Jitendra et al., 2007). For example, Fuchs et al. (2021) tested the effects of word-problem intervention on 416 at-risk first graders' WPS performance. They also tested the efficacy of schema-based instruction including Total, Difference, and Change schemas at first grade, as part of the *Pirate Math* schema-based intervention. Their results revealed the efficacy of schema-based WPS intervention with students

receiving intervention outperforming students in a BAU comparison condition on WPS measures with an effect size advantage of 1.08 *SDs* over the BAU group.

In an example at Grade 3, Jitendra et al. (2007) randomly assigned 88 third-grade students to two conditions: schema-based instruction (SBI) and general-strategy instruction to assess the differential effects for WPS. In the SBI condition, students received SBI in solving one-step, and two-step Total, Difference, and Change word problems. From pre-to posttest, students in the SBI condition outperformed students in the general-strategy condition, with an ES of 0.52. The same measure, administered 6 weeks after posttest, showed students in the SBI condition outperformed general-strategy condition students (ES = 0.69). Therefore, their results revealed that SBI to be more effective than general strategy instruction in enhancing students' mathematical WPS at posttest and maintenance.

Also at Grade 3, Powell et al. (2020) examined the word-problem performance and strategies utilized by 3rd-grade students ($n = 111$) experiencing MD. They compared the word-problem performance of students with MD who received schema intervention ($n = 50$) to students with MD who received general education classroom instruction ($n = 60$). Their results demonstrated that students with MD who received the word-problem schema intervention outperformed students with MD who received general education classroom instruction. Findings in their study suggested students with MD benefitted from use of meta-cognitive strategies and explicit schema instruction to solve word problems.

Furthermore, for secondary students with MD, Jitendra et al. (2017) investigated the efficacy of schema-based instruction on proportional WPS for students in Grade 7. Students in the intervention group received SBI for the Ratio/Proportion and Percent schemas. They found the intervention group scored, on average, 0.25 *SD* higher than their counterparts in control

classrooms on a posttest and delayed posttest of word-problem solving administered nine weeks later. Thus, schema-based instruction could be effective instruction on WPS for both elementary and secondary students.

Schemas are an essential element in helping students develop WPS skills; however, students with MD require additional instructional scaffolding to facilitate the development of schema (Hwang & Riccomini, 2016; Jitendra et al., 2002). According to Pape and Tchoshanov (2001), multiple representations, including hands-on and visual representation which are important components in CRA framework, are critical to the development of students' mathematical thinking. Furthermore, Milton et al. (2019) investigated the effects of a place value intervention with third-grade students with learning disabilities by using multiple-probe-across-students design. The intervention using the CRA framework, and their results showed a functional relation between CRA and completion of items requiring place value understanding.

CRA Framework

The CRA framework involves the concrete, representational, and abstract phases for learning. The CRA framework has been consistently endorsed to benefit students with disabilities, particularly students with high-incidence disabilities (Maccini & Gagnon, 2000; Miller et al., 2011; Miller & Hudson, 2006; Montague, 2005; Witzel & Allsopp, 2007). Researchers have suggested CRA as a Tier 2 and Tier 3 intervention within the response-to-intervention framework (Agrawal & Morin, 2016; Doabler & Fien, 2013; Powell & Fuchs, 2015).

Systematic reviews on the use of CRA to improve mathematical outcomes for students with and without disabilities determined similar conclusion: CRA framework is effective at improving mathematical outcomes (Bouck & Park, 2018; Lafay et al., 2019; Peltier et al., 2020).

Bouck and Park (2018) conducted a systematic review of effect of manipulative-based instruction including CRA on students' mathematics outcomes. For CRA approach, they determined that participants improved in dependent measures (e.g., early number sense, computation, algebra, word problem solving) across the 16 single case, and 13 group design studies. The majority of the effect sizes represented strong effects for use of the CRA approach. Then Lafay et al. (2019) extended Bouck and Park (2018) regarding participants' disability and learning outcomes. Lafay et al. (2019) specifically target children with MD, and they also examined the effects of CRA interventions on students' maintenance and transfer outcome. Both Bouck and Park (2018) and Lafay et al. (2019) recommended the use of the CRA instructional sequence to practitioners. However, they both found that there is a lack of high quality group design CRA studies based on Gerston et al. (2005). Bouck and Park (2018) determined 2 out of 13 group design CRA studies met all but one essential quality indicators, and Lafay et al. (2019) evaluated 3 out of 16 group design CRA studies as high quality.

Researchers have also conducted substantial empirical studies on the effect of CRA intervention on students' mathematics performance. Kim et al. (2015) investigated the effects of fraction WPS instruction involving explicit teaching of the CRA framework for three low-performing Asian English learners. Their results indicated all participants reached grade-level mastery on WPS, and maintained skills after the intervention ended. Flores and Hinton (2019) conducted multiple baseline design study for the effectiveness of CRA-SIM (strategic instruction model) on multiplication, and WPS across three students receiving tertiary interventions. Their results showed in addition to mastery of multiplication with regrouping, students applied their knowledge to discriminating between different operations when solving word problems.

CRA-I-Schema Instruction and WPS

Based on the research grounding of CRA and schema instructions, I will combine these two instructions to form a more explicit and organized intervention study. I will provide more reasoning on the combination of CRA and schema in my next section. Researchers show that using the concrete-representational-abstract (CRA) framework with explicit instruction improves students' computational skills (Mancl et al., 2012; Miller & Kaffar, 2011). Furthermore, researchers also show that schema instruction increases students' WPS performance (Fuchs et al., 2021). In current study, I combine CRA and schema instruction. The combination of CRA and schema may help translate words into equations more easily, and it could make WPS process more visualized.

Few researchers have combined CRA and schema intervention in WPS. In one study, Flores et al. (2016) implemented CRA and schema instruction to three third-grade students with MD. Authors included part-part-whole, comparison, and change schema in their study. They detected a functional relation between CRA within schema instruction and students' WPS performance. This study successfully showed CRA-I-Schema instruction can improve students WPS when students received individual tutoring. No published group design studies have been implemented by using the CRA-I-Schema instruction so far.

Potential Moderators

Given that students' prior knowledge of simpler concepts and procedures is essential to acquiring new knowledge of more difficult concepts and procedures, I will explore the prior knowledge as a moderator. Prior knowledge is one of the student characteristics that can impact the effectiveness of mathematics interventions involving CRA (Lafay et al., 2019). Osana et al.

(2018) determined that second graders' prior knowledge of numeration was correlated with the students' learning about the base-four positional system in an intervention that involved CRA, and their prior knowledge was also correlated with the ability to transfer the conceptual structure to novel problems. In current study, students' prior knowledge of WPS was evaluated by using the pretest of *Understanding of Word Problems* and *Word-Problem Solving*, and I would like to investigate the relationship between students' prior knowledge and WPS outcome.

Rationale for Using CRA-I-Schema in My Intervention

Even though CRA framework includes three phases, which are concrete, representation, and abstract, all three phases should be connected instead of isolated (Strickland, 2016). Students' explicit connections across phases is essential for learning. Without connecting all three phases, students may feel they are memorizing separate procedures to solve the same mathematical skill (Witzel et al., 2008). Students are likely to fail to succeed at the abstract level of the mathematical skill if they did not recognize all three phases were connected. Thus, students should be asked to put abstract equation by the side when they are solving the problem in concrete or representation phase. By doing this, the teacher can help students to connect between phases (Flores et al., 2020).

In my CRA framework intervention, I will add abstract (A) phase to each of the other two phases, which are concrete (C) and representational (R). For example, when tutors model the process of solving the addition computation ($23 + 45 = ?$) by using concrete base ten blocks, tutors would write out equation by the side as they make the two addends (23 and 45). Tutors would say "*Now I will use base ten blocks to make the first addend 23. Firstly, I am making the number in the tens place, which is 2.*" After tutors create the number 2 with base-ten blocks, tutors would write "2" on the paper. Then tutors will say "*Now, I will make the number in the*

ones place, which is 3.” Similarly with the tens place, tutors would write “3” in the ones place. Tutors would emphasize the two-digit number “23” on the paper. Tutors would use the same dialogue for the second addend of 45. By doing so, students could connect meaning of the concrete manipulatives with the abstract understanding of the equation ($23 + 45 = ?$).

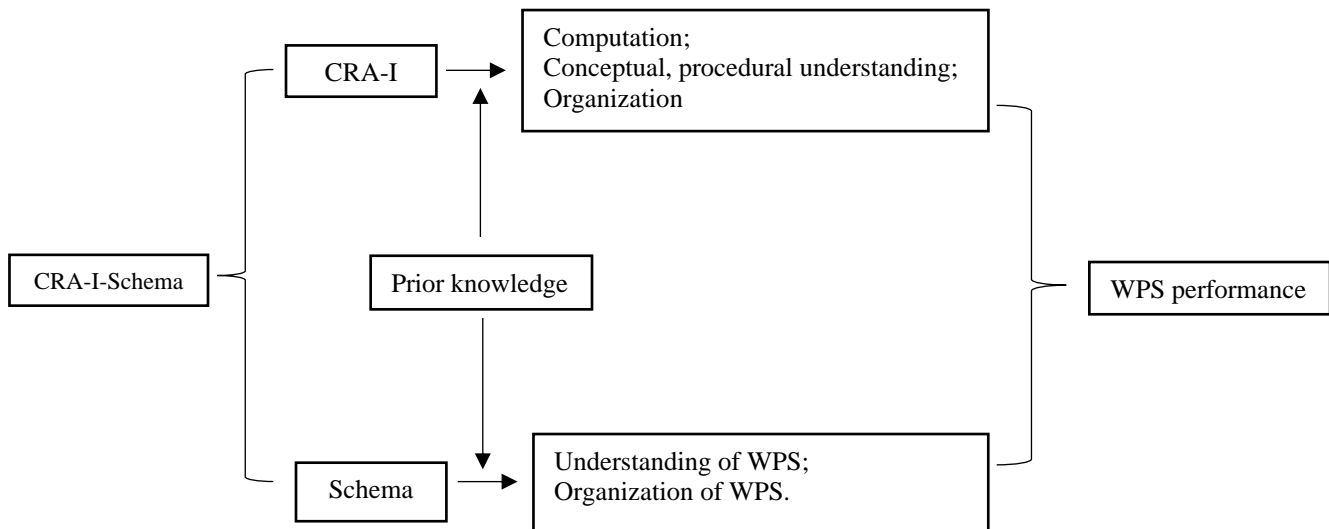
As aforementioned, schema instruction will be incorporated into the CRA framework in my study, which is named “CRA-I-Schema framework.” During the schema instruction, students will be taught the three schemas in word problems (Total, Difference, and Change; Powell, 2011). Often, word problems can be differentiated into types of problems that represent the conceptual background of the word problem. The problem type is determined by what is happening in the word-problem narrative. For example, students may be given the following information: “*There are seven red apples and four green apples.*” If students are asked, “*How many apples are there?*,” the problem type is Total because of the combining of the apples. If, however, students are asked, “*Five red apples were eaten, how many apples are left in the plate?*,” the problem type is Change because of the change in the number of apples. If students are asked, “*How many more red apples than green apples are there?*” the problem type is Difference because of the difference of red apples and green apples.

The schema instruction will happen before students start to represent the word problem using CRA framework. The key components of the CRA-I-Schema instructional lesson will include (1) teach the schema of Total, Difference, or Change in word problem, and (2) represent the word problem by using CRA framework. The hypothetical theoretical framework in my study has been illustrated in Figure 1. CRA may improve students’ computation performance, and schema instruction may increase students’ WPS performance. Then by combining these two effective strategies, I assume the CRA-I-Schema framework will be able to help students

facilitate a better conceptual understanding on WPS, than students who have not received the CRA-I-Schema instruction. In the theoretical model, I am also interested in examining if students' pretest performance (i.e., prior knowledge) moderates the effectiveness of CRA-I-Schema instruction.

Figure 1

Theoretical Framework



Note. CRA-I-Schema is the investigated independent variable, which has two essential components: CRA-I and schema; Prior knowledge is served as moderator of the effect of CRA-I-Schema on WPS performance; WPS performance is a dependent variable.

Chapter III: Method

In this dissertation, I investigated the efficacy of implementing CRA-I-Schema instruction for students with MD to improve their WPS skills. Additionally, I delved into the potential influence of students' initial WPS performance as a moderating factor on their response to the intervention. Finally, I examined students' satisfaction and acceptability of CRA-I-Schema and Schema-Alone instruction.

Participants

Teachers

I recruited Grades 2, 3, and 4 mathematics teachers from a school for students with dyslexia and mathematics difficulty in the Austin, TX area. The students in the classrooms ($n = 88$) of these recruited teachers were eligible for participation in the study. I sent out parents' consent forms to all the 88 students, and 36 students of parents signed the consent forms. Teachers were asked to fill out all students' demographic information. Classroom teachers received \$25 gift cards for their participation in this study.

Participating Students

To be eligible for my intervention study, students had to be current Grade 2, 3, or 4 students enrolled in the participating school. I applied to secure approval by the Institutional Review Board (IRB) at The University of Texas at Austin. I asked the teachers of the participating students to send home the caregiver consent/student assent forms. To participate this study, caregivers consented for their student's participation in the study, and students gave assent for study participation.

To qualify for participation in the study, students had to experience MD in WPS. According to a teacher survey regarding students' mathematics performance, 36 students among those returned consent and assent forms were experiencing difficulty with WPS. Then, I randomly assigned, blocking by grade level, 18 students to participate in CRA-I-Schema Instruction and 18 students to participate in Schema-Alone Instruction. By the end of the intervention, 14 students dropped out of the study due to different reasons as stated in next paragraph. In Table 1, I provide the demographic information for the 22 students who completed intervention and posttesting.

Students Attrition

I started working with 36 participants in the beginning of the study, however 14 students dropped out of the study because of various uncontrolled reasons. That is, 5 students left the study due to their after-school schedule change, and 4 students left the study because of family issue. One student was expelled by the school as a consequence of consistent school rule violation. Another 2 students left the study without giving specific reason. An additional 2 students did not attend the posttest session, therefore, I was not able to collect the posttest data from these 2 students. Hence, my final number of participants was 22 by the end of the 10-session intervention.

Table 1

Overall Participants Demographics Based on 22 Fully Participating Students

	CRA-I-Schema Instruction (N = 10)		Schema-Alone Control (N = 12)	
	<i>n</i>	%	<i>n</i>	%
Gender				
Male	4	40	5	42
Female	6	60	7	58
Race				

African American			2	14
White	8	80	9	64
Hispanic/Latino	2	20	2	14
Other			1	7
Grade Level				
Second	4	40	2	17
Third	3	30	6	50
Fourth	3	30	4	33
Singapore Math Level				
Below Grade Level	9	90	11	92
Equal to Grade Level	1	10	1	8
ELL Status	2	20		
WPS difficulties	10	100	12	100

Note. ELL = English Language Learner

Tutors

A team of three graduate research assistants (GRAs) and I conducted all pre- and posttesting. Together, we also conducted the intervention and review sessions (10 sessions). The other three GRAs were seeking a Ph.D. degree in special education at The University of Texas at Austin.

Research Design

With my dissertation study, I used a randomized controlled trial. As described, I randomized 36 students with MD with 18 students randomly assigned the CRA-I-Schema instruction group and 18 students to Schema-Alone instruction group (i.e., control group). By the end of the intervention, I had 10 students remaining in the CRA-I-Schema instruction group, and 12 students in the Schema-Alone control group. I conducted a priori power analysis by using G*Power 3.1.9.6 to evaluate whether the sample size was sufficient to detect an average effect size of 0.95 in this study. The effect size of 0.95 was calculated in a previous meta-analysis analyzing the efficacy of experimental intervention studies on WPS (Zheng et al., 2013). Software G*power was used for a *t*-test with alpha = 0.05, effect size = 0.95, and power = 0.8

requires $n = 38$. In my sample, the sample size remaining by the end of the intervention was 22. Based on G*Power, given sample size of 22, the power for this study is 0.73.

Research Questions

I asked the following research questions:

1. What is the efficacy of the CRA-I-Schema instruction compared to Schema-Alone instruction on a measure of WPS for students with MD?
2. What is the efficacy of the CRA-I-Schema instruction compared to Schema-Alone instruction on a measure of Addition and Subtraction for students with MD?
3. Do effects differ based on students' prior knowledge of WPS?
4. What is students' attitude towards CRA-I-Schema instruction and Schema-Alone instruction?

Materials and Procedures

Intervention Schedule

For the 10 students in the CRA-I-Schema instruction and the 12 students in the Schema-Alone instruction, I created a tutoring schedule in which they learned how to set up and solve Total, Difference, and Change word problems over 10 sessions. Each session lasted approximately 25 min in both CRA-I-Schema condition and Schema-Alone condition. Table 2 provides an overview of the calendar for the spring of 2023. Table 3 provides detail about which WPS schemas were taught during each lesson. Tutors delivered small-group instruction with 3-5 students in each group. These tutoring sessions were implemented after the school day was over in quiet classrooms of the school.

Table 2

Timeline for Intervention Implementation

Month	Activity
Early April	<ul style="list-style-type: none"> • Screening
April 17-21	<ul style="list-style-type: none"> • Pretesting (1 session) • First week intervention implementation (Lessons 1, 2, 3)
April 24-28	<ul style="list-style-type: none"> • Second week intervention implementation (Lessons 4, 5, 6, 7)
May 1-4	<ul style="list-style-type: none"> • Third week intervention implementation (Lessons 8, 9, 10) • Posttesting (1 session)

Table 3

CRA-I-Schema and Schema-Alone Lesson Overviews

Lesson	Days	CRA-I-Schema	Schema-Alone	Schemas
Instruction	1	C+R	A	Total
	2	C+R	A	Total
	3	C+R	A	Difference
	4	C+R	A	Difference
	5	C+R	A	Change
	6	C+R	A	Change
	7	R+A	A	Total
	8	R+A	A	Difference
	9	R+A	A	Change
Review	10	C+R+A	A	Total, Difference, and Change

Note. C = concrete manipulatives; R = representational drawings; A = abstract.

Word Problems

In the two conditions, students worked on the same word problems. It could have been a Total, Difference, or Change word problem with or without regrouping. For Total problems, students worked on Total problems with an unknown total. Students learned that Total problems involve parts put together for a total. For example: *“There are 23 bananas, and 35 apples on the dining table. How many fruits are on the dining table?”*

For Difference problems, students worked on Difference problems with an unknown difference. Students learned that Difference problems have a greater amount and a less amount that are compared for a difference. For example: *There are 23 bananas, and 35 apples on the dining table. How many more apples are there on the dining table than bananas?*

For Change problems, students learned about Change problems with an unknown end amount. Students learned about Change problems that have a start amount that either increases or decreases to a new end amount. For example: *There were 23 bananas, and 35 apples on the dining table. 4 apples are eaten by Jason. How many apples are left on the dining table now?*

Difference Between CRA-I-Schema instruction and Schema-Alone instruction

The primary difference between CRA-I-Schema instruction and Schema-Alone instruction was that students in CRA-I-Schema used concrete manipulatives and representational drawings to assist with their computation within word problems. Table 3 provides an overview of the CRA phases implemented in each of the 10 intervention sessions. Schema-Alone students learned how to solve word problems using only the abstract method.

Daily Intervention Activities

In the next sections, I describe daily activities that occur within the CRA-I-Schema and Schema-Alone instruction. In general, for each instructional lesson, students in the CRA-I-Schema and Schema-Alone conditions participated in 5 activities (a) advance organizer; (b) modeling; (c) guided practice; (d) independent practice; and (e) post organizer. During the review lessons, students were assigned the same five activities, but they were more self-guided. They were encouraged to model for one another, and the tutor offered immediate feedback as necessary.

Figure 2 Example Lesson Script for CRA-I-Schema Instruction.

LESSON 1 SCRIPT (CRA-Schema)

Advance Organizer (3mins)

T: Hi everyone, I am so happy to have the chance working with you in the next few weeks. Before I start to introduce our lessons, let me introduce myself. I am XX, a graduate student in University of Texas at Austin. My research is about how to help students to solve mathematics problems (change as appropriate). Now, tell me your name and one thing you like about math. (You are welcome to give students some examples: for example, addition, subtraction, multiplication, division, fraction, word problem, geometry, or anything else?)

T: Thank you, every one. It is nice to know all of your favorite things in math. Over the next few weeks, we're going to work on math word problems. Who can give me an example of word problem?

T: Excellent. Word problems are important for your daily life, or for your future employment. We will learn how to solve word problems. Today, we will learn about the problem type, and use it to solve word problems. Let's say "Total" together.

T: In a Total problem, we have part 1 and part 2 put together for a total.(also demonstrate with gesture.) Let's say that, "Part 1 and part 2 put together for a Total."

T: Here is the poster for Total problem type. Remember, that's parts put together for a Total. I can use this Total equation to solve Total problems. The equation is $P1 + P2 = T$. Let's say that together.

TOTAL

1. Write $P1+P2 = T$
2. Find **P1** or **P2**
3. Find **T**

T: P1 means part 1. P2 means part 2. What do you think the T means?

S: Total.

T: That's right. T means Total. We may have a question with "Total" missing or "P1 or P2" missing. And we will work on them later.

T: Now, let me introduce concrete base ten blocks to you, which is a type of manipulatives. (point to flats, rods, and ones of concrete base ten blocks to students). It has flats, rods, and units. One flat represents one hundred, one rod represents one ten, and one unit represents one.

T: What does one flat represent?

T: What does one rod represent?

T: What does one unit represent?

T: Then, there is another type of base ten blocks, which we can draw them on the paper. But we cannot actually touch it. We call them representational base ten blocks. I will draw one big squares to represent 100s. Draw a large square on your paper to show 100s.

S: (Draws.)

T: I will draw vertical lines to represent 10s. So one vertical line is one 10. Let's draw some tens together.

T: And I will draw circles to represent ones. One circle means one 1.

T: And this is place value mat. It has three columns, hundreds, tens, and ones. What are the columns?

S: Hundreds, tens, and ones.

T: When we make numbers by using concrete base ten blocks, we can put the base ten blocks in the corresponding columns. We will practice to make a number by using the concrete base ten blocks later. If we make numbers by using representational base ten blocks, we can draw squares, lines or circles to make numbers.

Modeling (6 min)

T: Let me give you one example of word problem in real life, and show you how to solve it.

Tasha and Mike went to the supermarket. Tasha bought 12 bottles of milk. Mike bought 7 bottles of juice. How many bottles of drink did Tasha and Mike buy together?

T: Now we need to think of a way to solve the word problem. Now, in this question, it is asking how many bottles of drink Tasha and Mike bought. So the question words are *how many bottles of drink*. I will circle them.

T: Then we can see Tasha bought 12 bottles of milk, and Mike bought 7 bottles of juice. Both milk and juice are counted as drinks. So there are two parts in this question. How many parts?

S: Two

T: And the question asks how many bottles did Tasha and Mike buy together. This asks about the total number of bottles. All this information tells me this is a Total problem. What kind of problem?

T: And it's a Total problem because we have two parts – milk and juice – that are put together for a Total. Why is it a Total problem?

T: The 1st part is *bottles of milk* Tasha bought, and the 2nd part is *bottles of juice* Mike bought. Let's mark them in the question (Circle bottles of milk, then mark as p1. Circle bottles of juice, then mark as p2)

T: Then the question is asking how many bottles in total. We will use this equation " $P1+P2 = Total$ " (Tutor show the poster to students again) to solve this word problem.

T: I can also use my hands to show a Total problem. See my hands. "I have part 1 in my left hand, and part 2 in my right hand. Then, I put two hands together to get the total." Can you follow my gesture and say it together with me?

T: Great job. Now let's put the total equation under the question: $P1+P2 = T$. What did I just circle for P1? What did I just circle for P2? So we plug in numbers to this equation. Now we get: $12+7 = T$. We just need to solve for T. What do we need to solve for?

T: As I said, we will also use base ten blocks to solve the word problem. Now, we already know which equation we are supposed to use. Let's take out the place value mat and concrete base ten blocks. I will model this for you. Pay attention. I will ask you to do it later.

T: Before we start to use base ten blocks to make numbers. Let's put the vertical form of the equation $12 + 7$ in the equation column. And make sure numbers in the ones place are lining with each other. Numbers in the tens place are lining with each other. Also add the plus sign.

T: We can use concrete base ten blocks to solve it. Firstly, let's make the number of 12. We start from the ones place. What is the number in the ones place?

T: Great job. I am going to put 2 units in the ones column in the place value mat. But we are not

done yet. This is two-digit number. So we also need to make the number in the tens place. What is the number in the tens place?

T: Yes, it is 1. But 1 represents one ten when it is in the tens place. In base ten blocks, which represent one ten?

T: That's right. We need 1 rod. So we put 1 rod in the tens column.

T: Now, we need to make the number for part 2. What is the number for part 2?

T: Nice thinking. Is 7 single digit number or double-digit number? So we only need 7 units to represent the number 7.

T: Now since we have made all numbers in both part 1 and part 2, we will need to add them together. Let's start from ones place. Let's count together how many units in the ones column.

T: It is 9. So we put the number 9 in the ones place. Then I will also write my number 9 in the vertical equation.

T: Now, we start to add up for tens column. How many rods do we have? Yes, correct. We have 1 rod in tens column. 1 rod means one ten. Then, we write 1 in the tens place. Then I also need to write 1 in the tens place of my vertical equation. So what is my final answer? Yes, I will plug 19 in the equation. Is it 19 cans? It is 19 what? Yes, it is 19 bottles. So make sure your put the unit for the number 19.

T: Now, let's check on our answer. 19 is greater than 12 and 7. So we say it is correct. In the end, let's write the final answer sentence. "Tasha and Mike bought 19 bottles of drink together."

Instruction Lesson Activities

During each session, tutors used explicit instruction to provide an advance organizer (verbally describe the task), demonstration (the tutor modeled the process), guided practice (the tutor and the students solved the problems together), independent practice (the students solved the WPs (word problems) without tutor's assistance), and a post-organizer (review task). For CRA-I-Schema instruction group, I describe the activities for concrete and representational lessons, representational and abstract lessons, and review lesson. For Schema-Alone control group, I describe the activities for abstract lessons.

Lessons 1-6

Advanced Organizer

As described, each lesson had five components. The first component was a 3-min advance organizer, in which tutor activated students' prior knowledge, and briefly introduced new information. First, tutor gave a real-life example for the *Total* WP. For example, the tutor said "If you have 7 pencils, and your friend has 8 pencils, how many pencils do you and your friend have together?" "This is a *Total* word problem. *Total* means I put parts together" Also, tutor demonstrated the *Total* schema by using gesture as in Pirate Math Equation Quest (<https://piratemathequationquest.com/group.html>). The tutor said: "In a Total problem, we have part 1 and part 2 put together for a *Total* (also demonstrated Total with a gesture.) Let's say that, "Part 1 and part 2 put together for a *Total*." Tutors also needed to show the Total equation and explain the meaning of each part of the equation. For example, tutor would say that "*P1 means part 1, P2 means part 2, and T means Total.*"

After that, for CRA-I-Schema group, the tutor started to introduce concrete base ten blocks or representational base ten blocks. Here, I use concrete base ten blocks as an example.

The tutor said “These are base ten blocks, and we will use them to solve addition word problems today.” Tutors also introduced what a flat, rod, and unit represented in concrete base ten blocks, and explained how to use place value mat (see Figure 3 and Figure 4).

For Schema-Alone control group, tutors also introduced Total, Difference, or Change schemas as in CRA-I-Schema instruction group, and they reviewed several addition mathematics facts with students. Tutors asked: “*What is 5 plus 8?*” Then tutor reviewed several more questions with students by using a “count up” strategy. As lessons proceeded, tutors reviewed more complex two-digit and two-digit addition and subtraction with and without regrouping questions by encouraging students to use the standard algorithm method.

Figure 3

Cubes, Flats, Rods, and Units in Concrete Base Ten Blocks

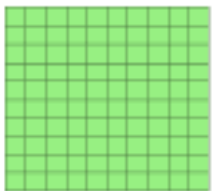


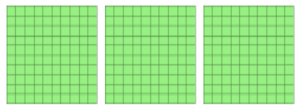
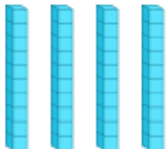

Hundreds	Tens	Units/Ones
Flat	Rod	Unit
		
100	10	1

Figure 4

Place Value Mat

Flats	Rods	Units
		

Modeling

The second component was 6-min modeling. The first step was for tutor to demonstrate how to identify the schema of the WP. The tutor showed students a two-digit and two-digit Total WP. For example: “*There are 23 bananas, and 35 apples on the dining table. How many fruits are on the dining table?*” The tutor started to read the problem, and underlined the question sentence of the problem. The tutor identified the schema of the WP and explained to students why it is a *Total* schema. Then tutor showed gesture of *Total* schema (i.e., “I have one part in my left hand, and a second part in my right hand. I put them together to get a *total*”). Then the tutor explained which was the first part (23 bananas) and second part (35 apples).

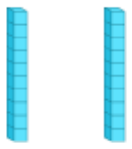
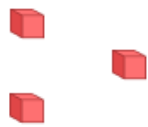
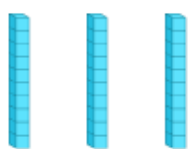
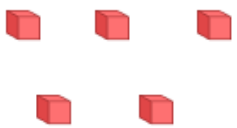
Then, starting from here, the modeling became different between CRA-I-Schema and Schema-Alone groups. For the CRA-I-Schema group, tutors demonstrated how to represent the WP with the place value mat by using concrete base ten blocks or representational base ten blocks. Here is an example of using concrete base ten blocks. The tutor said: “*We have known that there are two parts in this Total WP. The first part is banana, and the second part is apple. Now we need to use concrete base ten blocks to calculate the Total number of fruits.*” Then the tutor showed students the place value mat and also wrote the vertical equation “23 + 35” by the

side of the place value mat. A pictorial example of the procedure that a tutor used to solve a problem at the concrete level is shown in Figure 5. While the tutor represented the value of the ones place, the tutor pointed to the number they made. And once the tutor finished solving the addition in the ones place or tens place, the tutor wrote the sum in the vertical equation. By doing this, tutor showed students the connection between the abstract solution and the concrete method. For third step, after solving the problem, the tutors needed to check the answer. The tutors might have said: “Our answer is 58, 58 is greater than 23 or 35. So my answer is correct.” In the end, tutors needed to write, and speak aloud the answer sentence “There are 58 fruits on the dining table. Let’s say that together.” For the representational base ten blocks demonstration, tutors followed a similar procedure, but tutors used drawings on the place value mat to represent numbers instead of concrete base ten blocks.

For Schema-Alone control group, tutors identified the WP schema and explained why to use that schema using the same way as in CRA-I-Schema instruction group. Then tutor wrote the abstract equation $P1 + P2 = T$, and tutor explained to students what was P1 and P2. Then tutor circled P1 and P2 in the question, and substituted the numbers of 23 and 35 into the Total equation “ $P1 + P2 = T$ ” To solve the addition problem, tutors demonstrated how to use a traditional algorithm to solve the equation. Tutor lined up the two numbers of 23 and 35 vertically and added up from ones digit to tens digit. While tutor solved the computation, the tutor explained to students what she was doing.

Figure 5

Solve “There are 23 bananas, and 35 apples on the dining table. How many fruits are on the dining table?” by Using Concrete Base Ten Blocks.

Abstract Equation	Hundreds	Tens	Ones
$\begin{array}{r} 23 \\ +35 \\ \hline 58 \end{array}$			
			

Guided Practice

The third component was 5-min guided practice, in which tutors guided students to solve a similar word problem as in the modeling. In this activity, tutors gave instant feedback while students solved the word problem. In the first few lessons, tutors needed to guide students step by step. Tutor said “*What is the first step to solve this WP?*” Then, students solve the problem with the tutor. As students got more familiar with the lesson content, the tutor led the students, and encouraged students to solve the problem alone. This was helpful for students transitioning to independent work. In general, tutors and students worked together in this guided practice activity.

Independent Practice

The fourth component was 6-min independent practice, where students led themselves to solve the WP. In this activity, tutors also encouraged peer students to guide each other, but gave feedback whenever needed. Tutors provided two or three WP for the independent practice, however, students were not required to complete all of them because of time constraints. For

CRA-I-Schema group, students used either concrete or representational base-ten blocks to solve the WP. For Schema-Alone group, students were only allowed to solve the WP by using traditional algorithm.

Post Organizer

The last component was a 5-min post organizer, during which tutor reviewed lesson content with students. Tutor provided students a worksheet (See Table 4-1 for review lessons for CRA-I-Schema instruction group. Table 4-2 shows the review lessons for Schema-Alone control group). Students were given 3 min to finish questions on the worksheet. Then tutors reviewed the answers with students in the last 2 min. After that, tutors gave a brief preview for the next lesson.

Table 4-1

Review Questions for CRA-I-Schema Instruction Group

Question	Your Answer	Grade (1 pt for each)
1. Write the number made by tutor. (43)		
2. Make the number 78 using concrete base ten blocks. Ask the tutor to see your work after you are done.		
3. Solve the equation “ $23 + 4 =$ ” by using either concrete or representational base ten blocks.		
4. What is the equation for Total word problem?		
5. We can use Total equation to solve this word problem. “ <i>Katie bought 8 pairs of socks. She gave 2 pair of socks to her sister. How many pairs of socks does Katie have?</i> ”	True or False	

Table 4-2

Review Questions for Schema-Alone Control Group

Mark has 42 toy cars and his sister has 7 toy cars. How many toy cars do they have in total?
Kate donated 46 pencils to purple sage elementary school, and her brother donated 3 erasers to the same school. How many school supplies did Kate and her brother donate to purple sage elementary school?

Lessons 7-9





For the CRA-I-Schema instruction group, representational and abstract lessons (7-9) also used explicit instruction principles throughout the whole lesson. However, tutors started to fade out the concrete manipulatives, and gradually transitioned to representational and abstract methods. Tutors mainly used representational and abstract instructions for modeling activities. So, in these lessons, students were taught how to solve WP by using the schema and representational and abstract strategies. Thus, tutors used representational manipulatives (i.e., drawings) instead of concrete manipulatives to teach the whole lesson. In this study, tutors used vertical lines to indicate *rods*, and circles to represent *units*. In Figure 6, it shows how the representational manipulatives represent the equation “23+35.”

While lesson 9 was initially intended to be representational and abstract lesson, a significant number of students were ready for the abstract approach. Consequently, the majority of the time was devoted to tutors offering instructions on solving problems by using the abstract. Nevertheless, tutors continued to incorporate representational modeling into their instruction, tailoring their approach to individual students’ performance.

For the Schema-Alone control group, during lesson 7-9, students continued to solve Total, Difference, and Change word problems by using the abstract method. These lessons also followed an explicit teaching format, but students were not instructed or allowed to use any types of manipulatives throughout the whole lesson. In lessons 7-9, students encountered more complicated calculations involving regrouping. Therefore, they could use some strategies assisting with calculation (e.g., count-up or count-down strategies.)

Figure 6

Solve “There are 23 bananas, and 35 apples on the dining table. How many fruits are on the dining table?” by Using Representational Base Ten Blocks.

Abstract Equation	Hundreds	Tens	Ones
$\begin{array}{r} 23 \\ +35 \\ \hline 58 \end{array}$			
			

Review Lesson (Lesson 10)

In the final review lesson, tutors reviewed mathematics content students had learned in the past 9 lessons. Together, they reviewed different WP schemas, gestures of schemas, representation of WPs with concrete and representational manipulatives (CRA-I-Schema instruction group), and solving WP by the abstract (CRA-I-Schema instruction group and Schema-Alone control group). During the review lesson, tutors demonstrated the WP process to students whenever they encountered difficulties.

Training of Tutors

I trained the tutoring team on pretesting and posttesting, as well as using the lesson guides and lesson materials. The other three tutors also completed reliability checks with me for any measure that was administered to students. In the 3-hour in-person training, tutors became familiar with lesson guides and lesson materials. Before each tutor’s first tutoring session, tutors

completed a pseudo session with me. I provided immediate feedback for each tutor, and I sent them their fidelity checklist. Tutors met with me at the end of first week and at the end of the third week to discuss tutoring and resolve any teaching or behavior problems they encountered.

Measures

In this dissertation project, the research team collected data for pretesting, posttesting, and progress monitoring. The research team also collected demographic data about the participating students' gender, race/ethnicity, grade level, Singapore math level, dual-language status, and difficulty in mathematics.

Pretesting and Posttesting

The pretest battery consisted of six assessments, including *Understanding of Word Problems*, *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, *Two-Digit Subtraction*, and *Word-Problem Solving*. See Figures 7-12 for each of the measures. The posttest included all of the assessments as at pretest. However, I rearranged the questions in the posttest to prevent students from answering based on memory.

The *Understanding of Word Problems* test (Assessment #1) (Figure 7) tested students background knowledge of word-problem solving, schemas, and manipulatives. It included 5 questions, and students had 5 min to answer the questions. Tutor read the questions for students aloud. Each correct response was given 1 point with a total of 5 points. This test was developed by the author, and Cronbach's alpha was calculated as 0.78.

Addition Facts (Assessment #2) and *Subtraction Facts* (Assessment #3) (Figure 8 and 9) included mathematics single-digit addition and subtraction questions (Fuchs et al., 2003). The *Addition Facts* measure was comprised of 25 problems with sums from 0 to 18, and the *Subtraction Facts* measure was comprised of 25 subtraction fact problems with minuends from 0

to 18. These tests evaluated students' accuracy and fluency of foundational math facts, and they had 1 min to finish each of them. Each correct response was awarded 1 point. Cronbach's alpha was 0.87 for single-digit addition and 0.84 for single-digit subtraction.

Two-Digit Addition (Assessment #4) and Two-Digit Subtraction (Assessment #5) (Figure 10 and 11) involved two-digit and two-digit addition and subtraction computation problems with or without regrouping (Fuchs et al., 2003). They had 3 min to complete 20 problems on each of the two tests. Each correct response was given 1 point. Cronbach's alpha for *Two-Digit Addition* was 0.96, and the Cronbach's alpha for *Two-Digit Subtraction* was 0.87 (Powell et al., 2015).

Word-Problem Solving (Assessment #6) (Figure 12) concerned word problems with different types of schemas, including *Total, Difference, and Change*. There were 10 questions in assessment #6, and students had 15 min to complete assessment #6. Each question was graded based on students' understanding of word problems, and also based on computation. The grading rubric for *Word-Problem Solving* test is in Figure 13. The *Word-Problem Solving* test was modified from Powell et al. (2015), in which the Cronbach's alpha was 0.87.

Figure 7

Assessment #1, Understanding of Word Problems. (5min)

Questions	Circle or write your answer
1.What is an example of word problem?	Write your answer:
2.What is the type of this word problem? “Ms.Lopez bought 3 boxes of crayons for art class, and she bought 7 boxes of erasers for math class. How many boxes of crayons and erasers did Ms.Lopez buy in total?”	Circle your answer: A.Total. B.Difference. C.Change.
3.What is the type of this word problem? “David had 9 pens. Then, he gave 6 pens to his friend. Now, how many pens does David have?”	Circle your answer: A.Total. B.Difference. C.Change.
4.What is the type of this word problem? “Tommy has 14 blue shirts, and 34 red shirts. How many fewer blue shirts does Tommy have than red shirts?”	Circle your answer: A.Total. B.Difference. C.Change.
5.Have you used any types of manipulatives (for example, toys or objects) to solve any types of problems? List all that you know.	Write your answer:

Figure 8

Assessment #2, Addition Facts(1min).

$\begin{array}{r} 8 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ + 7 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ + 9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ + 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ + 2 \\ \hline \end{array}$
$\begin{array}{r} 4 \\ + 8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ + 9 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ + 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ + 1 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ + 7 \\ \hline \end{array}$
$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ + 6 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ + 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ + 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ + 9 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ + 8 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ + 5 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ + 7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ + 4 \\ \hline \end{array}$
$\begin{array}{r} 9 \\ + 7 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ + 5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ + 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$

Figure 9

Assessment #3, Subtraction Facts(1min)

$\begin{array}{r} 12 \\ - 8 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ - 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ - 6 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ - 7 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ - 8 \\ \hline \end{array}$
$\begin{array}{r} 16 \\ - 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ - 3 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ - 2 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ - 3 \\ \hline \end{array}$
$\begin{array}{r} 18 \\ - 9 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ - 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ - 4 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ - 5 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ - 4 \\ \hline \end{array}$
$\begin{array}{r} 17 \\ - 8 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ - 7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ - 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ - 6 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ - 8 \\ \hline \end{array}$
$\begin{array}{r} 10 \\ - 4 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ - 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ - 0 \\ \hline \end{array}$	$\begin{array}{r} 16 \\ - 9 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ - 1 \\ \hline \end{array}$

Figure 10

Assessment #4, Two-Digit Addition (3 min).

$\begin{array}{r} 32 \\ + 22 \\ \hline \end{array}$	$\begin{array}{r} 20 \\ + 77 \\ \hline \end{array}$	$\begin{array}{r} 35 \\ + 37 \\ \hline \end{array}$	$\begin{array}{r} 58 \\ + 29 \\ \hline \end{array}$
$\begin{array}{r} 68 \\ + 23 \\ \hline \end{array}$	$\begin{array}{r} 27 \\ + 49 \\ \hline \end{array}$	$\begin{array}{r} 33 \\ + 50 \\ \hline \end{array}$	$\begin{array}{r} 74 \\ + 22 \\ \hline \end{array}$
$\begin{array}{r} 30 \\ + 23 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ + 18 \\ \hline \end{array}$	$\begin{array}{r} 37 \\ + 52 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ + 47 \\ \hline \end{array}$
$\begin{array}{r} 39 \\ + 29 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ + 62 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ + 38 \\ \hline \end{array}$	$\begin{array}{r} 72 \\ + 20 \\ \hline \end{array}$
$\begin{array}{r} 42 \\ + 57 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ + 35 \\ \hline \end{array}$	$\begin{array}{r} 29 \\ + 28 \\ \hline \end{array}$	$\begin{array}{r} 42 \\ + 32 \\ \hline \end{array}$

Figure 11

Assessment #5, Two-Digit Subtraction (3 min).

$\begin{array}{r} 83 \\ - 52 \\ \hline \end{array}$	$\begin{array}{r} 90 \\ - 27 \\ \hline \end{array}$	$\begin{array}{r} 59 \\ - 32 \\ \hline \end{array}$	$\begin{array}{r} 88 \\ - 49 \\ \hline \end{array}$
$\begin{array}{r} 93 \\ - 55 \\ \hline \end{array}$	$\begin{array}{r} 42 \\ - 19 \\ \hline \end{array}$	$\begin{array}{r} 73 \\ - 50 \\ \hline \end{array}$	$\begin{array}{r} 68 \\ - 22 \\ \hline \end{array}$
$\begin{array}{r} 51 \\ - 23 \\ \hline \end{array}$	$\begin{array}{r} 82 \\ - 68 \\ \hline \end{array}$	$\begin{array}{r} 77 \\ - 41 \\ \hline \end{array}$	$\begin{array}{r} 99 \\ - 23 \\ \hline \end{array}$
$\begin{array}{r} 51 \\ - 23 \\ \hline \end{array}$	$\begin{array}{r} 82 \\ - 68 \\ \hline \end{array}$	$\begin{array}{r} 77 \\ - 41 \\ \hline \end{array}$	$\begin{array}{r} 99 \\ - 23 \\ \hline \end{array}$
$\begin{array}{r} 71 \\ - 29 \\ \hline \end{array}$	$\begin{array}{r} 94 \\ - 35 \\ \hline \end{array}$	$\begin{array}{r} 81 \\ - 47 \\ \hline \end{array}$	$\begin{array}{r} 92 \\ - 30 \\ \hline \end{array}$

Figure 12

Assessment #6, Word-Problem Solving Test (15min).

1. Jordan and Michelle have 23 computer games. Jordan has 14 computer games. How many computer games does Michelle have?
2. Martina spent \$15 on ice cream and \$12 on candy. How much money did Martina spend?
3. Sofia bought 37 paints at the paint store. For a painting, she used some of the paints. Now, she has 18 paints. How many paints did Sofia use?
4. Mr.Lopez sold 3 boxes of apples at the Farmer’s Market, and he also 7 boxes of peaches. How many more boxes of peaches did Mr.Lopez sell than apples?
5. Destiny spent \$18 on dresses. She also bought a pair of shoes for \$29. How much money did Destiny spend on dresses and shoes together?
6. Mateo baked some raisin cookies. Then, he ate 13 of the cookies. If Mateo has 28 cookies left, how many cookies did he bake?
7. The “STAR” animal park has 68 tigers and some elephants. Then 14 of the total number of tigers and elephants were transported to a different animal park. How many tigers and elephants are left in the “STAR” animal park?
8. Amari has 23 toy cars. She gave 8 of them to Darius. How many toy cars does Amari have now?
9. The principle in Doss Elementary school bought 12 packages of hot dogs, and coach bought 23 packages of hot dogs. How many more packages of hot dogs did the coach buy than the principle?
10. Emma planted some flowers and vegetables in her backyard. She planted 18 roses, 35 lilies, and 20 tomatoes sprouts. How many flowers in total did Emma plant in her backyard?

Figure 13

Grading Rubric for Word-Problem Solving (Assessment #6)

Grading Items	1pt/Item
Identify relevant/irrelevant information.	
Identify correct schema.	
Write correct equation.	
Demonstrate use of computation no matter answer was correct or not.	
Write correct answer for the equation.	
Write correct label for the number answer, e.g., 5 buckets instead of 5.	
Use drawings to help with the WPS.	

Data Collection

For pretest, we collected test data within a 30-min pretest session. Then during the three weeks' intervention sessions, we also conducted progress monitoring tests (see Table 4-1 and Table 4-2) at the end of each session. For the posttest, we administered the same measures as administered at pretest. However, during the posttest, I modified the order of the problems so students were not solving them based on their memorization. All tests were delivered through a group size of 3-5 students.

Fidelity of Implementation

Kovalesky (2007) determined that when research-based interventions were implemented with fidelity, a determination of their efficacy can be made with more reliability compared to when the intervention was not delivered with fidelity. I gathered data on the fidelity of implementation during the pretest, posttest, and tutoring sessions. I checked audio recordings for 40% of the tutoring sessions. The assessment of fidelity was conducted using checklists specific to each lesson. See Figure 14 for an example of implementation fidelity checklist for lesson 1.

Figure 14

Implementation Fidelity Checklist

Lesson 1	
Tutor: _____	School: _____
Teacher: _____	Student: _____
Reliability completed by: _____	Percentage: _____

<p>Behavior Demonstrated: ✓ Behavior Not Demonstrated: Blank Not Applicable: NA</p>

Advance Organizer

- ____ 1. Tutor asked students to review total and difference schema, poster, and gesture.
- ____ 2. Tutor introduced total schema and poster.
- ____ 3. Tutor displayed the hand gesture of change schema.
- ____ 4. Tutor reviewed the concrete base ten blocks with students.
- ____ 5. If students responded correctly, tutor reinforced correct response.
- ____ 6. If students responded incorrectly, tutor provided corrective feedback.

Modeling

- ____ 7. Tutor directed students to the problem she/he will model, and read the problem.
- ____ 8. Tutor identified question sentence with students, and circled the question phrase.
- ____ 9. Tutor identified the 1st and 2nd part in the word problem, and circled them.
- ____ 10. Tutor provided reasoning on which schema to be used.
- ____ 11. Tutor showed the total schema poster to students, and demonstrated by gesture.
- ____ 12. Tutor wrote down the total equation for the word problem, and plugged numbers to P1 and P2.
- ____ 13. Tutor put the vertical form of the equation in the equation column.
- ____ 14. Tutor demonstrated how to make the first addend starting from the ones place.
- ____ 15. Tutor demonstrated how to make the second addend.
- ____ 16. Tutor demonstrated how to add the two addends together by using concrete base ten blocks.
- ____ 17. Tutor emphasized 10 ones equal to 1 ten, and replaced 10 ones with 1 rod.
- ____ 18. Tutor wrote down corresponding answer in ones and tens place while she/he demonstrated the calculation using manipulatives.
- ____ 19. Tutor plug the final answer 20 in the equation, and wrote units.
- ____ 20. Tutor demonstrated how to check if the answer 20 is correct or not, and wrote the answer sentence for the word problem.

Behavior Demonstrated: ✓
Behavior Not Demonstrated: Blank
Not Applicable: NA

Guided Practice

- ___ 21. Remind students that tutor and students will work together on the word problem this time.
- ___ 22. Direct students to read the second word problem.
- ___ 23. Ask students to identify question sentence, schema, gesture.
- ___ 24. Ask students to write the equation for total schema.
- ___ 25. Ask students to label part 1 and part 2, plug numbers into equation.
- ___ 26. Lead students to solve the total equation by using base ten blocks.
- ___ 27. Direct students start from ones place, then move on to tens place.
- ___ 28. Replace 10 ones by 1 one ten/rod.
- ___ 29. Write the vertical form of the total equation in the equation column.
- ___ 30. Once finish solving addition on ones or tens place, tutor should direct students to write the answer in equation correspondingly.
- ___ 31. Tutor directed students to check on the correctness of the answer.
- ___ 32. Tutor directed students to write down the answer sentence.
- ___ 33. Tutor encouraged students to think what is the next step instead of telling them the next step.
- ___ 34. Tutor provided appropriate feedback after students' response

Independent Practice

- ___ 35. Tutor reminded students that they will work on their own now for the next word problem.
- ___ 36. Tutor asked students to read the question quietly on their own.
- ___ 37. Tutor only provided guidance whenever students get stuck.
- ___ 38. Tutor provided praise whenever appropriate.
- ___ 39. Tutor provided corrective feedback whenever needed.

Post Organizer

- ___ 40. Tutor gave a brief review on what they have learned in the first lesson.
- ___ 41. Tutor reviewed the total schema, gesture.
- ___ 42. Tutor reviewed what a flat, rod, and unit represents.
- ___ 43. Tutor asked students to make a two-digit number.
- ___ 44. Tutor set the timer for the review questions.
- ___ 45. Tutor did not give any help or clue to students.
- ___ 46. Tutor reviewed each of the questions with students.
- ___ 47. Tutor gave a brief overview on what they will learn the next day.

Data Analysis Strategies

To address the first two research questions, I conducted two-tailed t -tests using R software to compare of the mean difference (posttest-pretest) between CRA-I-Schema and Schema-Alone groups to test if there was significant improvement of students' WPS and addition and subtraction performance after CRA-I-Schema intervention. Subsequently, I interpreted the results based on the p value from the R output. When the p -value was less than 0.05, it indicated a significant impact on students' performance following the CRA-I-Schema instruction. Conversely, when the p -value was equal to or greater than 0.05, it signified a non-significant difference between the CRA-I-Schema and Schema-Alone groups. This suggested the CRA-I-Schema instruction did not lead to significant improvements in students' performance. Given the relatively small sample size ($N = 22$) in my study, I calculated effect sizes by using Hedges' g in R, which can correct for small sample sizes bias (Lakens, 2013; Marfo & Okyere, 2019).

For my third research question, I used the pretest measures of *Understanding of Word Problems* and *Word-Problem Solving* to determine students' prior knowledge of WPS. I conducted a moderation analysis by using fixed effects multiple linear regression model (MLR). This model is represented in equation (1), in which I hypothesized the posttest score of *Word-Problem Solving* was predicted by pretest of *Understanding of Word Problems* ($puwps$), pretest of *Word-Problem Solving* ($pwps$), and group assignment (g). Each symbol of the model is explained following (1).

$$Y = \beta_0 + \beta_1 puwps + \beta_2 pwps + \beta_3 g \quad (1)$$

$puwp$: Pretest of *Understanding of Word Problems*;

$pwps$: Pretest of *Word-Problem Solving*;

g : CRA-I-Schema or Schema-Alone group;

β_0 : The expected score for WPS posttest when pretest of Understanding of Word Problems and pretest of Word-Problem Solving equal to 0 in the Schema-Alone group;

β_1 : The expected change in WPS posttest score for a one-point increase on pretest of Understanding Word Problems after controlling for pretest of Word-Problem Solving across CRA-I-Schema group and Schema-Alone group;

β_2 : The expected change in WPS posttest for a one-point increase on pretest of Word-Problem Solving after controlling for pretest of Understanding of Word Problems across CRA-I-Schema group and Schema-Alone group;

β_3 : The mean difference score of WPS posttest between CRA-I-Schema group and Schema-Alone group.

To understand the social validity of the CRA-I-Schema and Schema-Alone instruction (i.e., my fourth research question), I administered a social validity questionnaire to the CRA-I-Schema and Schema-Alone students at the end of the intervention study. The social validity questionnaire comprised 9 statements in CRA-I-Schema condition (Table 8), and 7 statements in Schema-Alone condition (Table 9) that inquired about the importance, efficacy, willingness to use, and confidence improvement on the CRA-I-Schema instruction or Schema-Alone instruction. Students were offered three responses for each of the items to gauge their feedback, “No,” “I don’t know,” or “Yes.” Subsequently, I computed the percentage for each of the responses. Then, I analyzed and evaluated the social validity questionnaire to understand the satisfaction and impact of the intervention on participating students.

Chapter IV: Results

Preliminary Analysis

With this dissertation project, I recruited 36 participants but 12 students left the study due to various reasons during the intervention sessions, such as conflicting schedule, transferring to a different school, or family issues. Another 2 students did not attend the posttest session; therefore, I was not able to collect the posttest data from those students. My final number of participants for data analysis was 22 by the end of 10-session intervention. For this analysis, there were 10 students in CRA-I-Schema group, and 12 students in Schema-Alone group. Table 1 presents all participants' demographic data by condition.

All 22 participants attended a private local school, and they were all diagnosed of dyslexia. Among these 22 participants, 9 were male and 13 were female. Specifically, the CRA-I-Schema group was comprised of 6 females and 4 males; the Schema-Alone group included 5 males and 7 females. The ethnicity of participants consisted of White, Hispanic and African American. Within CRA-I-Schema group, 8 students identified as White, while 2 students identified as Hispanic. In Schema-Alone group, 9 students were White, 2 students were African American, 2 students were Hispanic/Latino, and 1 student did not identify their ethnicity. Participants' grade levels spanned from second to fourth grades. In CRA-I-Schema group, there were 4 Grade 2 students, 3 students at Grade 3, and 3 students at Grade 4. In Schema-Alone group, there were 2 Grade 2 students, 6 students at Grade 3, and 4 students at Grade 4. Two out of 22 students were identified as English Learners (ELs).

The participating school used Singapore Math curriculum (Cai, 2003; Gross & Merchlinsky, 2002; Hu, 2010; Leinwald & Ginsburg, 2007). Teachers used Singapore Math-

aligned assessments to determine each student's level of mathematics proficiency. Only 2 out of 22 students performed at grade level, while 20 students performed below grade level expectations. According to a survey conducted with teachers, all students struggled with WPS, including understanding of WPS and difficulty in WPS fluency.

There were 6 assessments administered at pre- and posttest: *Understanding of Word Problems*, *Word-Problem Solving*, *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, and *Two-Digit Subtraction*. The pre- and posttests included the same questions, but I reordered some of the questions on the posttest to avoid students answering questions based on their memory. At pretest, the order of the assessment was: *Understanding of Word Problems*, *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, and *Two-Digit Subtraction*, *Word-Problem Solving*. At posttest, the order of the assessments was: *Word-Problem Solving*, *Understanding of Word Problems*, *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, and *Two-Digit Subtraction*. I, in collaboration with three GRAs, conducted the pretesting prior to 10-session intervention. Then we administered the posttest after 10-session intervention.

To answer my research questions, first, I calculated the mean of pre- and posttest scores on all six measures. This information is provided in Table 5. Then, I computed the mean difference score between post-and pretest across all 6 measures (see Table 6). Furthermore, I conducted two-tailed *t*- tests to examine the statistical difference between pre- and posttest on each measure. I also calculated the mean difference score of the CRA-I-Schema group compared to the Schema-Alone group for all measures (see Table 6). I calculated Hedges' *g* values to describe the magnitude of treatment effects (What Works Clearinghouse, 2017) (see Table 6). Finally, I performed descriptive analysis on students' social validity data for the CRA-I-Schema group and the Schema-Alone group (see Table 8 and 9).

What is the Efficacy of the CRA-I-Schema Instruction Compared to Schema-Alone Instruction on a Measure of WPS for Students with MD?

To investigate research question 1, I used two assessments: *Understanding of Word Problems* and *Word-Problem Solving*, to measure students' performance on WPS at pre- and posttest. The means, standard deviations, *t*-test results, and Hedges' *g* are presented in Table 5. I computed the mean difference scores between CRA-I-Schema and Schema-Alone group across all the measures, and this information is provided in Table 6.

First, on the *Understanding of Word Problems* measure, the CRA-I-Schema group had a mean posttest score of 4.10 (*SD* = 2.42), which was significantly ($p < 0.05$) higher than their mean pretest score of 2.00 (*SD* = 1.63), and the effect size (Hedges' $g = 0.92$) was large and favored the posttest score. In the Schema-Alone group, the mean posttest score of 3 (*SD* = 1.60) was not significantly ($p > 0.05$) higher than that ($M = 3.25$, *SD* = 3.36) at pretest. The effect size was negligible (Hedges' $g = -0.09$).

Then, for the assessment of *Word-Problem Solving*, the CRA-I-Schema group had a mean posttest score of 22.2 (*SD* = 8.37), which was significantly ($p < 0.05$) higher than the mean pretest score of 4.60 (*SD* = 3.50). Here, Hedges' g was 2.33. This result indicated the CRA-I-Schema intervention had a large impact on students' WPS performance at posttest. The Schema-Alone group had a mean posttest score of 24.33 (*SD* = 13.16), which was also significantly ($p < 0.05$) higher than the mean pretest score of 13 (*SD* = 8.12). The effect size (Hedges' g) was 1.00, and the Scheme-Alone intervention had a large impact on students' WPS performance at posttest.

Furthermore, I conducted a comparison of the mean difference score of CRA-I-Schema group and Schema-Alone group for both two word-problem measures. For *Understanding of*

Word Problems, results demonstrated that the mean difference score from pre- to posttest ($M = 2.10$, $SD = 2.60$) in CRA-I-Schema was not significantly ($p > 0.05$) higher than that ($M = -0.25$, $SD = 3.36$) in Schema-Alone group. However, the effect size (Hedges' $g = 0.74$) between the mean difference score in CRA-I-Schema and Schema-Alone group was medium to large, suggesting the CRA-I-Schema intervention had a medium to large effect on students' understanding of WPS compared to the Schema-Alone group.

For the *Word-Problem Solving* measure, the mean difference score ($M = 17.60$, $SD = 7.82$) was not significantly ($p > 0.05$) larger in CRA-I-Schema group than in the Schema-Alone group ($M = 11.33$, $SD = 12.62$). Nevertheless, the effect size (Hedges' $g = 0.56$) between the mean difference score in CRA-I-Schema group and Schema-Alone group was medium to large, demonstrating the CRA-I-Schema intervention had a medium to large effect on students' understanding of WPS relative to Schema-Alone intervention.

What is the Efficacy of the CRA-I-Schema Instruction Compared to Schema-Alone Instruction on a Measure of Addition and Subtraction for Students with MD?

To investigate this question, I administered 4 assessments (i.e., *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, and *Two-Digit Subtraction*) to measure students' competence in addition and subtraction. To analyze the effect of CRA-I-Schema instruction on students' addition and subtraction performance, I conducted comparative analysis utilizing pairwise t -test between pretest and posttest performance (Table 5). Furthermore, I also examined the mean difference scores (posttest-pretest) between CRA-I-Schema group and Schema-Alone group across all 4 assessments by using two-sample independent t -test (Table 6). In addition, I computed Hedges' g effect sizes for each of the 4 assessments, with the corresponding effect size values presented in Table 5 and 6.

Addition Facts

Within CRA-I-Schema group, there was no significant difference ($p > 0.05$) when I compared the *Addition Facts* pretest ($M = 9.58, SD = 5.11$) to *Addition Facts* posttest ($M = 9.33, SD = 5.99$). In the Schema-Alone group, there was an average increment of 2 points from pretest score ($M = 9.58, SD = 5.11$) to posttest score ($M = 13.58, SD = 7.06$), but this increase was not significant ($p > 0.05$) (Table 5). When I compare the mean difference scores between the CRA-I-Schema and Schema-Alone, there was no significant ($p > 0.05$) difference between these two conditions.

Subtraction Facts

Within CRA-I-Schema group, there was a slight decrease from pretest ($M = 10.00, SD = 7.26$) to posttest ($M = 9.32, SD = 5.99$), but this decrease was not significant ($p > 0.05$). In the Schema-Alone group, there was not significant ($p > 0.05$) change from pretest score ($M = 14.30, SD = 8.41$) to posttest score ($M = 9.58, SD = 5.11$). Subsequently, upon comparing the mean difference scores between the CRA-I-Schema group and Schema-Alone group, there was no significant ($p > 0.05$) difference between these two conditions ($g = -0.07$).

Two-Digit Addition

Within CRA-I-Schema group, there was a significant ($p < 0.05$) decrease from pretest assessment ($M = 9.70, SD = 7.27$) to posttest assessment ($M = 6.40, SD = 7.17$). Correspondingly, in the Schema-Alone group, there was also a significant ($p < 0.05$) decrease from pretest ($M = 9.56, SD = 7.13$) to posttest ($M = 6.32, SD = 7.05$). By comparing the mean difference score between CRA-I-Schema ($M = -3.30, SD = 4.40$) and Schema-Alone group ($M = -0.50, SD = 9.64$), there was not a significant difference ($p > 0.05$; $g = -0.35$).

Two-Digit Subtraction

Within CRA-I-Schema group, there was a significant ($p < 0.05$) decrease from pretest ($M = 5.83, SD = 4.93$) to posttest ($M = 2.40, SD = 2.63$). Similarly, for Schema-Alone group, there was also a significant ($p < 0.05$) reduction from pretest ($M = 10.67, SD = 7.58$) to posttest ($M = 3.25, SD = 5.17$). By comparing the mean difference score between CRA-I-Schema ($M = -4.60, SD = 3.20$) and Schema-Alone groups ($M = -7.41, SD = 8.10$), there was not a significant difference ($p > 0.05$). Hedges' g was 0.37 between mean difference score of CRA-I-Schema and Schema-Alone group, indicating the CRA-I-Schema instruction had a smaller impact on *Two-Digit Addition* compared to Schema-Alone group.

Do Effects Differ Based on Students' Prior Knowledge of WPS?

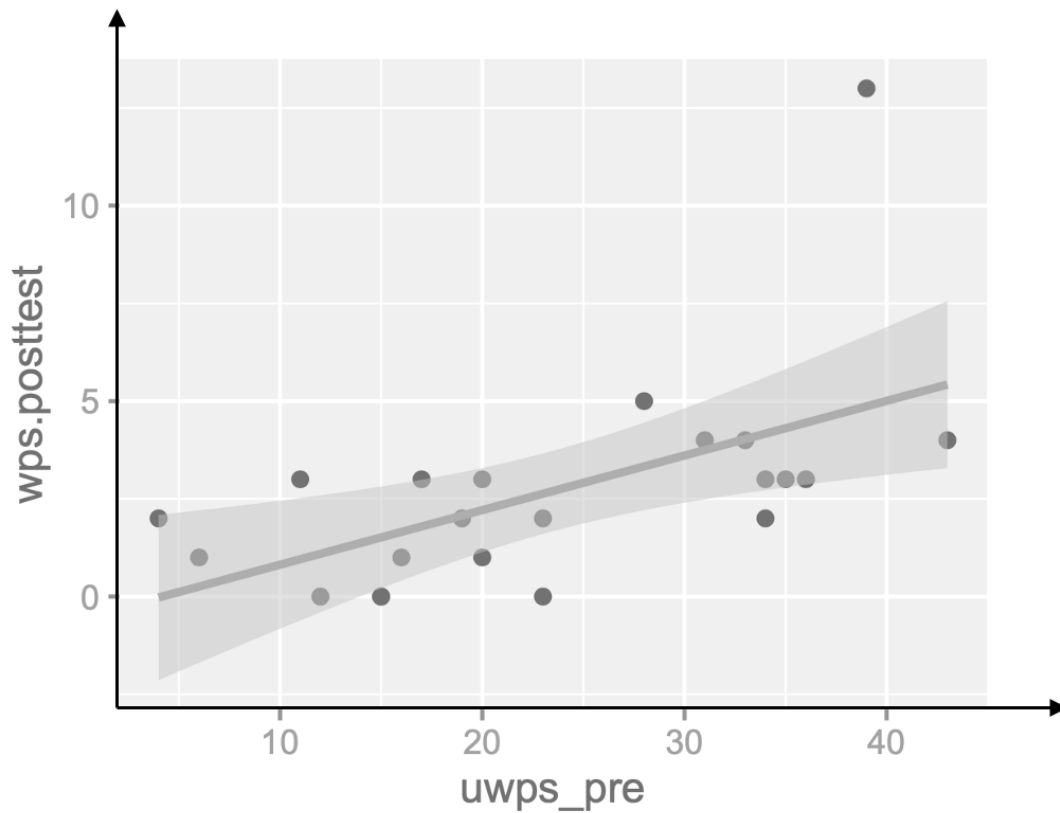
The multiple linear regression analysis result is summarized in Table 7, which presents the regression coefficients in the model (1). The unstandardized beta coefficients indicated the pretest scores of *Understanding of Word Problems* significantly predicted the posttest score of *Word-Problem Solving* ($\beta = 2.03, t = 2.45, p < 0.05$; Table 7). More precisely, a 1-point increase on pretest score of *Understanding of Word Problems* resulted in a 2.03-point increase of *Word-Problem Solving* posttest score, controlling for the pretest score of *Word-Problem Solving* across CRA-I-Schema group and Schema-Alone groups. This relationship is illustrated in Figure 15. As the pretest score of *Understanding of Word Problems* increased, the posttest score of *Word-Problem Solving* also increased.

Also, as shown in Table 7, the pretest score of *Word-Problem Solving* did not significantly predict the posttest score of *Word-Problem Solving* ($\beta = 0.36, t = 1.03, p > 0.05$). The unstandardized beta coefficients indicated that the posttest score would improve by 0.36 points for every 1-point increase on pretest score of *Word-Problem Solving* controlling for the pretest score of *Understanding of Word-Problem Solving* across CRA-I-Schema group and

Schema-Alone group. This trend was demonstrated in Figure 16, in which the posttest score of *Word-Problem Solving* improved as the pretest score of *Word-Problem Solving* increased.

Figure 15

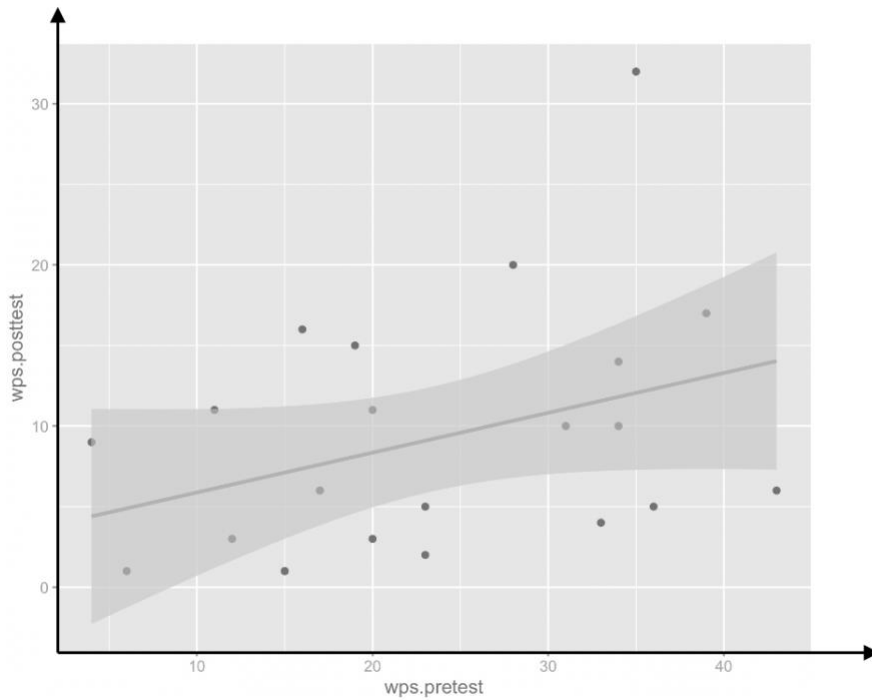
The Relationship between Pretest of Understanding of Word-Problem Solving and Posttest of Word-Problem Solving



Note. uwps_pre: pretest of *Understanding of Word-Problem Solving*;

wps.posttest: posttest of *Word-Problem Solving*.

Figure 16 *The Relationship between Pretest of Word-Problem Solving and Posttest of Word-Problem Solving.*



Note. wps.pretest = pretest of *Word-Problem Solving*;
wps.posttest = posttest of *Word-Problem Solving*.

Table 5

Comparison between Pretest and Posttest Scores for CRA-I-Schema and Schema-Alone Group across All the Measures

Measures	Pretest		Posttest		<i>t</i> -test	Hedges' <i>g</i>
	Mean	SD	Mean	SD		
CRA-I-Schema Instruction						
<i>Understanding of Word Problems</i>	2.00	1.63	4.10	2.42	0.03*	0.92
<i>Word-Problem Solving</i>	4.60	3.50	22.20	8.37	< 0.001***	2.33
<i>Addition Facts</i>	9.58	5.11	9.33	5.99	0.86	-0.04
<i>Subtraction Facts</i>	10.00	7.26	9.32	5.99	0.86	-0.09
<i>Two-Digit Addition</i>	9.70	7.27	6.40	7.17	0.04*	0.42
<i>Two-Digit Subtraction</i>	5.83	4.93	2.40	2.63	0.02*	-1.14

Schema-Alone						
<i>Understanding of Word Problems</i>	3.25	3.36	3.00	1.60	0.80	-0.09
<i>Word-Problem Solving</i>	13.00	8.12	24.33	13.16	0.02*	1.00
<i>Addition Facts</i>	9.58	5.11	13.58	7.06	0.06	0.60
<i>Subtraction Facts</i>	14.30	8.41	14.90	8.79	0.67	0.06
<i>Two-Digit Addition</i>	9.70	7.27	6.40	7.17	0.04*	-0.42
<i>Two-Digit Subtraction</i>	10.67	7.58	3.25	5.17	0.01*	-1.05

Note. * p is less than 0.05; ** p is less than 0.01; *** p is less than 0.001.

Table 6

Comparison of Mean Difference Score between CRA-I-Schema and Schema-Alone Group across All the Measures

Measures	CRA-I-Schema		Schema-Alone		t -test	Hedges' g
	Mean Diff	SD	Mean Diff	SD		
<i>Understanding of Word Problems</i>	2.10	2.60	-0.25	3.36	0.17	0.74
<i>Word-Problem Solving</i>	17.60	7.82	11.33	12.62	0.17	0.56
<i>Addition Facts</i>	-0.25	4.33	4.00	6.56	0.16	0.58
<i>Subtraction Facts</i>	-0.67	5.03	-0.30	4.47	0.85	-0.07
<i>Two-Digit Addition</i>	-3.30	4.40	-0.50	9.64	0.38	-0.35
<i>Two-Digit Subtraction</i>	-4.60	3.20	-7.41	8.10	0.34	0.37

Note. Mean Diff = mean difference of posttest score and pretest score.

Table 7

Fixed Effects Multiple Linear Regression Analysis of Moderation Effects of Prior WPS

Knowledge.

Variables in Model	Unstandardized Coefficients			
	β	SE	t	p value
Constant	13.05	5.25	2.49	0.02*
Pretest of <i>Understanding of Word Problems</i>	2.03	0.83	2.45	0.02*
Pretest of <i>Word-Problem Solving</i>	0.36	0.35	1.03	0.32
CRA-I-Schema vs. Schema-Alone group	3.43	4.95	0.69	0.50

Note. Dependent variable: Posttest score of *Word-Problem Solving*; $F(3, 18) = 3.34$, $p < 0.05$, $R^2 = 0.36$

What is Students' Attitude towards CRA-I-Schema Instruction and Schema-Alone

Instruction?

Regarding the CRA-I-Schema condition, a substantial percentage of students, ranging from 50% to 90%, consistently indicated a positive response by selecting "Yes" for all 9 statements. On average, approximately 68% of students rated "Yes" for each individual question. The high percentage of students rating "Yes" reflected a high level of acceptability and satisfaction with the CRA-I-Schema instruction. Importantly, 90% of participants agreed with statement 8: *"I am more confident at solving word problems now than before the math tutoring."* Additionally, 80% of participants expressed agreement with both statement 3 (*"Concrete and drawing base ten blocks helped me solve word problems better"*) and 7 (*"I would like to use concrete or drawing base ten blocks to solve other types of word problems"*). Furthermore, 70% of students rated "Yes" on statement 5: *"I am better at solving word problems now."* Then, 60% of students indicated "Yes" on statements 2 (*"I can tell the difference between the Total, Difference, and Change word problems"*) and 9 (*"I hope to have more time studying in this math tutoring class."*). Finally, 50% of students agreed on statements 4 (*"Concrete and drawing base ten blocks helped me solve computation problems better."*) and 6 (*"I would like my classroom teacher to use the same methods as in math tutoring to teach me solve word problems."*)

Within the Schema-Alone condition, a range of 50% to 83% of students rated as "Yes" for all 7 statements, with an average of approximate 62% of students rated as "Yes". On statement 6 (*"I hope to have more math tutoring like this in the future"*), 83% of students rated this statement as a "Yes." Overall, 66% of students concurred with statement 1 *"The math tutoring lessons helped me to understand how to solve math word problems."*, statement 2 *"I can tell the difference among a Total, Difference, and Change word problem."*, and statement 7 *"I*

hope to have more time using these math tutoring strategies in my math class.” Then, 50% of students affirmed statement 3 “After this math tutoring, I am better at solving word problems now.”, statement 4 “I would like my classroom teacher to use the same methods as in math tutoring to teach me solve word problems.” and statement 5 “I am more confident at solving word problems now than before the math tutoring.”

Table 8

Social Validity of CRA-I-Schema Instruction for Participants (N = 10)

Statements	No (%)	I don't know (%)	Yes (%)
1.The math tutoring lessons helped me to understand how to solve math word problems.	10	20	70
2.I can tell the difference between the total, difference, and change word problem.	20	20	60
3.Concrete and drawing base ten blocks helped me solve word problems better.	10	10	80
4.Concrete and drawing base ten blocks helped me solve computation problems better.	20	30	50
5.I am better at solving word problems now.	10	20	70
6.I would like my classroom teacher to use the same method as in math tutoring to teach me solve word problems.	20	30	50
7.I would like to use concrete or representational base ten blocks to solve other types of word problems.	10	10	80
8.I am more confident at solving word problems now than before the math tutoring.	0	10	90
9.I hope to have more time studying in this math tutoring class.	20	20	60

Table 9*Social Validity of Schema-Alone Instruction for Participants (N = 12)*

Statements	No (%)	I don't know (%)	Yes (%)
1.The math tutoring lessons helped me to understand how to solve math word problems.	0	34	66
2.I can tell the difference among a Total, Difference, and Change word problem.	17	17	66
3.After this math tutoring, I am better at solving word problems now.	17	33	50
4.I would like my classroom teacher to use the same methods as in math tutoring to teach me solve word problems.	33	17	50
5.I am more confident at solving word problems now than before the math tutoring.	0	50	50
6.I hope to have more math tutoring like this in the future.	0	17	83
7.I hope to have more time using these math tutoring strategies in my math class.	17	17	66

Note. Only 6 out of 12 participants in Schema-Alone condition submitted social validity questionnaires.

Chapter V: Discussions

Developing mathematics proficiency in elementary school is critical to later academic and life success, as early mathematics difficulties are predictive of lower academic achievement in later grades (Nelson & Powell, 2018). Thus, the development of effective early mathematics interventions designed to close academic achievement gap is crucial (Frye et al., 2013; Gersten et al., 2009). To understand the intricacies of mathematics intervention, researchers have begun to explore the comparisons between different interventions and variables that may influence the efficacy of interventions (Fuchs & Fuchs, 2019; Miller et al., 2014).

Therefore, the main purpose of this study was to investigate the efficacy of the CRA-I-Schema instruction on mathematical WPS performance for students with MD, and I further compared the efficacy of CRA-I-Schema instruction to the Schema-Alone condition. Additionally, I examined the impact of the CRA-I-Schema instruction on students' addition and subtraction performance. Concurrently, I conducted moderation analyses to analyze if students' WPS performance varied based on their prior knowledge of WPS. In the end, to evaluate students' satisfaction on my CRA-I-Schema instruction, I scrutinized the response from students' social validity questionnaire. In the next sections, I provide a thorough discussion pertaining to each of the research questions.

Research Question 1: Efficacy of the CRA-I-Schema Instruction Compared to Schema-Alone Instruction on a Measure of WPS for Students with MD

With the first research question, I investigated the efficacy of the CRA-I-Schema instruction, and I compared it with Schema-Alone instruction on a measure of WPS. In this study, I used *Understanding of Word Problems* and *Word-Problem Solving* measures to assess

students' performance of WPS at both pre- and posttest. For the *Understanding of Word Problems*, students in CRA-I-Schema condition showed greater gains from pre- to posttest over the Schema-Alone students ($g = 0.74$), which suggested the CRA-I-Schema instruction had a larger effect on students' comprehension of WPS relative to the Schema-Alone group. Similarly, on the *Word-Problem Solving* measure, the advantage for CRA-I-Schema instruction students over Schema-Alone students was medium to large ($g = 0.56$), indicating the CRA-I-Schema instruction had medium to large effect on students' *Word-Problem Solving* relative to Schema-Alone students.

This finding is consistent with my initial hypothesis that CRA-I-Schema instruction would yield a greater improvement in the WPS performance compared to Schema-Alone instruction. This could be explained by the more structured and organized nature of the CRA-I-Schema instruction. Furthermore, this study extends existing literature regarding the efficacy of the CRA approach when integrated with schema instruction for students with MD. In mathematics, students can use schemas to organize information from a word problem in ways that represent the underlying structure of a problem type. In my study, CRA assisted the computational aspect of WPS, which could further help to translate texts into equations more easily, and it could enhance the visualization of the WPS process.

My study was structured as a randomized-controlled trial, serving as an extension of prior SCD investigations on the efficacy of the CRA approach in improving WPS performance. Kim et al. (2015) investigated the effects of fraction WPS instruction involving explicit teaching of the CRA sequence with three low-performing students, and they established a functional relation in all three participants. They further replicated the functional relation in the teaching of a second type of fraction word problems. All participants reached grade-level mastery on both types of

word problems, maintained skills after the intervention ended. Flores et al. (2016) implemented CRA and schema instruction to three third-grade students with MD. They also detected a functional relation between CRA within schema instruction and students' WPS performance.

The result of pre- and posttest comparison across CRA-I-Schema and Schema-Alone conditions revealed noteworthy findings. Specifically, the effect size from *Understanding of Word Problems* pre- to posttest was larger in CRA-I-Schema condition ($g = 0.92$) than in Schema-Alone condition ($g = -0.09$). Similarly, on the *Word-Problem Solving* measure, the effect size from pre- to posttest exhibited a marked disparity between these two conditions, with CRA-I-Schema condition showing a substantially larger effect size ($g = 2.33$) in contrast to the Schema-Alone condition ($g = 1.00$).

This finding aligns with both my hypothesis and the findings from prior studies. The CRA approach could make the WPS process more visualized and tangible, which could further improve the comprehension of the computation process of WPS. Milton et al. (2018) investigated the effects of alternating CRA on students' mastery of unknown facts and on their conceptual understanding. They demonstrated a functional relation between CRA instruction and students' computation performance. Additionally, concerning the conceptual understanding of mathematics facts, they also noted that students were able to use more words, sentences, and drawings that meaningfully demonstrated how they understood the computation after intervention.

Remarkably, the CRA-I-Schema group displayed a significant growth in *Word-Problem Solving* performance from pre- to posttest, which is similar to results such as in Flores et al., (2016). In their study, they employed a combination of CRA and schema-based instruction for three students receiving tertiary interventions in mathematics. Their study established a

functional relationship between this instructional approach and the WPS performance for these three students. Thus, my pilot group design study contributes a valuable addition to the existing literature

It is important to note that when I compared the mean difference score of CRA-I-Schema and Schema-Alone conditions on *Understanding of Word Problems* and *Word-Problem Solving*, I did not identify significant differences ($p > 0.05$). This may be attributed to the relatively small sample size ($N = 22$), which can cause insufficient statistical power of my study, and potentially lead to type II error (Columb & Atkinson, 2016). Type II error might have occurred when the effect of the CRA-I-Schema instruction was deemed insignificant when in fact the intervention could have been effective. With this consideration, my study may serve as a preliminary exploration of the initial comparative efficacy between CRA-I-Schema and Schema-Alone instruction. Thus, I would like to investigate this more by using larger sample size to provide more definitive insights.

Research Question 2: Efficacy of the CRA-I-Schema Instruction Compared to Schema-Alone Instruction on Measures of Addition and Subtraction for Students with MD

To answer the second research question, I examined the efficacy of the CRA-I-Schema instruction compared to Schema-Alone instruction on various measures of addition and subtraction, encompassing *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, and *Two-Digit Subtraction*. My findings revealed a noteworthy impact on *Addition Facts*. There was a medium to large effect ($g = 0.58$) on *Addition Facts* by comparing the mean difference score (posttest-pretest) of CRA-I-Schema group to the Schema-Alone group. However, for *Two-Digit Subtraction*, I noted a smaller effect ($g = 0.37$) in favor of CRA-I-Schema group over Schema-Alone group. This result was within expectation considering the difficulty level of the *Addition*

Facts measure and *Two-Digit Subtraction* measure. *Addition Facts* is relatively less challenging, so students can master the addition facts within a shorter time frame. However, the *Two-Digit Subtraction* assessment involving subtraction problems with and without regrouping, which may demand more practice for proficiency.

As for the measures of *Subtraction Facts* and *Two-Digit Addition*, the influence of CRA-I-Schema instruction may not be considered, as evidenced by negative effect sizes ($g = -0.07$, and $g = -0.35$, respectively). This unexpected discovery may be attributed to two primary factors. First, students in my study only received relatively brief CRA-I-Schema instruction consisting of only 10 sessions that mainly focused on WPS instead of addition and subtraction computation. Compared to previous mathematics computation efforts that targeted on a wide range of addition or subtraction competencies (e.g., Flores & Hinton, 2022; Kaya & Yildiz, 2023), the present intervention involved fewer sessions and only focused on single-digit and two-digit addition and subtraction.

Second, while the content between the pre- and posttests remained consistent, I modified the sequence of assessments on the posttest. This alteration was used to prevent students relying on their memory-based responses. At pretest, the order of the assessments followed this order: *Understanding of Word Problems*, *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, *Two-Digit Subtraction*, and *Word-Problem Solving*. In contrast, at posttest, the order of the assessments was: *Word-Problem Solving*, *Understanding of Word Problems*, *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, and *Two-Digit Subtraction*. Thus, at posttest, I placed the measures of addition and subtraction after the measure of *Word-Problem Solving* to make sure students had sufficient time to complete all the questions in the *Word-Problem Solving* assessment. However, one consequence for this order adjustment was that some students had

time constraints at posttest to finish the addition and subtraction assessments, which led to incomplete responses in the addition and subtraction assessments. Consequently, this alteration in sequence contributed to lower score on *Addition Facts*, *Subtraction Facts*, *Two-Digit Addition*, and *Two-Digit Subtraction* measures. In the future, I may consider to extend the pre- and posttest time to make sure students have sufficient time to try on all the questions.

Research Question 3: Moderation Effects of Students' Prior Knowledge of WPS

With my third research question, I explored whether students' prior knowledge of WPS functioned as a moderating factor on WPS performance at posttest. In this study, I utilized the pretest measures of *Understanding of Word Problems* and *Word-Problem Solving* to evaluate students' prior knowledge of WPS. I learned that the impact of the CRA-I-Schema instruction was significantly moderated by the pretest of *Understanding of Word Problems*, while the pretest of *Word-Problem Solving* did not exhibit a significant moderating effect on the intervention. In other words, students demonstrated analogous progress in WPS performance across CRA-I-Schema and Schema-Alone conditions regardless of their pre-intervention performance on the measure of *Word-Problem Solving*.

Conversely, students showed significantly different growth in WPS performance across CRA-I-Schema and Schema-Alone conditions based on their pre-intervention performance on the measure of *Understanding of Word Problems*. More specifically, a 1-point increase on pretest score of *Understanding of Word Problems* resulted in a 2.03-point increase of *Word-Problem Solving* posttest score, controlling for the pretest score of *Word-Problem Solving* across CRA-I-Schema group and Schema-Alone groups. This finding illuminated the interplay between students' prior knowledge of WPS and the efficacy of the CRA-I-Schema instruction.

Understanding of Word Problems

I determined there was a significant moderation impact of pretest of *Understanding of Word Problems* on students WPS performance at posttest. This finding aligned with my initial hypothesis that prior knowledge is one of the students' characteristics that can impact the effectiveness of mathematics interventions involving CRA (Lafay et al., 2019). In the measure of *Understanding of Word Problems*, students answered questions based on their conceptual understanding of word problems instead of solving actual word problems. It is pertinent to note that all of the student participants in this study exhibited comorbid difficulties in two domains, dyslexia and mathematics WPS. Dyslexia is considered to be a phonological-based reading disorder in which the relation between orthography and phonology is hampered, not caused by a lack of intelligence or education (Lyon et al., 2003). Decoding remains effortful, and reading speed remains slow. These decoding problems may have a severe impact on students' reading comprehension (Lyon et al., 2003; Roitsch & Watson, 2019). Kirby et al. (2008) showed that students with dyslexia experienced more difficulty in deriving key points from a text than students without dyslexia. In my study, students needed to comprehend the word problems in the measure of *Understanding of Word Problems* in order to derive accurate answers. However, students' dyslexia may have prevented them understanding the word problems, which further impacted the performance on the measure of *Understanding of Word Problems*.

Having low reading comprehension and WPS difficulties in my participants significantly influenced their understanding of WPS which aligned with findings of previous researchers (Fuchs et al., 2004; 2019). Among the third-grade students, Fuchs et al. (2004) investigated the responsiveness to mathematical problem-solving instruction and compared the conceptual underpinnings of WPS between students at risk for both reading and mathematics disability ($n = 32$), students at risk for mathematics disability only ($n = 13$), students at risk for reading

disability only ($n = 27$), and students not at risk ($n = 129$). Their exploratory regression analyses suggested that students at risk for both reading and mathematics disabilities improved less than other groups of students on WPS conceptual underpinnings.

Word-Problem Solving

In addition to the pretest measure of *Understanding of Word Problems*, I also included *Word-Problem Solving* pretest as another way to account for students' prior word-problem knowledge. In the pretest of *Word-Problem Solving*, students solved 10 word problems. Based on the regression analysis, the pretest of *Word-Problem Solving* did not moderate the effect of the CRA-I-Schema instruction. In other words, students' posttest performance on WPS did not differ significantly based on their pretest score of *Word-Problem Solving*. This finding conflicts with my hypothesis and other existing literature.

Wang et al. (2019) explored the efficacy of fractions intervention with and without an embedded self-regulation component on fraction WPS for third-grade students at risk for mathematics disabilities. Within the condition without self-regulation component, they found pretest word-problem performance moderated the posttest score of fraction word problems. More specifically, the posttest score of fraction word problems became higher as the pretest score of word problems increased. Comparably, Jitendra et al.(2013b) compared the effects of small-group tutoring on mathematics WPS using the schema-based instruction and standards-based curriculum. Their study encompassed 136 third-grade participants who had been identified as at-risk for mathematics difficulties. Their results revealed that students with higher pretest scores of WPS in the schema-based instruction group outperformed their counterparts in the standards-based curriculum group on a measure of word-problem solving.

While a growing body of research suggests that students's prior knowledge is predictive of learning outcomes, the moderation effects of student variables (e.g., status of comorbidity disabilities) have not been studied comprehensively in the MD population. In my study, all of the participants were diagnosed of MD and dyslexia. Consequently, the potential moderating influence of the pretest performance in *Word-Problem Solving* on WPS outcomes may also need to account for the influence of dyslexia on students' learning processes and its subsequent impact on their posttest *Word-Problem Solving* results. Furthermore, it is worth noting that, given the relatively small sample size of 22, my study may not have sufficient power to detect the moderation effect of pretest of *Word-Problem Solving*.

Research Question 4: Students' Attitude towards CRA-I-Schema Instruction and Schema-Alone Instruction

The findings regarding social validity revealed that students rated the CRA-I-Schema instruction as important, satisfying, acceptable, and effective. This qualitative result was consistent with other quantitative analysis findings in my study. As indicated by responses to the social validity questionnaire, it is evident that the CRA-I-Schema instruction resulted in a notable enhancement of 90% participants' confidence level at solving word problems by the end of the intervention. Furthermore, a substantial 80% of participants acknowledged the utility of concrete materials and drawing base-ten blocks in improving their WPS abilities, expressing their willingness to incorporate these tools into their future problem-solving endeavors.

Limitation and Future Research

Before concluding, I identify several limitations to this study. First, the sample size in my study was relatively small, which caused insufficient statistical power of my study, and potentially lead to type II error. Additionally, the reduced sample size contributed to an

underpowered moderation analysis. Considering these constraints, my study's finding should be considered as preliminary. In the future, researchers may consider the use of a larger sample size to facilitate more comprehensive exploration of the research topic.

Second, regarding the design of the intervention, it is noteworthy that students' comorbidity disability status was not considered sufficiently prior to the intervention. All students had dyslexia, which may have meant additional effort reading the WPS questions. Then for larger group (e.g., 4-5 students per group), tutors may have difficulties in allocating sufficient time to read the questions multiple times for each student given the high demand of the whole group. To address this, future researchers may want to extend the duration of each intervention session or reduce the group size to ensure implementers have sufficient time for students who have reading difficulties. Finally, it is important to acknowledge that due to limited number of student participants, this study exclusively incorporated two instructional conditions (i.e., CRA-I-Schema and Schema-Alone conditions). In future, it may be valuable to include a business-as-usual group to further investigate the efficacy of CRA-I-Schema and Schema-Alone instruction in comparison to a business-as-usual.

Implication

First, given students' conceptual understanding of WPS could impact their WPS performance at posttest in this study, I recommend classroom teachers to provide more comprehensive and contextual explanations during instruction. It is essential for students to move beyond a mere focus on numerical elements within the word problems, and strive for a comprehension of the problem in its entirety. In the beginning of my study, many students in third grade encountered challenges in articulating the question sentence and identifying the relevant information connected with the problem. Without knowing what information is related

to the question, students may merely perform calculation without thoughtful consideration, which could potentially lead to erroneous solutions.

Furthermore, considering the substantial effect size observed in the CRA-I-Schema condition compared to the Schema-Alone condition on the measure of *Word-Problem Solving*, educators are encouraged to integrate additional components of the CRA approach into their WPS instructional strategies. These supplementary components may encompass the utilization of concrete manipulatives, visual representation (like drawings), and abstract WPS techniques. Moreover, according to the result of social validity questionnaire, 80% of students agreed with the statements about hands-on tools and drawings (“*Concrete and drawing base ten blocks helped me solve word problems better;*” “*I would like to use concrete or representational base ten blocks to solve other types of word problems.*”), which could further confirm that students are also willing to use the CRA method in their WPS. While many students had used manipulatives or drawings in their classroom activities, they often lacked a clear understanding of the relationship between the tools and the problems they solved. I would recommend for educators to allocate additional time to explain the connections between manipulatives WPS tasks. In addition, educators may find it beneficial to provide written explanations of the computational procedures alongside the use of manipulatives or drawings during WPS activities.

Finally, it is essential to recognize that while the CRA-I-Schema instruction was primarily developed for students with MD, it is imperative to account for the diverse range of students’ needs when implementing it in classroom. For instance, students in my study also had dyslexia, which requires special accommodations including individual reading support. Students might find it beneficial to make supplementary notations on their work to help them recall information during the process of solving word problems. As for future researchers, they may

want to design the intervention in a way that could reach a fuller range of learners. One potential avenue for improvement could involve the inclusion of mathematical vocabulary within WPS, which could help participants with reading difficulties comprehend the problems better. This also underscores the intricate relationship between reading proficiency and mathematics WPS skills. Thus, I call for researchers to investigate the role of student characteristics more systematically in future studies to provide more inclusive and effective instructions.

Conclusion

My finding on the initial efficacy of the pilot CRA-I-Schema intervention indicates the CRA-I-Schema instructions could lead to improved WPS performance for students with MD across Grade 3 to 5. Considering the research of CRA-I-Schema instruction that employed a group design is scarce, my findings provide significant implications for forthcoming researchers. My study could offer valuable insights on instructional components, experimental design, and intervention procedure. Furthermore, students who received CRA-I-Schema instruction outperformed students in Schema-Alone condition, even with relatively brief intervention sessions. Hence, schools may consider incorporating more explicit CRA-I-Schema instruction into classroom teaching, and emphasize the connections between CRA-I-Schema instruction and conceptual understanding of WPS. In conclusion, students with different levels of WPS performance may benefit differently from the CRA-I-Schema instruction. Future researchers may want to design the intervention method to make it more targeted on addressing specific challenges of students.

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