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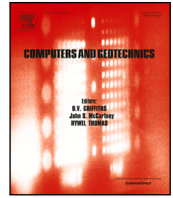
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Research paper

Semi-analytical predictive model for natural and artificial thawing of circular ground-ice walls

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ABSTRACT

Artificial ground freezing (AGF) is a ground improvement technique enabling the construction of underground structures in challenging geological conditions. After constructing an underground structure within the ground-ice cofferdam, the soil undergoes a thawing process that can impact the structure stability and waterproofing properties of the lining. Minimizing or preventing potential damage, as well as avoiding delays in construction, can be achieved through a rational design of thawing regimes. In this paper, we present a semi-analytical model for the thermal behavior of ice-wall during its natural or artificial thawing. The process is described by three independent one-dimensional mathematical problems: the thawing of the outer surface of the ice wall, the thawing of its inner surface, and the thawing of soils around the freeze pipes (in the case of artificial thawing). The proposed approach facilitates the calculation of natural and artificial thawing times and the power required for artificial thawing. The efficiency of the model is demonstrated by comparison with numerical simulation results. This makes the approach suitable and desirable for engineering practice. Importantly, the model allows for seamless analysis of several combinations of influencing factors to select thawing parameters aligned with the requirements of different construction projects.

1. Introduction

Artificial ground freezing (AGF) is a ground improvement technique for enhancing geological hydro-mechanical conditions by temporarily freezing the soil. This modification is crucial for ensuring control over groundwater and maintaining the safety of construction works. Widely employed in both civil (Viggiani and de Sanctis, 2009; Mandolini and Viggiani, 2017; Cai et al., 2022; Nikolaev et al., 2022; Xiang et al., 2023; Nikolaev et al., 2024a) and mining (Alzoubi et al., 2019; Levin et al., 2021; Aparin and Sankovsky, 2022; Semin et al., 2024; Chen et al., 2024) industries, the AGF has proven to be versatile. Further discussion of AGF applications is provided in a recent review by Zhou et al. (2022).

The AGF operation typically progresses through three stages (Fedukin, 1968; Trupak, 1974). The initial stage involves 'active freezing' or 'ice wall formation', where a frozen wall is rapidly created. Active freezing continues until the frozen wall reaches the designed thickness based on mechanical strength criteria. The second stage, 'passive freezing' or 'ice-wall holding/maintenance', aims to sustain the frozen wall until a waterproof lining is built to cover the shaft/tunnel

walls. The third stage starts after the lining construction. For indirect freezing methods, the refrigeration plant is stopped, while for direct freezing methods, the loading of refrigerants into the freeze pipes ceases. Consequently, the frozen wall begins to thaw. Thawing can occur naturally, driven by the inflow of heat from the surrounding soils and the shaft or tunnel itself. Alternatively, it can be done artificially, where the common approach involves heating a liquid and circulating it through the freeze pipes. Natural thawing of the frozen wall may extend over several years, while artificial thawing can be completed in just a few weeks or months (Fedukin, 1968; Trupak, 1974; Harris, 1995; Andersland and Ladanyi, 2004).

During thawing, the mechanical properties of soils change, potentially causing damage to the lining (Nasonov et al., 1992; Harris, 1995). The process may be accompanied by additional development of frost heat and thaw settlement. As the ice wall partially melts, grouting works are performed, which must be accurately integrated into the construction schedule to prevent delays. After the cofferdam is compromised, water starts penetrating through it, revealing defects in the grouting and lining's insulation properties. These must be

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fixed immediately to prevent flooding of the newly built underground structure.

Predictions of ice wall thawing require an accurate mathematical description of the dynamics of the phase change boundary. However, as indicated in the review by [Ngo et al. \(2022\)](#), modern studies on soil thawing mostly consider thawing of seasonally or permanently frozen soils, while there are no reliable models for thawing of artificially frozen soils, see also [Levin et al. \(2021\)](#). For example, in a study by [Cai et al. \(2023\)](#), a laboratory-scale test was conducted to examine soil thawing, revealing dynamic variations in the ice wall along the depth due to moisture migration. [Levin et al. \(2021\)](#) analyzed the ice wall thawing during the construction of a mine shaft. Additionally, [Zhou and Tang \(2015\)](#) investigated the effect of thawing on artificially frozen soil properties using centrifuged modeling. An engineering analysis of energy conservation within the ice wall during thawing was undertaken by [Parshakov et al. \(2021\)](#) based on the classical studies of [Trupak \(1954, 1974\)](#) as a supplementary tool to justify the results of numerical simulations. The process of natural thawing was also studied during laboratory tests conducted by [Ma et al. \(2023\)](#), focusing on the construction of a subway cross passage for an underground system. Various thermal insulation methods were applied to reduce the heat flux from the ice wall to the tunnel. The influencing factors on thawing are further discussed by [Zou et al. \(2023\)](#).

The common method for numerical thermal analysis of ice-wall formation and thawing is the finite element method ([Zhou and Meschke, 2013](#); [Vitel et al., 2016](#); [Casini et al., 2016](#); [Tounsi et al., 2020](#); [Zheltnin et al., 2022](#); [Hou et al., 2022](#); [Bublik and Semin, 2023](#); [Li and Li, 2023](#)). However, numerical simulations are typically time-consuming, especially when a large number of simulations must be performed to optimize freezing and thawing parameters for a large number of soil layers intersected by a vertical shaft or a tunnel. Additionally, numerical simulations alone do not provide an in-depth understanding of the parameters that contribute to the process development. It is essential to have analytical methods that can be used in engineering practice for preliminary analysis to expedite the attainment of the most reliable project solution.

While there are several analytical and semi-analytical models to describe the ice wall formation ([Trupak, 1954](#); [Baholdin, 1963](#); [Frvik et al., 1980](#); [Trupak, 1974](#); [Sanger and Sayles, 1979-4](#); [Fangzheng and Mingping, 2004](#); [Jiang et al., 2009](#); [Zhang et al., 2012](#); [Nikolaev, 2022](#)), and some simplified cases of soil thawing ([Zhang et al., 2012](#); [Cai et al., 2015](#)), a full model that can be used to engineering thermal design of the ice wall thawing is not available. This study aims to develop and test such a model for one of the most popular cases in the engineering practice — thawing of a circular ice wall around a vertical shaft. This model is based on the recent semi-analytical model of the ice-wall formation proposed by [Nikolaev et al. \(2022\)](#). While the paper discusses the thawing of a circular ice wall around a vertical shaft, the results can be applied to cases involving horizontal circular ice walls created around tunnels, as well as modified for the case of linear ice walls.

The paper is structured as follows. The semi-analytical model for soil thawing is derived in Section 2. Results of several verification tests are given in Section 3 and Appendix A to demonstrate the accuracy of the developed approach. The model is used to analyze the influence of various technological and geological properties on the dynamics of ice wall thawing with results presented in Section 4. The conclusions are drawn in Section 5.

2. Mathematical description of ice wall thawing

2.1. Differential equations of heat transfer

Saturated soils contain three major components: solid matrix particles, liquid water, and solid water (ice). Their physical parameters are denoted by subscripts s , w , and i , respectively. In the unfrozen state, the soil does not contain ice, whereas frozen soils can contain both liquid

and solid water. In the present paper, to simplify the derivation of the model, the presence of liquid water in the frozen soils will not be taken into account. The properties of soils in the unfrozen and frozen states are denoted by the subscripts un and f , respectively.

The soil's thermal behavior is governed by the energy conservation law. The differential equation describing this law is constructed within the framework of the volume average method, which involves averaging microscopic properties of soil components over a representative elementary volume ([Vitel et al., 2016](#)). According to [Grenier et al. \(2018\)](#), [Stuuroop et al. \(2021\)](#), in the case of conductive heat transfer, the energy conservation equation can be presented in the plane Cartesian coordinate system (x, y) as

$$C_V^{eq} \frac{\partial T}{\partial t} = \nabla \cdot \mathbf{q} = \left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} \right] \quad (1)$$

where $T(\mathbf{x}, t)$ is temperature; $C_V^{eq}(\mathbf{x}, t) = C_V + L_w n \frac{\partial S_w}{\partial T}$ is the 'equivalent' ('apparent') heat capacity, that takes into account the release of the volumetric latent heat of water solidification L_w ; $S(\mathbf{x}, t)$ is the saturation degree, for which $S_w + S_i = 1$; C_V is the volumetric heat capacity of soils; n is the porosity; t is the time; ∇ is the mathematical operator that defines the divergence of a vector field; and $\mathbf{q}(\mathbf{x}, t)$ is the heat flux due to heat conduction with the component q_x and q_y along axis x and y of the considered Cartesian coordinate system respectively.

The dependence of the liquid water content on temperature below the freezing point is defined by the soil freezing characteristic curve (SFC) ([Kurylyk and Watanabe, 2013](#); [Stuuroop et al., 2021](#)). According to it, the soil state can be divided into unfrozen and frozen. In this paper, a simple piece-wise SFC is used:

$$S_w(\mathbf{x}, t) = \begin{cases} 0, & T < T_f - \Delta T_m \\ \frac{(T - T_f) - \Delta T_m}{\Delta T_m}, & T_f - \Delta T_m \leq T \leq T_f \\ 1, & T_f < T \end{cases} \quad (2)$$

Here, T_f represents the freezing temperature at which the freezing of pore water begins, and ΔT_m denotes the temperature range of the transition (mushy) zone between frozen and unfrozen soils.

The heat flux due to the heat conduction is described by Fourier's law, which for a two-dimensional plane domain can be written as:

$$\mathbf{q}(\mathbf{x}, t) = -\lambda_{av} \nabla T = -\lambda_{av} \left[\frac{\partial T}{\partial x} \mathbf{i} + \frac{\partial T}{\partial y} \mathbf{j} \right], \quad (3)$$

where $\lambda_{av}(\mathbf{x}, t)$ is the average heat conductivity of soil, ∇ is the mathematical operator that defines the gradient of a scalar field, and $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors along axis x and y of the considered Cartesian coordinate system respectively.

The volumetric heat capacity of the three-phase medium can be defined as an arithmetic average:

$$C_V = n S_w \rho_w C_w + n (1 - S_w) \rho_i C_i + (1 - n) \rho_s C_s \quad (4)$$

where ρ_w , ρ_i , ρ_s and C_w , C_i , C_s represent the density and specific heat capacity of soil components (liquid and solid water and soil grains), respectively.

Several mathematical models exist for the average heat conductivity of different types of soils ([Cox et al., 1999](#); [He et al., 2021](#)). In the present study we define it following [Cote and Konrad \(2005\)](#), by

$$\lambda_{av} = \lambda_w^{n S_w} \lambda_i^{n(1-S_w)} \lambda_s^{(1-n)}. \quad (5)$$

where λ_w , λ_i and λ_s represent the heat conductivity of liquid and solid water and soil grains, respectively.

It should be noted that soil freezing and thawing behavior is a significantly more complicated process that cannot be fully described by Eqs. (1)–(5). The presence of water filtration, due to both temperature and water gradients, as well as the formation of ice lenses and associated frost heave, can alter the dynamics of phase change propagation; for more details, see [Li and Li \(2023\)](#), [Li et al. \(2023a,b\)](#), [Zhang et al. \(2023\)](#), [Nikolaev et al. \(2023, 2024b\)](#). However, the application

of advanced mathematical models can be time-consuming and may require a significant number of parameters, defined through laboratory experiments for every soil layer crossed by the ice wall, which may not always be available. Therefore, it is important to develop relatively simple engineering methods that reliably assess the dynamics of the ice wall and determine freezing and thawing times.

The solution of the system of Eqs. (1) to (5) is usually performed numerically, e.g., by the finite element method. In the following sections, we will use such a numerical approach for the prediction of the ice wall formation and thawing. However, for some problems, analytical and semi-analytical solutions exist. In such cases, the domain is typically divided into two distinct regions with constant thermal properties and without a transition zone between them, i.e., $T_m \rightarrow 0$.

In the case of heat transfer around a cylindrical heat source, the Eqs. (1)–(5) can be conveniently expressed in a plane cylindrical coordinate system (r, θ) . If the heat transfer is axisymmetrical, this can be further simplified (Hahn and Özisik, 2012), representing a one-dimensional heat transfer with phase change in a semi-infinite domain:

$$\frac{\partial T_f}{\partial t} = \alpha_f \left(\frac{\partial^2 T_f}{\partial r^2} + \frac{1}{r} \frac{\partial T_f}{\partial r} \right) \quad (6)$$

$$\frac{\partial T_{un}}{\partial t} = \alpha_{un} \left(\frac{\partial^2 T_{un}}{\partial r^2} + \frac{1}{r} \frac{\partial T_{un}}{\partial r} \right) \quad (7)$$

where T_f and T_{un} are the temperature distributions in frozen and unfrozen cylindrical regions of soils $\alpha_f = \lambda_f / (c_{V,f})$ and $\alpha_{un} = \lambda_{un} / (c_{V,un})$ is a thermal diffusivity coefficient of frozen and unfrozen soils.

The properties of frozen and unfrozen soils are defined by substitution of Eq. (2) into (4) and (5). This yields the corresponding values of the volumetric heat capacity $C_{V,f}$ and $C_{V,un}$:

$$\begin{aligned} C_{V,f} &= n\rho_i C_i + (1-n)\rho_s C_s, \\ C_{V,un} &= n\rho_w C_w + (1-n)\rho_s C_s \end{aligned} \quad (8)$$

and the corresponding values of the heat conductivity λ_f and λ_{un} :

$$\begin{aligned} \lambda_f &= \lambda_i^n \lambda_s^{(1-n)}, \\ \lambda_{un} &= \lambda_w^n \lambda_s^{(1-n)}. \end{aligned} \quad (9)$$

On the moving boundary between frozen and unfrozen cylindrical regions of soils, $R_{ph}(t)$, the following boundary conditions are defined:

$$T_f(t, R_{ph}) = T_{un}(t, R_{ph}) \quad (10)$$

$$\left(\lambda_f \frac{\partial T_f}{\partial r} - \lambda_{un} \frac{\partial T_{un}}{\partial r} \right) \bigg|_{r=R_{ph}(t)} = L \frac{dR_{ph}(t)}{dt} \quad (11)$$

where $L = L_{w,n}$ represents the latent heat of water solidification in a unit volume of soil. This condition defines that the difference between the amounts of heat transferring to the moving phase change boundary from both sides per unit of time must account for the latent heat of solidification released during the movement of this boundary per the same time interval.

At an infinite distance from the heat source/sink, the temperature is constant and equal to the initial temperature of the soil $T_{in}(r \gg 0)$.

The system of Eqs. (6)–(11), supplemented by a condition at the boundary of the semi-infinite domain, can be employed for the analytical description of ice-wall formation and thawing. An analytical solution for the Neumann (constant heat flux) boundary condition is well-known (Hahn and Özisik, 2012). A semi-analytical solution for Dirichlet (constant temperature) boundary condition is presented in the work of Nikolaev et al. (2022). This formulation establishes an effective approach for analyzing ice-wall thawing.

2.2. Semi-analytical model of ice wall thawing

The life cycle of the ice wall can be divided into three periods: active freezing (formation of the ice wall), passive freezing (maintenance of the ice wall shape), and thawing. The thermal state of the ice wall differs during these stages.

During active freezing, the average temperature of the ice wall is at its minimum and can be roughly assessed to be 0.4 of the temperature of the freeze pipe surface (Fedukin, 1968), or accurately calculated based on the analytical solution by, e.g. Hu et al. (2018), Shao et al. (2020). Once the ice wall reaches its projected size, active freezing concludes, and passive freezing begins. The purpose of this stage is to maintain the projected shape of the ice wall until the construction works are accomplished. During passive freezing, the freezing plant operates at reduced power (approximately 30%–50% of the power during active freezing), leading to an increase in the average temperature of the ice wall. After the shaft or tunnel is constructed, and waterproof lining is installed, the ice wall can be liquidated. Thawing of the ice wall can occur naturally or artificially by pumping a hot liquid through the freeze pipes. Regardless of the thawing method, the average temperature of the ice wall increases until it reaches the freezing temperature of the pore water. This stabilization of the temperature is observed shortly after the freeze pipe is turned off. The uniformity of temperature within the ice wall during thawing was demonstrated, for example, by Levin et al. (2021). The fact that the temperature of the ice wall remains constant throughout the entire thawing period will be considered the basis for the developed semi-analytical model.

The ice wall thawing is shown schematically in Fig. 1. It is assumed that the inner and outer surfaces of the ice wall are cylindrical and are moving inwards from their positions at the end of passive freezing. The temperature of the ice wall before thawing is assumed to be constant and equal to the freezing temperature of the water. For simplicity, the water solidification temperature is considered to be 0 °C.

In the case of artificial thawing of the ice wall, in addition to the propagation of the inner and outer surfaces, there is a thawing zone around every freeze pipe. This thawing zone has a cylindrical shape with the outer boundary moving outwards from the freeze pipe. This shape is maintained until the neighboring zones are merged. After that, the forms of these cylinders as well as the temperature distribution in them are distorted. However, to simplify the mathematical representation of the artificial soil thawing, let us assume that in the main plane, see Fig. 1, the dynamics of the thawing zone around every freeze pipe will continue to align with the same mathematical law as defined before the thawing cylinders are merged.

In this case, the problem of ice wall thawing can be divided into three independent one-dimensional problems:

- Heat transfer to the ice wall from the hot liquid circulated in the pipes.
- Heat transfer through the inner cylindrical surface from the shaft lining.
- Heat transfer through the outer cylindrical surface from the surrounding soils.

In the case of natural thawing, only heat transfer to the inner and outer surfaces of the ice wall must be considered. The thawing is complete when the inner and the outer boundaries meet. In the case of artificial thawing, the thawing is complete when the thawing zone around the freeze pipes meets both the inner and the outer boundaries.

The thawing around the freeze pipes is independent, as both initial and boundary conditions are accurately defined, regardless of the previous freezing history. This can be reasonably assumed also for the thawing in the inner region. In both cases, the ‘semi-infinite’ domains have an initial temperature of 0 °C. For the thawing of the outer region, however, the initial temperature of the soil is not defined, and it depends on the previous history of the ice wall. In other words, the

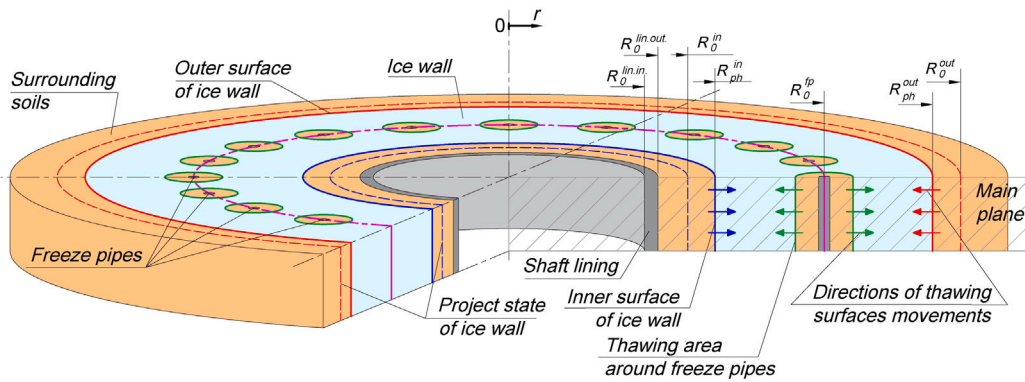


Fig. 1. The model representation of the ice-wall thawing.

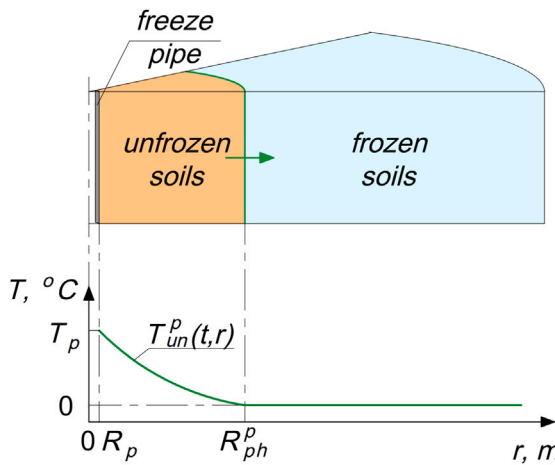


Fig. 2. The formation of the thawing region around the freeze pipe during artificial thawing of soils.

longer the ice wall exists, the lower the temperature around the ice wall, and the larger the influence zone of the ice wall in the surrounding soils. As a result, the solutions to the problems will be examined at different time scales. The first two problems will be considered without taking into account the time of active and passive freezing stages, while the last problem will be solved for the time that is counted from the beginning of the freezing process. To establish the time from the initiation of thawing, we will represent time with the letter t , while employing the letter $\tilde{t}$ to denote the overall time from the start of freezing works. These two time variables will be connected by the relation $t = \tilde{t} - \tilde{t}_{in}$, where $\tilde{t}_{in}$ signifies the time at which the thawing process begins.

2.3. Thawing of soils around a freeze pipe

The thawing of soils around the freeze pipe can be described by a solution of the system (6)–(11). In this case, the ice wall is considered as a semi-infinite body with the initial temperature of $0^\circ C$. The boundary of the domain corresponds to the position of the freeze pipe axis. The problem is shown in Fig. 2.

The freeze pipe with circulating hot liquid requires an additional boundary condition to the system (6)–(11). Three types of boundary conditions can be defined to represent different regimes of liquid flow:

- Dirichlet boundary condition with constant temperature of the pipe equal or close to the temperature of the liquid;
- Robin (convective) boundary condition with convective heat transfer coefficient defined by the liquid flow regime;

- Neumann boundary condition with heat flux from the pipe to the soils defined by the power of the heater used to heat the liquid.

The choice should be based on the equipment used for heating the liquid in the considered construction project. The implementation of these types of boundary conditions in the model is given as follows.

2.3.1. Dirichlet boundary condition

The solution to the problem of soil thawing due to the presence of a heat source with constant temperature can be obtained using the semi-analytical model by Nikolaev et al. (2022) that was developed to describe freezing of soils. Based on it, if the outer radius of the freeze pipe wall is denoted by R_p and its positive constant temperature as T_p , the temperature distribution in the unfrozen region $T_{un}^p(t, r)$ around the freeze pipe can be described as:

$$T_{un}^p(t, r) = T_p \left\{ 1 - \frac{\left[E_1 \left(\frac{R_p^2}{4\alpha_{un}t} \right) - E_1 \left(\frac{r^2}{4\alpha_{un}t} \right) \right]}{\left[E_1 \left(\frac{R_p^2}{4\alpha_{un}t} \right) - E_1 \left(\frac{R_{ph}^2}{4\alpha_{un}t} \right) \right]} \right\}, \quad \text{if } R_p \leq r \leq R_{ph}^p \quad (12)$$

where $E_1(x) = \int_x^\infty (e^{-p}/p) dp$ is an integral exponential function. The temperature of the ice wall is assumed to be constant and equal to zero and can be mathematically represented as $T_f^p(t, r) = 0^\circ C$.

This relation can be used to define the heat flux to the 1 m freeze pipe Q_p at any time instance t by applying the Fourier Law, Eq. (3), at the boundary of the freeze pipe, $r = R_p$. This gives:

$$Q_p = \frac{-4\pi\lambda_{un}T_p \exp \left\{ -\frac{R_p^2}{4\alpha_{un}t} \right\}}{\left[E_1 \left(\frac{R_p^2}{4\alpha_{un}t} \right) - E_1 \left(\frac{R_{ph}^2}{4\alpha_{un}t} \right) \right]}. \quad (13)$$

Following Nikolaev et al. (2022), the position of the phase change boundary is defined by the relation:

$$R_{ph}^p(t) = \beta_p t^{\omega_p} + R_p \quad (14)$$

where ω_p and β_p are constants that can be numerically calculated from the transcendental equation obtained by substitution of (14) into the conditions on the boundary between frozen and unfrozen region, Eq. (11), considered at two relatively large time instances. For the problem of the formation of thawing zone around the freeze pipe, we use 3 and 50 days; however, any larger instances, e.g., 50 and 500 days, provide nearly the same results. This condition can be presented in the following form:

$$\lambda_{un} \frac{T_p \exp \left[-\frac{(\beta_p t^{\omega_p} + R_p)^2}{4\alpha_{un}t} \right]}{E_1 \left(\frac{R_p^2}{4\alpha_{un}t} \right) - E_1 \left[\frac{(\beta_p t^{\omega_p} + R_p)^2}{4\alpha_{un}t} \right]} = \frac{\omega_p \beta_p L}{2} \left(\beta_p t^{2\omega_p-1} + R_p t^{\omega_p-1} \right) \quad (15)$$

After ω_p and β_p are found from Eq. (15), the position of the phase change boundary and the heat flux to the ice wall can be found by relations (13) and (14), respectively.

2.3.2. Robin boundary condition

In the absence of a relatively simple analytical or semi-analytical solution for the current problem with the Robin boundary condition, we employ the model of a single ice cylinder proposed by Nikolaev et al. (2022) to develop a practical solution suitable for engineering applications. The model allows us to consider heat transfer in multi-layer domains, thereby taking into account the thermal resistance caused by the presence of the layer of the freeze pipe wall. This type of boundary condition involves a special function that governs the heat flux from the freeze pipe to the circulating liquid within it. To characterize this heat transfer, the temperature of the liquid T_l and the convection heat transfer coefficient η_p are defined as follows:

$$Q_p(t) = \eta_p [T_l - T_p(t)] \quad (16)$$

In this boundary condition, the temperature of the freeze pipe, T_p , and the heat flux, Q_p , are functions of time.

To solve the system of differential Eqs. (6) to (11) in the presence of the boundary condition (16), we assume that the heat transfer between the liquid and the freeze pipe wall can be considered as steady-state. Additionally, we assume that the thermal resistance caused by the presence of the convective heat transfer coefficient can be accounted for by modifying the heat conductivity of the material of the freeze pipe wall and considering this thermal resistance in the solution.

The convective heat transfer resistance χ_l between the freeze pipe wall and its internal liquid and the conductive heat transfer resistance of the cylindrical wall of the freeze pipe χ_p with the thickness Δ_p have the following forms:

$$\chi_l = \frac{1}{\eta_p 2\pi R_p h}, \quad \chi_p = \frac{\ln \frac{R_p}{R_p - \Delta_p}}{\lambda_p 2\pi h} \quad (17)$$

where h is the height of the pipe and λ_p is the heat conductivity of the freeze pipe material.

The effective heat conductivity λ_{ef}^p is related to the effective heat transfer resistance by $\chi_{ef} = \ln [R_p / (R_p - \Delta_p)] / (\lambda_{ef}^p 2\pi h)$, where:

$$\chi_{ef} = \chi_l + \chi_p \quad (18)$$

Substitution of Eq. (17) into Eq. (18) gives λ_{ef}^p as:

$$\lambda_{ef}^p = \frac{\lambda_p}{1 + \frac{\lambda_p}{\eta_p (R_p - \Delta_p) \ln [R_p / (R_p - \Delta_p)]}} \quad (19)$$

Following the approach by Nikolaev et al. (2022), the temperature distribution within the freeze pipe wall $T_w^p(t, r)$ with the heat conductivity defined by the effective heat conduction coefficient (19) can be described as:

$$T_w^p(t, r) = T_l + \frac{(T_p(t) - T_l) \left[E_1 \left(\frac{(R_p - \Delta_p)^2}{4\alpha_{ef}^p t} \right) - E_1 \left(\frac{r^2}{4\alpha_{ef}^p t} \right) \right]}{\left[E_1 \left(\frac{(R_p - \Delta_p)^2}{4\alpha_{ef}^p t} \right) - E_1 \left(\frac{R_p^2}{4\alpha_{ef}^p t} \right) \right]}, \quad \text{if } R_p - \Delta_p \leq r \leq R_p \quad (20)$$

where α_{ef}^p is the effective thermal diffusivity of the freeze pipe wall; $T_p(t)$ is the temperature of the outer surface of the freeze pipe wall.

The temperature distribution within the unfrozen region around the freeze pipe and the position of the phase change boundary can be defined by Eqs. (12) and (14), used for the case with the Dirichlet boundary condition, however, the value $T_p(t)$ is variable and defined

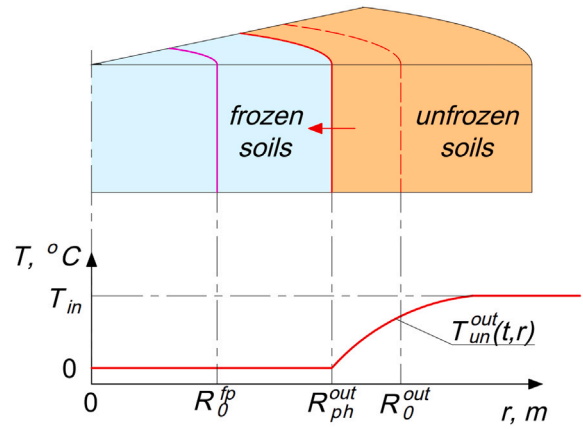


Fig. 3. Thawing of soils around the cylindrical ice wall.

by the relation:

$$T_p(t) = \left\{ \lambda_{ef}^p T_l \exp \left[-\frac{R_p^2}{4\alpha_{ef}^p t} \right] \left[E_1 \left(\frac{R_p^2}{4\alpha_{un}^p t} \right) - E_1 \left(\frac{R_{ph}^2}{4\alpha_{un}^p t} \right) \right] \right\} \times \left\{ \lambda_{ef}^p \exp \left[-\frac{R_p^2}{4\alpha_{ef}^p t} \right] \left[E_1 \left(\frac{R_p^2}{4\alpha_{un}^p t} \right) - E_1 \left(\frac{R_{ph}^2(t)}{4\alpha_{un}^p t} \right) \right] + \lambda_{un} \exp \left[-\frac{R_p^2}{4\alpha_{un}^p t} \right] \left[E_1 \left(\frac{(R_p - \Delta_p)^2}{4\alpha_{ef}^p t} \right) - E_1 \left(\frac{R_p^2}{4\alpha_{ef}^p t} \right) \right] \right\}^{-1} \quad (21)$$

The heat flux to the freeze pipe is described by Eq. (13).

2.3.3. Neumann boundary condition

If the heat flux Q_p to the freeze pipe is given, i.e., Neumann boundary conditions, the classical solution presented in Hahn and Özisik (2012) can be applied. The temperature distribution in the thawing region around the freeze pipe can be described by:

$$T_{un}^p(t, r) = \frac{Q_p}{4\pi \lambda_{un}} \left[E_1 \left(\frac{R_{ph}^2(t)}{4\alpha_{un}^p t} \right) - E_1 \left(\frac{r^2}{4\alpha_{un}^p t} \right) \right] \quad (22)$$

The solution of this equation when $r = R_p$ determines the temperature of the pipe surface. In practical calculation, this temperature can be considered approximately equal to the temperature of the heat transfer liquid within the pipe. The position of the phase change boundary is defined by the relation (14) considering that $\omega_p = 0.5$ and the coefficient β_p is defined as a root of the transcendental equation:

$$Q_p \exp \left[-\frac{\beta_p^2}{4\alpha_{un}^p} \right] = \pi \beta_p^2 L \quad (23)$$

2.4. Thawing of soils in the outer region of a cylindrical ice wall

The temperature of soils far away from the ice wall is assumed constant T_{in} . The ice wall is considered a solid cylinder, with an axis through the origin of the coordinate system. The problem is shown in Fig. 3. Heat is transferred to the boundary between frozen and unfrozen regions due to the temperature gradient around the ice body. Since the temperature of the ice wall is constant, there is no heat flux inside the wall.

The temperature distribution in the outer region of unfrozen soils $T_{un}^{out}(\tilde{t}, r)$ around the ice wall can be described by the following relation (Hahn and Özisik, 2012):

$$T_{un}^{out}(\tilde{t}, r) = T_{in} \left\{ 1 - \frac{E_1\left(\frac{r^2}{4\alpha_{un}\tilde{t}}\right)}{E_1\left(\frac{R_{ph}^{out2}(\tilde{t})}{4\alpha_{un}\tilde{t}}\right)} \right\} \quad (24)$$

By differentiating this equation with respect to r , we find the heat flux to the 1 m height of the ice wall at the point $r = R_{ph}^{out}$:

$$Q_{out}(\tilde{t}, R_{ph}^{out}) = 2\pi R_{ph}^{out} \lambda_{un} \frac{dT_{un}^{out}(\tilde{t})}{dr} = 4\pi \lambda_{un} \frac{T_{in} \exp\left[-\frac{R_{ph}^{out2}(\tilde{t})}{4\alpha_{un}\tilde{t}}\right]}{E_1\left[\frac{R_{ph}^{out2}(\tilde{t})}{4\alpha_{un}\tilde{t}}\right]} \quad (25)$$

These equations can be used to describe the temperature distribution and the heat flux during the formation of a single ice cylinder around a freeze pipe with constant temperature (Cai et al., 2018; Nikolaev et al., 2022) or constant heat flux on its surface (Hahn and Özisik, 2012). However, such simple steady boundary conditions are only observed during active ground freezing. The non-linear boundary conditions typical for ‘passive’ ground freezing, as well as the stage of the thawing of the ice wall, make these equations inadequate.

For developing approximate relations for the dynamics of the outer surface of the ice wall, we adopt the methodology proposed by Trupak (1974). This approach uses the principle of energy conservation to describe the dynamics of the ice wall. It states that the heat flowing from the surrounding soils to the ice wall is used to increase the enthalpy of the ice wall, thus contributing to its thawing. More details about the application of energy conservation to the ice-wall thawing are presented by Parshakov et al. (2021).

Given that the heat flux to the ice wall depends on its previous dynamics, directly applying Eq. (25) to describe the heat flux at each time instance cannot provide accurate results. However, by assuming a linear change in its value during thawing, we can determine an average value, Q_{out}^{av} , that characterizes the entire thawing process. We define the average heat flux during thawing as the heat flux to the cylindrical surface at the position of the ice wall’s center, which approximately equal to the position of the freeze pipes R_0^{fp} , at the time required to reach this point, $\tilde{t}_{fp}$, calculated from the beginning of the freezing, i.e. $Q_{out}^{av} = Q_{out}(\tilde{t}_{fp}, R_0^{fp})$.

To define $\tilde{t}_{fp}$, we consider the thawing of half of the ice wall thickness from the position $R_{ph}^{out}(\tilde{t}_{in}) = R_0^{out}$, where $\tilde{t}_{in}$ is the time of the beginning of thawing, to the position $R_{ph}^{out}(\tilde{t}_{fp}) = R_0^{fp}$. Following Trupak (1974), the change in enthalpy of half of the ice wall during thawing can be expressed as:

$$\Delta H = \pi \left[R_0^{out2} - R_0^{fp2} \right] H_{un,th} \quad (26)$$

where $H_{un,th}$ represents the change in the enthalpy of a unit volume of soils, associated with the absorption of heat leading to a temperature increase from 0 °C to $\tilde{t}_{in}$. In other words, $H_{un,th} = L + \tilde{t}_{in} C_{V_{un}}$.

The heat, ΔH , must be transferred to the ice wall from the surrounding soils at a rate determined by the average heat flux Q_{out}^{av} . This can be expressed as:

$$\Delta H = Q_{out}^{av} (\tilde{t}_{fp} - \tilde{t}_{in}) \quad (27)$$

This equation establishes the connection between the time instance $\tilde{t}_{fp}$ needed to melt half of the ice wall and the following alteration of its outer radius from R_0^{out} to R_0^{fp} , if the change in enthalpy ΔH is known. Consequently, by setting Eqs. (26) and (27) equal to each other, we

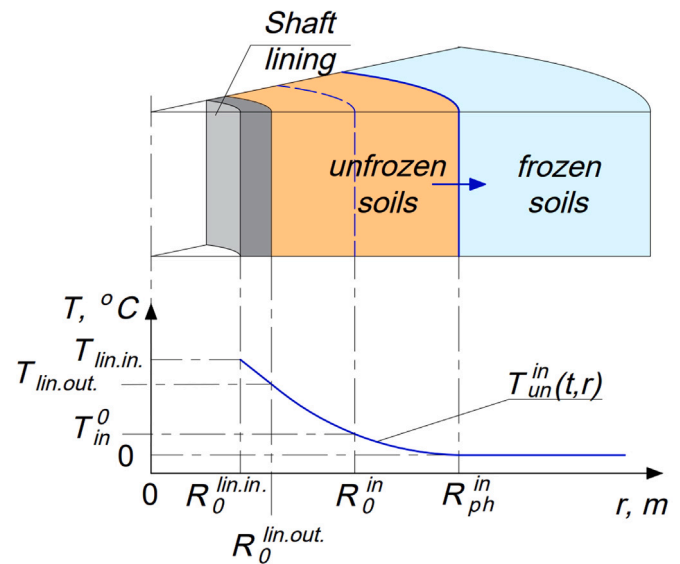


Fig. 4. Thawing of soils within the cylindrical ice wall.

obtain:

$$\left[R_0^{out2} - R_0^{fp2} \right] H_{un,th} = 4\lambda_{un} \frac{T_{in} \exp\left[-\frac{R_0^{fp2}}{4\alpha_{un}\tilde{t}_{fp}}\right]}{E_1\left[\frac{R_0^{fp2}}{4\alpha_{un}\tilde{t}_{fp}}\right]} (\tilde{t}_{fp} - \tilde{t}_{in}) \quad (28)$$

The solution of this transcendental equation provides the time $\tilde{t}_{fp}$ at which the heat flux to the ice wall equals the average heat flux over the entire thawing period.

Since the dynamics of the ice wall during the thawing cannot be directly described within the framework of this theory, we assume that the position changes linearly. This can be expressed as:

$$\begin{aligned} R_{ph}^{out}(t) &= R_0^{out} - \beta_{out}(\tilde{t}_{in} - \tilde{t}) \\ &= R_0^{out} - \frac{R_0^{out} - R_0^{fp}}{\tilde{t}_{fp} - \tilde{t}_{in}} (\tilde{t}_{in} - \tilde{t}) \end{aligned} \quad (29)$$

Here, β_{out} can be considered as the rate of change of the outer position of the ice wall, or in other words, its speed.

2.5. Thawing of soils in the inner region of a cylindrical ice wall

The thawing of the inner surface of the ice wall can be conceptualized as heat transfer in a multi-layer cylindrical wall, with one of the inter-layer boundaries associated with the phase change (see Fig. 4). It is important to account for the disparity in thermal properties between the shaft lining material and soils, as well as the presence of the unfrozen region between the lining and the ice wall. The influence of warm moving air within the shaft can be considered by either setting the temperature of the inner lining surface $T_{lin.in.}$ to a constant value equal to the temperature of the air T_{air} or defining Robin (convective) boundary conditions through the additional definition of the convective heat transfer coefficient from the inner lining surface to the air within the shaft $\eta_{lin.}$. In most cases, this problem can be simplified, as there is usually no unfrozen zone between the lining and the ice wall, i.e., $R_0^{lin.out.} = R_0^{in.}$, and/or by neglecting the presence of the lining.

Robin boundary condition can be applied by the procedure described in Section 2.3. In this case, the heat conductivity of the lining material is modified to incorporate the convective thermal resistance between the inner surface of the lining and the air within the shaft.

The effective heat conductivity can be defined by an equation identical to the Eq. (19):

$$\lambda_{ef}^{lin} = \frac{\lambda_{lin}}{1 + \frac{\lambda_{lin}}{\eta_{lin} R_0^{lin.in.} \ln \left[\frac{R_0^{lin.out.}}{R_0^{lin.in.}} \right]}} \quad (30)$$

where $R_0^{lin.in.}$ and $R_0^{lin.out.}$ represent the inner and outer radii of the shaft lining, respectively, and λ_{lin} is the heat conductivity of the lining material.

The heat transfer in a multi-layer cylindrical wall can be described using the approach outlined by Nikolaev et al. (2022). The temperature distribution within the lining is given by:

$$T_{un}^{lin}(t, r) = T_{air} + (T_{lin.out.}(t) - T_{air}) \times \frac{\left[E_1 \left(\frac{R_0^{lin.in.2}}{4\alpha_{ef}^{lin} t} \right) - E_1 \left(\frac{r^2}{4\alpha_{ef}^{lin} t} \right) \right]}{\left[E_1 \left(\frac{R_0^{lin.in.2}}{4\alpha_{ef}^{lin} t} \right) - E_1 \left(\frac{R_0^{lin.out.2}}{4\alpha_{ef}^{lin} t} \right) \right]}, \quad \text{if } R_0^{lin.in.} \leq r \leq R_0^{lin.out.} \quad (31)$$

where, $T_{lin.out.}(t)$ represents the temperature of the outer surface of the lining, and α_{ef}^{lin} denotes the effective thermal diffusivity of the shaft lining, which considers the presence of convective thermal resistance on the inner lining surface.

Similarly, the temperature distribution within the unfrozen region between the lining and the maximum project size of the ice wall R_0^{in} is given by:

$$T_{un}^{in}(t, r) = T_{lin.out.}(t) + (T_{in}^0(t) - T_{lin.out.}(t)) \times \frac{\left[E_1 \left(\frac{R_0^{lin.out.2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{r^2}{4\alpha_{un}^{in} t} \right) \right]}{\left[E_1 \left(\frac{R_0^{lin.out.2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{R_0^{in2}}{4\alpha_{un}^{in} t} \right) \right]}, \quad \text{if } R_0^{lin.out.} \leq r \leq R_0^{in} \quad (32)$$

where, $T_{in}^0(t)$ represents the temperature of the unfrozen soils at the position R_0^{in} .

Finally, the temperature distribution within the thawed region of the ice wall is given by:

$$T_{un}^{in}(t, r) = T_{in}^0(t) \times \left\{ 1 - \frac{\left[E_1 \left(\frac{R_0^{in2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{r^2}{4\alpha_{un}^{in} t} \right) \right]}{\left[E_1 \left(\frac{R_0^{in2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{R_{ph}^{in2}}{4\alpha_{un}^{in} t} \right) \right]} \right\}, \quad \text{if } R_0^{in} \leq r \leq R_{ph}^{in} \quad (33)$$

The position of the phase change boundary is defined by the relation:

$$R_{ph}^{in}(t) = \beta_{in} t^{\omega_{in}} + R_0^{in} \quad (34)$$

where, ω_{in} and β_{in} are constants that can be numerically defined from the conditions at the interface between the thawed and frozen regions, as given by Eq. (11). According to Nikolaev et al. (2022), these parameters are generally variable and time-dependent; however, at large time scales, their influence becomes negligible. For the problem of the formation of the thawing zone around the vertical shaft, we can assume that the parameters become nearly constant when $t > 10^3$ days. Therefore, in the calculations, we define the values of ω_{in} and β_{in} by considering the two solutions at the time instances of 10^3 and 10^4 days. In this case, the condition (11) can be expressed in the following form:

$$\lambda_{un} \frac{T_{in}^0(t) \exp \left[-\frac{(\beta_{in} t^{\omega_{in}} + R_0^{in})^2}{4\alpha_{un}^{in} t} \right]}{E_1 \left(\frac{R_0^{in2}}{4\alpha_{un}^{in} t} \right) - E_1 \left[\frac{(\beta_{in} t^{\omega_{in}} + R_0^{in})^2}{4\alpha_{un}^{in} t} \right]} = \frac{\omega_{in} \beta_{in} L}{2} (\beta_{in} t^{2\omega_{in}-1} + R_0^{in2\omega_{in}-1}) \quad (35)$$

The solutions of the transcendent equation when $t = 10^3$ and $t = 10^4$ days provide the values of the constants ω_{in} and β_{in} .

The temperature $T_{in}^0(t)$ at the surface within the unfrozen soils at the position R_0^{in} is given by:

$$T_{in}^0(t) = \left\{ T_{lin.out.}(t) \left[E_1 \left(\frac{R_0^{in2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{R_{ph}^{in2}}{4\alpha_{un}^{in} t} \right) \right] \right\} \times \left\{ E_1 \left(\frac{R_0^{in2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{R_{ph}^{in2}}{4\alpha_{un}^{in} t} \right) \right\} + \left[E_1 \left(\frac{R_0^{lin.out.2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{R_0^{in2}}{4\alpha_{un}^{in} t} \right) \right]^{-1} \quad (36)$$

The temperature $T_{lin.out.}(t)$ at the outer surface of the shaft lining is given by:

$$T_{lin.out.}(t) = \left\{ \lambda_{ef}^{lin} T_{air} \exp \left[-\frac{R_0^{lin.out.2}}{4\alpha_{ef}^{lin} t} \right] \left[E_1 \left(\frac{R_0^{lin.out.2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{R_{ph}^{in2}}{4\alpha_{un}^{in} t} \right) \right] \right\} \times \left\{ \lambda_{ef}^{lin} \exp \left[-\frac{R_0^{lin.out.2}}{4\alpha_{ef}^{lin} t} \right] \left[E_1 \left(\frac{R_0^{lin.out.2}}{4\alpha_{un}^{in} t} \right) - E_1 \left(\frac{R_{ph}^{in2}}{4\alpha_{un}^{in} t} \right) \right] \right\} + \lambda_{un} \exp \left[-\frac{R_0^{lin.out.2}}{4\alpha_{un}^{in} t} \right] \left[E_1 \left(\frac{R_0^{lin.in.2}}{4\alpha_{ef}^{lin} t} \right) - E_1 \left(\frac{R_0^{lin.out.2}}{4\alpha_{ef}^{lin} t} \right) \right]^{-1} \quad (37)$$

The dynamics of the thawing of the inner surface of the ice wall is found by simultaneous solution of Eqs. (31)–(37).

In the case of Dirichlet boundary condition on the shaft wall and without considering the presence of the shaft lining and an unfrozen layer between it and the ice wall, we can neglect Eqs. (36) and (37). Additionally, we replace the temperature $T_{lin.out.}(t)$ in Eq. (35) with a constant value T_{air} . These simplifications make it possible to derive the values of ω_{in} and β_{in} by solving two transcendental equations obtained by substituting two different values of t into Eq. (35).

3. Verification

To assess the accuracy of the developed approach, the results of the semi-analytical model of ice-wall thawing are compared with the numerical solutions obtained through the finite element method (FEM).

Before that, each one-dimensional model element was tested individually: the thawing of soils around a single freeze pipe, the thawing of the ice wall due to heat flow from surrounding soils, and the thawing of the ice wall due to heat flow from the shaft. The discussion of these verification calculations and their results are presented in Appendix A. It is demonstrated that these model elements can reproduce the results of numerical simulation and, therefore, they can be used as parts of the developed model of ice-wall thawing.

In this section, we consider three two-dimensional verification examples that demonstrate the ability of the model to describe the natural and artificial thawing of ice walls. Since the developed semi-analytical solution depends on the initial history of the ice wall creation, in the numerical model, we will consider not only ice wall thawing but also its creation.

The following types of saturated soils are used: soft clay as discussed by Hu et al. (2017); loam, with properties presented by Li et al. (2021); and sand, as considered by Pimentel et al. (2012). The thermophysical properties of these materials are presented in Table 1.

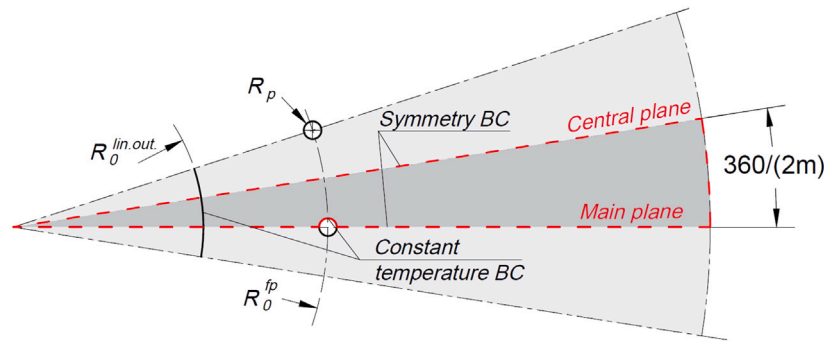


Fig. 5. The scheme of the two-dimensional freezing and thawing numerical experiment with indicated boundary conditions (BC). The red dashed line is the simulation domain.

Table 1
Thermophysical properties of the considered soils.

Property	Clay	Loam	Sand
ρ , kg/m ³	1670	1500	1980
λ_f , W/(m °C)	2.06	1.63	3.2
λ_{un} , W/(m °C)	1.65	1.28	1.9
C_f , J/(kg °C)	1720	1371	1063
C_{un} , J/(kg °C)	3400	1639	1499
L_f , MJ/m ³	122	73.7	136.9

The finite element analyses were performed by COMSOL Multiphysics software. Since this numerical method cannot provide a solution in the case without the mushy zone between frozen and unfrozen soil, the SFC is defined by Eq. (2) with $\Delta T_m = 0.5$ °C. The phase change temperature of groundwater, T_f , is set to be 0 °C, unless another value is specified.

In the FEM model, a two-dimensional problem is solved in which a sector of a circle with a size of $360^\circ/(2m)$ is studied, where m is the number of freeze pipes simulated in the example. The side boundaries of the domains are associated with symmetrical boundary conditions. The length of the sector side boundary was 30 m. The scheme of the problem is presented in Fig. 5. It is worth noting that in some literature sources, the central plane is also referred to as the 'locking plane' (Nasonov et al., 1992).

The table detailing the model parameters for various scenarios is available in Table 2. The first scenario reflects typical conditions for Brine ground freezing, the second pertains to SCD ground freezing, while the last one is dedicated to LN ground freezing. Additionally, Table 1 provides information on the properties of the soils involved.

For all considered scenarios, we apply Dirichlet boundary conditions to specify the temperature of the freeze pipe surface and the temperature of the outer surface of the shaft lining. During both active and passive freezing, the presence of the shaft is neglected. When thawing begins, an additional boundary condition is introduced to the model to represent the presence of the shaft wall with a constant positive temperature.

3.1. Scenario 1: Freezing and thawing of the ice wall using the brine ground freezing method

In this example, we explore the creation of an ice wall with a thickness of 2.5 m using the brine ground freezing method. This ice wall is formed over 90 days of active freezing with a constant freeze pipe temperature of -30 °C. Subsequently, there is 90 days of passive freezing during which, to maintain the size of the ice wall, the freeze pipe temperature is increased to -8 °C. Following this, the thawing process begins, encompassing both natural and artificial freezing simulations.

The shape of the ice wall at different time instances that was provided by the FEM simulation is shown in Fig. 6.

Table 2
Freezing and thawing scenarios considered in model verification.

	Scenario 1	Scenario 2	Scenario 3
Active freezing duration, days	90	30	15
— with T_p , °C	-30	-70	-120
Passive freezing duration, days	90	60	45
— with T_p , °C	-8	-5	-10
Artificial thawing T_p , °C	40	30	60
Soil type	Clay	Loam	Sand
R_0^{fp} , m	5	7	6
$R_0^{lin.out}$, m	3	5	4
R_p , m	0.05	0.065	0.06
m (distance between pipes, m)	24 (1.31)	26 (1.69)	22 (1.71)
T_{in} , °C	12	8	10
T_{air} , °C	10	20	4

The first three panels illustrate different stages of active freezing. It is noticeable that until the merging of neighboring ice cylinders, their growth inward and outward is similar. However, after that point, the wall experiences much more intense growth inside. This aligns with the classical representation indicating that 60% of the ice wall thickness is located inside the freeze pipe plane (Nasonov et al., 1992).

The fourth figure depicts the shape of the ice wall at the end of the passive freezing. It should be noted that during passive freezing, the size of the ice wall increases. However, this growth is attributed to the increase of the wall in the inward direction, while the outer radius of the ice wall remains unchanged.

For natural thawing, shown in panels five and six, the temperature becomes uniform approximately two weeks after the start of thawing and maintains its value until the end of the thawing process.

During the artificial thawing of soils, depicted in panels seven and eight, the shape of the thawing zone around the freeze pipe was nearly circular until two neighboring thawing zones merged. After that, the dynamics of thawing increased. It should be noted that by the end of passive freezing, most of the ice wall area has a temperature higher than -5 °C, which, just after 10 days of thawing, approaches nearly a constant value of -0.5 °C.

The dynamics of the freezing and thawing fronts, based on numerical simulation and predictions by the developed semi-analytical model, are presented in Fig. 7. As seen, the overall agreement between the two calculation methods is relatively good. In both thawing regimes, the developed approach was able to predict both the dynamics of the thawing and the total time required for it with reasonable accuracy for engineering calculations.

The presented results also prove that the dynamics of the outer surface of the ice wall is indeed linear during nearly the entire thawing time. Only when the thickness of the ice wall becomes less than 0.4 m, the change in thickness becomes more rapid. For artificial thawing, the model enables us to describe the dynamics of freezing only in the main plane. However, as thawing can be considered complete when at least

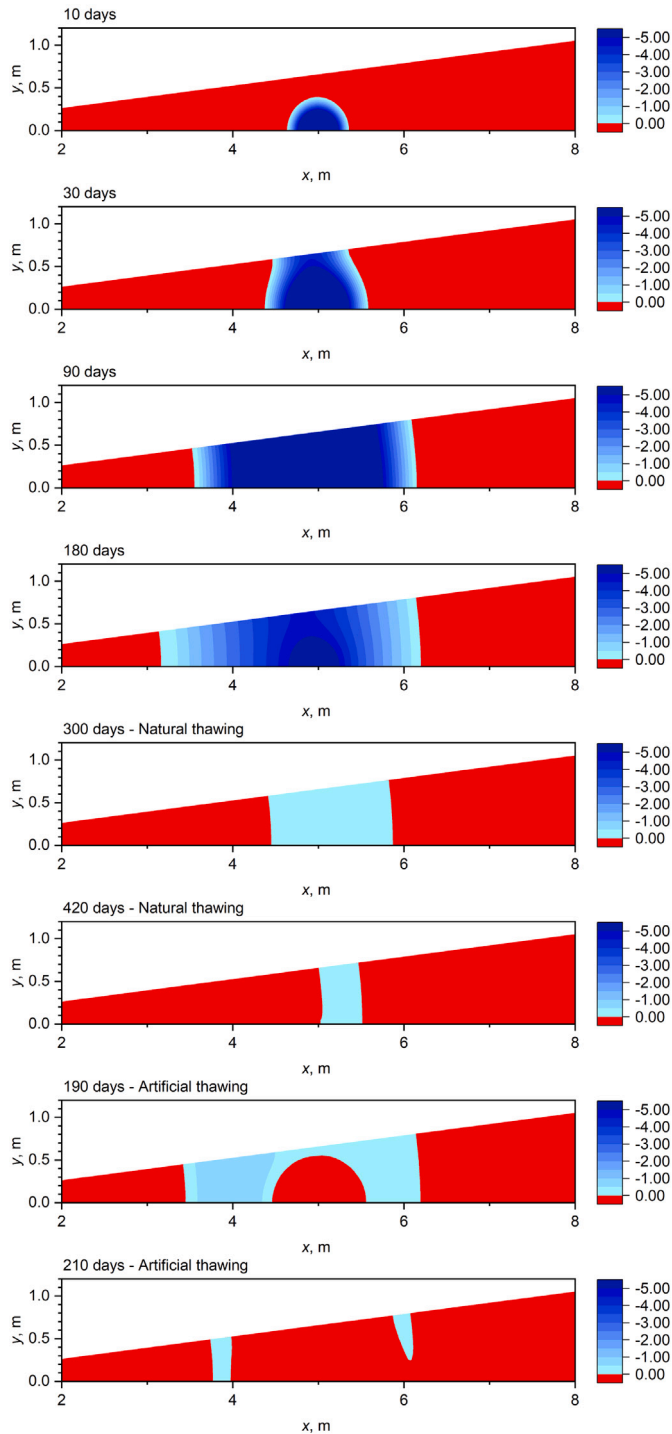


Fig. 6. The dynamics of the ice wall during brine ground freezing and thawing based on FEM simulation following Scenario 1.

one area of the cofferdam is compromised, and this always occurs in the main plane, therefore we can conclude that the predictive ability of the model is still sufficient.

The results also demonstrate that during artificial thawing, the ice wall was divided into two rings. According to Levin et al. (2021), such behavior of the ice wall should not affect the stability of the shaft lining if there are no defects in it.

The change in ice wall thickness, as predicted by two calculation methods, is illustrated in Fig. 8, showing reasonable agreement between their predictions. It should be noted that the duration of natural thawing was 262 days, whereas artificial thawing could be completed in 34 days, or even faster if the temperature of the liquid circulated in the freeze pipes were higher. Due to this, for civil engineering projects, artificial thawing should be considered a more reasonable option.

The model allows us to analyze the dynamics of thawing around the freeze pipe and assess the heat flux to it. The calculation results are presented in Fig. 9. In this figure, the FEM results are presented for the main plane in the outward direction, as the dynamics in the inner direction are nearly identical. It is clear that after approximately 12 days since the start of thawing, the neighboring thawed cylinders merged, and the dynamics of thawing changed as the influence of the freeze pipes overlapped. However, even after 40 days since the start of thawing, the discrepancy between the two solutions was less than 0.1 m. Therefore, if the duration of thawing is less than this time, we can expect relatively good predictive ability. If the distance between freeze pipes were larger, we should expect a prolongation of this period.

The model was able to predict the heat flux to the freeze pipe quite well during the entire thawing period. Based on this, we can accurately define the power of the heating station required for artificial thawing.

3.2. Scenario 2: Freezing and thawing of the ice wall using the SCD ground freezing method

In this example, we explore the creation of an ice wall with a thickness of 3.0 m using the solid carbon dioxide (SCD) ground freezing method. This ice wall is formed over 30 days of active freezing with a constant freeze pipe temperature of -70°C . Subsequently, there is 60 days of passive freezing during which, to maintain the size of the ice wall, the freeze pipe temperature is increased to -5°C . Following this, the thawing process begins, encompassing both natural and artificial freezing simulations.

The shape of the ice wall at different time instances, as provided by FEM simulation, is shown in Fig. 10. The dynamics of the ice wall demonstrate behavior qualitatively similar to what was observed in the first scenario. However, due to the much lower temperature of the air within the shaft, the final stage of artificial thawing was different. In the first example, the outer part of the ice wall thawed first, and only after that did the inner part melt. In this scenario, however, the outer part was the last to melt.

The dynamics of the freezing and thawing fronts, based on numerical simulation and predictions by the developed semi-analytical model, are presented in Fig. 11. As seen, the overall agreement between the two calculation methods is good. In both thawing regimes, the developed approach was able to predict both the dynamics of the thawing and the total time required for it with reasonable accuracy for engineering calculations.

As in the first example, the semi-analytical method fail to describe the final stage of the ice-wall formation when its thickness becomes less than 0.4 m. Due to that, the developed model should be supplemented by the condition, that the thawing is considered completed when the thickness of the ice wall becomes less than 0.4 m.

The change in the thickness of the ice wall, as predicted by the two calculation methods, is shown in Fig. 12. The calculation results agree quite well. Natural thawing took 177 days, while artificial thawing only took 54 days.

The model lets us analyze how thawing progresses around the freeze pipe and evaluate the heat flux to it. The calculation results, presented in Fig. 13, follow a similar pattern to what was discussed in the first scenario.

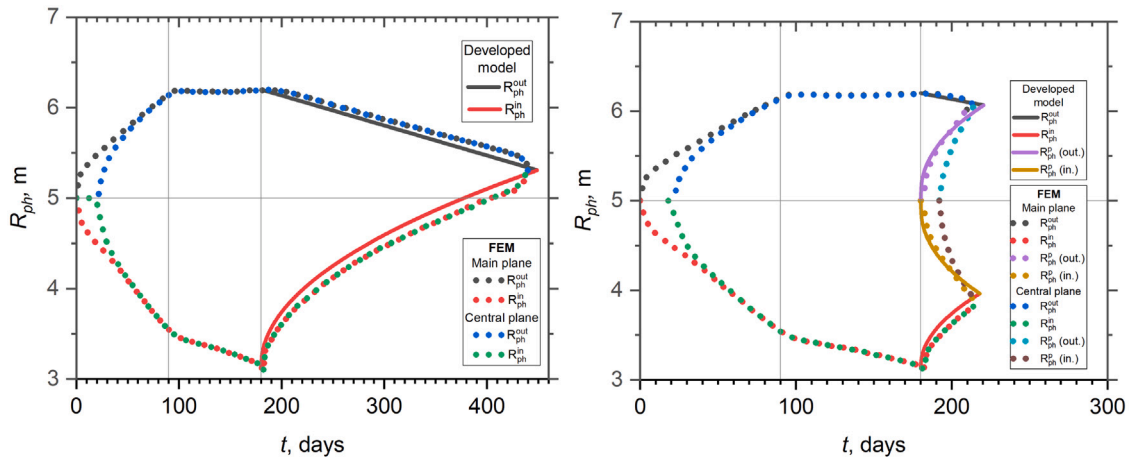


Fig. 7. The position of the phase change front during freezing and thawing following Scenario 1. Left panel – natural thawing; right panel – artificial thawing.

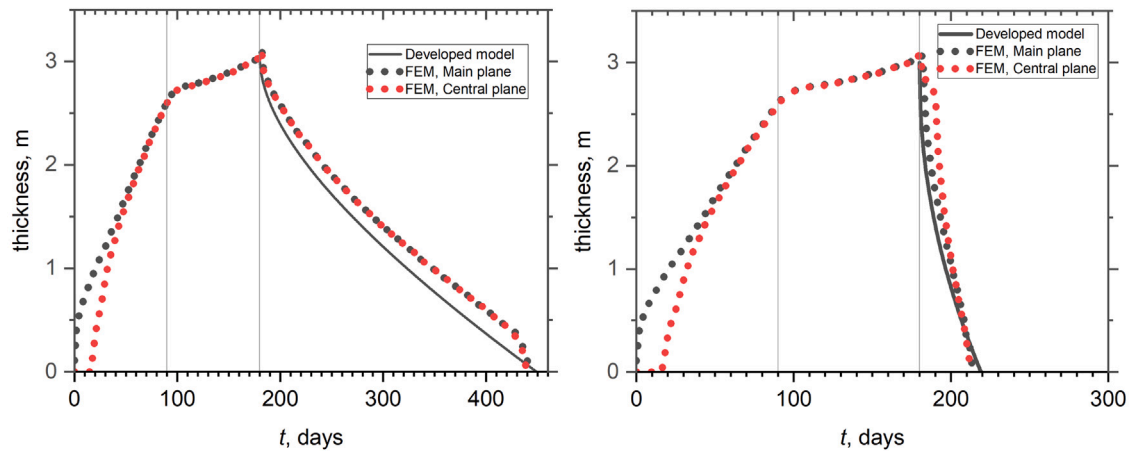


Fig. 8. The dynamics of the ice-wall thickness during brine ground freezing following Scenario 1. Left panel – natural thawing; right panel – artificial thawing.

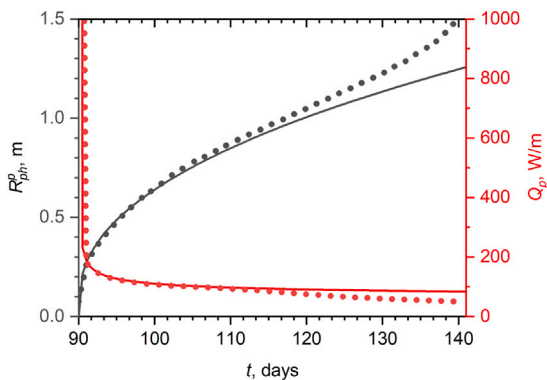


Fig. 9. The dynamics of the soil thawing around the freeze pipe and the change in the heat flux to it following Scenario 1. The dotted lines represent the results of FEM simulation; the solid lines denote the results of the developed model.

3.3. Scenario 3: Freezing and thawing of the ice wall using the LN ground freezing method

In this example, we explore the creation of an ice wall with a thickness of 2.9 m using the liquid nitrogen (LN) ground freezing method. This ice wall is formed over 15 days of active freezing with a constant freeze pipe temperature of -120°C . Subsequently, there is 45 days of passive freezing during which, to maintain the size of the ice

wall, the freeze pipe temperature is increased to -10°C . Following this, the thawing process begins, encompassing both natural and artificial freezing simulations.

The shape of the ice wall at different time instances, as provided by FEM simulation, is shown in Fig. 14. The dynamics of the freezing and thawing fronts, based on numerical simulation and predictions by the developed semi-analytical model, are presented in Fig. 15. The change in the thickness of the ice wall, as predicted by the two calculation methods, is shown in Fig. 16. The calculation results agree quite well. Natural thawing took 578 days, while artificial thawing only took 36 days.

The model lets us analyze how thawing progresses around the freeze pipe and evaluate the heat flux to it. The calculation results, presented in Fig. 17, follow a similar pattern to what was discussed in the previous scenarios.

The presented verification examples demonstrate that the developed semi-analytical model is capable of describing the dynamics of thawing soils.

In the following section, the influence of the main model parameters on the thawing parameters will be studied.

4. Analysis of the ice-wall thawing and its design

In this section, we consider the influence of some geological and technological factors on the thawing of the ice wall created in saturated soils.

The thawing process relies on the thermophysical properties of unfrozen soils. According to Eqs. (4) and (5), heat capacity and heat

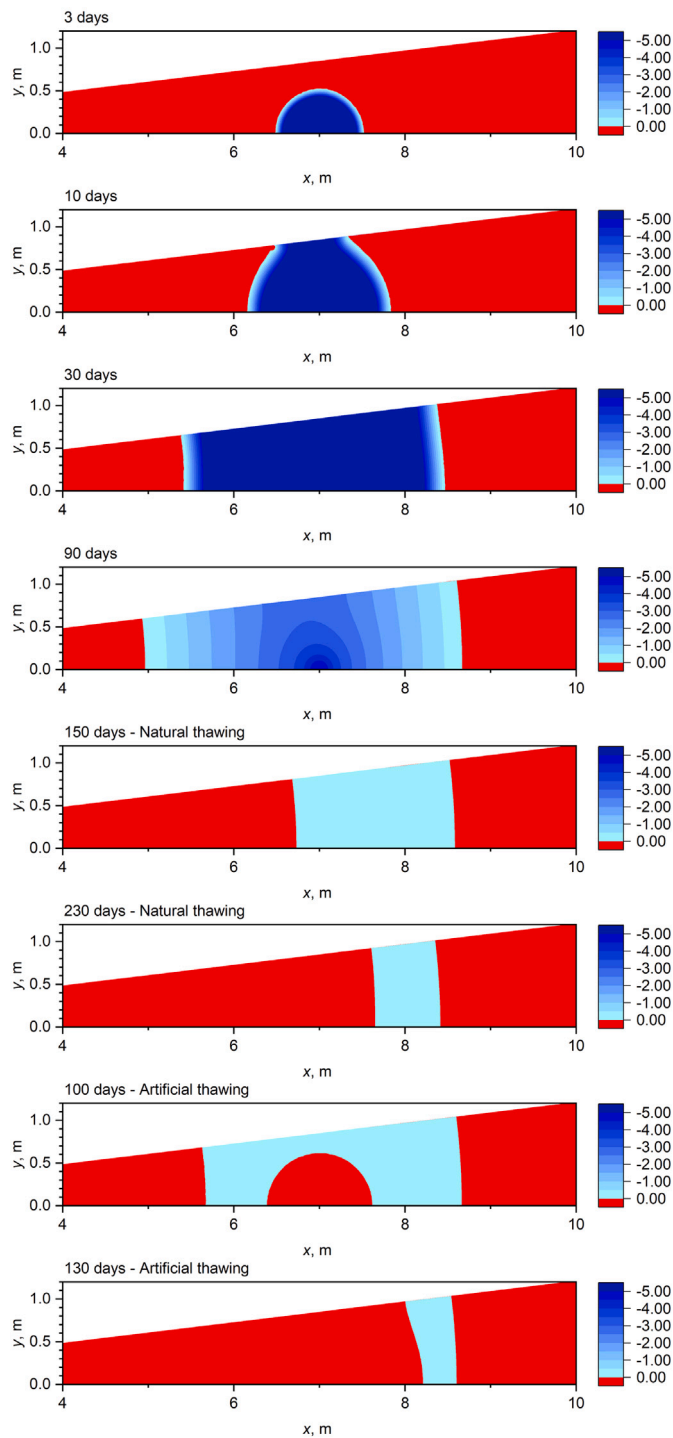


Fig. 10. The dynamics of the ice wall during solid carbon dioxide ground freezing and thawing based on FEM simulation following Scenario 2.

conductivity can be defined based on the volumetric content of the soil components. Similarly, soil density can be also defined. The unfrozen saturated soils contain only two components: soil grains and liquid water. The properties of water remain nearly constant in the considered temperature range. The heat capacity and density of soil grains also do not vary significantly depending on the type of soil. For example, the analysis of available literature data performed by Waples and Waples (2004) indicated that most common minerals have a heat capacity between 700 and 800 J/(kg °C), e.g., for quartz it is 740 J/(kg

°C). The typical values of soil particle densities are indicated around 2650 kg/m³, e.g., for quartz 2660 kg/m³, see also Andersland and Ladanyi (2004). Therefore, we can conclude that the parameters to be studied should include the heat conductivity of the soil grains and porosity. For the considered case, we use the following parameters:

- heat conductivity of soil grains – 1, 3 and 5 W/(m °C)
- porosity – 0.3, 0.4, 0.5

The other model parameters are:

- heat capacity of soil grains – 750 J/(kg °C)
- heat capacity of water – 4182 J/(kg °C)
- density of soil grains – 2650 kg/m³
- density of water – 1000 kg/m³
- heat conductivity of water – 0.6 W/(m °C)
- latent heat of water solidification – 334 kJ/kg

As technological factors, we consider the diameter of the freeze pipes, the thickness of the ice wall, and the time at which thawing is initiated.

In engineering of the artificial freezing process, understanding the relationship between the temperature of the liquid pumped into the freeze pipe and the corresponding heat flux is crucial. This knowledge allows for the estimation of the required heating power and ensures that the liquid temperature remains below a critical value.

As demonstrated in Appendix A.1 during the verification example with Neumann boundary conditions, when the heat flux to the pipe is constant, the temperature of the freeze pipe wall increases. However, this temperature increase undergoes a significant alteration approximately 10–15 days after the initiation of thawing.

To further investigate, we calculate the heat flux from a 1 m section of a freeze pipe with a diameter of 0.12 m for various soil parameters and heat fluxes. The temperature of the liquid is assessed at 10, 30, and 50-day time instances. These values cover the majority of civil engineering projects. In our calculations, we also provide the value of β_p that allows us to calculate the position of the phase front using Eq. (14) at any time instance.

The calculation results are presented in Fig. 18. The graphs can be used to determine both the thawing regime and thawing duration. Based on the known heat conductivity of soil grains and the known porosity of soils, one can choose the heat flux from the pipe and define the temperature of the liquid to ensure that this value is suitable for the application from the panels (a) – (c). After that, the value of the parameter β_p can be also defined, which can be used to determine the time of thawing of the soils around the freeze pipe by referring to the graph in Figure (d).

The dynamics of the outer surface of the ice wall is described by Eq. (29). According to it, the position of the ice wall surface depends on the dimension parameter β_{out} . This parameter is a function of both the physical properties of soils and the technological parameter of the AGF works.

To further investigate the properties of this parameter, let us define its value for the considered soils. As the technological parameter, we will vary the time of the beginning of thawing, $\tilde{t}_{in}$, from 60 to 220 days with a step of 40 days, and the initial position of the ice wall surface: $R_0^{out} = 4.7$ m and $R_0^{out} = 5.4$ m for $R_0^{fp} = 4$ m; and $R_0^{out} = 5.7$ m and $R_0^{out} = 6.4$ m for $R_0^{fp} = 5$ m. The calculations are performed for two values of the initial temperature of soils: 8 and 12 °C. The results are presented in Fig. 19.

The results show that the value of β_{out} , which can be considered as the speed of the outer surface movement during thawing can vary significantly both due to the change of the technological and geological factors. The most important factor that affects the dynamics is the initial temperature of the soil. The increase of its value from 8 to 12 leads to an increase in the speed by at least 50%.

The results of the simulation also reveal that the influence of $\tilde{t}_{in}$ on β_{out} depends on the thickness of the ice wall. If the ice wall is thin

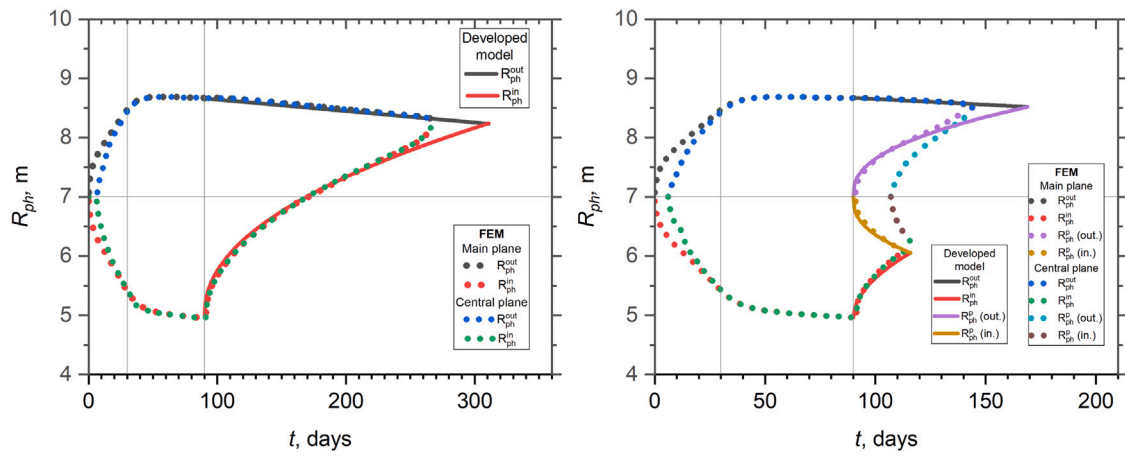


Fig. 11. The position of the phase change front during freezing and thawing following Scenario 2. Left panel – natural thawing; right panel – artificial thawing.

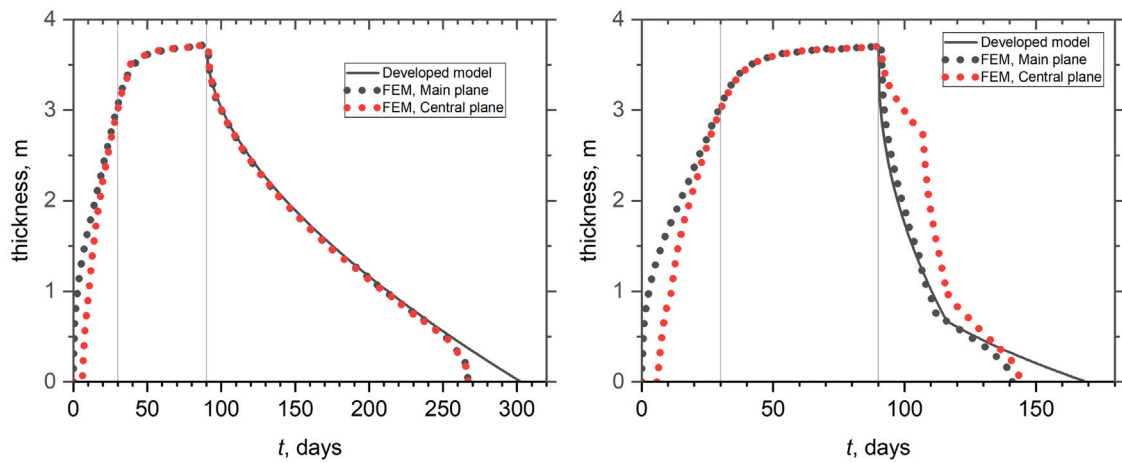


Fig. 12. The dynamics of the ice-wall thickness during liquid nitrogen ground freezing following Scenario 2. Left panel – natural thawing; right panel – artificial thawing.

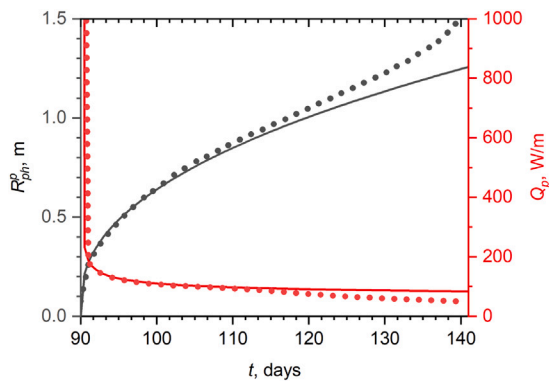


Fig. 13. The dynamics of the soil thawing around the freeze pipe and the change in the heat flux to it following Scenario 2. The dotted lines represent the results of FEM simulation; the solid lines denote the results of the developed model.

($R_0^{out} - R_0^{fp} = 0.7$ m for the considered case), the increase of $\tilde{t}_{in}$ leads to a decrease in β_{out} by up to 40%. At the same time, when the ice wall is thick ($R_0^{out} - R_0^{fp} = 1.4$ m for the considered case), the parameter β_{out} becomes independent of $\tilde{t}_{in}$. Moreover, for thin walls, we can also observe the dependence of β_{out} on R_0^{fp} , whereas for thick walls, this dependence is not observed. This phenomenon can be explained by the fact that the thawing time of the thick wall can reach several hundred

days, leading to the temperature field in soils reaching a quasi-steady state, ensuring nearly constant heat inflow to the ice wall.

The dynamics of the inner surface of the ice wall is described by Eq. (34). According to it, the position of the ice wall surface depends on two parameters β_{in} and ω_{in} . These parameters are functions of both the physical properties of soils and the thermal regime within the shaft during the ice wall thawing.

To further investigate the properties of these parameters, let us define their values for the considered soils. The shaft wall will be represented in the model by defining the constant temperature on the domain boundary with the radius $R_0^{lin.out.} = R_0^{in.}$, equal to 3, 4, and 5 m. The influence of the shaft lining material is not considered. The boundary temperature, which represents the temperature of the air within the shaft T_{air} , is changing from 4 to 28 °C with a step of 4 °C.

The calculation results are presented in Fig. 20. They indicate that both parameters behave quite similarly. The increase in air temperature and the heat conductivity of soil grains leads to larger values. The greater porosity values result in decreased parameter values. It is worth noting that while parameter β_{in} can vary significantly depending on model properties, the value of ω_{in} remains relatively stable, changing only within the range of 0.44 to 0.47. This slight variation can significantly impact calculation results only for large time scales, as illustrated by Fig. 21, depicting the dynamics of the phase front propagation as a function of parameters β_{in} and ω_{in} .

Importantly, when the air temperature drops below 8 °C, its influence on ω_{in} becomes nearly independent.

Nevertheless, the major impact on the phase front dynamics is due to parameter ω_{in} . For the engineering design of artificial thawing, we

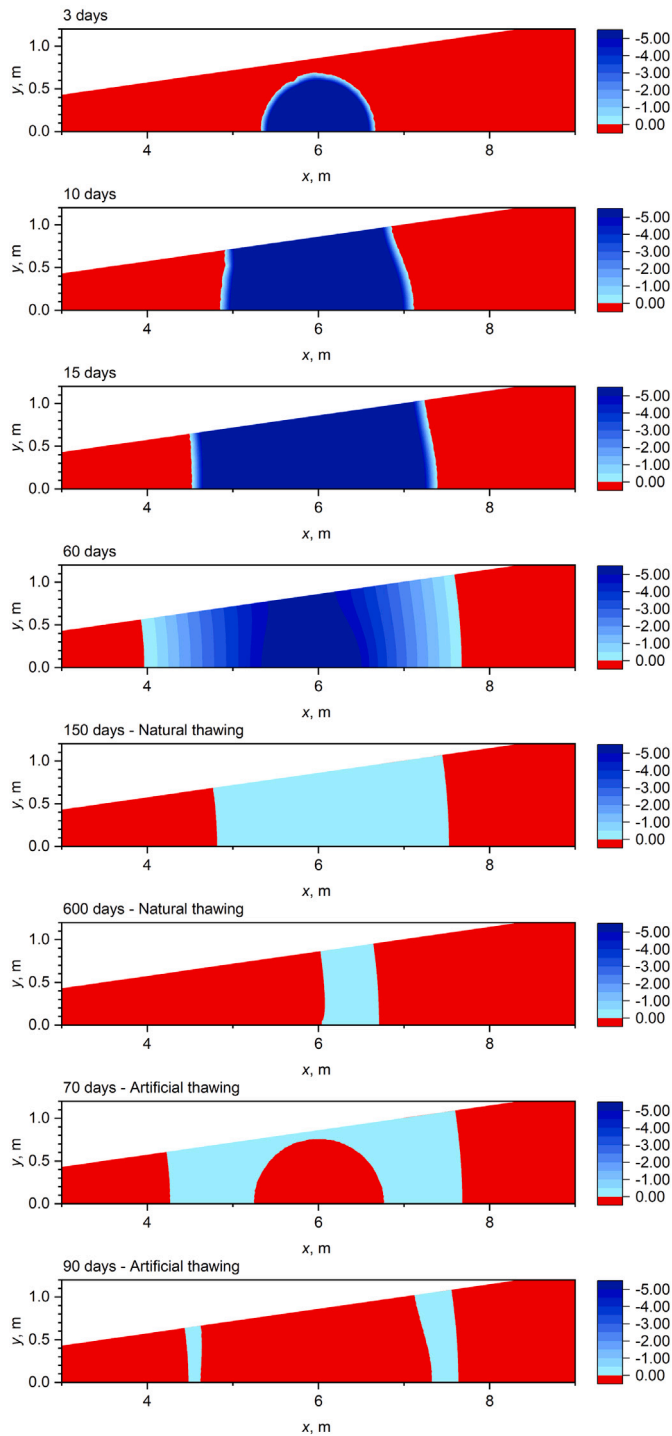


Fig. 14. The dynamics of the ice wall during liquid nitrogen ground freezing and thawing based on FEM simulation following the Scenario 3.

can apply this value a priori as a constant in the range between 0.44 and 0.47, depending on the shaft radius. In this case, solving the system of transcendental equations from (31) to (37) becomes simpler. However, for natural thawing design, the system of equations must be fully solved, as even a small variation of ω_{in} can lead to a significant under- or overprediction of the phase change position after several hundred days of process development.

It is important that during the artificial thawing of soils, the primary contribution to thawing comes from the growth of the thawed area

around the freeze pipes. Consequently, for ice walls with a small thickness of up to 1.5 m, only this process needs consideration, as the thawing induced by natural heat flow will not play a substantial role in the overall thawing.

5. Conclusions

In this paper, a novel semi-analytical model is formulated to mathematically describe the dynamics of the ground ice wall formed around a vertical shaft or tunnel during natural or artificial thawing. This model simplifies the intricate process by dividing it into three independent one-dimensional heat transfer problems, making it amenable to solutions using general mathematical software or various programming languages.

Each component of the model undergoes thorough verification to ensure its accuracy across a broad range of initial conditions and model parameters. Validation through one and two-dimensional test cases demonstrates the model's effectiveness as a tool for describing ice wall thawing. It proves valuable for predicting the artificial thawing time of soils and determining the necessary heating power for the plant responsible for heating the heat transfer liquid under typical geological conditions. For natural thawing, the model performs well in predicting the dynamics of the ice wall when the soil temperature is approximately below 15 °C.

The model allows us to perform an extensive parametric study to reveal the influence of main technological and geological factors on the ice wall thawing within their variation range typical for civil engineering projects. Based on these calculations, a series of graphs were plotted, upon which an engineer can roughly assess the thawing time without performing complex mathematical calculations. Based on this analysis, it can be concluded:

- For the artificial thawing of soil, the dynamics of the thawing zone can be defined by Eq. (14), depending on the parameter β_p . The assessed values of β_p mainly depend on the thermal power Q_p of the heating station. Depending on the soil grain heat conductivity and soil porosity, β_p takes values between 0.08 and 0.12 for $Q_p = 50$ W/m, between 0.14 and 0.19 for $Q_p = 150$ W/m, and between 0.17 and 0.27 for $Q_p = 300$ W/m. Between these values, β_p can be considered to change linearly. It should be noted that an increase in the soil grain heat conductivity from 1 to 5 W/(m °C), as well as a decrease in the soil porosity from 0.5 to 0.3, can lead to a growth of β_p by less than 25%.
- The thawing dynamic of the outer surface of the ice wall is defined by Eq. (29) and depends on the parameter β_{out} . The performed parametric study shows that this parameter can vary significantly due to changes in technological and geological factors. The most important factor that affects the dynamics is the initial temperature of the soil. An increase in its value from 8 to 12 leads to an increase in speed by at least 50%.
- The position of the thawing front at the inner area of the ice wall is defined by Eq. (34) and depends on the parameters β_{in} and ω_{in} . Both parameters behave quite similarly. An increase in air temperature and the heat conductivity of soil grains leads to larger values, while greater porosity values result in decreased parameter values. It is worth noting that while parameter β_{in} can vary significantly depending on model properties, the value of ω_{in} remains relatively stable, changing only within the range of 0.44 to 0.47.

The paper offers a comprehensive understanding of the physics behind ice-wall thawing and also provides a powerful tool for conducting precise engineering calculations that are essential for the safe delivery and liquidation of the ice-wall in the multi-layered soil geological profile of underground structures. Furthermore, by integrating the model into the optimization solution, ground-freezing costs can be minimized through the strategic selection of ground-freezing layout and regime.

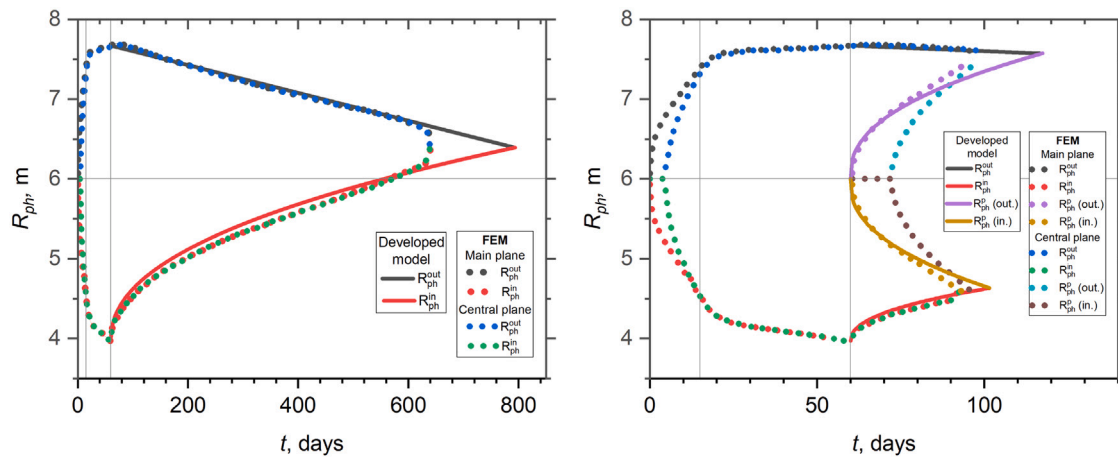


Fig. 15. The position of the phase change front during freezing and thawing following Scenario 3. Left panel – natural thawing; right panel – artificial thawing.

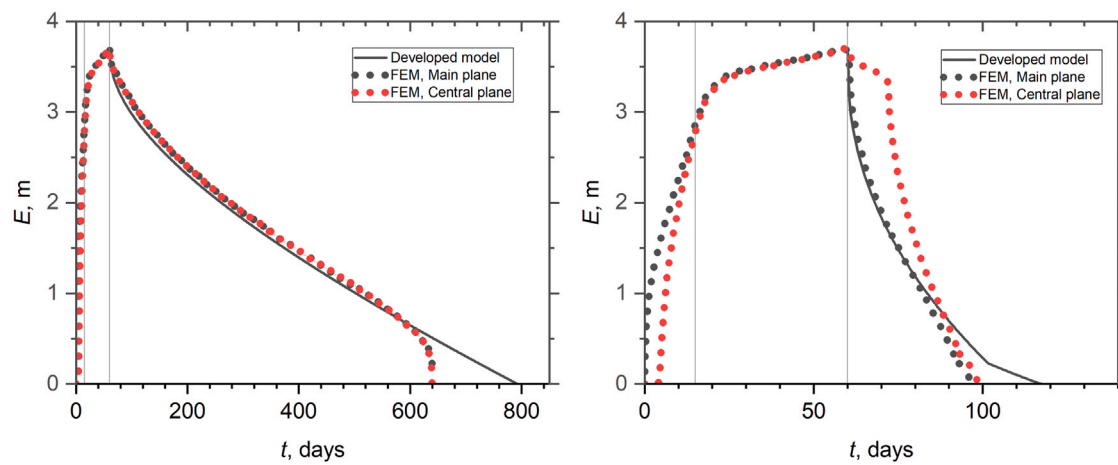


Fig. 16. The dynamics of the ice-wall thickness during liquid nitrogen ground freezing following Scenario 3. Left panel – natural thawing; right panel – artificial thawing.

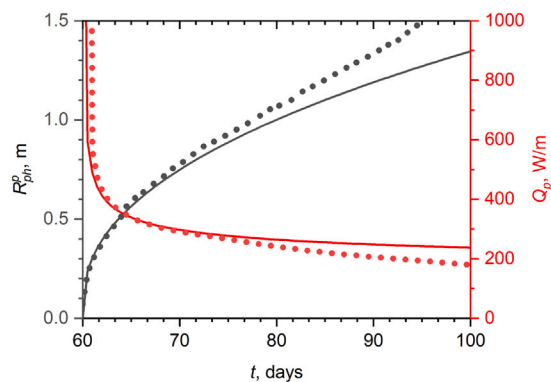


Fig. 17. The dynamics of the soil thawing around the freeze pipe and the change in the heat flux to it following Scenario 3. The dotted lines represent the results of FEM simulation; the solid lines denote the results of the developed model.

CRediT authorship contribution statement

Petr Nikolaev: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Project administration, Methodology, Investigation, Formal analysis, Data curation. **Andrey Jivkov:** Writing – review & editing, Writing – original draft, Formal analysis. **Hamid Rajabi:** Writing – review & editing, Writing – original draft, Formal analysis. **Huaxiang Yan:** Writing – review & editing, Writing

– original draft, Formal analysis. **Xiaoming Zhu:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software. **Majid Sedighi:** Writing – review & editing, Writing – original draft, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data supporting the findings of this study are available within the paper.

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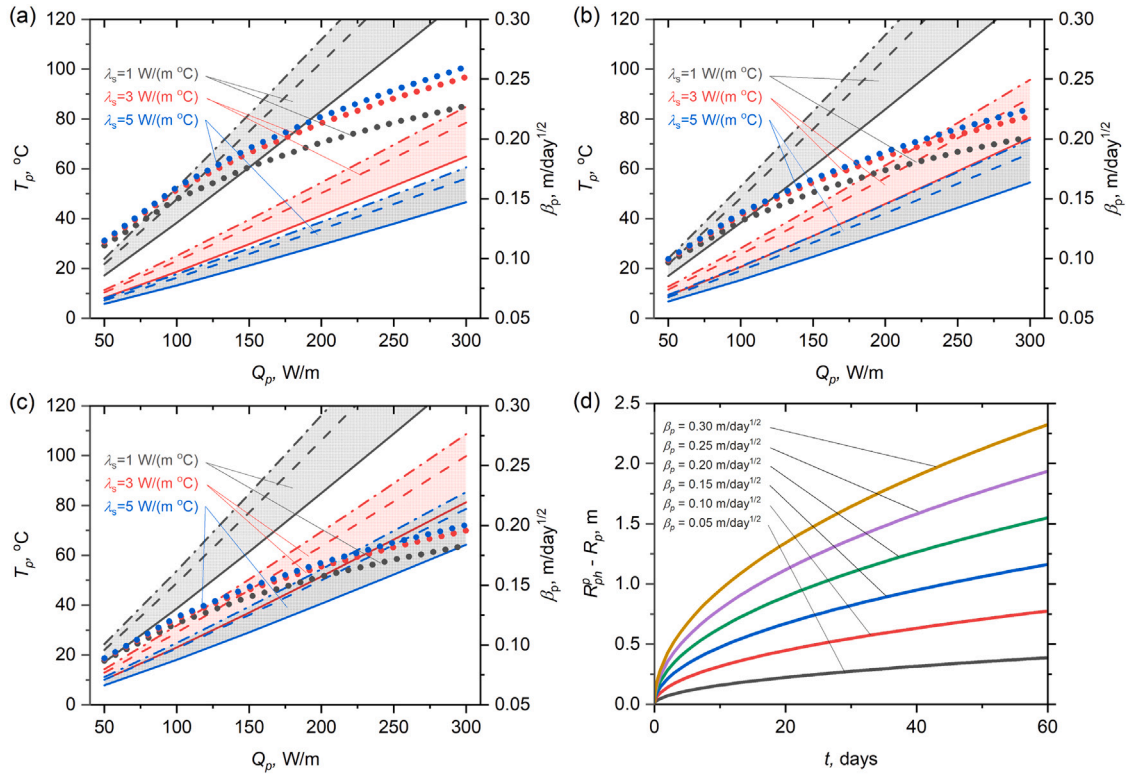


Fig. 18. The panels (a) – (c) illustrate the freeze pipe surface temperature varying with the heat flux from the freeze pipe at time instances: 10 (solid lines), 30 (dashed lines), and 50 (dash-dotted lines) days. The results are presented for different values of heat conductivity of the soil grains and various porosity levels (0.3 (a), 0.4 (b), and 0.5 (c)). The dotted lines are the corresponding values of the parameter β_p . Panel (d) depicts the dynamics of phase change front propagation for various values of the parameter β_p .

Appendix A. Verification of the one-dimensional model elements

A.1. Thawing of ice wall around a single freeze pipe

Three pipe outer diameters of 0.1, 0.13, and 0.16 m are considered. Dirichlet, Robin, and Neumann boundary conditions are used. The ice wall is formed in clay and loam.

For the Dirichlet boundary condition, the temperature of the freeze pipe is assumed constant equal to 40, 60, and 80 °C.

The results from the developed model and the FEM are presented in Fig. A.1, illustrating the dynamics of the thawing front according to Eq. (14) and the heat flux calculated by Eq. (13).

Good agreement between the two methods is demonstrated. The slight discrepancy may be explained by the presence of the mushy zone in the FEM model. This example can also be considered as additional verification of the semi-analytical approach presented by Nikolaev et al. (2022).

For the case with Neumann boundary conditions, three freeze pipe sizes with outer diameters of 0.1, 0.13, and 0.16 m, and three heat fluxes of 75, 150, and 225 W/m are considered. The analysis is performed for the same two soils: clay and loam. The results are presented in Fig. A.2, illustrating the position of the phase front according to Eq. (14) and the temperature of the heating pipe surface calculated by Eq. (22) for $r = R_p$.

As before, good agreement between the two methods is demonstrated, and the discrepancy may be explained by the presence of the mushy zone in the FEM model. Notably, the pipe temperature does not increase significantly during the entire thawing period and approaches a constant value. Based on this, it can be suggested that the Neumann boundary condition can be used in engineering design, as it provides

a direct link between the electric power of the heater used to warm the heat transfer liquid and the dynamics of the thawing. The only condition to consider is that the temperature of the liquid should remain significantly lower than its boiling temperature throughout the entire thawing time.

For the case with Robin boundary condition, the temperature of the liquid is $T_l = 70$ °C for the clay and $T_l = 40$ °C for the loam. The heat transfer coefficient η_p is 20, 60, and 100 W/(m² °C). The thickness of the steel heating pipe wall is 8 mm, and the outer diameters are 0.1, 0.13, and 0.16 m. The thermophysical properties of the pipe are $\lambda_p = 50$ W/(m °C), $C = 460$ J/(kg °C), and $\rho = 7850$ kg/m³. The results are presented in Fig. A.3, illustrating the dynamics of the ice wall defined by Eq. (14), the heat flux to the pipe given by Eq. (16), and the pipe temperature given by Eq. (21). Again, agreement between the two methods is demonstrated.

Notably, this example demonstrates the possibility of implementing the Robin (convective) boundary condition in the semi-analytical model by Nikolaev et al. (2022). Therefore, the Robin boundary condition can be used in the future to describe the process of soil freezing.

A.2. Natural thawing of ice wall due to the presence of the outer heat flux

To demonstrate the ability of the developed approach to describe the thawing of the outer surface of the ice wall, we consider the following problem that involves active freezing, passive freezing, and the subsequent thawing of the ice wall around the cylindrical surface. The solution tests the assumption that the average temperature of the ice wall increases to 0 °C shortly after the end of passive freezing. To represent the presence of the freeze pipes in the one-dimensional problem (Fig. 3), for the active freezing, we set some negative temperature to the cylindrical surface with a radius R_0^{in} equals to 4, 5, and 6 m.

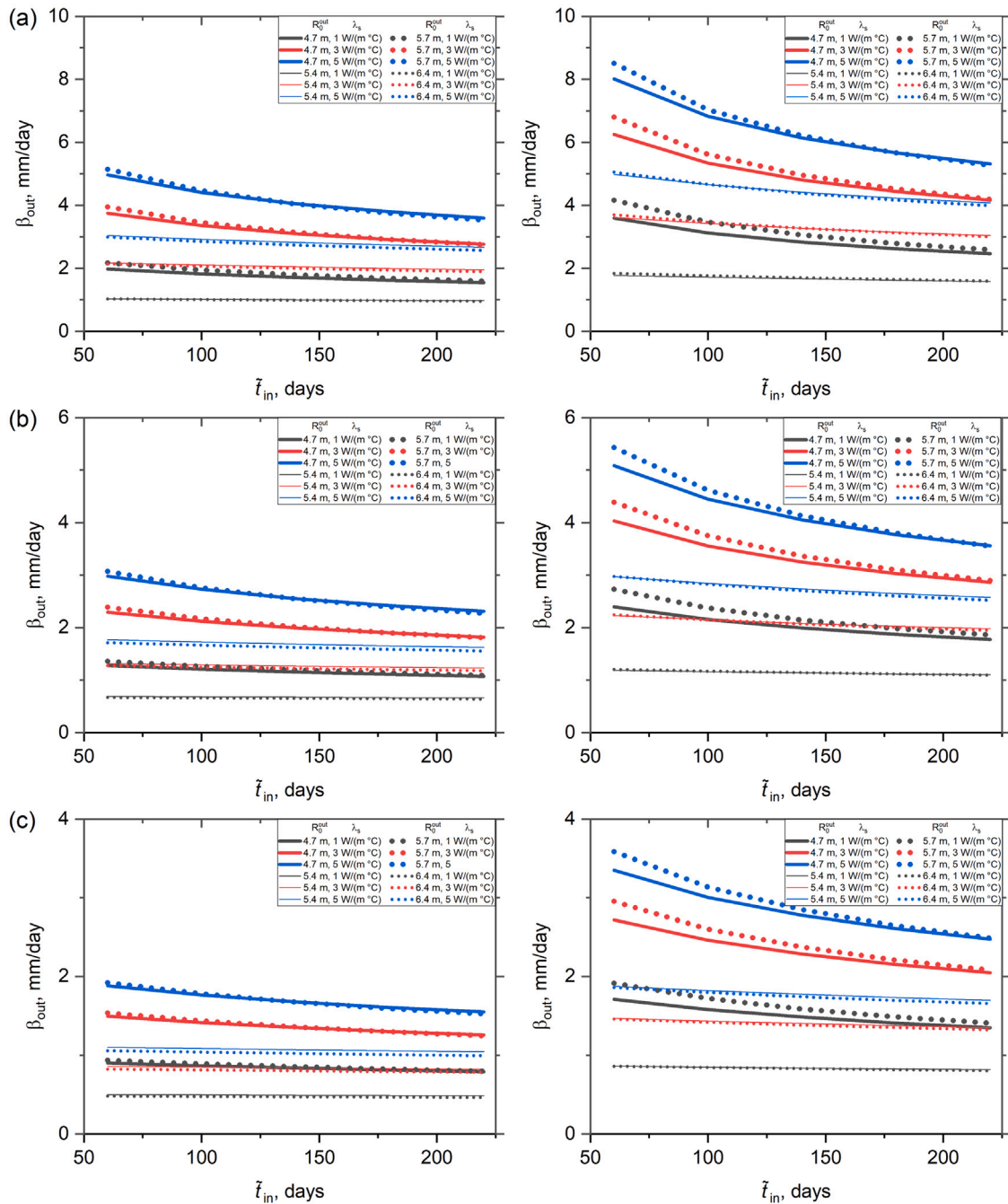


Fig. 19. The variation of the parameter β_{out} depending on the time of the beginning of thawing $\tilde{t}_{in}$ and properties of soils: $n = 0.3$, panels (a); $n = 0.4$, panels (b); $n = 0.5$, panels (c). The left panels represent $T_{in} = 8$ °C; the right panels represent $T_{in} = 12$ °C.

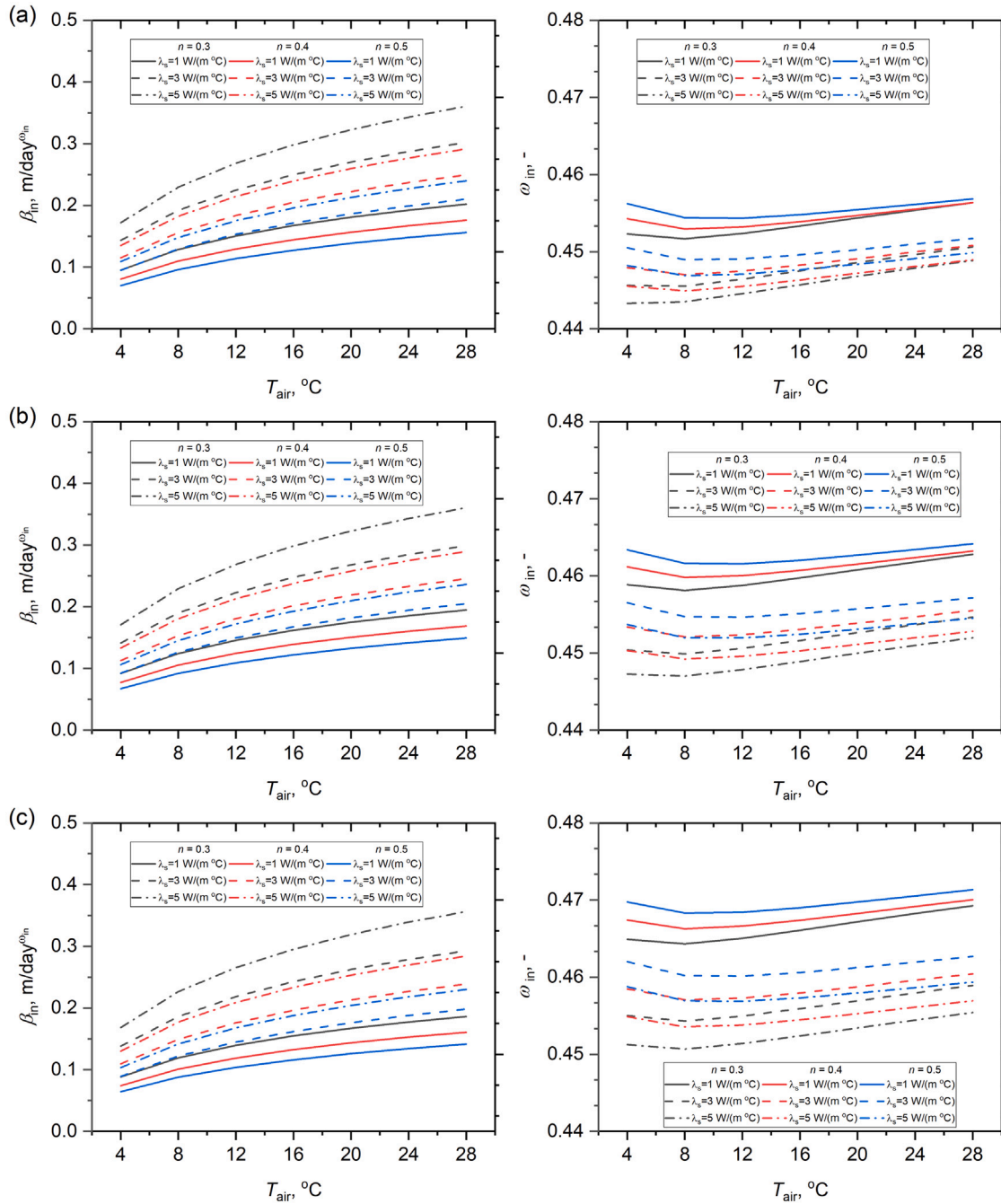


Fig. 20. The variation of the parameter β_{in} (left panels) and ω_{in} depending on the temperature within the shaft T_{air} and the thermophysical properties of soils.

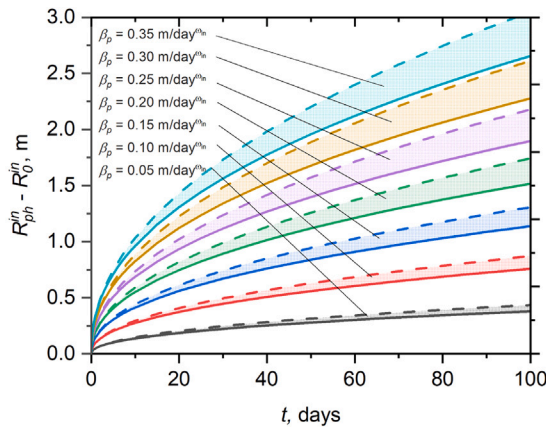


Fig. 21. The dynamics of phase change front propagation for various values of the parameters β_{in} and ω_{in} . The solid lines corresponds to $\omega_{in} = 0.44$, the dashed lines are for $\omega_{in} = 0.47$.

Table A.3

Periods of the ice wall formation and natural thawing.

Freezing method	Active freezing, days	Passive freezing, days	Natural thawing, days
Brine freezing	0–60	60–180	180–720
SCD freezing	0–30	30–210	210–720
LN freezing	0–20	20–90	90–720

Then, the temperature of this surface increases to halt the growth of the ice wall, representing passive freezing, and finally, we define the zero heat flux boundary condition on this surface to start the thawing. As soils, we applied clay and loam with initial temperatures of 8 and 20 °C. The first value can be considered typical for civil underground construction projects, whereas the second value represents rare extreme cases.

To represent three freezing methods, we assume that the temperature of the cylindrical surface with R_0^{in} is –20 °C, which is typical for brine ground freezing, –50 °C for solid carbon dioxide ground freezing, and –100 °C for liquid nitrogen ground freezing. After the time devoted to active ground freezing is over, the temperature of the cylindrical surface increases to –8 °C (in the case of the initial soil temperature of 20 °C) and –5 °C (in the case of the initial soil temperature of 8 °C) to represent the phase of passive freezing. Depending on the freezing method, three freezing scenarios are considered and presented in Table A.3.

In the semi-analytical model, the start of the thawing is considered at the time instance $\tilde{t}_{in}$, which corresponds to the beginning of the thawing period presented in Table A.3. For the radius of the ice wall at the beginning of the ice wall thawing $R_{ph}^{out}(\tilde{t}_{in}) = R_0^{out}$, we take the value of the ice wall thickness obtained in the numerical simulation for the considered time. This value can also be defined as the projected thickness of the wall, reached during the ice wall formation. However, due to the possible growth of the ice wall during the passive stage, the radius can become slightly larger or smaller than the project value.

The calculation results are presented in Fig. A.4. They demonstrate relatively good agreement between the two calculation methods, confirming that the initial assumption that the dynamics of the ice wall during thawing is linear is valid.

For brine and SCD freezing methods, the agreement is tangibly better than for the case of LN freezing. This discrepancy arises because in the considered scenarios thawing starts relatively early after the start of freezing due to the short active and passive freezing stages, resulting in warmer surrounding soils. As a result, the temperature gradient at the phase change boundary is much higher, leading to more intense heating of the outer surface of the ice wall and a larger rate

of ice wall shrinkage, that cannot be predicted by the proposed semi-analytical approach. It should also be noted that the accuracy of the method decreases with the increase in the initial temperature of the soils, especially for ice walls of larger radius. Nevertheless, it can be concluded that during the first month of thawing for unusually large initial temperatures of soils and several months for typical values of the initial temperature, regardless of other initial conditions, the accuracy of the proposed approach can be considered acceptable, ensuring that the method can be effectively used for the design of artificial soil thawing.

It is worth noting that in this example, the temperature of the frozen soils is not set to be 0 °C manually before the start of thawing, but it reaches this value naturally during the simulation. The relatively good agreement with the theoretical model indirectly confirms that the initial assumption that the temperature of the ice wall before thawing is close to the phase change temperature is valid.

A.3. Thawing of ice wall due to the presence of the inner heat flux

Let us analyze the model's ability to describe the thawing of soils in the inner region of the ice wall. To do that, both types of boundary conditions that were discussed in Section 2.5 are considered. For both cases, the initial inner radius of the ice wall, denoted as R_0^{in} , is chosen as 3, 4, and 5 m. In both cases, we are studying the thawing of the ice wall formed in clay and loam soils.

For the Dirichlet boundary conditions, shaft lining is not taken into account. The constant temperature boundary condition is specified on the surface with the radius of R_0^{in} . Three temperature values are considered: 5, 15, and 30 °C, approximately corresponding to the air temperature that could circulate in the shaft during different seasons: winter, spring/autumn, and summer respectively.

The calculation results, in the form of the dependence of the position of the phase change boundary on time from the beginning of thawing, as defined by Eq. (34), are presented in Fig. A.5.

The comparison demonstrates that the accuracy of the proposed semi-analytical model is quite good, even if it is slightly worse compared to the results achieved for a similar problem of thawing around the freeze pipe which was discussed in Appendix A.1. This difference may indicate a limitation of the model proposed by Nikolaev et al. (2022) in capturing the dynamics of the phase front in soils around larger cylinders. Nevertheless, the demonstrated agreement can be considered reasonable for engineering practice, as the predicted radii by the two methods differ by less than 10%. Moreover, for artificial thawing, which should have a relatively short duration (up to 2–3 months) with ice wall thicknesses typical of civil engineering projects, this difference becomes even more acceptable.

For the second task of this verification exercise, we consider Robin boundary conditions, which are incorporated into the model by modifying the heat conductivity of the concrete shaft lining using Eq. (30). The thickness of the lining is set to be 0.2 m and is adjacent to the inner surface of the ice wall with the same three values of the radius. The temperature of the air within the shaft is set to be 20 °C, while the convective heat transfer coefficient η_{lin} is defined to be equal to 5, 15, and 30 W/(m² °C).

The calculation results are presented in Fig. A.6. The agreement between the two calculation methods is fairly good, especially at the initial stage. In most cases, the developed approach was able to reproduce the influence of η_{lin} on the ice wall dynamics.

The presented verification examples demonstrate that every component of the developed semi-analytical model is capable of describing the required problems with acceptable accuracy.

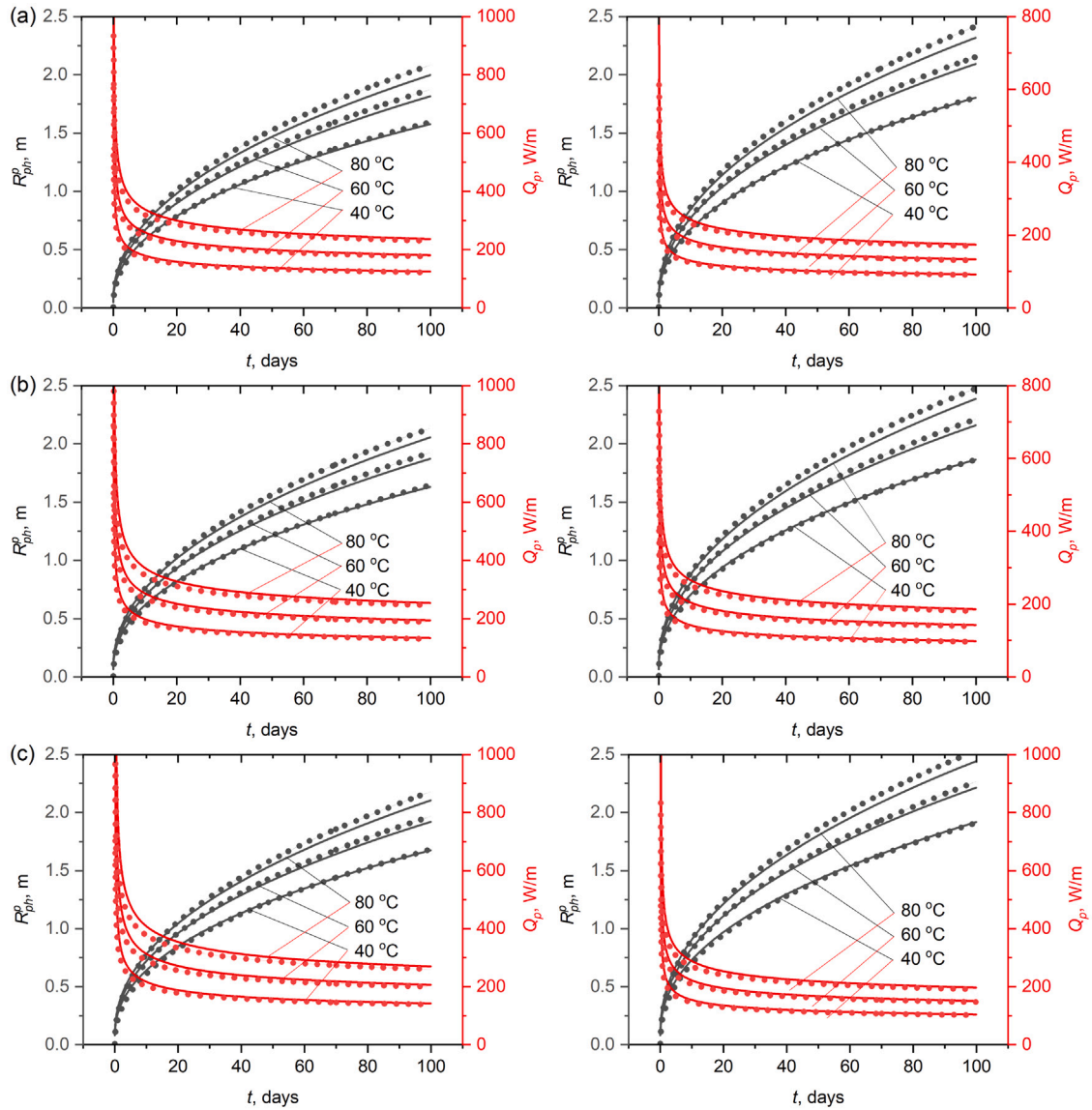


Fig. A.1. Position of the phase change front around a freeze (heating) pipe and change in heat flux to the pipe during artificial thawing of ice wall. Left figures — soft clay; right figures — loam. The outer diameters of the pipe are 0.1 (a), 0.13 (b), and 0.16 (c) m. Dotted lines represent the FEM simulations; solid lines represent the proposed semi-analytical model.

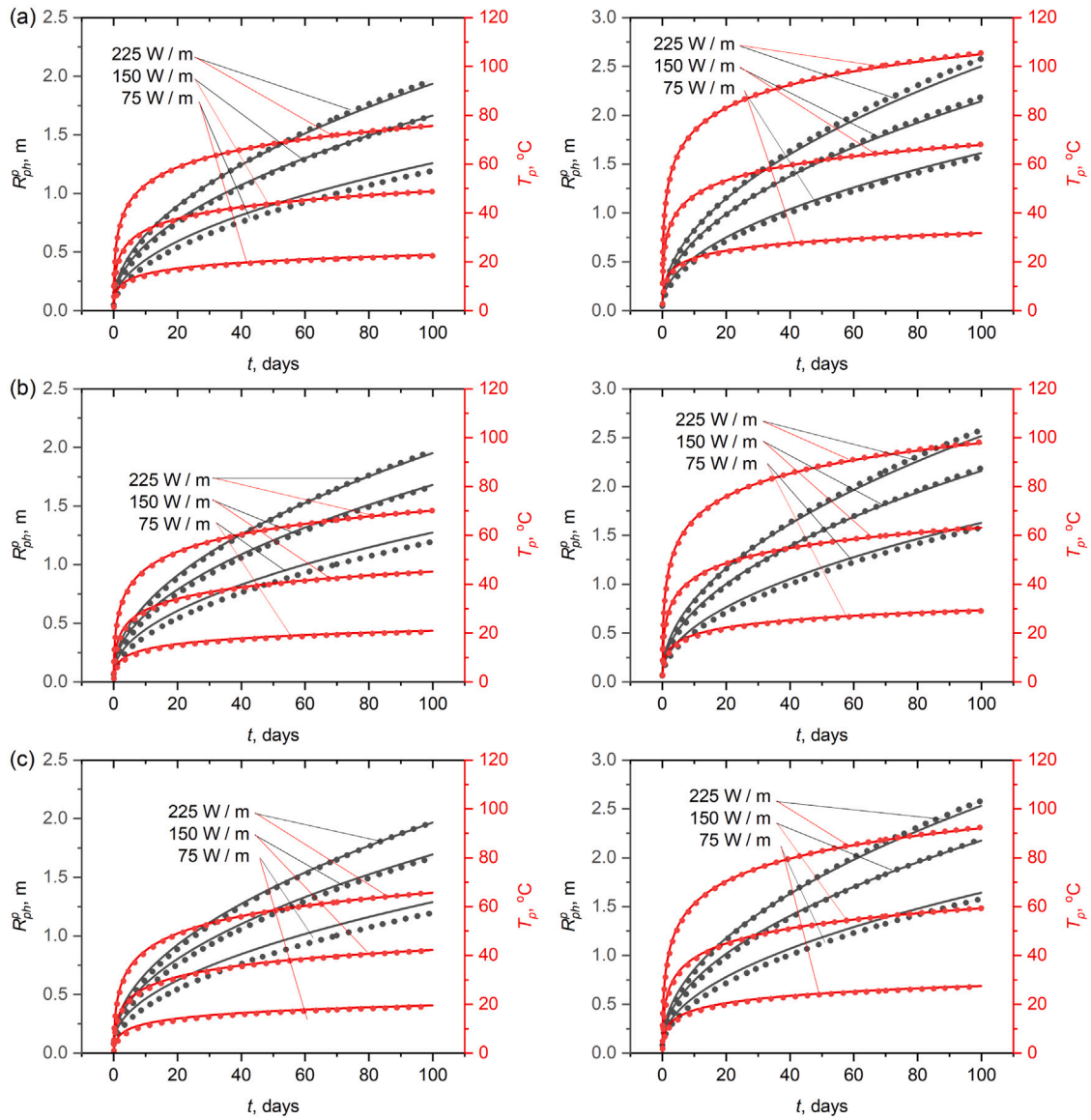


Fig. A.2. Position of the phase change front around a freeze (heating) pipe and its temperature during artificial thawing of the ice wall. Left figures — soft clay; right figures — loam. The outer diameters of the pipe are 0.1 (a), 0.13 (b), and 0.16 (c) m. Dotted lines represent the FEM simulations; solid lines represent the proposed semi-analytical model.

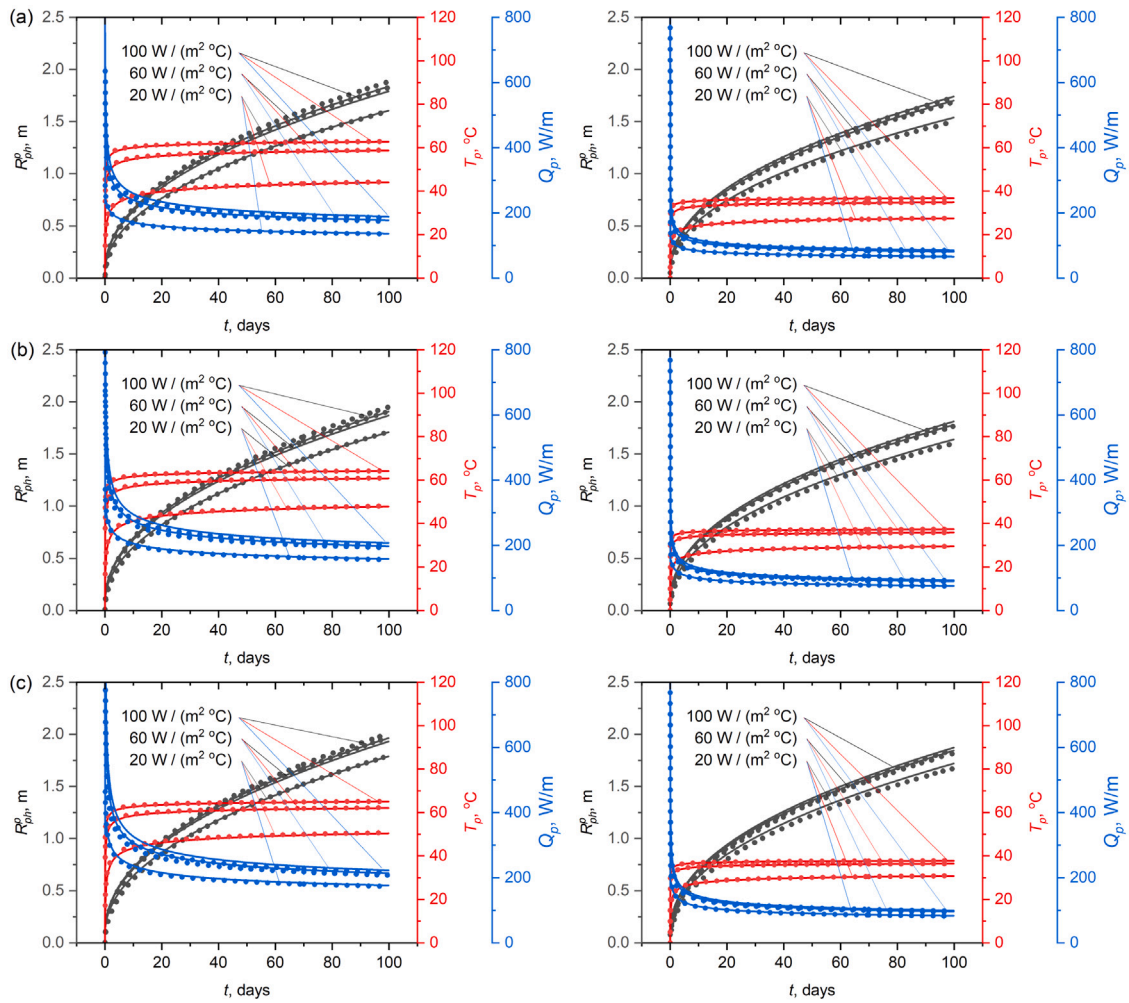


Fig. A.3. Position of the phase change front around a freeze (heating) pipe, the pipe temperature, and the heat flux to the pipe during artificial thawing of ice-wall. Left figures — soft clay; right figures — loam. The outer diameters of the pipe are 0.1 (a), 0.13 (b) and 0.16 (c) m. Dotted lines represent the FEM simulations; solid lines represent the proposed semi-analytical model.

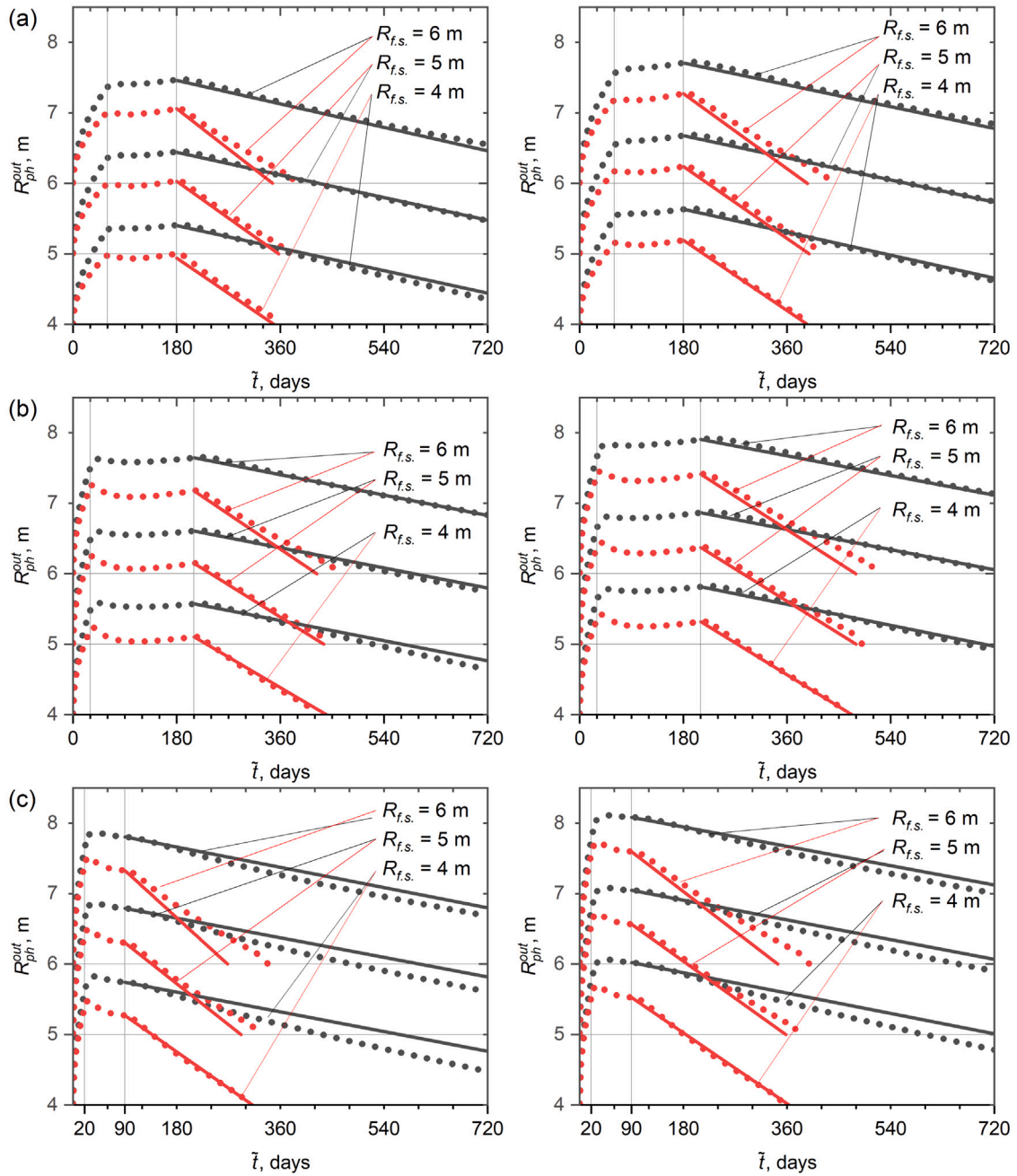


Fig. A.4. The position of the phase change front during the formation and natural thawing of the ice wall for different freezing methods: brine freezing — (a), solid carbon dioxide (SCD) freezing — (b), and liquid nitrogen (LN) freezing — (c). Left figures — clay; right figures — loam. The black lines are the initial soil temperature of 8 °C, the red lines are the initial soil temperature of 20 °C. Dotted lines represent the FEM simulations; solid lines represent the proposed semi-analytical model.

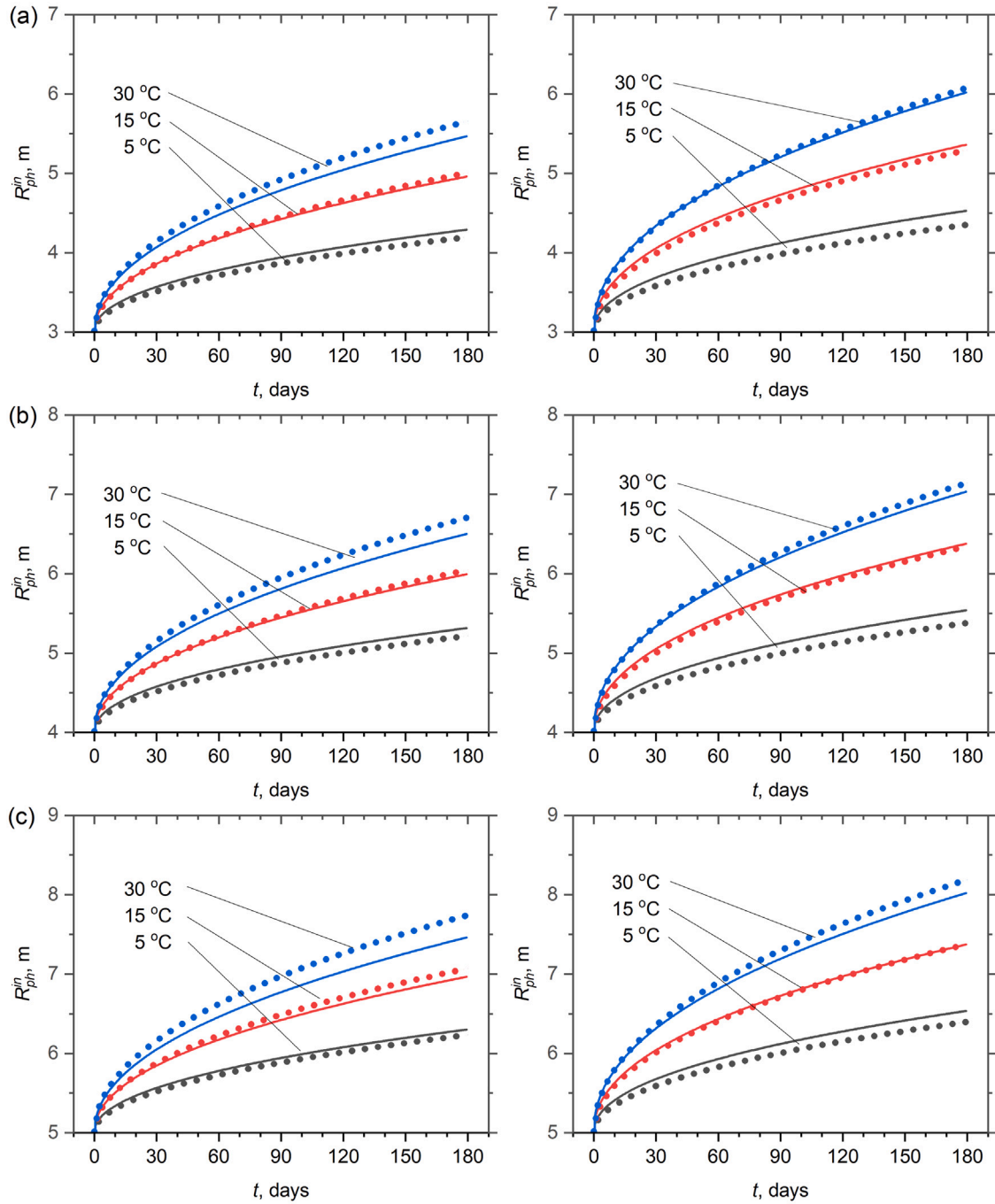


Fig. A.5. The position of the phase change front during the thawing of the inner part of the cylindrical ice-wall in the presence of Dirichlet boundary conditions for different initial radii of the ice wall R_0^n : 3 m–(a); 4 m–(b); 5 m–(c). Left figures — clay; right figures — loam. Dotted lines represent the FEM simulations; solid lines represent the proposed semi-analytical model.

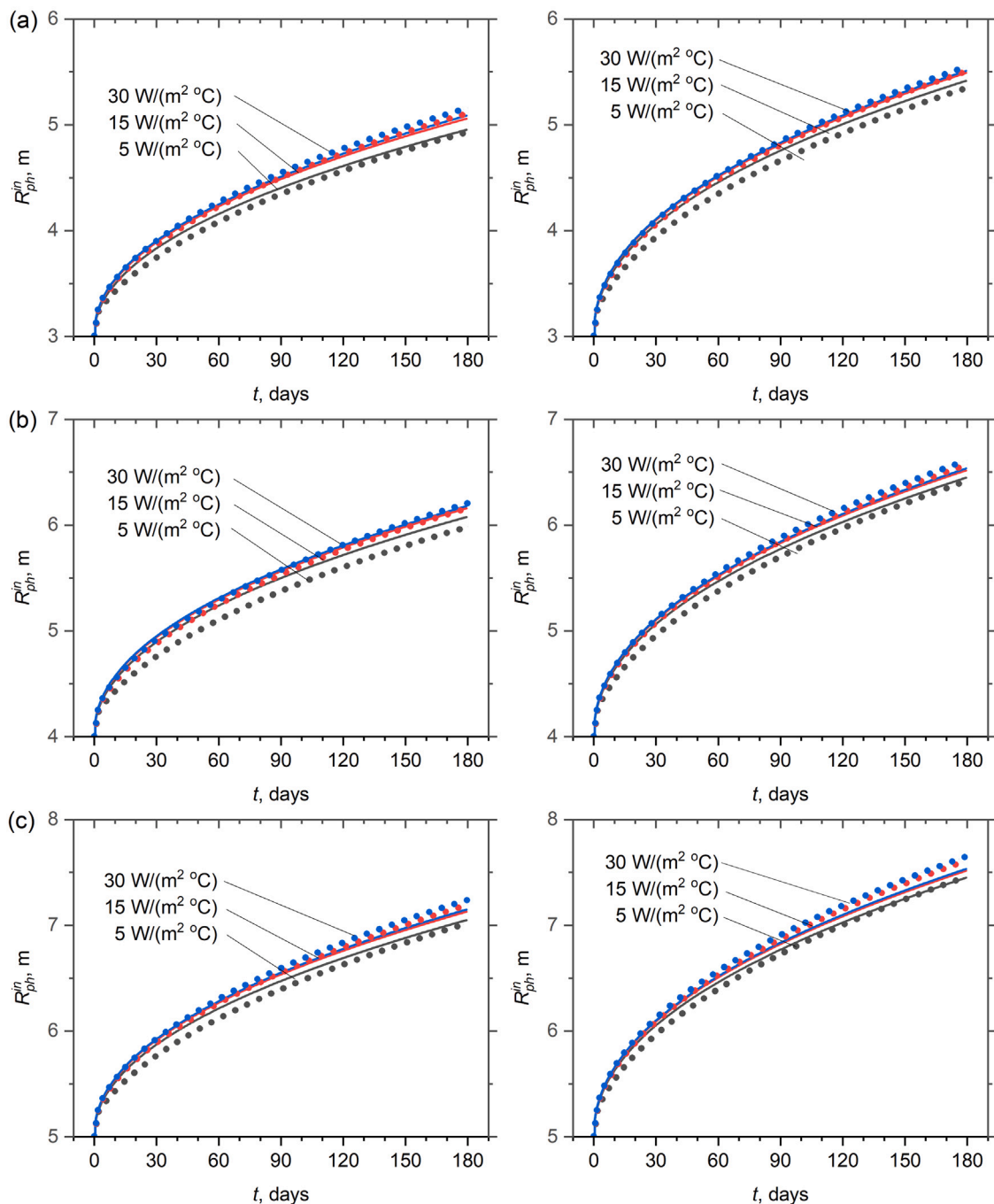


Fig. A.6. The position of the phase change front during the thawing of the inner part of the cylindrical ice-wall in the presence of Robin boundary conditions for different initial radii of the ice wall R_0^n : 3 m–(a); 4 m–(b); 5 m–(c). Left figures — clay; right figures — loam. Dotted lines represent the FEM simulations; solid lines represent the proposed semi-analytical model.

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