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Fatigue life prediction of mistuned steam turbine blades subjected to deviations in blade geometry

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Abstract

The blades of the steam turbine are subjected to bending of the steam flow, centrifugal loading, vibration response, and structural mistuning. These factors mentioned contribute significantly to the fatigue failure of steam turbine blades. Low pressure (LP) steam turbines experience premature blade and disk failures due to the stress concentrations in the root location of the blade of its bladed disk. This study of mistuned steam turbine blades subjected to variation in blade geometry will be of great significance to the electricity generation industry. A simplified, mistuned, scaled-down steam turbine bladed disk model was developed using ABAQUS finite element analysis (FEA) software. The acquisition of the vibration characteristics and steady-state stress response of the disk models was carried out through FEA. Such studies are very limited. Subsequently, numerical stress distributions were acquired and the model was subsequently exported to Fe-Safe software for fatigue life calculations based on centrifugal and harmonic sinusoidal pressure loading. Vibration characteristics and response of the variation of the geometric blade of the steam turbine were investigated. Natural FEA frequencies compared well with the published literature of real steam turbines, indicating the reliability of the developed FEA model. The study found that fatigue life is most sensitive to changes in blade length, followed by width and then thickness, in this order. Analytical life cycles and Fe-Safe software show a percentage difference of less than 4.86%. This concludes that the numerical methodology developed can be used for real-life mistuned steam turbine blades subjected to variations in blade geometry.

Keywords: *blade geometry, low-pressure, fatigue life, fe-safe, finite element analysis, turbine blades*

1. Introduction

Several power plants suffer from fatigue failures of the steam turbine blades. The leading cause of the unavailability of large fossil fuel turbines worldwide has been recognised as the result of turbine blade failures and blade damage [1,2]. A single-blade failure of a steam turbine can cause significant component damage and financial losses [3]. Fatigue damage in the engineering industry has cost several percentages of the gross domestic product. Prediction of the life of steam turbine blades subjected to varying blade geometries attached to a common disk is a great benefit in fossil fuel power plants [2]. During turbine operation, the blades are subjected to high pressure flow bending, centrifugal stress, and high vibration response induced by alternating stresses.

Uncertainties of the geometric deviations of the steam turbine blades affect aerodynamic performance. Therefore, it affects the working conditions of the entire steam turbine component. The design and optimisation methods of steam turbine blades are deterministic; therefore, the approach method for life prediction is both the development of probabilistic methodology and deterministic. Focused on the edges of the blade and the roughness of the surface.

Experience has shown that fatigue most often results in cracks and fractures in the root structure of the blade under consistent operating conditions. Structural integrity has been a barrier to industrial development. Multiple research literature on different aspects over over one hundred years has been published; currently, fatigue is

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still being studied. Fatigue is the process of accumulation of damages due to the presence of cyclic stresses. Each component subjected to alternating stress and loading experiences fatigue. The method studied cannot predict all of the fatigue lifespans.

Fatigue is identified as one of the main damage mechanisms that cause significant losses. Turbine blade fatigue is classified as high cycle fatigue (HCF) or low-cycle fatigue (LCF). LCF is usually associated with fewer load cycles with a much larger strain range, corrosion, or high temperature [4]. HCF is commonly associated with moderate mean stress levels and high dynamic stresses.

During the assembly of the mistuning among the blades, there is a change in modal characteristics [5]. The uneven distribution of the blade amplitudes of a model shape can result in premature failure of the steam turbine blades due to the high fatigue amplitude on the blades.

Blade model development involved 3D ABAQUS software. The FE method is used to develop a numerical mode that includes all geometric variations of the blade. Prediction of the fatigue life of mistuned steam turbine blades is complex due to some uncertain variables such as material fatigue properties, excitation force size, dimensions, stress state, and vibration response. Therefore, the selection of material, elastic modulus, and density to ensure that the properties of the model correlate with the experimental object is a necessity.

The FEM of a free-standing LP steam turbine blade is used to measure the vibration characteristics and stress state of the blade. Model analysis is used to characterise the natural frequencies of the blade model. Natural frequencies of the model are validated using the literature.

1.1. Research problem

Turbomachine fatigue failures occur mainly in mistuned blades due to cyclic stress and mean stress, vibration resonance, and the harsh working environment to which they are exposed [6]. Most fatigue cracks start at the root of the side of the concave blade and spread later [1]. Mistuning is any irregularity in the bladed disk system that makes the response of the system asymmetric [7]. These irregularities can be initiated by structural and aerodynamic means. Structural irregularities arise as a result of the geometric variations from blade to blade [4].

Variations in boundary conditions and the fixation of the blades to the disk or shrouding may also complicate the variation in the geometric dimensions of the blade [8]. Turbomachinery is generally designed to attain perfect cyclic symmetry conditions; random deviations among subdivisions caused by manufacturing tolerances, material defectiveness, nonuniform assembly, and wear characterise real blades [9].

Traditional research on the HCF of steam turbine blades is mainly based on S-N curves and nominal stress approaches. Several methods have been established to predict the fatigue life of steam turbine blades, along with comparative studies of the various methods. Blade life estimation has traditionally been performed using deterministic models, which often require excessively conservative assumptions [7]. Incorporate probabilistic modelling, which could eliminate conservative assumptions and allow uncertainty and variability in key variables to be taken into account [8]. These works develop a probabilistic procedure that can be used to assess the HCF of steam turbine blades. Hence, inspired by the problems encountered with steam turbine blades, this research presents a numerical methodology for the prediction of the life of mistuned steam turbine blades subjected to variations in blade geometry as such studies have not yet been contemplated.

1.2. Important aspect of the study

There are a large number of power plants worldwide that suffer from fatigue failures of the steam turbine blades. Steam turbine blades operate at a high temperature, in a moist and corrosive environment. The steam turbine blades are also subjected to bending of the steam flow, centrifugal loading, vibration response, and structural mistuning. The steam turbine disk becomes asymmetric due to wear in service caused by deviation in blade geometry. These factors mentioned contribute significantly to the fatigue failure of steam turbine blades. Driven by the problems encountered by the electricity generation utilities of steam power plants with respect to fatigue failure of steam turbine blades, it is hypothesised that the study of mistuned steam turbine blades subjected to variation in blade geometry will be of great significance for the electricity generation industry.

Steam turbine blade disks are originally designed to be cyclically symmetric, and inherent factors, such as manufacturing tolerances, material inhomogeneity, and wear in service cause small dissimilarities to appear between each of their elementary sectors [10]. The phenomenon is known as mistuning of steam turbine blades and is a major concern in the steam electricity generation industry. Mistuning greatly affects the performance and life of steam turbine blades; therefore, it needs to be dealt with well in advance. Mistuned blades subjected to variation in blade geometry factors indicate that HCF in steam turbine blades is a problem that needs further research and development of solutions.

Consistency inspections and rapid replacement of damaged blades are required to manage the lifespan and maintenance of steam turbine blades. Replacement of blades too early or too late is insufficient [11]. It is unknown when fatigue will occur, which will cause great loss to the power generation industry. The following reasons support the mentioned blade replacement factors.

i. Late replacement of damaged blades ultimately leads to shutdown of the entire steam turbine with implications in time and cost.

ii. If the damaged blade is not replaced immediately, it leads to more blade failures and greater losses.

iii. During steam turbine outage, troubleshooting and inspection, as well as the replacement process and commissioning. Time and money are wasted without electricity production.

iv. Significant time is also wasted when ordering replacement blades from the original equipment manufacturer.

Very little research has been reported on mistuned steam turbine blades resulting from variations in blade geometry. This study uniquely contributes to the concern of the prediction methodology for the life of mistuned steam turbine blades, including the effects of geometric variation of the blade.

The approach of probabilistic technique to the prediction of fatigue life of simplified, real-value mistuned blade model has not been adequately explored. It is envisaged that a reliable methodology can be realistically developed upon integration of the above hypotheses. The approach needs to be undertaken with high precision since it has not been adequately investigated. The Brown-Miller algorithm in Fe-Safe software is then employed. The results findings will positively contribute to the safety, stability, efficiency, availability, and financial gain of the power plant.

2. Review of the literature

2.1. Effects of mistuning of a bladed steam turbine

Mistuning refers to variations in the characteristics of each sector of a system that is expected to be ideally cyclically symmetric [5]. The accounting of uncertainties and irregularities in mechanical systems is now commonly recognised as an important part of a successful design process [5]. Irregularities can be due to structural or aerodynamic means. These irregularities arise due to geometric variations in dimensions from blade to blade. They can also be complicated due to boundary conditions and variations in the fixity of the blades to disk or shrouding. Steam turbine blade disks are designed to reproduce perfect cyclic symmetry [8].

2.2. Identification mistuning

F. Pichot et al [10] presents a method for mistuning identification for steam turbine bladed disks by measuring the system modes and natural frequencies. Mistuning identification aims to find the mistuning parameters to correct the model so that it properly describes the dynamics of a real steam turbine bladed disk. To do so, global measurements of a system of modes are necessary and are taken as a reference. The procedure is similar to the model update technique. To accurately assess the responses, mistuning parameters must be identified to update the models. This can only be done through experimental investigations. The identification of mistuning also allows the designer to check whether mistuning of a given bladed disk is within manufacturing tolerances [10]. A. Lange et al [12] used a method to transfer the geometric uncertainties of the compressor blades into a numerical simulation. They found a method to capture geometric variations of measured blades by typical profile parameters. An optical measurement procedure using structured light was applied to scan the compressor blades to obtain a 3-D point cloud of the measured blade. The evaluation of this method is based on curves of constant spanwise coordinates between the hub and the casing. In their study, the design geometry is adapted by a special reconstruction algorithm to consider geometric uncertainties in the numerical simulation. Furthermore, the variances between the measured aerofoil and the design geometry are computed through the profile parameters. Finally, the 3-D blade was reconstructed by assigning the parameters to the spanwise coordinate.

2.2.1. Characteristics of Vibration on Mistuned Steam Turbine Blades

Y.J. Yan et al [13] investigated the vibration mechanism of a sprocket that was mistuned using the natural frequencies and modes of the sprocket. They found that the natural frequency of a mistuned bladed disk is denser and has a wider scatter compared to that of the tuned harmonious one. Figure 1 shows the natural frequencies found on tuned and mistuned bladed disks. This indicates that more resonance may occur on the mistuned bladed disk. Based on Figure 1 the mode shape of the tuned bladed disk is either regularly distributed in a cosine wave or the constant, but the mode shape of the mistuned bladed disk is very anomalous. Furthermore, their study concluded that the mode shapes of the mistuned bladed disk are characterised by extremely large amplitudes in a few blades. These blades certainly become the most susceptible to fatigue failure.

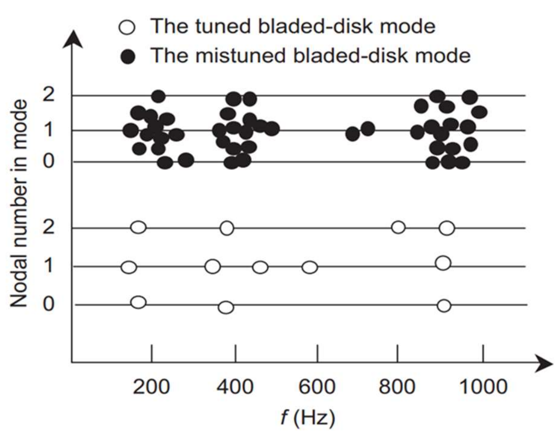

Figure 1: Natural frequencies of tuned and mistuned bladed disks [13].

2.2.2. Forced Response Characteristics of the Steam Turbine Blade Disk

The maximum forced response increases with mistuning of strengths up to a certain critical level [14]. Frequently, demonstrating a maximum response at a low degree of mistuning, beyond which a further escalation in mistuning causes the forced response to drop and roll out. It is assumed that the amplitude magnification factor affects the mistuning of the blade after it initially rises by dropping it [15]. Furthermore, recent studies have shown that numerical approaches have been tried in the development of new systems for the accurate prediction of the maximum amplification factor.

Furthermore, mistuning arises due to manufacturing tolerances, material nonuniformities, small imperfections, rubbing, and other similar factors as mentioned in Section 2.2. Small mistuning levels can lead to an extreme increase in the maximum forced response level, which may be many times greater than the tuned turbo machine system [16]. This simply elaborates that; the vibrating energy is not evenly distributed along all the steam turbine blades but confined to a restricted number of mistuned blades. H. Liao et al [16] concluded that the manifestation of the vibration localisation phenomenon is the result of mistuning.

2.3. High cycle fatigue (HCF)

The HCF describes fatigue cycles in a range of 10^4 times to 10^8 times [17]. The accumulation of HCF damage occurs over 10,000 cycles [1]. For each start and stop, the number of vibratory load cycles accumulates and can cause fatigue failure [1]. Hence, HCF is used in this study.

Dynamic stresses are influenced by the magnitude and source of excitation, the damping of the blade, and the degree of geometrical mistuning of the turbomachinery blade [18]. Dimensional imperfections and deviations are due to tolerances and manufacturing process, material inhomogeneity, and wear in service [19]. These deviations lead to mistuning and can increase the forced response vibration amplitude, increasing the HCF; this is commonly called mistuning. It affects the blade variations, which leads to mistune of the blade's natural frequencies, which affects the entire steam turbine component. Due to the geometric variation of the blade, mistuning cannot be ignored when there is a coupling among the blades.

2.3.1. Blade fatigue life prediction

Various factors were shown to affect the fatigue life of steam turbine blades, more specifically, mistuned geometric blades. The main factors that affect the lifespan of the blade are the static and dynamic stress distributions generated on the blade during operation [20]. This results in mean stresses and alternating stresses. According to the literature cited, the life of blades is threatened by uncertainty in blade material fatigue properties, blade mistuning in terms of geometric variations, and harsh working conditions during operation (elevated temperatures). N.S. Vyas et al [20] confirmed that mistuning of the steam turbine blades in the form of geometric variations and harsh working conditions during operation (elevated temperatures) affect the life of the components of the steam turbine. Furthermore, several researchers proposed different fatigue life calculation methods which can be applied to determine the life of the blades.

A study by [20] presented a life prediction algorithm that can be used to estimate the total fatigue life of a steam turbine bladed disk. The proposed methodology for the prediction of total life is based on the use of strain life concepts to determine the crack initiation life and the propagation life. The methodology enables us to use the application of linear elastic fracture mechanics. Numerous case scenarios in which steam turbine blade disk cracks are numerically modelled in the blade root or hot spot are considered.

2.3.2. Dang van Fatigue Criterion

Dang van criterion is a stress-based multiaxial fatigue criterion used to predict the HCF life of mechanical components [8]. It relates the variation of the stress state in a material point to a critical parameter that should not be reached. The study by [21] used the concept above on a roller bearing for windmill applications, the influence of hardness variations and different residual stresses. M. Cerullo [21] found that, according to the concept mentioned above, the highest damage factor is reached below the surface of the structure, regardless of the safe location used. This suggests that failure is more likely to initiate in the material a little below the surface, which is consistent with the literature that reports subsurface failures in steam turbine applications.

3. Mathematical Modelling

Relevant mathematical theories used to calculate the life of the mistuned steam turbine blades are shown below. The correct formulation of the mathematical models, which adequately represents both the physics and the mistuned blade behaviour, is imperatively applied.

3.1. Vibration equations for a disk

During energy production, the blades of the steam turbine are in dynamic motion. Applications of equation of motion relate are useful as they relate to the excitation forces acting on the entire mistuned-bladed steam turbine system. A study by [22] presented the vibration dynamics of the mistuned bladed disk and found that the dynamics are influenced by the excitation to solve the direct dynamic. Therefore, the equation is more useful in the time function [13].

A typical structural model of the bladed disk of an aeroengine compressor is generalised in Figure 2.(a) and (b). Where (a) is a framework figure of experimental bladed-disk and (b) is the model of a bladed-disk.

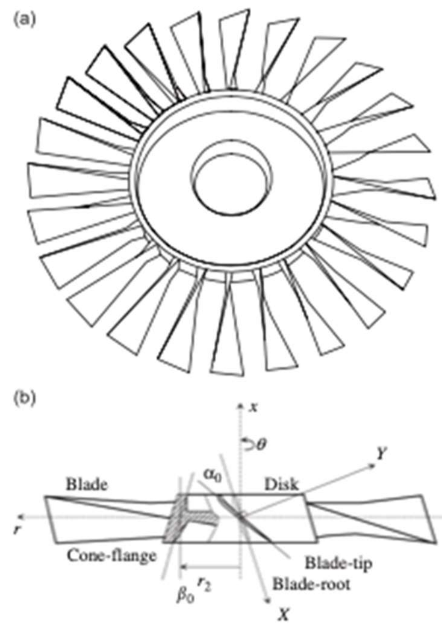

Figure 2. Mechanical model of a typical bladed disk [13].

The disk is simplified and divided into three parts as substructures: (1) a variable-thickness disk with a fixed centre, (2) a cone flange connected to the disk rim with thickness as a variable, (3) multiple blades twisted along the centred blade disk.

All blades are assumed to be fixed on the cone flange to the external rim of the disk. In this study, only the blade calculations are considered. Hypothetically, the longitudinal flex of the cone flange is ignored due to the reinforced effect of multiple blade roots. The experimental mode analysis and mode correction have been carried out. With multiple modes of vibration of the blade and the disk, the mode synthesis technique is applied. Y.J. Yan et al [13] finally provided the forced response vibration equation of motion for the mistuned bladed disk presented in Eq. (1).

$$[M][\ddot{q}] + [C][\dot{q}] + [K][q] = [F(t)] \quad (1)$$

3.2. Stress due to centrifugal load

Loads affecting the rotating bladed disk are centrifugal force [23]. The centrifugal forces depend on the rotational speed and the distance of each element from the rotational axis. The rotating steam turbine bladed disk operates at high rotational velocities that lead to the development of the concept mentioned above. However, this study does not go deep into centrifugal forces; it is focused on HCF. Mistuned centrifugal forces of the steam turbine blade are used to determine if yielding will occur or not. Figure 3 illustrates the action of the centrifugal force on the blade of the steam turbine.

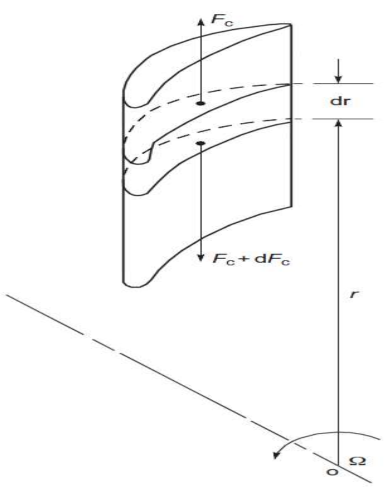

Figure 3. Centrifugal forces acting on a blade of a steam turbine [23].

The centrifugal force and normal stress are estimated with an applied operational speed of 3000 r/min with a set of Eq. (2) [7,24],

$$F = m\omega^2 r \quad (2)$$

where m is the mass of the bladed disk, ω is the rotational speed, and r is the distance from the centre of gravity of the blades. The above Eq. 2 can be simplified to Eq. (3).

$$F_C = (\frac{W}{g})(\frac{2\pi N}{60})^2(r) \quad (3)$$

where W is the weight of the blade, g is the gravity constant, N is the rotational speed of the turbomachine, and r is the radius of the blade to the stress concentration point. The centrifugal stress at the root of the blade with the variables of Table 1 is calculated using Eq. (4).

$$\sigma_{CF} = \frac{F}{A} \quad (4)$$

Table 1. Variables of the steam turbine blade [6].

Area (A)	1.7×10^{-4} m^2
Gravity constant (g)	9.81 m/s^2
Model mass (m)	2.798 kg
Elasticity Modulus (E)	200 GPa
Poisson ratio (v)	0.3-
Centrifugal Stress (σ_{CF})	121.8 MPa

The concept mentioned on section 2.3.2 proposed a method to assess HCF for components subjected to multiple stresses. Therefore, the theory of the above mentioned concept is useful for stress-based multiaxial creation for the prediction of the life of HCF. The method relates the variation of the stress state in the material points to a critical parameter that should not be reached.

The Dang van multiaxial mathematical method is set and applied with parameter equations that algorithm in Fe-Safe software for fatigue life prediction of mistuned steam turbine blades subjected to variation in blade geometry [22]. The studied method uses a simplified formula in Eq. (5),

$$\tau_{max}(t) + \alpha_{DV}\sigma_H(t) \leq \tau_w \quad (5)$$

where $\tau_{max}(t)$ is the instantaneous value of the Tresca shear stress, σ_w is the fatigue limit in pure bending, the α_{DV} is a constant that depends on the material fatigue limits, and $\sigma_H(t)$ is the instantaneous hydrostatic component of the stress tensor.

4. Numerical modelling

4.1. Development of geometric models

A scaled-down of the model of a complete LP steam turbine bladed disk assembly used in practise was developed. The ultimate purpose of the numerical model is to economically simulate the HCF life of mistuned steam turbine blades. Furthermore, the model comprises eight simplified straight cantilever blades with dimensional variations as shown in Figure 4 and Figure 6. During model development, 5 mm fillets were used at the blade roots that corresponded to the experimental model [8] to reduce the severe stress concentration.

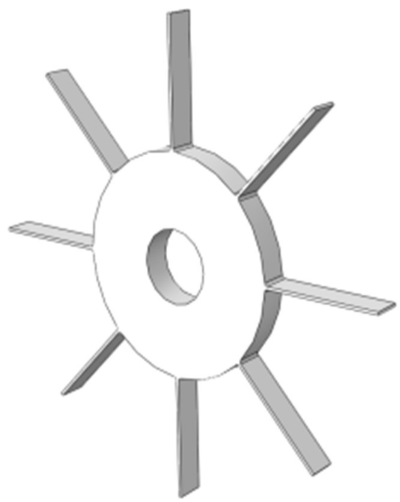

Figure 4. FE model of a mistuned steam turbine bladed disk.

4.2. Material Selection

The ASTM 304 stainless steel material is the most widely used for steam turbine blades. The material has a relatively high ductility characteristic, which is ideal for steam turbine blades that are subjected to high centrifugal and bending loads. This ductile allows the root contact to undergo slight plastic deformation to better share the loads to eliminate failure and crack [3]. Therefore, ASTM 304 was the material chosen for this study. The material and fatigue properties of ASTM 304 stainless steel were obtained from [24] and validated by [25]. The properties are shown in Table 2.

Table 2. Mechanical Properties of 304 stainless steel [24].

304 stainless steel	Typical value
Tensile Strength (MPa)	600
Yield Strength, (Offset 0.2 %) (MPa)	310
Elongation (Percent in 50 mm)	60
Hardness (Brinell)	170
Fatigue Endurance Limit (MPa)	240
Density (kg/m^3)	7 900

4.3. Finite element mesh selection

The generation of mesh is a critical aspect in simulation studies, it controls the FE model calculation time and the accuracy of the FEA results. During the analysis of the steam turbine bladed disk, the model was meshed using continuum, three-dimensional, first-order reduced integration brick elements type (C3D8R), with a homogeneous mass distribution. Therefore, a 1 mm mesh size was selected as the best mesh to use at the roots of the blade and the default mesh size on the rest of the FE model.

4.4. Mass Check

ASTM 304 stainless steel material was used to manufacture the eight bladed steam turbine disk. The prototype disk model manufactured was subjected to variation in geometric dimensions, which means that the thickness, length, and width were altered to be different in a symmetric pattern.

The mass of the FE model was found to be 2.798 kg and the experimental prototype manufactured using the weight of the computer numerical control (CNC) wire cutter model was measured to be 2.729 kg [8] as illustrated in Table 3.

Table 3. FE and Physical Model Mass Comparison.

FE Model Mass	2.798 kg
Physical Model Mass	2.729 kg
Percentage Error	2 %

Figure 5 and Figure 6 illustrate the geometry of the simplified, scaled-down mistuned blade disk model in 3-dimensional and 2-dimensional dimensions, respectively. The 2-dimensional drawing units are in millimetres. When calculating the percentage error, Eq. (6) was used. Where a is the FE Modal Mass and b is the Physical Model Mass from Table 3.

$$\left[\frac{|a-b|}{\frac{a+b}{2}}\right] \times 100 \tag{6}$$

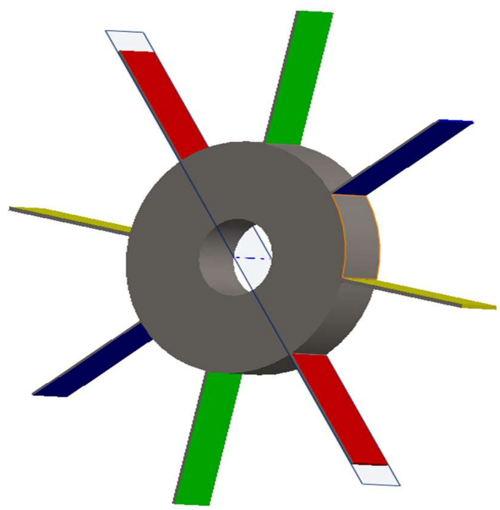

Figure 5. Simplified Scaled-down 3-Dimentional (CAD) Steam Turbine Bladed Disk Model.

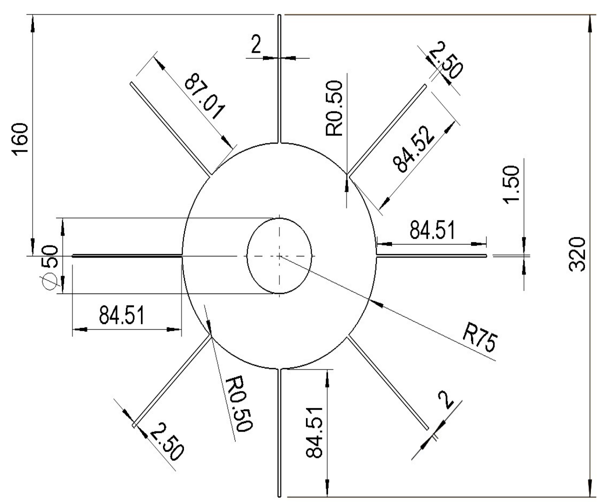

Figure 6. Dimensional Mistuned Steam Turbine Bladed Disk Model.

The boundary condition of the finite element model was initially assigned by applying rotational constraints to the centre hub surface to prevent axial movements within the simulation. Therefore, a rotational speed of 314.159 rad/s angular velocity was applied in an axial direction acting along the centre hub. The stress analysis distribution was acquired in this step. The von Mises stress is equivalent to the mean stress [6].

The maximum mean stress from the FE simulation at the blade roots is anticipated to be 175.2 MPa, as illustrated in Figure 7. The mathematical centrifugal was calculated to be 121.8 MPa.

Both stresses are compared to the material yield strength of 310 MPa; this can be concluded that no yielding takes place as both the analytical and the finite element results are below the maximum yield strength of the material. The mean stress results, which are depicted in Figure 7 also reveal that the peak stress location is in the blade root, which is in agreement with the initial anticipation.

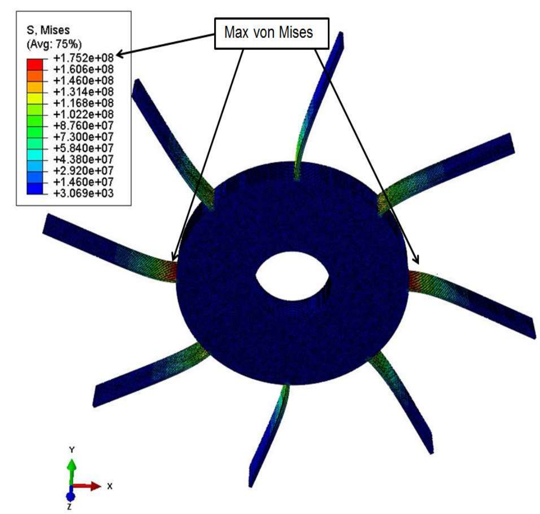

Figure 7. Mean Stress Distribution

4.5. Modal Analysis Results

Mistuned steam turbine blades subjected to geometric variation blades were explored and simulated in 20 different cases of the FE model to study the behaviour of their modal parameter. 19 are different mistuned and 1 is a tuned reference case as illustrated in Table 6, where *t* is the thickness of the blade, *L* is the length of the blade, and *w* is the width of the blade.

4.6. Natural Frequencies

Natural frequencies are critical for this study, and hence eigen frequencies were extracted in the FE modal analysis. Due to the asymmetrical nature caused by our 8 simplified cantilever blades of a mistuned steam turbine, there are unrepeated natural frequencies [8]. Table 4 summarises the first eight cases of the natural frequencies of the mode with the corresponding operational rotation speed in the FE analysis and partially validated with experimental frequencies in the literature of [8,11].

Table 4. Summary of the modes of natural frequencies.

Mode	Natural Frequencies (Hz) FEA	Experimental Naturalistic Frequencies (Hz) [8]	Percentage difference (%)	Mode shape
1	233	222	4.84	Bend
2	1430	1352	5.61	Bend
3	1878	1770	5.92	Bend + Twist
4	1960	1935	1.28	Bend
5	2263	2256	0.30	Bend
6	4002	3997	0.13	Bend
7	5780	5421	6.41	Bend
8	6163	6544	5.99	Bend + Twist

The highest natural frequency difference is observed in modes 7 and 8 due to blade twisting and bending. The mode shapes of the mistuned bladed disk are characterised by extreme large amplitudes in few blades, and these blades potentially become the ones at the most risk of fatigue failure [8]. Hence, mode 3 and 8 are bend + Twist as illustrated in Table 4. The amount of stress distribution in the modal simulation used to identify the model hot spots. Figure 7 with an arrow representing the location of the initial crack. Moreover, Figure 7 confirms that the root of the cracks starts at the blade with the literature of [8,11].

4.7. Model Boundary Conditions and Rotation

The boundary condition of the finite element model was initially assigned by applying rotational constraints to the centre hub surface to prevent axial movements within the simulation. Therefore, a rotational speed of 3000 r/min converted to 314.159 rad/s angular velocity was applied in y-direction as shown in Figure 7 acting along the central hub. The y-axis represents the radial direction in the hub.

The shaft hole of the steam turbine model is restricted by the type of displacement / rotation restriction to prevent axial movement. Table 5 illustrates the degree of freedom of the FE model and is shown in Figure 7.

Table 5. Degree of freedom.

U1	Motion in z-direction
UR1	Rotation in the x-direction
UR2	Rotation in the y-direction

5. Life prediction

In this process, the maximum dynamic stress distribution results acquired from the steady-state dynamic stress analysis are later exported as an.obd (file) from the ABAQUS software into Fe-Safe software.

Fe-Safe software was used to run all 8 cases of fatigue life models for arbitrary geometric mistuned patterns. The fatigue life of a tuned cyclic bladed disk model was calculated to have a minimum of 4.58730 on the LOGlife scale which is equivalent to 4.58730×10^6 cycles. The cycles occur at node 45 490 as shown in Figure 8.

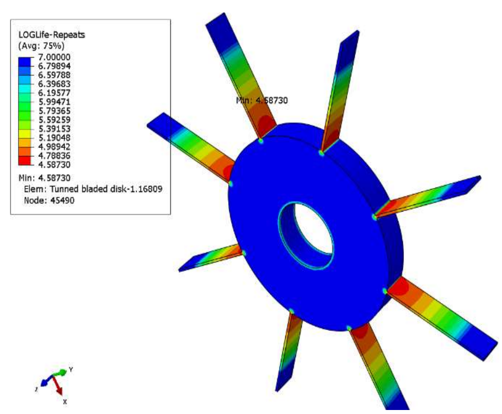

Figure 8. Fe-Safe fatigue life of the tuned model.

It can be concluded that fatigue failure occurs at the stress-concentrated point on the blade root location depicted in Figure 8 and previously projected by the FEA results in Figure 7. Furthermore, Table 6 illustrates the detailed fatigue life cycles due to length, width and thickness changed in 20 cases in decreasing order.

Table 6. Sensitivity Analysis.

Case	Model Description	Fatigue Life Cycles
1	Reference/Tuned model	4.587×10^6
18	Decreased blade length of 2 adjacent (-1 mm, L)	4.574×10^6
19	Increased blade width of 2 adjacent (+1 mm, w)	4.188×10^6
20	Decreased blade width of 2 adjacent (-1 mm, w)	4.130×10^6
9	Mixed mistuned pattern (+/- L, t, & w)	4.097×10^6
6	Increased blade width of 2 opposing (+1 mm, w)	4.093×10^6
7	Decreased blade width of 2 opposing (-1 mm, w)	4.090×10^6
10	Mistuned pattern (rubbing/rotor misalignment) (-L)	4.090×10^6
8	Mixed mistuned pattern (+/- L,t & w)	4.043×10^6
12	Mixed mistuned pattern (+/- L& t)	4.020×10^6
5	Decreased blade length of 2 opposing (-1 mm, L)	4.016×10^6
13	Mixed mistuned pattern (+/- w & t)	4.012×10^6
2	Increased blade thickness of 2 opposing (+1 mm, t)	3.997×10^6
11	Mixed mistuned pattern (+/- L& w)	3.980×10^6
4	Increased blade length of 2 opposing (+1 mm, L)	3.960×10^6
17	Increased blade length of 2 adjacent (+1 mm, L)	3.955×10^6
15	Increased blade thickness of 2 adjacent (+1 mm, t)	3.911×10^6
16	Decreased blade thickness of 2 adjacent (-1 mm, t)	3.999×10^6
3	Decreased blade thickness of 2 opposing (-1 mm, t)	3.907×10^6
14	Mixed mistuned pattern (+/- L,t & w)	3.966×10^6

Table 7. Analytical and Simulated Fatigue Life Different.

Paragraph	Analytical fatigue life (cycle)	Simulated fatigue life (cycle)	Percentage (%)
1	4.435×10^6	4.587×10^6	3.37
18	4.357×10^6	4.574×10^6	4.86

Table 7 illustrates the comparison and validation between the analytical and simulated life cycle results of randomly chosen cases. Case 1 is the tuned while case 18 is the mistuned. The two cases percentage error are close to each other. The Brown-Miller strain-life equation gives a realistic life estimation and the life $2N_f$ are calculated by solving Eq. (7) for analytical fatigue life cycles.

Where N_f is the number of cycles to failure, $\Delta\varepsilon_n$ is the nominal strain range for the cycle, $\Delta\gamma_{max}$ is the maximum shear strain range or amplitude for the cycle, $\sigma'_f = 1057$ MPa is the fatigue strength coefficient (from the 304 stainless steel fatigue properties), $b = -0.0385$ is the fatigue strength exponent (from the 304 stainless steel fatigue properties), ε'_f is the fatigue ductility coefficient and c is the fatigue ductility exponent (ranging from -0.5 to -0.7 for metals). Moreover, Eq. (6) was applied to calculate the percentage error.

$$\frac{\Delta\gamma_{max}}{2} + \frac{\Delta\varepsilon_n}{2} = 1.65\frac{\sigma'_f}{E}(2N_f)^b + 1.75\varepsilon'_f(2N_f)^c \tag{7}$$

6. Conclusions

Engineering approximations of numerical data have been made when necessary due to insufficient information on mistuned blades, turbine performance, and material fatigue data. Analytical life cycles and Fe-Safe software show a percentage difference of 3.37 % to 4.86 %. The procedure was to individually manipulate the blade geometries based on these geometric configurations and their respective fatigue cycles to failure.

The maximum simulated fatigue life cycle shows to be 4.587×10^6 on tuned bladed disk and then 4.574×10^6 in a mistuned bladed disk in length adjustment. Fatigue starts at the root of the blade. Fatigue life is most sensitive to changes in blade length, followed by width and then thickness, in this order.

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Nomenclature

A	Area
B	Fatigue strength
C	Fatigue ductility exponent
E	Young's modulus
F	Force
FEA	Finite Element Analysis
FEM	Finite Element Modelling
f	Excitation Frequency
g	Gravitational acceleration
L	Blade length
LP	Low-Pressure
M	Mass matrix
N	Rotation velocity
N_f	Number of cycles to failure
P	Pressure
r	Radius
t	Blade thickness
V	Poisson's ratio
w	Blade width
ε'_f	Fatigue ductility coefficient
$\Delta_{\varepsilon n}$	Normal strain range
τ	Shear stress
τ_w	Fatigue limit in reversed torsion
$\tau_{max}(t)$	Instantaneous value of Tresca shear stress
σ_w	Fatigue limit in pure bending
$\sigma_H(t)$	Instantaneous hydrostatic component
ω	Rotational velocity
$\Delta\gamma_{max}$	Maximum shear strain range

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