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The Role of Ambiguity in Financial Markets
Applications to Return, Volatility and Economic Prediction

So, Ha Yan

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PhD Thesis

The Role of Ambiguity in Financial Markets:

Applications to Return, Volatility and Economic Prediction

Raymond Ha Yan So

King’s College London

University of London

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To Sarah

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Chapter 1.

Introduction

Risk and uncertainty are of crucial importance to financial decision making and investment. The complexity in understanding the implications of uncertainty to the financial markets and potentially the real economy relates to the fact that agents are not always certain about the exact probability of future outcomes. While the notion of ambiguity or uncertainty beyond probabilistic risk is well defined in Knight (1921), applications to financial economics especially in the empirical finance area are still scarce. Following major disruptions to the financial system and damages to the real economy resulting from the Great Financial Crisis of 2007-2009, academic attention has shifted towards understanding subjective behavior in the financial markets and the implications of uncertainty to the stability of the financial system and the real economy. Despite the need to study the actual impact of Knightian uncertainty (ambiguity) on the financial markets and real economy, empirical research in this area remains rare mainly due to the difficulty of quantifying investors' ambiguity ex ante. Motivated by the need to empirically study the role of Knightian uncertainty in financial markets and the real economy, and the lack of evidence in this area, this research examines the information dynamics related to option market extracted ambiguity and its associations with market excess returns, economic activity and stock market volatility.

The research focuses on the information content of option market ambiguity extracted using an ambiguity-adjusted option pricing model developed by Driouchi, Trigeorgis and So (2016). With the inherently forward looking nature of financial options, a rich set of information can be obtained to improve our understanding of the investor-market information dynamics. The information gathered is then tested and applied to several important domains in the finance and economics literatures.

Chapter 2 investigates the information content of option market ambiguity in the financial market by looking at the relationship between ambiguity and ex post market returns in the United States and eight other countries with active option trading activities. We find that option market ambiguity robustly predicts market excess returns and contains extra information in addition to that of other existing return predictors both in-sample and out-of-sample. More importantly, the chapter unveils empirically the long sought-after positive risk-return tradeoff as predicted by Merton (1973) and confirms the theoretical predictions of Cao et al (2005) regarding the dynamics among risk, ambiguity and equity premium under limited market participation. The results show a clear positive risk-return trade-off when ambiguity is controlled for. Our findings further suggest, as predicted by Cao et al. (2015), risk (ambiguity) carries a positive (negative) premium and that ambiguity dominates risk in the determination of the equity premium. In an additional analysis extending the tests to eight additional countries, we find that the predictive power of ambiguity generally holds globally.

Witnessing the aftermath of the Great Financial Crisis of 2007-2008 in the real economy, huge attention has been drawn to studying the relationship between financial market uncertainty and real economic activity. Chapter 3 is devoted to understanding the information efficiency of option market ambiguity in predicting ex post economic activity. A number of theories, including precautionary saving theory, real option theory, and financial friction theory, suggest uncertainty depresses real economic activity. By comparing it to established measures of aggregate uncertainty, option market ambiguity is significantly correlated to thirteen out of fourteen established macroeconomic uncertainty measures. This suggests option market ambiguity tends to also be a good proxy for aggregate uncertainty, and might even be a source of macroeconomic uncertainty. We analyze the causal relationship between option market ambiguity and economic activity indicators covering production, employment, consumption, and overall economic output using a vector autoregressive

(VAR) model. Results from VAR suggest a unidirectional causal relationship between option market ambiguity and economic activity, in which option market ambiguity significantly Granger-cause each of the economic indicators. The finding confirms the negative relationship between uncertainty, as inferred from the option market, and real economic activity. Guided by the theories relating uncertainty to depressed economic activity and confirmation from the VAR results, we investigate the information efficiency of option market ambiguity in predicting economic activity indicators. In an extensive analysis covering eight economic indicators for production, employment, consumption, and overall output, we find superior predictive power afforded by option market ambiguity for all eight indicators for all horizons up to eight quarters ex post. The superior predictive power of option market ambiguity holds even when other established predictors of economic activity are controlled for.

Inspired by the heterogeneous behavior of individuals towards situations of gains and losses, and the shifts in ambiguity attitudes observed in laboratory settings, we also investigate to what extent ambiguity attitudes inferred from option prices could improve the accuracy of volatility modeling. From a volatility forecasting point of view, Chapter 4 examines if investors' attitudes to ambiguity could improve our ability to forecast volatility in-sample, out-of-sample, and generate economic value in risk management and investment. Considering investors' behavior in the gain and loss domains, GARCH-class models that take into account the information of investors' ambiguity attitudes robustly outperform their unambiguous or ambiguity-free counterparts both in-sample and out-of-sample. Economic significance analysis based on two simple volatility-timing trading strategies further highlights the importance of considering investors' ambiguity attitudes in volatility forecasting.

Having investigated the role of ambiguity in market return, economic activity and volatility dynamics, this research further examines in Chapter 5 the role of ambiguity in the estimation of variance risk premium. Considered to be a measure of risk aversion, with some even viewing it as a proxy for Knigntian uncertainty, the variance risk premium has emerged rapidly as a fundamental indicator of aggregate uncertainty in the empirical finance literature in the last decade. The estimation of the variance risk premium is not without limitations however. Attempts to improve the estimation of the variance risk premium have mainly focused on capturing a more accurate conditional variance component, which essentially reduces the measurement exercise to a mere volatility forecasting problem. Chapter 5 tackles the (mis)estimation problem of variance risk premium (and its two variance components) from an ambiguity point of view and investigates if a subjective adjustment factor, inferred from traded option prices according to the aforementioned ambiguity-adjusted option pricing model, is able to improve the forecasting accuracy of variance risk premium to financial market returns, economic activity, and financial instability. Results from the chapter suggest that the consideration of ambiguity in the estimation of the variance risk premium and its two market variance components can deliver additional information content in financial and economic predictions. In an additional analysis, we also find that the subjective adjustment factor improves the detection of international volatility spillovers. This suggests investors and policy makers can be better informed about spillover risks when ambiguity is explicitly accounted for.

To summarize, this research presents novel empirical evidence on the importance of considering ambiguity in financial prediction, economic prediction, and volatility modeling, and contributes to the empirical financial economics literature on market behavior under Knightian uncertainty. There are four major contributions from this research. Chapter 2 validates the theoretically predicted relationship between ambiguity and market returns, and is the first study to uncover a positive risk-return trade-off empirically when ambiguity is controlled for. Chapter 3 is the first study to validate option market ambiguity as an appropriate proxy for macroeconomic uncertainty, confirms that Knightian uncertainty has a negative impact on economic activity, and shows that ambiguity contains superior information in predicting every aspect of real economic activity. Chapter 4 contributes to the literature by unveiling the importance of investors' ambiguity attitudes in volatility modeling both in-sample and out-of-sample, and showing that excess trading returns can be earned through ambiguity-based volatility-timing strategies. Chapter 5 is the first study to address the problem of (mis)estimation of variance risk premium and its variance components from an ambiguity point of view, and shows that subjectively adjusting the variance risk premium and its market variance components for Knightian uncertainty improves predictive power to market returns, economic activity and financial instability.

Chapter 2.

Option Market Ambiguity and Excess Returns

ABSTRACT

This chapter examines the informational efficiency of market ambiguity in predicting market excess returns and the equity premium internationally. Empirical results show a strong predictive ability of option-implied, and sentiment-based, ambiguity for U.S. stock market returns for up to three years. The study also provides evidence of return predictability in eight other countries. Proxying for dispersion in beliefs, our ambiguity measures provide additional information to that of standard return predictors and recent economic uncertainty indicators. We document a negative relation between ambiguity and the equity premium in line with limited market participation theory (Cao, Wang and Zhang, 2005), and confirm a positive intertemporal risk-return trade-off after controlling for ambiguity.

2.1. Introduction

Predictability of excess market returns or the equity premium has been an important area of inquiry in financial economics (e.g., Campbell, 1987; Fama and French, 1988; Hodrick, 1992; Ang and Bekaert, 2007). Excess return prediction spans a wide range of empirical contexts (e.g., Pesaran and Timmermann, 1995; Patelis, 1997; Cohen and Frazzini, 2008; Rapach, Strauss and Zhou, 2013). These include financial market efficiency, profitable hedge portfolio strategies and financial "propagation" mechanisms (e.g., Bernanke and Gertler, 1989). Much research on market return prediction focused on predictor variables relating to accounting or financial ratio information and fundamental financial variables such as book-to-market, price-to-earnings or aggregate dividend yield (e.g., Campbell, 1987; Fama and French, 1988; Campbell and Shiller, 1988; Hodrick, 1992; Lewellen,1999, 2004; Ang and Bekaert, 2007; Menzly and Ozbas, 2010).

The recent financial crisis of 2007-2009 has additionally brought attention to subjective investor behavior, changing uncertainty attitudes and sentiment affecting the real economy and markets. The rapid collapse of the equity markets globally has renewed interest in investor "irrationality" or behavioral characteristics and potential links to market excess returns. To help understand the role of investor perceptions of uncertainty in relation to equity market returns, many researchers turned to the forward-looking options markets (e.g., Cremers and Weinbaum, 2010; Bali and Murray, 2013; An et al., 2014). Option market-extracted information (such as risk-neutral implied volatility or skewness, deviations from put-call parity, and open interest) harbors valuable information in predicting market returns. Many option market-extracted predictors, such as CBOE's volatility index (VIX) or the variance risk premium (VRP), rely on the standard option pricing assumption of risk-neutrality and thus can only tell part of the full story. To overcome this restriction, model uncertainty, miscalibration and subjective investor behavior were also examined and shown to contribute to excess market return prediction (e.g., Bondt and Thaler, 1985; Avramov, 2002; Simsek, 2013).

Despite a rising need to understand the relation between subjective investor ambiguity perceptions and market excess returns, literature on this issue has been relatively scarce (e.g., Jeong et al., 2015) partly due to the difficulty in inferring investors' ambiguity and divergence in uncertainty beliefs from market data. This chapter helps fill this gap by establishing an empirical link between option market-extracted ambiguity and equity market excess returns. Using a quarter century of financial market data (from 1990 to 2012), we show that option-market ambiguity inferred from VIX data robustly predicts aggregate excess returns on the S&P 500 index for horizons up to 36 months. This finding is robust to an alternative ambiguity specification that is sentiment-based (i.e., dispersion between bullish and bearish sentiment indices) and not reliant on option information. Our option market ambiguity measure, which also captures divergence in heterogeneous ambiguity beliefs among representative option investors, has favorable predictive power and consistency compared to other predictor variables and ambiguity proxies based on both in-sample and out-of-sample tests. The predictive power of option market ambiguity remains significant after controlling for traditional predictor variables used in extant research. To ascertain that our measure of option market ambiguity is an empirically robust predictor of excess market returns, we extend our study to eight other major countries in addition to the U.S. stock market. To our knowledge, this is the first study to examine the informational efficiency of option market ambiguity in predicting market excess returns and equity premia in an international context.

Knight (1921) early on highlighted the distinction between risk and ambiguity in economic phenomena. Unlike risk, which involves making reasonable probabilistic estimates of possible outcomes, ambiguity refers to situations in which economic agents are unsure about the very distribution of such outcomes. Empirical implications of the role of ambiguity in human decision-making and economic choice have been corroborated since Ellsberg (1961). With subjective behavior constituting an important factor in economic decisions, risk-neutral based pricing models may be incomplete in capturing the full implications of the uncertainty surrounding market return dynamics (Epstein, 1999; Jeong et al., 2015). Accounting for ambiguity in asset pricing is particularly relevant in turbulent economic times (e.g., Dieckmann, 2011; Guidolin and Liu, 2014; Baker et al., 2016).

CBOE's volatility index or VIX, known as the "fear gauge", is deemed to contain information regarding S&P 500 (SPX) option investors' attitudes toward risk and possibly ambiguity (Miao, Wei and Zhou, 2012; Bekaert, Hoerova and Lo Duca, 2013). More recently the variance risk premium or VRP (e.g., see Carr and Wu, 2009), capturing the difference between option implied variance (IV or

VIX[2]) and stock index realized variance (RV) and considered as a proxy for investor risk aversion, has been proposed as a potential indicator of ambiguity aversion (e.g., see Drechsler, 2013). Bollerslev, Tauchen and Zhou (2009) document a strong and robust power for VRP in predicting short-term U.S. stock market excess returns. Bekaert, Hoerova and Lo Duca (2013) further suggest that VIX itself contains information about investors' risk aversion attitudes, other non-linear pricing effects and potentially Knightian uncertainty or ambiguity. The above studies have shown the richness of predictive information contained in VIX and the options markets. While Bollerslev, Tauchen and Zhou (2009) show the promise of directly extracting investors' uncertainty attitudes from the market, a large portion of VRP's predictive ability seems driven by investors' time-varying risk-aversion rather than by ambiguity attitudes (Bekaert, Hoerova and Lo Duca, 2013). Despite the above notable efforts to harvest ambiguity information from the VIX, to date it has not been feasible to isolate investors' ambiguity perceptions from an option-based indicator, and establish market ambiguity as a robust excess return predictor in- and out-of-sample.

This chapter contributes to extant literature by extracting investors' heterogeneous ambiguity beliefs from the options market and showing how dispersion in such beliefs can be used to predict subsequent stock market excess returns and the equity premium in a global context even when consumption-based risk aversion is controlled for. We find that our option market ambiguity measure robustly predicts stock market excess returns and equity premia in the nine countries examined. Specifically, we infer option market ambiguity from the VIX in line with rank-dependent utility theory based on an ambiguity-adjusted option pricing model (A-OPM) and examine the informational efficiency of option-market implied ambiguity in predicting excess returns in the U.S. stock market, as well as in Belgium, France, Germany, Hong Kong, Japan, Netherlands, Switzerland and the U.K. We show that our ambiguity measure predicts ex post market excess returns beyond VIX, VRP and other established predictors of excess stock market returns in the U.S. and internationally. We particularly help unveil the intertemporal relationship between risk, ambiguity and market returns by decoupling ambiguity from risk. We empirically confirm the existence of a positive and significant intertemporal risk-return tradeoff as predicted by ICAPM theory (Merton, 1973) once ambiguity is explicitly controlled for. Moreover, we provide robust evidence that the relation between ambiguity and the equity premium is negative, in line with the limited market participation theory predictions of Cao, Wang and Zhang (2005). This negative relation between ambiguity and returns is also consistent with disagreement theory (e.g., Baker et al., 2016) and recent evidence on the relationship of survey- and statistical-based uncertainty and disagreement proxies with excess returns (e.g., Diether, Malloy, and Scherbina, 2002; Yu, 2011; Kim, Ryu and Seo, 2014). This finding is robust to an alternative sentiment-based ambiguity measure of dispersion in beliefs among optimistic (bullish) and pessimistic (bearish) investors that does not rely on our A-OPM, option data or the VIX.

2.2. Background Literature and Theoretical Predictions

Many studies examining the predictability of market returns focus or benchmark on the intertemporal risk-return trade-off. Merton's (1973) intertemporal capital asset pricing model (ICAPM) posits that the conditional expectation of excess returns on the stock market should be positively related to the market's conditional variance:

$$E_t[r_{t+1}] = \psi + \gamma V_t \tag{2.1}$$

where $E_t[r_{t+1}]$ is the expected market return conditional on the information set at time t, ψ is the constant term, V_t is the conditional variance of market returns, and γ captures economic agents' relative risk-aversion. Similar to Anderson, Ghysels and Juergens (2009) and Hedegaard and Hodrick (2016), we include a constant term to account for possible model misspecification, capturing the influence of other potential state variables. Despite a clear theoretical prediction of ICAPM concerning a positive sign for γ, the empirical evidence has been inconclusive and validation of a significant positive risk-return relation has not been straight-forward (e.g., see Campbell, 1987; Bollerslev, Tauchen and Zhou, 2009; Nyberg, 2012). French, Schwert and Stambaugh (1987) using GARCH-in-mean find a positive relationship between expected market premium and the volatility of stock returns. Bali and Peng (2006) find a significant positive risk-return association using intra-day high frequency data. Not many other empirical studies, however, find evidence of a significant positive relationship using non-parametric estimation of volatility and low frequency (e.g., daily or monthly) data. Several studies find an insignificant relation (e.g., Nelson, 1991; Campbell and Hentschel, 1992), while others even document a significant negative association (e.g., Glosten et al., 1993). If ambiguity is an important driver (but so far an omitted variable), a way to help address this empirical puzzle is to explicitly account for ambiguity as an incremental explanatory factor or as a control. Leippold, Trojani and Vanini (2008) find evidence of ambiguity potentially explaining the weak relationship between excess returns and conditional variance in low frequency data. Yet no clear and robust evidence has been provided to date documenting a positive risk-return relationship while controlling for ambiguity. This chapter addresses this issue by investigating the role of ambiguity in asset pricing.

Anderson, Ghysels and Juergens (2009) extend Merton's ICAPM framework to test the role of ambiguity, beyond risk, in determining expected excess returns as follows:

$$E_t[r_{t+1}] = \psi + \gamma V_t + \theta A_t \tag{2.2}$$

where θ captures economic agents' attitudes to ambiguity and A_t is the amount (degree) of ambiguity prevailing in the economy at time *t*. If there is no ambiguity (i.e., A_t = 0) or if agents are neutral (i.e., neither ambiguity averse nor seeking ambiguity such that θ = 0), the relationship between return and risk/ambiguity of Eq. (2.2) reduces to Merton's ICAPM of Eq. (2.1). We examine the above risk/uncertainty vs. expected return relationship by empirically testing the validity of Eq. (2.2). We confirm that a positive relationship exists between risk and return ($\gamma > 0$) as predicted by Merton (1973) when the effect of ambiguity is explicitly considered.

The theoretical sign of the ambiguity coefficient (θ) in above Eq. (2.2) is not as clear-cut as it is influenced by economic agents' heterogeneous beliefs and their degree of participation in the market when faced with ambiguity (Ui, 2011; Epstein and Schneider, 2007). Since a large fraction of individuals do not participate in the financial market,[1] the dynamics between ambiguity and the equity premium, through the mechanism of market nonparticipation, is too important to ignore (Ui, 2011). While earlier work suggests the equity premium should rise as market participation decreases (Basak and Cuoco, 1998), Guiso, Haliassos and Jappelli (2003) document a positive relationship between the equity premium and market participation in empirical data. One explanation is that agents are

[1] According to the U.S. Consumer Expenditure Survey, less than one third of U.S. households had investments in either stocks or bonds between 1982 and 1995 (Paiella, 2007). Guiso, Haliassos and Jappelli (2003) show that stock market participation in Europe is limited, ranging from 15% in Italy to 54% in Sweden.

ambiguous about the distribution of future market returns, which affects their willingness to participate in the market and the premium they require for exposure to heightened uncertainty. Dow and Werlang (1992) and Epstein and Schneider (2007) examine theoretically how ambiguity affects agents' decision to participate in the market. Chen and Epstein (2002) recommend the equity premium be decomposed into a risk premium and an ambiguity premium component, with the risk premium derived from standard expected utility and the ambiguity one from agents' ambiguity attitudes.

Cao, Wang and Zhang (2005, CWZ thereafter) show in a general equilibrium model that increased levels of perceived ambiguity dispersion leads to limited market participation and raises the required risk premium as fewer investors bear the market risk. On the other hand, the ambiguity premium decreases as relatively more ambiguity-tolerant (or ambiguity-seeking) investors remain in the market requiring a lower ambiguity premium. Since the two effects are in conflict, their net effect on the equity premium depends on their relative dominance. CWZ and Easley and O'Hara (2009) suggest the ambiguity effect tends to dominate the risk effect in driving the equity premium. We test and confirm empirically the hypothesized negative (positive) relationship between ambiguity (risk) and returns, confirming the relative dominance of ambiguity among the two effects.

Testing the above relationship between market excess returns, risk and ambiguity requires a reliable measure of ambiguity. Although ambiguity cannot be directly observed ex ante, researchers have tried to extract ambiguity-related information from financial market data. Relying on a multiple-priors specification (accommodating multiple growth rate scenarios), Drechsler (2013) suggests that the difference between implied and realized variance captured by the VRP contains information regarding investors' ambiguity aversion (in addition to risk aversion). Anderson et al. (2009) employ disagreement among forecasters as a proxy for market ambiguity. Driouchi et al. (2016) infer ambiguity aversion from observed equity option prices using Choquet expected utility. Andreou et al. (2015) extract stock market ambiguity from the dispersion of trading volume of S&P 500 index options. Brenner and Izhakian (2015) extract ambiguity from intra-day trading data of an exchange-traded fund (SPY) on the S&P500 index. Our ambiguity measure is different from Andreou et al. (2015) as it does not rely on intraday trading volume. Our measure differs from Driouchi et al. (2016) as their measure captures average ambiguity aversion whereas ours measures the divergence in ambiguity beliefs among heterogeneous market investors (i.e., averse and seekers). Our measure is also different from Brenner and Izhakian (2015), who associate their ambiguity indicator with volatility of volatility, as we do not extract ambiguity from ETF (which essentially tracks the underlying asset) but instead we extract ambiguity from the VIX.[2] For robustness we also use a sentiment-based measure, involving dispersion between bullish and bearish sentiments, that captures ambiguity information independently from option data.

We particularly examine the informational efficiency of option-market implied ambiguity in predicting stock market excess returns and the equity premium in the U.S. and eight other countries globally during 2000 to 2012 (for which implied volatility indices data are available). We help unveil the intertemporal relationship between risk, ambiguity and market return by separating ambiguity from risk. In inferring ambiguity from the options market we rely on an ambiguity-extended option pricing model (A-OPM) based on the rank-dependent utility framework proposed by Chateauneuf et al. (1996) and applied to option pricing via Choquet-Brownian stochastic processes by Driouchi et al. (2015). While the multiple-priors ambiguity specification used previously to explain VRP dynamics incorporates parametric uncertainty in the drift of the Brownian motion (i.e., the growth rate of market returns), the Choquet or rank-dependent specification underlying our A-OPM extends the notion of ambiguity to accommodate multiple scenarios in both the drift and the volatility of the underlying process and captures heterogeneous investor beliefs. We specifically infer the divergence in ambiguity beliefs among representative ambiguity averse and ambiguity-seeking S&P 500 investors by extracting the ambiguity information directly from the VIX and testing whether option-market implied ambiguity predicts market excess returns in short and long horizons. We further test three predictions made by Cao, Wang and Zhang (2005) based on limited market participation theory:

(P1) ambiguity is a significant factor in determining the equity premium;

[2] The study of Brenner and Izhakian (2015) differs in two additional dimensions: 1) they focus on contemporaneous explanatory power of ambiguity with excess returns without considering predictive power and thus the intertemporal nature of the relationship between risk, ambiguity and expected return as predicted by Merton (1973) and Cao, Wang and Zhang (2005); 2) in their risk-uncertainty-return analysis they do not separate out ambiguity from risk. They show a positive coefficient for conditional volatility with an interaction term between conditional volatility and ambiguity.

(P2) ambiguity and risk have opposite effects on the equity premium with ambiguity (risk) being negatively (positively) related to the equity premium; and

(P3) ambiguity dominates risk in the determination of the equity premium.

2.3. Ambiguity Measurement and Testing Methodology

We follow Chateauneuf et al. (1996) and Driouchi et al. (2016) in deriving option market ambiguity using an ambiguity-extended option pricing model (A-OPM) based on rank-dependent utility theory and probability weighting. The model relies on a Choquet modification of the geometric Brownian motion (GBM) allowing for ambiguity and subjective behavior in probability judgment and valuation that accounts for multiple scenarios in both the drift and the volatility of the Brownian motion.[3] The underlying asset process takes the form:

$$\frac{dS}{S} = (\mu + m\sigma)dt + s\sigma dz \qquad (\forall\, m \in\,]-1,1[\,, \forall\, s\ \in]0,1]\,) \tag{2.3}$$

where $S \equiv S_t$ is the price of the underlying asset (here the S&P 500 index) at time t accommodating multiple mean drifts $\mu + m\sigma$ and standard deviation scenarios $s\sigma$ in above, m and s are the mean and standard deviation of a general Wiener process W with $dW = mdt + sdz$, z being a standard Wiener process. Parameters $m \in\,]-1,1[$ and s $\in]0,1]$ are functions of a capacity variable c, with 0 < c < 1, allowing uncertainty in model parameters that represents investors' perceived ambiguity.[4] Based on the above Choquet (distorted) Brownian motion, the current price P_t^C of a European call option at time t with strike price K and maturity T under A-OPM is (see also Supplementary Appendix):

$$\begin{aligned} P_t^C = {} & S_t e^{-\delta' T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + \left(r' - \delta' + 0.5(s\sigma)^2\right)T}{s\sigma\sqrt{T}}\right) \\ & - K e^{-r' T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + \left(r' - \delta' - 0.5(s\sigma)^2\right)T}{s\sigma\sqrt{T}}\right) \end{aligned} \tag{2.4}$$

[3] Related to the uncertain expected utility theory of Gul and Pesendorfer (2014), Choquet utility has been validated by Kast and Lapied (2010) and Kast et al. (2014) in the context of decision theory with applications to asset pricing.

[4] Parameters $m\sigma$ and s entertain (multiple states of) uncertainty in the mean and variance of the process; these are functions of a capacity variable c, with 0 < c < 1 summarizing the degree of investors' perceived ambiguity: c < 0.5 indicates investor ambiguity aversion, c = 0.5 risk/ambiguity neutrality, and c > 0.5 ambiguity-seeking attitudes.

where:

$$r' = r + m\frac{[r-(\mu+m\sigma)]}{s^2\sigma}, \delta' = \delta - \frac{(m+s^2\sigma-s\sigma)[(\mu+m\sigma)-r]}{s^2\sigma},$$

$$m = 2c - 1 \text{ and } s = \sqrt{4c(1-c)} \quad (\forall\, c\ \in]0,1[)$$

where P_t^C is the theoretical ambiguity-adjusted call option price, *K* is the strike price of the option, r is the risk-free interest rate, r' is the subjective discount rate, δ is the dividend yield, δ' is the subjective dividend yield, *T* is time to maturity, σ is the stock index return volatility, *c* is the capacity variable capturing ambiguity aversion, and μ is the subjective investor required return on the S&P500 index.

Given the above theoretical price of a call option on the stock market index (SPX) under ambiguity, we can invert the market observed or equivalent market price to infer the market investors' subjective attitudes to ambiguity as captured by capacity variable c. We use VIX as the source of ambiguity information extraction as it is reliable, widely used and publicly available, incorporates information from the volatility surface, and captures implied volatility dynamics with a one-month maturity eliminating potential bias of the term structure of implied volatility. With capacity variable *c* capturing aggregate investors' ambiguity attitudes, market investors' degree of ambiguity can be extracted from the implied volatility or market price of S&P 500 options. Parameter $c < 0.5$ (or $c > 0.5$) indicates ambiguity aversion (or ambiguity seeking); for $c = 0.5$ the above A-OPM of Eq. (4) reduces to the classical Black-Scholes European call option formula. Investors' degree of implied ambiguity aversion (*IAA*) or implied ambiguity-seeking (*IAS*) is then obtained by minimizing the absolute error between the equivalent market price (as implied by *VIX*)[5] and the theoretical A-OPM price given by Eq. (4) restricting $0 < c \leq 0.5$ and $0.5 \leq c < 1$, respectively:

$$IAA_t \equiv c_{AA,t} = \underset{c|0<c\leq 0.5}{\arg\min}\left[\left|P_t^C(S_t, K, r_f, T, \sigma_t, \mu_t, c_t) - P_t^{Mkt}(S_t, K, r_f, T, VIX_t)\right|\right] \tag{2.5}$$

$$IAS_t \equiv c_{AS,t} = \underset{c|0.5\leq c<1}{\arg\min}\left[\left|P_t^C(S_t, K, r_f, T, \sigma_t, \mu_t, c_t) - P_t^{Mkt}(S_t, K, r_f, T, VIX_t)\right|\right] \tag{2.6}$$

[5] We also infer option market ambiguity from traded option prices (rather than extract it from the VIX) and find relatively similar predictive power for market excess returns. In our price-based extraction, we select option contracts according to the CBOE's standard methodology for computing VIX. This allows us to include liquid option contracts covering different moneyness and maturity levels. Hodrick's (1992) reverse regressions results for price-based IA are reported in the Appendix Table A2.2.

In Eqs. (2.5) and (2.6) above, P_t^C is the theoretical ambiguity-adjusted call option price from Eq. (2.4), P_t^{Mkt} is the equivalent market price estimated from the standard Black-Scholes option model using VIX_t as volatility input, S_t is the closing level of the S&P 500 index on day t, K is the strike price of the option, r_f is the risk-free interest rate, T is time to maturity in years, σ_t is the stock index return volatility, c_t is the capacity variable capturing ambiguity attitudes, μ_t is the subjective investor required return on the S&P500 index, and VIX_t is the closing level of VIX on day t.

The resulting capacity estimates (c_t) from the minimization of Eqs. (2.5) and (2.6) for ambiguity averse and ambiguity-seeking investors, respectively, are referred to as *IAA* and *IAS*. Here, the equivalent market price of an S&P 500 option, obtained through at-the-money (ATM) Black-Scholes mapping from implied volatility using *VIX*, is an intermediate vehicle in a two-step ambiguity extraction process. A similar approach using IV-price conversion is followed by Jiang and Tian (2005) in a curve-fitting procedure for model-free implied volatility estimation, and by Cremers and Weinbaum (2010) in estimating put-call parity deviations. This procedure is based on the idea that the option price obtained through Black-Scholes OPM mapping, using VIX as the volatility input, is representative of the observed or equivalent market price.[6] Eqs. (2.5) and (2.6) extract ambiguity (via c_t) from this equivalent market price in the second step by inverting the A-OPM of Eq. (2.4). This two-step approach is equivalent to mimicking (or reverse-engineering) the equivalent market price of options over an extended time window (1990 to 2012) and extracting option-based ambiguity information accordingly. The enhanced prediction ability of our ambiguity measure over standard OPM implied volatility or VIX arises from the explicit recognition of multiple mean ($\mu + m\sigma$) and volatility ($s\sigma$) scenarios in our A-OPM. Our implied ambiguity measure IA (see Eqs. (2.7)-(2.8) below)

[6] Our extraction of ambiguity information from the VIX can be described intuitively as follows. We first input the VIX as a volatility measure into the Black-Scholes OPM for one-month options (T = 1 m) with strike (K) at the S&P level (at-the-money) to recover an equivalent market price (premium) of ATM options on the index. Using VIX as the source of primary information extraction is equivalent to inferring the price of ATM index options over a 23 year period. This is confirmed by the very high correlation between the VIX and its predecessor (VXO) series since 1990 ($\rho = 0.987$). Initially (from 1990-2003) the VIX was obtained by inverting the Black-Scholes OPM for ATM options, a procedure we are reverse-engineering. We then use our ambiguity-adjusted OPM of Eq. (2.4) equated to this equivalent price of ATM index options to extract the degree of investor ambiguity aversion (capacity measure c) on each day t. Since Black-Scholes OPM is nested in our A-OPM with $c = 0.5$ corresponding to risk/ambiguity neutrality, daily deviations of estimated c_t from 0.5 (risk/ambiguity neutrality) track investors' time-varying implied subjective ambiguity aversion ($c < 0.5$) or ambiguity seeking ($c > 0.5$) attitudes.

captures ambiguity (Knightian uncertainty), beyond risk, as the sum of deviations from being ambiguity-averse or -seeking from risk/ambiguity neutrality which is equivalent to the divergence in beliefs between ambiguity averse and -seeking investors.

Specifically, with investor ambiguity attitudes obtained as described above, we estimate our option-market implied ambiguity (IA) measure as the divergence in ambiguity beliefs among different representative option market investor groups (averse and seekers) by taking the sum of absolute differences of each of the IAA and IAS groups from the risk/ambiguity neutral benchmark (c = 0.5). We construct two measures to proxy for implied ambiguity dispersion between the ambiguity averse (IAA) and ambiguity-seeking (IAS) investor groups (no assumption is made about the specific weights of these groups in the market):

$$IA_t = (|IAS_t - 0.5| + |IAA_t - 0.5|) \cdot 100 \tag{2.7}$$

$$IA_t^* = (|IAS_t - 0.5| + 0.5) \cdot (|IAA_t - 0.5| + 0.5) \cdot 100 \tag{2.8}$$

The above IA_t measures overall option-market implied ambiguity and divergence in beliefs on day t obtained through the summation of absolute differences from risk/ambiguity neutrality. For robustness purposes, we also estimate the alternative measure IA_t^*. IA_t^* represents option-market implied ambiguity estimated through the product of the re-based absolute deviations from risk/ambiguity neutrality.[7] Given the distorted (multiple) returns $\mu' = \mu + m\sigma$ in Eq. (3) and the subjective parameter $m = 2c - 1$ in Eq. (4), it also follows from Eq. (2.7) that:

$$IA_t = (m_{IAS} - m_{IAA}) \cdot \frac{100}{2} = \frac{(\mu_t + m_{IAS}\sigma_t) - (\mu_t + m_{IAA}\sigma_t)}{\sigma_t} \cdot \frac{100}{2} \tag{2.9}$$

$$= \frac{\mu'_{IAS} - \mu'_{IAA}}{\sigma_t} \cdot \frac{100}{2}$$

where m_{IAS} and m_{IAA} are m for ambiguity-seeking and averse investors, respectively. That is, IA_t measures the belief heterogeneity or divergence in ambiguity attitudes[8] among representative ambiguity averse and seeking investors through their difference in beliefs about the distorted risk-

[7] Re-basing of the absolute deviations avoids that the IA* measure collapses to zero when either IAS or IAA is at the risk-neutral benchmark level of 0.5. Under risk/ambiguity neutrality, IA reduces to a base value of zero, while IA* reduces to a base value of 25.

[8] We thank an anonymous referee for this suggestion.

adjusted returns μ' and their subjective adjustments to the unobserved drift μ. While our main focus is on the IA_t measure that is option-based, we also use for robustness an imperfect alternative proxy that roughly matches the divergence in beliefs description of Eq. (2.9), namely the dispersion among the Bull and Bear sentiment indices from Investor Intelligence (Fisher, 2000). In line with Brenner and Izhakian's (2015) intraday-based ambiguity measure, we also use the volatility of VIX ($\mathrm{Vol}_{\mathrm{VIX}}$) as another alternative ambiguity proxy that does not rely on our A-OPM for robustness.

The above relations (i.e., Eqs. (2.7-2.9)) are analogous to Abdellaoui et al. (2011) who capture Knightian uncertainty through deviations from Bayesian expected utility (rational behavior). They are also a related notion to the VRP, being the difference between "risk-neutral" expected stock market variance (VIX^2) (corresponding to c = 0.5 in our A-OPM) and (actual or "physical") realized variance (RV) reflecting investor risk-aversion attitudes, but extended here to ambiguity-aversion involving $c < 0.5$ (aversion) or $c > 0.5$ (seeking). VRP has also been suggested to contain some uncertainty or ambiguity information (e.g., Drechsler, 2013). However, although both IA and VRP rely on information from implied volatility (VIX or VIX^2) and realized variance (RV), allowing to infer investors' required premium, VRP simply relies on their difference whereas IA uses a different channel (distinct from VRP) capitalizing on our specific ambiguity-adjusted OPM (A-OPM) of Eq. (2.4) with parametric uncertainty in both drifts and volatilities (see Eq. (2.3)) and on the heterogeneous ambiguity beliefs specification in Eq. (2.9). Effectively, using VIX to infer a Black-Scholes option price equivalent and then using the ambiguity-based model (A-OPM) on that equivalent market price given the estimated realized variance and other inputs enables extracting more ambiguity-related information than the one contained in VRP. Our aim here is to derive measures of option market ambiguity based on the deviations of investors' ambiguity attitudes from the risk/ambiguity neutral Black-Scholes world benchmark.[9] Because IAS and IAA capture ambiguity information in both the drifts and volatilities of the Brownian motion described in Eq. (2.3), we expect

[9] We here refer to the Black-Scholes model as "risk-neutral" in the sense that it assumes investors do not demand a risk premium for risks unrelated to equity market returns. The implied volatility inverted from option prices (through the Black-Scholes model) is the true expectation of future volatility if investors are risk-neutral. When investors are not exhibiting risk- (and ambiguity-) neutral behavior, this model is miscalibrated. Our IA measure captures this miscalibration information through our ambiguity-adjusted option model (A-OPM) of Eq. (2.4).

the predictive power of our implied ambiguity measures (IA and IA*) to be more consistent than that of other known predictors. This includes interpretations of VRP as an ambiguity proxy based on multiple-priors explanations (i.e., involving uncertainty in drift only) and Vol_{VIX} based on unknown unknowns (i.e., uncertainty in volatilities).

The predictive power of the above option-market implied ambiguity measures is then tested according to the following standard long-horizon predictive regression specification (e.g., see Fama and French, 1989; Lewellen, 2004) using monthly observations:

$$r_{t+k} = \alpha + \beta x_t + \varepsilon_{t+k} \tag{2.10}$$

where r_{t+k} is the annualized geometric excess return of the S&P 500 index over the risk-free interest rate measured over a horizon of k month(s) from time t, with x_t being a 1 x h row vector of explanatory variables (excluding the intercept term) and β is an h x 1 vector of slope coefficients. The use of monthly observations in testing the predictive power of each predictor variable beyond a one-month horizon is adjusted for the fact that returns are overlapping. Although standard adjustments for the computation of standard errors for overlapping observations, such as Hansen and Hodrick (1980) and Newey and West (1987), are widely used (see for e.g., Neal and Wheatley, 1998), Hodrick (1992) proposes an alternative standard error (labelled 1B)[10] for long-horizon regressions and shows this new estimator to be less biased in small sample analyses than the above traditional estimators. Ang and Bekaert (2007) further show that some predictability evidence in the literature disappears when using the more conservative Hodrick (1992) standard errors. We employ the more conservative Hodrick (1992) standard error for more robust inference.

2.4. Data and Variables Description

[10] In addition to this standard error estimator, Hodrick (1992) proposes a reverse regression approach that consists of regressing one-period returns onto the sum of predictors over the previous k periods. This shares similar characteristics with the alternative standard error estimator. We test the predictive power of each predictor variable with Hodrick's reverse regressions. Results are similar to the t-statistics reported in the various tables. We present the Hodrick reverse regression results for option-market implied ambiguity extracted from the VIX in Appendix Table A2.1; and extracted from traded option prices in Table A2.2.

2.4.1. Option Market Ambiguity

Our ambiguity measures, IA and IA*, are extracted from the monthly closing level of the VIX index from the Chicago Board Options Exchange (CBOE) website. Our dataset spans the period from January 1990 to December 2012. We employ the new VIX index as a proxy for the 30-day implied volatility of S&P 500 index options. The old version, relabeled by CBOE as VXO in 2003, had the S&P 100 index as its underlying. Using the new VIX also enables comparisons with other related studies, e.g., Bollerslev, Tauchen and Zhou (2009) and Miao, Wei and Zhou (2012).[11]

Besides the VIX, several other inputs are needed to extract ambiguity from the option market. We use the one-month US dollar LIBOR as the risk-free rate from Thomson Reuters Datastream. In line with Gonzalez-Rivera, Lee and Mishra (2004), we use as volatility measure employed in the computation of the theoretical A-OPM price of Eq. (2.2) the Risk Metrics EWMA estimate.[12] As common in empirical asset pricing (e.g., Fama and French, 1992), we use the historical market return (in this case the one-year geometric return)[13] on the S&P 500 index as proxy for investors' required return (μ) in Eqs. (2.5) and (2.6). Our results are robust to the use of alternative proxies for investors' required return, volatility and riskless interest rate.

In extending our investigation globally to eight other countries (Belgium, France, Germany, Hong Kong, Japan, Netherlands, Switzerland and the United Kingdom), the underlying equity market indices for these countries are: BEL20, CAC 40, DAX 30, HSI, NIKKEI 225, AEX, SMI 20 and

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[11] We also infer option-market ambiguity from traded option prices and find similar predictive power for market excess returns. The choice of information extraction domain, either VIX-implied equivalent option prices or traded SPX option prices, does not crucially affect the predictive power of option market ambiguity.

[12] A popular industry practice to estimate volatility, RiskMetrics EWMA has been shown to perform as well as other more sophisticated parametric models in option pricing (Gonzalez-Rivera, Lee and Mishra 2004) and asset allocation (Harris and Nguyen 2013). For robustness, we also use out-of-sample GARCH(1,1) with a three-year rolling estimation window and historical volatility computed as the standard deviation of the past 22 daily returns. Prediction results, presented in Table 2.5, are generally not affected by the choice of volatility measure.

[13] The choice of investors' required rate of return (μ) is partly related to memory considerations. Barberis, Huang and Santos (2001) underline the importance of investors' memory, specifically how far back an investor's mind stretches when recalling past gains and losses in determining required returns, and show that investors tend to have a short memory. We use the past 12-month return as our proxy for the required rate of return (μ) as this time frame both fits the short memory constraint and gives reasonable sample size (252 trading days). Robustness results in Table 2.5 show that the choice of sampling length (up to 3 years) does not alter our findings regarding the predictive power of IA. For further robustness we also used the risk-free rate for μ and found the predictive power of IA still holds.

FTSE 100, respectively. For the international evidence our sample spans from January 2000 to December 2012 due to limited availability of implied volatility data in these countries.[14]

2.4.2. Other Predictor Variables

In addition to option market ambiguity, we consider a number of known predictor variables and alternative ambiguity or economic uncertainty proxies. Traditional predictor variables include the consumption wealth ratio measured as the ratio of consumption to wealth (*CAY*, Lettau and Ludvigson, 2001), credit spread (*CRE*, Fama and French, 1989; Pontiff and Schall, 1998) calculated as the difference between Moody's AAA and BAA corporate bond yields, aggregate dividend yield (*DY*, Campbell and Shiller, 1988; Fama and French, 1988; Hodrick, 1992), S&P 500 index price-earnings ratio (*PER*, Campbell and Shiller, 1988) , stochastically detrended risk-free rate (*RREL*, Campbell, 1991) calculated as the difference between one-month treasury bill rate and its 12-month moving average, CBOE's implied skewness (*SKEW*), and the term spread (*TERM*) between 10-year treasury bond and 3-month treasury yield.

We also consider a set of alternative proxies for heightened uncertainty and ambiguity used in the literature, including the dispersion (*DISP*) between bullish (*IBULL*) and bearish sentiment indices (*IBEAR*, Fisher, 2000) from Investors Intelligence Advisors, related to our Eq. (2.9), the Consumer Confidence Index (*CCI*, Lemmon and Portniaguina, 2006), Economic Policy Uncertainty Index (*PUI*, Baker, Bloom and Davis, 2013), University of Michigan's Consumer Sentiment Index (*UMCSI*, Lemmon and Portniaguina, 2006), and the variance risk premium (*VRP*, Bollerslev, Tachen and Zhou, 2009; Drechsler, 2013) capturing the difference between ex-ante risk-neutral expectation of future return variance (measured by implied variance IV) and ex-post realized variance (estimated by 5-minute intra-day realized variance, RV).

PUI, *IBEAR*, *IBULL*, *CCI* and *UMCSI* data were obtained from Thomson Reuters Datastream. Monthly data on S&P 500 dividend yield and price-earnings ratios (*PER*) were obtained from Robert Shiller's website. Data on the term spread and credit spread were downloaded from the website of the

[14] Since VBEL was discontinued in 2010, we use a sample period from January 2000 to November 2010 for Belgium. VHSI does not have backdated data to 2000 and so we use a sample period from January 2001 to December 2012.

Federal Reserve Bank of St. Louis. Intra-day 5-minute realized variance (RV) data for BEL20, HSI, FTSE100, DAX30, CAC40, AEX, SMI20 and NIKKEI225 were obtained from the Oxford-Man Institute's website. Monthly total market capitalizations for each equity index and government bond yield data used in the calculation of market excess returns are obtained from Thomson Reuters Datastream. *CAY* data were obtained from Martin Lettau's website. One-month Treasury bill rates are obtained from Kenneth French's website. Monthly *VRP* data were obtained from Hao Zhou's website.

Figure 2.1.Option-market Implied Ambiguity and Annualized Excess Returns on S&P 500 (1990-2012)

Figure 2.1 plots the time-varying S&P 500 annualized excess return and option market implied ambiguity (IA). The upper figure shows the excess return of the S&P 500 index over the 3-Month T-bill in annualized percentages. The lower figure shows the option market ambiguity measure IA. The plot for IA*is very similar. The shaded grey bands show (from left to right) the periods of the "dot-com bubble", the 2007-2009 Great Financial Crisis, and the European debt crisis, respectively. The sample covers monthly observations from January 1990 to December 2012.

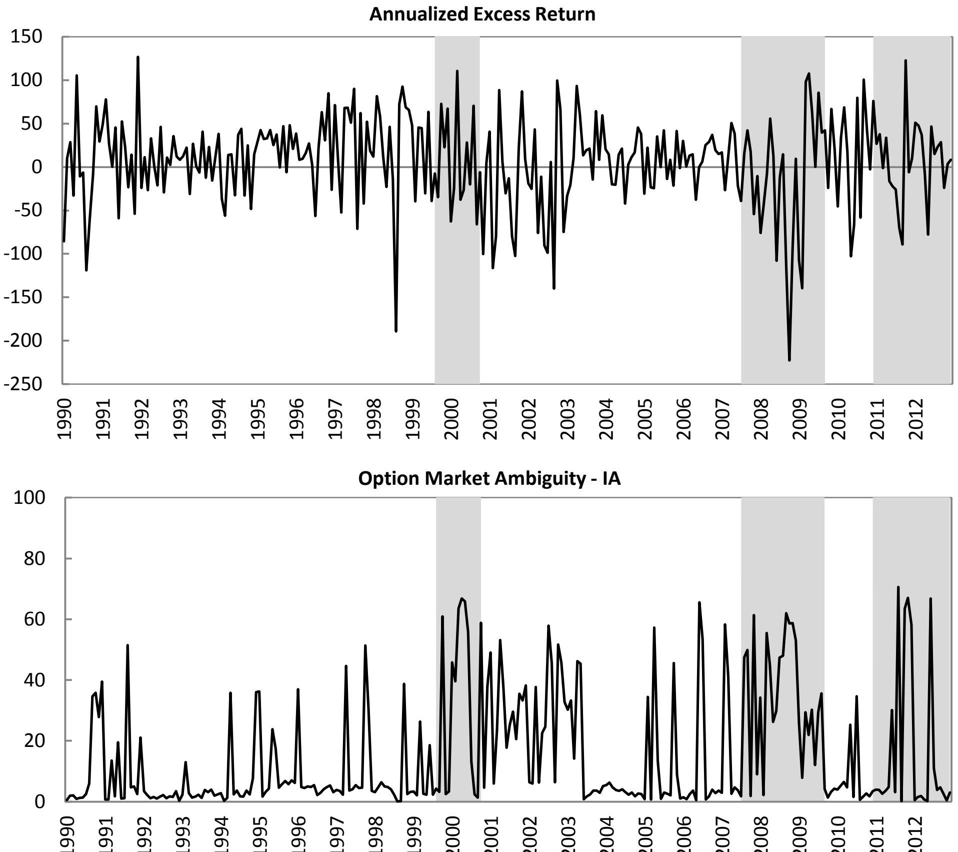


Figure 2.1 plots annualized excess returns of the S&P 500 index over the 1990 to 2012 period, also showing the time evolution of our option-based ambiguity measure IA (the pattern of IA^* is very similar). Distinctive periods with prolonged high levels of ambiguity are observed, such as during the 1999 to 2000 "dot-com bubble". IA peaked in April 2000, around the "dot-com bubble" as the NASDAQ 100 index reached an all-time high in late-March, followed by three years of relative turmoil (IA from 2001 to 2003 is about 25 compared to an all-time average of 15). By the end of 2002,

the NASDAQ 100 index declined by more than 75% from its peak of 4816 points. Another prolonged period of high ambiguity surrounds the 2007 to 2009 global financial crisis. A more recent period of prolonged high ambiguity is the European sovereign debt crisis manifesting investors' concern about a possible exit of an EU country from the Eurozone. In August 2011, the IA indicator reached an all-time high of 70. In that month, the European Commission President warned of the risk the sovereign debt crisis spreading beyond the periphery of the Eurozone. The yields on government bonds from Greece, Spain and Italy rose sharply, while Germany's (considered as safe haven) fell to record lows in the same month.

Table 2.1. Descriptive Statistics and Correlations of S&P 500 Excess Return and Predictor Variables

This table presents descriptive statistics and correlation coefficients of the S&P 500 excess return and predictor variables. $R_M - R_f$ is the excess return of the S&P 500 calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bills. IA and IA* measure option market implied ambiguity extracted from VIX as defined in equations (2.7) and (2.8). Seven traditional predictors are used for comparison. CAY is the consumption-wealth ratio. CRE is the credit spread between Moody's AAA and BAA bond yield indices. DY represents the aggregate dividend yield on the S&P500 index. PER is the price/earnings ratio. RREL is the stochastically detrended interest rate. SKEW denotes CBOE's SKEW index. TERM denotes the term spread between 10Y T-bond and 3M T-bill. Alternative ambiguity proxies include CCI, DISP, PUI, UMCSI and VRP, denoting the Consumer Confidence Index, Dispersion (DISP) between Investors Intelligence Sentiment Indices – Bearish and Bullish, Policy Uncertainty Index, University of Michigan's Consumer Sentiment Index, and the variance risk premium calculated as the difference between implied variance (IV) and realized variance (RV). Variables are denoted as annualized percentages. The sample period covers monthly observations from January 1990 to December 2012.

Panel A. Descriptive Statistics

		Ambiguity		Risk Measures		Traditional Predictors							Alternative Ambiguity Proxies				
	R_M-R_f	*IA*	*IA**	*RV*	*IV*	*CAY*	*CRE*	*DY*	*PER*	*RREL*	*SKEW*	*TERM*	*CCI*	*DISP*	*PUI*	*UMCSI*	*VRP*
Mean	6.03	15.35	33.94	21.32	39.79	0.27	0.97	2.10	25.15	-0.08	116.17	1.88	90.67	17.67	106.59	86.41	18.47
Std. Dev.	52.51	19.53	12.33	37.47	35.61	1.57	0.42	0.66	16.00	0.33	5.06	1.16	28.12	10.34	35.18	13.29	20.35
Skewness	-0.77	1.29	1.52	7.86	3.34	0.04	3.06	0.66	4.18	-0.39	0.39	-0.15	0.01	0.18	1.02	-0.25	-2.48
Kurtosis	1.57	0.29	1.08	85.26	16.31	-0.99	12.23	-0.48	20.07	0.10	-0.22	-1.14	-0.90	-0.84	0.55	-0.62	35.17
AR(1)	0.07	0.36	0.35	0.65	0.80	0.95	0.96	0.99	0.97	0.83	0.55	0.98	0.97	0.61	0.86	0.95	0.26

Panel B. Correlation Coefficients

	R_M-R_f	*IA*	*IA**	*RV*	*IV*	*CAY*	*CRE*	*DY*	*PER*	*RREL*	*SKEW*	*TERM*	*CCI*	*DISP*	*PUI*	*UMCSI*	*VRP*
R_M-R_f	1.00																
IA	-0.02	1.00															
*IA**	-0.02	0.99	1.00														
RV	-0.38	0.43	0.44	1.00													
IV	-0.42	0.34	0.33	0.85	1.00												
CAY	0.04	-0.04	-0.07	0.03	0.10	1.00											
CRE	-0.13	0.30	0.27	0.59	0.65	-0.06	1.00										
DY	-0.02	-0.11	-0.12	0.05	0.04	0.45	0.29	1.00									
PER	0.03	0.15	0.11	0.25	0.40	-0.03	0.59	0.04	1.00								
RREL	0.10	-0.13	-0.11	-0.24	-0.31	-0.10	-0.41	-0.20	-0.31	1.00							
SKEW	0.12	-0.06	-0.06	0.00	0.00	-0.31	0.05	-0.11	-0.05	0.16	1.00						
TERM	-0.04	-0.08	-0.10	0.08	0.06	0.17	0.27	0.35	0.22	-0.36	-0.08	1.00					
CCI	0.04	-0.03	-0.02	-0.21	-0.19	-0.02	-0.56	-0.60	-0.17	0.34	-0.03	-0.67	1.00				
DISP	0.14	-0.16	-0.16	-0.12	-0.16	-0.42	-0.05	-0.37	-0.04	0.09	0.20	0.01	0.12	1.00			
PUI	-0.14	0.18	0.17	0.37	0.44	-0.01	0.51	0.28	0.15	-0.29	0.14	0.42	-0.70	-0.08	1.00		
UMCSI	0.09	-0.15	-0.14	-0.29	-0.28	0.10	-0.62	-0.54	-0.15	0.37	-0.10	-0.46	0.92	0.13	-0.70	1.00	
VRP	-0.03	-0.20	-0.24	-0.36	0.19	0.11	0.04	-0.02	0.24	-0.10	0.00	-0.03	0.05	-0.05	0.09	0.05	1.00

Panel A of Table 2.1 reports descriptive statistics for all the predictor variables. For the option-based ambiguity measures, IA and IA*, first-order autocorrelations are relatively low. For most traditional predictors, except SKEW, first-order autocorrelations are generally very high. This includes one of the most widely tested predictor variables, dividend yield. In the set of alternative ambiguity proxies considered, autocorrelations are also generally high, except for VRP. In light of serious inference issues characterizing highly auto-correlated predictors, adjusted R^2 for regressions involving highly persistent return predictors need to be interpreted carefully. Low autocorrelation in IA and IA^* reduces potential concerns associated with excess return predictions using highly auto-correlated predictor variables. Panel B of Table 2.1 summarizes the correlations among the variables. Correlations between the traditional predictor variables and contemporaneous monthly excess returns are weak, except for IV which is significantly negatively correlated. Our option market ambiguity measures IA and IA^* show insignificant correlation with contemporaneous excess returns (-0.02).

2.5. Predicting Market Returns

The predictive power of various market return predictors is examined with standard long-horizon prediction regressions with various lags according to Eq. (2.10). We find that the predictive power of option-market ambiguity is significant even when controlling for well-established predictor variables and alternative ambiguity proxies. When controlling for ambiguity, implied variance (IV) shows a positive and significant association with future excess returns, validating the risk-return trade-off relation proposed by ICAPM theory (Merton, 1973) but not readily confirmed in low-frequency empirical studies (Bali and Peng, 2006). We show that our uncertainty-related measures, both option ambiguity extracted from VIX and bull-bear sentiment dispersion (DISP), robustly predict future excess market returns with a negative sign. As a second alternative proxy for ambiguity, Vol_{VIX} also shows a negative sign but its predictive ability (consistent with Brenner and Izhakian's (2015)) is weaker than that of IA and DISP. Our findings confirm the predictions of Cao, Wang and Zhang (2005) regarding the determinants of the equity premium under ambiguity.

2.5.1. Ambiguity and Market Return Prediction

Table 2.2 reports our main empirical results on market return predictability for the U.S. Our main measures of ambiguity (IA and IA^*) show strong and consistent predictive power for future aggregate excess returns (on S&P 500) for all prediction horizons considered. The coefficient of IA is consistently negative for all horizons in line with recent studies reporting a negative relationship between proxies of

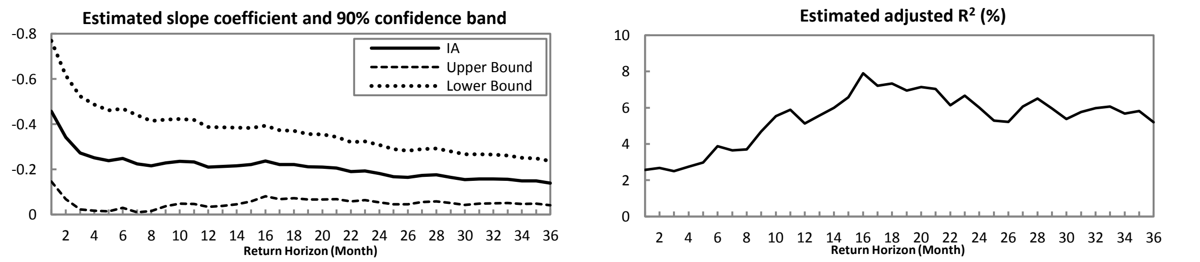
Table 2.2. Predictive Regressions for Option-market Implied Ambiguity

Table 2.2 shows predictive regression results for option market implied ambiguity measures IA and IA* on S&P 500 excess returns. Predictive regressions are specified according to equation (2.10). The sample covers monthly observations from January 1990 to December 2012. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Return Horizon (k)	*Cst*			*IA*			*Adj. R^2 (%)*	*Return Horizon (k)*	*Cst*			*IA**			*Adj. R^2 (%)*
1	13.58	(3.72)	***	-0.46	(-2.41)	**	*2.57*	1	31.37	(3.15)	***	-0.73	(-2.40)	**	*2.61*
6	10.19	(3.45)	***	-0.25	(-1.86)	*	*3.87*	6	20.14	(3.07)	***	-0.41	(-1.95)	*	*4.13*
12	9.77	(3.66)	***	-0.21	(-1.96)	*	*5.14*	12	18.10	(3.50)	***	-0.34	(-2.03)	**	*5.37*
18	9.59	(3.77)	***	-0.22	(-2.44)	**	*7.33*	18	18.71	(4.31)	***	-0.37	(-2.63)	***	*8.00*
24	9.00	(3.65)	***	-0.18	(-2.35)	**	*6.01*	24	16.77	(4.46)	***	-0.31	(-2.62)	***	*7.00*
30	8.54	(3.53)	***	-0.15	(-2.25)	**	*5.38*	30	15.37	(4.47)	***	-0.27	(-2.55)	**	*6.58*
36	8.21	(3.46)	***	-0.14	(-2.34)	**	*5.21*	36	14.39	(4.60)	***	-0.25	(-2.64)	***	*6.41*

Figure 2.2. Estimated Slope Coefficients and Adjusted R^2 of Implied Ambiguity from Predictive Regressions

Figure 2.2 shows the estimated slope coefficient and the adjusted R^2 of implied ambiguity (IA) from the predictive regression specification according to equation (2.10). The figure on the left shows the slope coeffient of IA from the regressions of k-month S&P 500 excess returns. The figure on the right shows the adjusted R^2 values from the corresponding regressions. The sample covers monthly observations from January 1990 to December 2012. The plots for IA* are similar.



ambiguity and the equity premium (Brenner and Izhakian, 2015; Andreou et al., 2015). This is also later evidenced in univariate predictability findings based on the sentiment dispersion (DISP) index (Table 2.3, Panel C). Hodrick (1992) robust t-statistics using one-month prediction horizons for IA and IA* are significant (at -2.41 and -2.40, respectively) and remain significant for longer prediction horizons up to 36 months. Adjusted R^2 for the 36-month horizons ranges from 2.57% to 7.33% for IA and from 2.61% to 8% for IA*. The predictive power provided by IA* is generally slightly superior to that of IA. Our empirical findings in Table 2.2 show a strong and persistent predictive power by option-market implied ambiguity measures in both short and long horizons. Figure 2.2 shows a time-consistent predictive power for our ambiguity measures including slope coefficients, confidence bands and adjusted R^2 for various prediction horizons. Exhibiting a consistently negative coefficient of option-implied ambiguity across all prediction horizons, our findings are in line with recent theoretical ambiguity and asset pricing literatures on the

Table 2.3. Predictive Regressions for Option-market Implied Ambiguity and Other Predictor Variables and Alternative Ambiguity Proxies

Table 2.3 shows predictive regression results for option-market implied ambiguity and other predictor variables on S&P 500 excess returns. IA and IA* measure option market implied ambiguity. CAY is the consumption-wealth ratio. CRE is the credit spread. DY represents the aggregate dividend yield on the S&P500 index. PER is the price/earnings ratio. RREL is the stochastically detrended interest rate. SKEW denotes CBOE's SKEW index. TERM denotes the term spread between 10Y T-bond and 3M T-bill. CCI, DISP, PUI and UMCSI, denoting the Consumer Confidence Index, Dispersion (DISP) between Investors Intelligence Sentiment Indices – Bearish and Bullish, Policy Uncertainty Index, and University of Michigan's Consumer Sentiment Index respectively. VRP is the variance risk premium. Predictive regressions are specified according to equation (2.10). The sample covers monthly observations from January 1990 to December 2012. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Panel A. Option Market Ambiguity

k	Cst		IA		Adj. R^2 (%)
1	13.58	(3.72)***	-0.46	(-2.41)**	2.57
12	9.77	(3.66)***	-0.21	(-1.96)*	5.14
24	9.00	(3.65)***	-0.18	(-2.35)**	6.01
36	8.21	(3.46)***	-0.14	(-2.34)**	5.21

k	Cst		IA*		Adj. R^2 (%)
1	31.37	(3.15)***	-0.73	(-2.40)**	2.61
12	18.10	(3.50)***	-0.34	(-2.03)**	5.37
24	16.77	(4.46)***	-0.31	(-2.62)***	7.00
36	14.39	(4.60)***	-0.25	(-2.64)***	6.41

Panel B. Traditional Predictor Variables

k	Cst		CAY		Adj. R^2 (%)
1	5.47	(1.76)*	3.99	(2.42)**	1.08
12	4.90	(1.62)	5.10	(3.47)***	21.18
24	4.30	(1.50)	5.61	(3.74)***	44.04
36	3.81	(1.40)	5.39	(3.68)***	59.62

k	Cst		CRE		Adj. R^2 (%)
1	16.05	(1.45)	-9.79	(-0.81)	0.27
12	6.42	(0.92)	0.07	(0.01)	-0.38
24	5.13	(0.90)	1.19	(0.22)	-0.25
36	6.00	(1.20)	0.05	(0.01)	-0.42

k	Cst		DY		Adj. R^2 (%)
1	-6.28	(-0.55)	6.11	(1.17)	0.23
12	-11.21	(-1.14)	8.43	(2.06)**	10.13
24	-12.18	(-1.42)	8.75	(2.62)***	19.53
36	-12.45	(-1.60)	8.75	(2.94)***	29.61

k	Cst		PER		Adj. R^2 (%)
1	5.25	(0.84)	0.05	(0.21)	-0.34
12	6.65	(1.62)	-0.01	(-0.04)	-0.38
24	5.25	(1.53)	0.04	(0.34)	-0.18
36	5.88	(2.00)**	0.01	(0.08)	-0.41

k	Cst		RREL		Adj. R^2 (%)
1	8.18	(2.73)***	21.48	(2.27)**	1.52
12	7.89	(2.88)***	17.70	(2.01)**	11.49
24	6.59	(2.49)**	4.12	(0.76)	0.74
36	6.00	(2.30)**	-0.58	(-0.16)	-0.38

k	Cst		SKEW		Adj. R^2 (%)
1	75.40	(1.02)	-0.59	(-0.93)	-0.04
12	-20.01	(-0.42)	0.23	(0.57)	0.05
24	31.87	(1.02)	-0.22	(-0.84)	0.26
36	70.52	(3.03)***	-0.56	(-2.82)***	5.35

k	Cst		TERM		Adj. R^2 (%)
1	10.10	(1.78)*	-1.88	(-0.75)	-0.19
12	3.92	(0.74)	1.36	(0.60)	0.47
24	-1.19	(-0.22)	4.01	(2.05)**	12.23
36	-2.00	(-0.39)	4.48	(2.50)**	22.08

Panel C. Alternative Ambiguity Proxies

k	Cst		CCI		Adj. R^2 (%)
1	8.28	(0.64)	-0.02	(-0.14)	-0.35
12	11.14	(1.05)	-0.05	(-0.44)	0.30
24	18.14	(2.09)**	-0.13	(-1.30)	6.57
36	21.06	(2.83)***	-0.16	(-1.81)*	14.22

k	Cst		DISP		Adj. R^2 (%)
1	15.58	(2.34)**	-0.51	(-1.75)*	0.66
12	10.73	(2.62)***	-0.24	(-1.75)*	1.71
24	12.65	(3.88)***	-0.37	(-3.58)***	7.74
36	12.12	(3.90)***	-0.35	(-3.94)***	10.28

k	Cst		PUI		Adj. R^2 (%)
1	8.53	(0.74)	-0.02	(-0.16)	-0.35
12	4.73	(0.64)	0.02	(0.26)	-0.28
24	0.30	(0.04)	0.06	(1.04)	1.28
36	-1.41	(-0.21)	0.08	(1.51)	3.02

k	Cst		UMCSI		Adj. R^2 (%)
1	-5.50	(-0.21)	0.14	(0.47)	-0.24
12	2.28	(0.10)	0.05	(0.19)	-0.24
24	15.92	(0.88)	-0.11	(-0.53)	0.72
36	20.01	(1.28)	-0.16	(-0.86)	2.79

k	Cst		VRP		Adj. R^2 (%)
1	-4.81	(-0.92)	0.62	(2.48)**	5.39
12	3.50	(1.12)	0.16	(2.18)**	3.23
24	4.37	(1.50)	0.10	(1.80)*	1.95
36	5.41	(1.92)*	0.03	(0.71)	-0.01

negative association between ambiguity and the equity risk premium. These findings corroborate CWZ's prediction (P1) that ambiguity is important in asset pricing.

Table 2.3 presents comparative univariate regression results based on seven traditional market predictor variables and five alternative ambiguity proxies using monthly prediction horizons for up to 3 years. For easier comparison, we pool the predictor variables according to their category (option market ambiguity in Panel A, traditional predictors in Panel B, and alternative proxies for economic uncertainty or ambiguity in Panel C). Among the known market predictor variables, consumption wealth ratio CAY[15] has the strongest predictive power for future market excess returns in both short and long horizons. Aggregate dividend yield also predicts market excess returns quite well, consistent with earlier findings by Hodrick (1992) and Cochrane (2007). The stochastically-detrended interest rate (RREL) only predicts at short horizons, while SKEW and TERM predict only at long horizons. With regard to alternative ambiguity proxies, DISP predicts returns fairly well in line with Eq. (2.9). VRP predicts short and medium term returns only. Overall, option market ambiguity (IA and IA*) and DISP along with CAY are the only predictors that predict well over the entire range of return horizons considered. Table 2.3 confirms that, overall, six other predictor variables also harbor future market excess returns prediction information, besides our option ambiguity measures.

To investigate further whether our option market ambiguity measures contain different or incremental predictive power (beyond the six variables that also capture predictive excess return information), we examine joint predictions using bivariate and multivariate predictive regressions. Results are reported in Table 2.4. Due to similarities with IA* findings, we report only the more conservative IA results (IA* results are available from the authors). Panels A and B of Table 2.4 report bivariate regression results involving our option-market implied ambiguity (IA) measure jointly with each of the other known predictor variables and alternative ambiguity proxies, respectively. In Panel A, IA remains robust in predicting future market excess returns when each of the other established predictor variables is also

[15] Despite the strong in-sample predictive power of CAY, concerns have been raised in the literature regarding the use of this ratio as a predictor out of sample. Since CAY is constructed using ex post estimation regression coefficients, Welch and Goyal (2008) question the suitability of CAY as a return predictor as investors cannot estimate this ratio in real time without knowledge of ex post information. Campbell and Thompson (2008) also raise concerns about revision of data definitions for several variables used in the construction of CAY by the Bureau of Economic Analysis in 2003, making the definition of the variable itself uncertain. We do not deal with these issues here, using CAY merely as a control.

Table 2.4. Bivariate Predictive Regressions for Option-market Implied Ambiguity and Other Predictor Variables

Table 2.4 shows bivariate and multivariate predictive regression results for option-implied market ambiguity IA, other predictor variables and alternative ambiguity proxies on S&P 500 excess returns. IA and IA* measure option market implied ambiguity. CAY is the consumption-wealth ratio. CRE is the credit spread. DY represents the aggregate dividend yield on the S&P500 index. PER is the price/earnings ratio. RREL is the stochastically detrended interest rate. SKEW denotes CBOE's SKEW index. TERM denotes the term spread between 10Y T-bond and 3M T-bill. CCI, DISP, PUI and UMCSI, denoting the Consumer Confidence Index, Dispersion (DISP) between Investors Intelligence Sentiment Indices – Bearish and Bullish, Policy Uncertainty Index, and University of Michigan's Consumer Sentiment Index respectively. VRP is the variance risk premium. Predictive regressions are specified according to equation (2.10). Panel A shows bivariate regressions between IA and other known predictor variables, and panel C between IA and alternative ambiguity proxies. Panel C contains multivariate results among IA, CAY and either sentiment indexes (IBEAR/IBULL) or VRP. The sample covers monthly observations from January 1990 to December 2012. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Panel A. Traditional Predictor Variables

k	Cst		IA		CAY		Adj. R^2 (%)
1	12.35	(3.36) ***	-0.44	(-2.33) **	3.75	(2.26) **	3.48
12	7.88	(2.94) ***	-0.19	(-1.78) *	4.98	(3.43) ***	25.30
24	6.61	(2.64) ***	-0.15	(-2.02) **	5.49	(3.71) ***	48.19
36	5.41	(2.24) **	-0.10	(-1.78) *	5.29	(3.65) ***	62.35

k	Cst		IA		CRE		Adj. R^2 (%)
1	16.99	(1.53)	-0.43	(-2.30) **	-3.92	(-0.32)	2.30
12	6.91	(0.99)	-0.23	(-2.49) **	3.36	(0.48)	5.39
24	5.63	(0.99)	-0.21	(-3.01) ***	3.98	(0.74)	7.13
36	6.40	(1.29)	-0.16	(-2.83) ***	2.17	(0.49)	5.48

k	Cst		IA		DY		Adj. R^2 (%)
1	3.45	(0.28)	-0.44	(-2.28) **	4.70	(0.89)	2.56
12	-7.17	(-0.74)	-0.18	(-1.68) *	7.84	(1.91) *	13.79
24	-8.98	(-1.09)	-0.14	(-1.93) *	8.26	(2.51) **	23.19
36	-10.18	(-1.38)	-0.10	(-1.78) *	8.39	(2.88) ***	32.09

k	Cst		IA		PER		Adj. R^2 (%)
1	10.38	(1.62)	-0.47	(-2.49) **	0.14	(0.55)	2.38
12	9.04	(2.19) **	-0.21	(-2.05) **	0.03	(0.21)	4.86
24	7.17	(2.06) **	-0.19	(-2.64) ***	0.08	(0.71)	6.49
36	7.39	(2.48) **	-0.14	(-2.59) ***	0.03	(0.42)	5.06

k	Cst		IA		RREL		Adj. R^2 (%)
1	14.34	(3.95) ***	-0.42	(-2.22) **	18.31	(1.97) **	3.56
12	10.50	(3.95) ***	-0.17	(-1.76) *	16.42	(1.94) *	14.86
24	9.11	(3.69) ***	-0.17	(-2.46) **	2.80	(0.55)	6.15
36	8.13	(3.38) ***	-0.14	(-2.53) **	-1.64	(-0.49)	5.07

k	Cst		IA		SKEW		Adj. R^2 (%)
1	95.51	(1.31)	-0.47	(-2.49) **	-0.70	(-1.12)	2.67
12	-12.22	(-0.27)	-0.21	(-1.99) **	0.19	(0.48)	5.07
24	39.00	(1.27)	-0.18	(-2.41) **	-0.26	(-0.99)	6.53
36	75.30	(3.17) ***	-0.14	(-2.40) **	-0.58	(-2.89) ***	11.06

k	Cst		IA		TERM		Adj. R^2 (%)
1	18.53	(2.77) ***	-0.47	(-2.43) **	-2.53	(-0.98)	2.52
12	7.65	(1.46)	-0.20	(-1.90) *	1.08	(0.48)	5.30
24	1.64	(0.33)	-0.16	(-2.12) **	3.80	(1.95) *	16.89
36	0.11	(0.02)	-0.12	(-2.08) **	4.34	(2.44) **	25.95

Panel B. Alternative Ambiguity Proxies

k	Cst		IA		CCI		Adj. R^2 (%)
1	16.07	(1.29)	-0.46	(-2.43) **	-0.03	(-0.20)	2.23
12	14.97	(1.44)	-0.21	(-2.00) **	-0.06	(-0.49)	5.60
24	20.82	(2.40) **	-0.18	(-2.33) **	-0.13	(-1.29)	12.57
36	23.54	(3.14) ***	-0.14	(-2.42) **	-0.16	(-1.84) *	19.96

k	Cst		IA		DISP		Adj. R^2 (%)
1	26.27	(3.71) ***	-0.67	(-2.29) **	-0.51	(-2.70) ***	3.92
12	15.65	(4.20) ***	-0.31	(-2.30) **	-0.24	(-2.21) **	8.18
24	16.71	(4.93) ***	-0.42	(-3.69) ***	-0.21	(-2.59) ***	15.98
36	15.26	(4.62) ***	-0.39	(-3.84) ***	-0.16	(-2.53) **	17.61

k	Cst		IA		PUI		Adj. R^2 (%)
1	10.70	(0.92)	-0.47	(-2.54) **	0.03	(0.26)	2.24
12	5.33	(0.73)	-0.23	(-2.10) **	0.05	(0.69)	5.49
24	1.11	(0.16)	-0.20	(-2.53) **	0.08	(1.40)	8.76
36	-1.47	(-0.22)	-0.17	(-2.78) ***	0.10	(2.01) **	10.94

k	Cst		IA		UMCSI		Adj. R^2 (%)
1	10.39	(0.40)	-0.45	(-2.43) **	0.04	(0.12)	2.22
12	10.02	(0.45)	-0.21	(-2.04) **	0.00	(-0.01)	4.78
24	22.05	(1.21)	-0.19	(-2.52) **	-0.15	(-0.70)	7.59
36	25.91	(1.62)	-0.16	(-2.62) ***	-0.20	(-1.05)	9.66

k	Cst		IA		VRP		Adj. R^2 (%)
1	1.65	(0.31)	-0.34	(-1.85) *	0.55	(2.23) **	6.61
12	7.02	(2.76) ***	-0.18	(-1.73) *	0.12	(1.78) *	6.88
24	7.41	(2.92) ***	-0.17	(-2.16) **	0.07	(1.32)	6.83
36	7.92	(3.03) ***	-0.14	(-2.24) **	0.01	(0.27)	4.87

Panel C. Multivariate Predictive Regressions

k	Cst		IA		CAY		DISP		Adj. R^2 (%)
1	22.60	(2.84) ***	-0.49	(-2.54) **	2.30	(1.22)	-0.51	(-1.56)	3.96
12	7.34	(2.03) **	-0.19	(-1.83) *	5.06	(3.32) ***	0.03	(0.22)	25.03
24	7.92	(2.49) **	-0.16	(-2.11) **	5.31	(3.48) ***	-0.07	(-0.79)	48.20
36	6.23	(1.91) *	-0.10	(-1.86) *	5.17	(3.41) ***	-0.04	(-0.54)	62.32

k	Cst		IA		CAY		VRP		Adj. R^2 (%)
1	1.21	(0.23)	-0.34	(-1.82) *	3.04	(1.79) *	0.52	(2.10) **	7.10
12	6.02	(2.36) **	-0.17	(-1.62)	4.87	(3.33) ***	0.09	(1.22)	26.01
24	5.84	(2.28) **	-0.14	(-1.88) *	5.45	(3.68) ***	0.04	(0.66)	48.28
36	5.99	(2.27) **	-0.11	(-1.77) *	5.33	(3.68) ***	-0.03	(-0.57)	62.46

included. Given the strong predictive power of CAY reported in Table 2.3, the robust joint predictive power of IA and CAY in the bivariate setup suggests IA contains meaningful incremental information beyond that of CAY. As further confirmed in Appendix Table A2.3, IA is the only predictor variable which remains significant for all horizons in bivariate regressions along with CAY. In Table 2.4 Panel

B, IA strongly and robustly predicts future market excess returns in all regressions when other ambiguity proxies are controlled for. The dispersion between bullish and bearish sentiments (DISP) motivated by our Eq. (2.9) is the only other ambiguity variable which remains significant in predicting future market excess returns in all horizons considered, suggesting that IA and DISP contain complementary ambiguity information. In both Panels A and B, the sign of IA is consistently negative implying that the negative relationship between IA and future market excess returns holds when other predictors are controlled for. This provides confirmation to CWZ's second prediction (P2) regarding a negative relation between ambiguity and ex post equity premium. In Panel C, we specify the first two sets of multivariate regressions based on those robust predictor variables which predict excess returns for all horizons. In the presence of IA and CAY, the predictive power of DISP disappears suggesting that the information content of IBEAR and IBULL divergence is likely to be subsumed in CAY and IA. By contrast, IA remains robust in the first two sets of regressions in Panel C. The predictive power and statistical significance of IA is maintained when included along DISP (with CAY). Given evidence that VRP might also partly capture investors' attitudes toward ambiguity, we further specify multivariate regressions involving IA, CAY and VRP. From the third set of regressions in Panel C, IA is significant in almost all horizons. The predictive power of VRP is also weakened and becomes insignificant at the 12-month horizon. In the medium term, information in IA and VRP seems partly absorbed by CAY (VRP is insignificant but IA remains significant).[16] The above suggests that CAY, though not a normative predictor, may also capture a portion of ambiguity information. Overall, the above results provide strong evidence that option market ambiguity contains incremental excess return information beyond that of other predictor variables and alternative ambiguity or economic uncertainty proxies.

[16] This finding can be explained by, and is consistent with, the fact that the ambiguity-related information of VRP is in line with a multiple-priors explanation (i.e., uncertainty in the drift only) while the information in IA (or IA*) follows the more general Choquet and rank-dependent utility specification (accommodating uncertainty or multiple scenarios in both drift and volatility).

Table 2.5. Robustness Results for Option-market Implied Ambiguity with Alternative Specifications for σ and μ

Table 2.5 presents predictive regression robustness results for market ambiguity IA estimated with different inputs for σ and μ. Volatility σ is estimated in three different ways: (i) from RiskMetrics EWMA, (ii) from out-of-sample GARCH(1,1) with 3-year rolling estimation window, and (iii) from 1-month (22-trading day) historical volatility. Investor's subjective required rate of return (μ) is estimated from historical returns over windows ranging from 12 to 36 months, as well as using the 3M risk-free rate. The sample covers monthly observations from January 2000 to December 2012. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

σ: RiskMetrics EWMA; μ: 12m return				***σ: GARCH(1,1); μ: 12m return***				***σ: HV 22-day; μ: 12m return***			
k	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*
1	13.58 (3.72)***	-0.46 (-2.41)**	*2.57*	1	13.89 (3.51)***	-0.41 (-2.43)**	*2.16*	1	11.38 (3.07)***	-0.27 (-1.58)	*0.88*
12	9.77 (3.66)***	-0.21 (-1.96)*	*5.14*	12	9.97 (3.50)***	-0.19 (-2.19)**	*4.62*	12	10.02 (3.79)***	-0.20 (-2.12)**	*5.42*
24	9.00 (3.65)***	-0.18 (-2.35)**	*6.01*	24	9.28 (3.54)***	-0.17 (-2.58)***	*5.92*	24	8.92 (3.54)***	-0.15 (-2.26)**	*5.17*
36	8.21 (3.46)***	-0.14 (-2.34)**	*5.21*	36	8.20 (3.30)***	-0.12 (-2.24)**	*4.06*	36	8.06 (3.30)***	-0.11 (-2.16)**	*4.12*

σ: RiskMetrics EWMA; μ: 18m return				***σ: GARCH(1,1); μ: 18m return***				***σ: HV 22-day; μ: 18m return***			
k	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*
1	13.46 (3.75)***	-0.44 (-2.33)**	*2.60*	1	13.84 (3.62)***	-0.41 (-2.41)**	*2.32*	1	10.50 (2.92)***	-0.23 (-1.32)	*0.55*
12	9.90 (3.75)***	-0.21 (-2.05)**	*5.83*	12	9.75 (3.49)***	-0.18 (-2.16)**	*4.33*	12	9.40 (3.58)***	-0.17 (-1.98)**	*3.99*
24	9.07 (3.71)***	-0.18 (-2.32)**	*6.75*	24	9.26 (3.61)***	-0.17 (-2.60)***	*6.33*	24	8.56 (3.42)***	-0.14 (-2.14)**	*4.29*
36	8.23 (3.45)***	-0.14 (-2.21)**	*5.64*	36	8.23 (3.33)***	-0.12 (-2.26)**	*4.62*	36	7.95 (3.25)***	-0.11 (-2.15)**	*4.11*

σ: RiskMetrics EWMA; μ: 24m return				***σ: GARCH(1,1); μ: 24m return***				***σ: HV 22-day; μ: 24m return***			
k	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*
1	12.70 (3.59)***	-0.40 (-2.14)**	*2.22*	1	13.23 (3.47)***	-0.38 (-2.21)**	*2.04*	1	10.02 (2.78)***	-0.20 (-1.16)	*0.37*
12	8.88 (3.35)***	-0.15 (-1.51)	*2.99*	12	9.11 (3.26)***	-0.15 (-1.76)*	*2.89*	12	8.92 (3.40)***	-0.14 (-1.66)*	*2.78*
24	8.52 (3.50)***	-0.15 (-2.00)**	*4.76*	24	8.85 (3.50)***	-0.15 (-2.30)**	*5.10*	24	8.29 (3.32)***	-0.12 (-1.92)*	*3.40*
36	7.88 (3.33)***	-0.12 (-1.98)**	*4.39*	36	8.02 (3.30)***	-0.11 (-2.09)**	*4.12*	36	7.76 (3.19)***	-0.10 (-1.97)**	*3.49*

σ: RiskMetrics EWMA; μ: 36m return				***σ: GARCH(1,1); μ: 36m return***				***σ: HV 22-day; μ: 36m return***			
k	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*	*k*	*Cst*	*IA*	*Adj. R^2 (%)*
1	11.93 (3.42)***	-0.36 (-1.92)*	*1.85*	1	13.12 (3.49)***	-0.39 (-2.28)**	*2.30*	1	10.72 (3.01)***	-0.25 (-1.49)	*0.87*
12	8.50 (3.11)***	-0.13 (-1.38)	*2.32*	12	8.65 (3.03)***	-0.13 (-1.63)	*2.19*	12	8.75 (3.27)***	-0.13 (-1.83)*	*2.82*
24	8.22 (3.32)***	-0.13 (-1.98)**	*3.97*	24	8.47 (3.28)***	-0.13 (-2.36)**	*4.20*	24	8.26 (3.27)***	-0.12 (-2.35)**	*3.97*
36	7.76 (3.26)***	-0.11 (-2.17)**	*4.35*	36	7.84 (3.19)***	-0.11 (-2.29)**	*3.91*	36	7.80 (3.18)***	-0.11 (-2.54)**	*4.49*

We conduct a variety of robustness tests and provide additional results confirming the predictive power of market ambiguity and its sensitivity to alternative input specifications. First, we examine robustness of the predictive power of IA to different volatility and alternative subjective investor required return (μ) estimations. IA is alternatively inferred using different volatility estimation methods, including the RiskMetrics EWMA, out-of-sample GARCH(1,1) with 3-year rolling estimation window, and using simple historical volatility measured as the sample standard deviation of daily returns over 22 trading days (one month). We further use different historical return estimation horizons as proxies for the investors' subjective required return (μ), namely 12-, 18-, 24-, and 36-month horizons. Related robustness results are reported in Table 2.5. Table 2.5 confirms that the predictive power of IA is generally preserved, particularly for longer horizons. Among the alternative volatility specifications considered, IA inferred from out-of-sample GARCH(1,1) seems to provide a comparable (or relatively better) predictive power as RiskMetrics EWMA, while a simple 22-day standard deviation of returns gives a slightly weaker predictive power. This confirms that accounting for the autoregressive nature of volatility is important in estimating IA. Concerning the subjective rate of return (μ) estimations, the use of 12- and 18-month return windows provides the best predictive power.[17] This is in line with the findings of Barberis, Huang and Santos (2001) that investors have a short memory when determining required returns. We have also controlled for relative risk aversion measured using consumption- and CAPM-based approaches and find the strong predictive power of IA still holds.

Since option market ambiguity may also contain aggregate economic uncertainty information, we compare the correlations between our IA measure and many established indicators of economic uncertainty. We consider the conditional variance of the Chicago Fed National Activity Index (CV_{CFNAI}) and of industrial production growth (CV_{IP}) estimated using GARCH (1,1) models (Bollerslev, 1986); the macroeconomic uncertainty index ($MUNC_{BBC}$) of Bali, Brown, and Caglayan (2014) based on principal component analysis (PCA); and the macroeconomic uncertainty measure ($MUNC_{JLN}^{1M}$ etc.) of Jurado, Ludvigson and Ng (2015) measured by a weighted conditional variance of financial and macroeconomic series with 1-, 3-, and 12-month forecasting horizons. We also examine survey-/media-based measures such as disagreement among economic forecasters from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF_{CQ}) with different forecasting horizons. IA shows significant correlations with each of the above uncertainty proxies, with the highest correlations with $MUNC_{JLN}^{12M}$ (ρ=0.405) and SPF_{4Q} (ρ=0.356). The correlation between IA and the first principal component formed by $MUNC_{JLN}^{12M}$ and SPF_{4Q} is higher (at 0.49). While the sign of the association between our ambiguity measures and excess returns is in line with CWZ, we sought to find further confirmatory empirical evidence for robustness. We use $MUNC_{JLN}^{12M}$, which measures dispersion in forecasting errors, as a predictor of ex post excess returns and find negative insignificant coefficients under the Hodrick (1992) standard error adjustment.

Regarding our extraction of option-implied ambiguity from VIX for our main IA measure, we perform robustness on other needed inputs, including the risk-free interest rate (r_f). For further

[17] Although not in line with Barberis, Huang and Santos (2001) short memory principle, we also used 60-month return windows to estimate the subjective rate of return for further robustness. Prediction results are qualitatively similar to those using 24- and 36-month horizons. The predictive power of IA is robust to alternative inputs of subjective rate of return.

validation we repeat the analysis in Table 2.2 adding direct controls for μ and r_f (one at a time as reported in Supplementary Appendix Table SA2.1). IA and IA* are significant in all horizons considered. The coefficients of IA and IA* are stable in line with the univariate results. Robust t-statistics generally improve when μ is directly controlled for. When r_f is also controlled, IA and IA* are again negative and significant in all horizons. The robust t-statistics of option market ambiguity are generally improved. This suggests the information content of our ambiguity measures is not directly attributed to μ and r_f.

To confirm that our results are not driven by the way we extract option-implied ambiguity from the VIX, we alternatively extract divergence in beliefs and option market ambiguity information from traded S&P 500 option prices with short average maturities of one month for the same sample period based on our A-OPM of Eq. (2.4). Despite lower data quality compared to that contained in CBOE's VIX, the predictive ability of IA extracted from traded call and put option prices on the S&P 500 index largely holds, more so when extracted from puts. In a robustness test that captures differences in ambiguity beliefs among representative ambiguity-averse and ambiguity-seeking investors without relying on option data or our A-OPM, the level of dispersion between bullish and bearish sentiment (DISP) predicts future excess returns for 34 out of 36 monthly horizons with a negative coefficient in our univariate settings (unreported).[18] This confirms in a robust way our finding that a negative association exists between measures of economic uncertainty based on divergence in heterogeneous ambiguity beliefs and ex post excess returns. The robust power of IA in predicting market excess returns and the equity premium holds regardless of the method we use to extract our ambiguity measure (whether from VIX, from traded SPX option prices or from DISP). Table A2.2 presents our additional results for the price-based IA predictors.

2.5.2. Ambiguity and the Risk-Uncertainty vs. Return Trade-off

To further examine the role and importance of market ambiguity in asset pricing, we next consider the information content of our IA measures by examining the risk-uncertainty-return trade-off through

[18] This predictive ability no longer holds, however, when CAY is controlled for.

multiple regression setups. Table 2.6 presents our predictive excess market return regression results for k = 6 to 18 months ahead. Panel A reports univariate regression results with implied variance (IV or VIX^2) as proxy for risk as the sole regressor. Risk as proxied by implied variance is not significantly related to future excess returns, in line with most empirical literature failing to document a significant positive risk-return trade-off in low-frequency data. When divergence in ambiguity beliefs is controlled for (adding IA in the bivariate regressions) in Panel B, the significance of IV generally improves (becoming significant at most intermediate horizons from 9 to 16 months). The coefficient of IA is stronger and significantly negative for all horizons in the presence of IV. Adjusted R^2 for the bivariate regressions is larger than (the sum of) adjusted R^2s for the separate univariate regressions. While IV does not exhibit market excess return predictability on its own, when including ambiguity it shows significant positive association with future excess market returns. This association is positive but not significant if we replace IA with DISP, confirming that IA captures more accurate and forward-looking ambiguity information.

As Merton (1973) assumes a market in which investors can borrow and lend at a common interest rate, we also consider the risk-uncertainty-return relation when the borrowing rate proxied by one-month USD Libor is controlled for. When controlling for the interest rate in Panel D, the risk-uncertainty-return trade-off of Eq. (2.2) holds concretely: the risk measure (IV) is significantly positively related to future market excess returns over most horizons (from 8 to 17 months) while the ambiguity measure (IA) is negative and significant in all horizons. The above confirms CWZ’s second prediction (P2) that risk is positively related, and ambiguity is negatively related, to future market excess return. To put our findings in broader perspective, note that option-implied ambiguity alone is significant in predicting market excess returns (Table 2.2) but implied variance (proxying for risk) in itself is not (Panel A of Table 2.6). The robust t-statistics for IA are higher than those for IV for all horizons. This corroborates CWZ’s third prediction (P3), that ambiguity dominates risk in the determination of the equity premium.

Additionally, Cao, Wang and Zhang (2005) suggest that when the level of ambiguity dispersion in the market increases, investors with high ambiguity aversion leave the market while those with low ambiguity aversion stay. The latter participants that dominate the market in high ambiguity regimes

Table 2.6. Risk-Uncertainty vs. Return Trade-off

Table 2.6 shows predictive regression results using implied variance (IV) as measure for risk, option-market implied ambiguity IA, and Libor as control for predicting S&P 500 excess returns. Predictive regressions are specified according to equation (2.10). The above are meant to test theoretical predictions by Merton's (1993) ICAPM and limited market participation theories whereby risk (IV) has a positive sign while ambiguity (IA) a negative association with retruns. Results in Panel D are obtained according to Eq. 2.11. The sample covers monthly observations from January 1990 to December 2012. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Panel A. Risk-return Tradeoff

k	*Cst*	*IV*	*Adj. R^2 (%)*
6	3.19 (0.79)	0.08 (0.77)	*1.03*
7	3.23 (0.85)	0.08 (0.83)	*1.18*
8	3.50 (1.00)	0.07 (0.83)	*1.11*
9	3.74 (1.12)	0.07 (0.84)	*1.06*
10	4.12 (1.29)	0.06 (0.78)	*0.84*
11	4.40 (1.43)	0.05 (0.75)	*0.68*
12	4.50 (1.50)	0.05 (0.75)	*0.66*
13	4.43 (1.50)	0.05 (0.81)	*0.79*
14	4.57 (1.59)	0.05 (0.77)	*0.65*
15	4.85 (1.71) *	0.04 (0.66)	*0.37*
16	4.92 (1.76) *	0.04 (0.64)	*0.32*
17	5.08 (1.83) *	0.03 (0.56)	*0.13*
18	5.23 (1.89) *	0.03 (0.49)	*0.02*

Panel B. Uncertainty-Risk vs Return Tradeoff

k	*Cst*	*IV*	*IA*	*Adj. R^2 (%)*
6	5.93 (1.42)	0.14 (1.41)	-0.33 (-2.83) ***	*7.48*
7	5.75 (1.46)	0.13 (1.49)	-0.31 (-2.62) ***	*7.46*
8	5.84 (1.60)	0.13 (1.56)	-0.29 (-2.76) ***	*7.52*
9	6.18 (1.79) *	0.12 (1.65) *	-0.31 (-2.97) ***	*8.77*
10	6.60 (2.01) **	0.11 (1.67) *	-0.31 (-3.04) ***	*9.47*
11	6.83 (2.16) **	0.11 (1.67) *	-0.30 (-2.94) ***	*9.59*
12	6.74 (2.19) **	0.10 (1.64)	-0.27 (-2.83) ***	*8.57*
13	6.71 (2.23) **	0.10 (1.73) *	-0.28 (-2.85) ***	*9.39*
14	6.82 (2.33) **	0.10 (1.72) *	-0.28 (-2.93) ***	*9.68*
15	7.09 (2.47) **	0.09 (1.62)	-0.28 (-3.07) ***	*9.80*
16	7.29 (2.59) ***	0.09 (1.65) *	-0.29 (-3.36) ***	*11.27*
17	7.21 (2.59) ***	0.08 (1.59)	-0.28 (-3.22) ***	*10.16*
18	7.34 (2.66) ***	0.08 (1.48)	-0.27 (-3.24) ***	*9.91*

Panel C. Uncertainty-Risk vs Return Tradeoff with DISP

k	*Cst*	*IV*	*DISP*	*Adj. R^2 (%)*
6	8.01 (1.52)	0.07 (0.66)	-0.25 (-1.48)	*1.82*
7	8.30 (1.65) *	0.06 (0.70)	-0.26 (-1.59)	*2.30*
8	8.72 (1.87) *	0.06 (0.69)	-0.27 (-1.71) *	*2.53*
9	8.50 (1.92) *	0.05 (0.70)	-0.25 (-1.62)	*2.34*
10	8.41 (2.00) **	0.05 (0.65)	-0.22 (-1.53)	*1.94*
11	8.49 (2.09) **	0.04 (0.62)	-0.21 (-1.51)	*1.76*
12	8.77 (2.21) **	0.04 (0.61)	-0.22 (-1.64)	*2.00*
13	8.64 (2.21) **	0.04 (0.66)	-0.22 (-1.69) *	*2.19*
14	9.00 (2.32) **	0.04 (0.60)	-0.23 (-1.82) *	*2.36*
15	9.86 (2.58) ***	0.03 (0.46)	-0.26 (-2.14) **	*2.84*
16	9.83 (2.60) ***	0.03 (0.44)	-0.25 (-2.18) **	*2.82*
17	10.09 (2.67) ***	0.02 (0.36)	-0.26 (-2.27) **	*2.92*
18	10.79 (2.85) ***	0.01 (0.27)	-0.29 (-2.54) **	*3.73*

Panel D. Uncertainty-Risk vs Return Tradeoff - Controlling for Borrowing/Lending Rate Libor

k	*Cst*	*IV*	*IA*	*r_f*	*Adj. R^2 (%)*
6	2.65 (0.42)	0.15 (1.51)	-0.34 (-2.83) ***	0.77 (0.59)	*7.67*
7	2.62 (0.43)	0.14 (1.60)	-0.31 (-2.62) ***	0.73 (0.56)	*7.66*
8	2.46 (0.43)	0.14 (1.71) *	-0.30 (-2.78) ***	0.79 (0.64)	*7.89*
9	2.83 (0.51)	0.13 (1.82) *	-0.31 (-2.99) ***	0.77 (0.65)	*9.20*
10	3.55 (0.65)	0.12 (1.84) *	-0.31 (-3.05) ***	0.70 (0.59)	*9.81*
11	4.22 (0.79)	0.12 (1.84) *	-0.30 (-2.95) ***	0.60 (0.51)	*9.79*
12	4.50 (0.86)	0.11 (1.80) *	-0.27 (-2.85) ***	0.51 (0.44)	*8.63*
13	4.85 (0.96)	0.11 (1.88) *	-0.28 (-2.86) ***	0.42 (0.37)	*9.34*
14	5.40 (1.10)	0.10 (1.85) *	-0.28 (-2.94) ***	0.32 (0.29)	*9.52*
15	6.15 (1.29)	0.09 (1.73) *	-0.28 (-3.09) ***	0.21 (0.20)	*9.53*
16	6.63 (1.42)	0.09 (1.75) *	-0.30 (-3.38) ***	0.15 (0.14)	*10.97*
17	7.05 (1.55)	0.08 (1.66) *	-0.28 (-3.23) ***	0.04 (0.04)	*9.82*
18	7.42 (1.68) *	0.08 (1.53)	-0.27 (-3.25) ***	-0.02 (-0.02)	*9.56*

Panel E. Price of Risk in High and Low Ambiguity Regimes

k	*Cst*	$IV \times (1 - D_{IA>\overline{IA}})$	$IV \times D_{IA>\overline{IA}}$	*IA*	*r_f*	*Adj. R^2 (%)*
6	0.62 (0.11)	0.21 (2.52) **	0.10 (0.76)	-0.24 (-1.59)	0.74 (0.57)	*8.00*
7	1.36 (0.24)	0.18 (2.33) **	0.11 (0.94)	-0.25 (-1.74) *	0.71 (0.55)	*7.62*
8	2.18 (0.40)	0.14 (2.01) **	0.13 (1.20)	-0.28 (-2.22) **	0.78 (0.64)	*7.56*
9	2.59 (0.48)	0.14 (2.05) **	0.13 (1.29)	-0.30 (-2.44) **	0.77 (0.64)	*8.87*
10	3.62 (0.67)	0.12 (1.87) *	0.13 (1.40)	-0.31 (-2.62) ***	0.70 (0.59)	*9.47*
11	4.66 (0.89)	0.10 (1.62)	0.13 (1.53)	-0.32 (-2.73) ***	0.61 (0.52)	*9.50*
12	5.33 (1.03)	0.08 (1.36)	0.13 (1.66) *	-0.32 (-2.82) ***	0.53 (0.46)	*8.50*
13	5.68 (1.12)	0.08 (1.38)	0.13 (1.76) *	-0.32 (-2.83) ***	0.44 (0.39)	*9.22*
14	6.32 (1.28)	0.07 (1.26)	0.12 (1.83) *	-0.33 (-2.98) ***	0.34 (0.31)	*9.48*
15	7.22 (1.50)	0.06 (1.01)	0.12 (1.85) *	-0.33 (-3.19) ***	0.24 (0.22)	*9.63*
16	7.80 (1.65) *	0.06 (0.93)	0.12 (1.96) **	-0.36 (-3.49) ***	0.18 (0.17)	*11.19*
17	8.30 (1.78) *	0.04 (0.75)	0.11 (1.95) *	-0.34 (-3.41) ***	0.08 (0.07)	*10.12*
18	8.73 (1.91) *	0.03 (0.60)	0.11 (1.89) *	-0.34 (-3.46) ***	0.03 (0.03)	*9.96*

require a lower ambiguity premium, so the sign of IA on market returns should be negative. At the same time, limited market participation results in a higher price of risk since less investors remain in the market to share the risk. This suggests that the risk premium (sign of IV) should be higher in higher ambiguity regimes than in lower ambiguity states.[19] To test the above hypothesis and assess empirically how long the gradual market participation shift takes as divergence in ambiguity beliefs rises, we specify the following regressions:

$$r_{t+k} = \alpha + \beta_1(IV \times D_{IA>\overline{IA}}) + \beta_2[IV \times (1 - D_{IA>\overline{IA}})] + \beta_3 IA + \beta_4 Libor + \varepsilon_{t+k} \quad (2.11)$$

where r_{t+k} is the ex post k-month excess return of the S&P 500 index, IV is the implied variance measuring risk, $D_{IA>\overline{IA}}$ is a dummy variable taking the value of 1 when IA is above or equal to its mean and zero otherwise. This setup allows obtaining comparable coefficients from IV under both high and low IA regimes while directly controlling for the level of implied market ambiguity (IA) itself.

Table 2.6 Panel E results corroborate Cao et al's (2005) prediction concerning the price of risk being higher under a high ambiguity regime. The switch adjustment from a relatively low risk premium to a high risk premium, depending on the time it takes for ambiguity averse investors to leave the market and for ambiguity-tolerant investors to dominate, takes about a year. Panel E shows that such a shift tends to happen after 11 months of rising ambiguity. IV is significant (and positively priced) under the low ambiguity regime prior to the 11-month horizon, and in the high ambiguity regime after the 11-month horizon. From a one-year horizon onwards, the price of risk in high ambiguity regimes is higher (ranging from 0.11 to 0.13 with mean of 0.12) than that under low ambiguity (ranging from 0.03-0.08 with mean 0.06) or the unconditional IV in Panel D (ranging from 0.08-0.11 with mean 0.09). This persists beyond the 18-month horizon. This finding confirms Cao et al's (2005) prediction concerning a higher risk premium under high ambiguity. The panel also highlights a gradual shift towards more ambiguity-tolerant investors. Interestingly, the theoretical (ICAPM) risk-return trade-off holds more clearly (significantly) for days with low or no ambiguity. That is, the IV x (1 - $D_{IA>\overline{IA}}$) variable capturing low regimes is more significantly positive in a trivariate specification of Eq. (2.10) without IA. The IV x $D_{IA>\overline{IA}}$ variable capturing high regimes is

[19] Cao et al's (2005) prediction of a higher risk premium when ambiguity is high depends on two factors: 1) the number of investors who participate in the market decreases (i.e., the number of ambiguity averse individuals who leave the market exceeds the number of ambiguity-tolerant investors who enter the market as ambiguity rises); and 2) the time period the process of reduced market participation takes for this gradual shift to materialize as ambiguity rises.

positive and significant only after controlling for IA. For further robustness, we divided our sample into states with above vs. below median IA (instead of mean) as well as into two sub-samples according to (i) NBER recessions and (ii) financial crisis periods based on the crisis classification of Reinhart and Rogoff (2009). Results (unreported) again confirm Cao et al's (2005) conjecture about a significant positive sign for IV and negative one for IA. Further, the coefficient of IV is higher during NBER recessions than non-NBER recession times and it is higher during financial crisis periods than non-crisis times. Finally, we tested the above relationships with IA extracted from traded option prices, with similar results.

While the above findings are in line with CWZ's predictions, it is also important to verify if increased ambiguity is indeed associated with limited stock market participation. Using data from the Survey of Consumer Finances published by the Federal Reserve, we find a significant negative correlation of -0.55 between changes in the proportion of household investment in stocks and the average level of IA over our sample period. This confirms a negative relationship between the level of ambiguity in the market and stock market participation as suggested by CWZ. This is also in line with the recent survey evidence of Dimmock et al. (2016). The above findings represent the first piece of empirical evidence on the intertemporal risk-uncertainty-return relation for low frequency nonparametric data.[20] Bollerslev, Tauchen and Zhou's (2009) attempt to reveal a positive and significant risk-return relation by including VRP was not effective. The risk-uncertainty-return relation revealed above by including IA (rather than VRP) suggests that the ambiguity information contained in IA operates through a different and more effective channel than VRP. This enables our approach to empirically uncover the sought-after risk-return trade-off based on nonparametric and low frequency conditional variance. This calls for further consideration of ambiguity measures in asset pricing models to help better understand the risk-return trade-off puzzle.

[20] As noted, Brenner and Izhakian (2015) provide partial evidence for a risk-uncertainty-return trade-off though not in an intertemporal setting as intended by Merton (1973). Their inclusion of an interaction term is useful but does not separate out ambiguity from risk.

2.5.3. Out-of-sample Prediction

Welch and Goyal (2008) raise the concern that many variables used to predict excess market returns fail to outperform simple historical averages in out-of-sample analysis. We next investigate whether the predictive performance of IA is robust out-of-sample.

2.5.3.1. Econometric Specification

To investigate the out-of-sample prediction performance of each of the predictor variables considered, we divide our sample into two sub-periods: an estimation period and a forecasting period. For robustness, we employ two estimation period specifications, one with a rolling estimation period and another with a recursive one. Under the rolling estimation period, the forecast on day *t* is based on observations from *t-m* to *t-1*, with *m* the number of observations in the estimation period; under recursive estimation, the sample used to estimate the forecast on day *t* is based on observations from *1* to *t-1*. Following Campbell and Thompson (2008), Welch and Goyal (2008), Rapach, Strauss and Zhou (2009), and Li, Ng and Swaminathan (2013), we use the historical average market excess return as the benchmark for comparison. For fairness, we assume the first day of our sample (January 1990) as the first piece of information in investors' information set. The first out-of-sample forecast using predictor variable x_t^i is obtained by:

$$\hat{r}_{t+1}^i = \hat{\alpha}_{m,t}^i + \hat{\beta}_{m,t}^i x_{m,t}^i \tag{2.12}$$

where $\hat{\alpha}_{m,t}^i$ and $\hat{\beta}_{m,t}^i$ are estimated using *m* observations under the fixed estimation period specification, estimated using all observations up to *t*.

2.5.3.2. Prediction Evaluation

To evaluate the performance of out-of-sample predictions by each predictor model, we compute an out-of-sample (*OS*) R^2 statistic (Campbell and Thompson, 2008) that can be compared to the in-sample R^2 statistics:

$$R_{OS}^2 = 1 - \frac{\sum_{t=1}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=1}^{T}(r_t - \bar{r}_t)^2} \tag{2.13}$$

Above r_t is the realized market excess return, $\hat{r}_t$ is the predicted value from the out-of-sample regression, and $\bar{r}_t$ is the historical average excess return. R_{OS}^2 is positive if the mean squared prediction error (MSPE) of forecasts from a predictor variable is smaller than the historical average forecasts. To test whether the advantage in out-of-sample prediction is statistically significant, we follow Clark and West (2007) in using the adjusted squared prediction error statistic (SPE_{adj}), as follows:

$$SPE_{adj} = (r_t - \bar{r}_t)^2 - [(r_t - \hat{r}_t)^2 - (\bar{r}_t - \hat{r}_t)^2] \tag{2.14}$$

The time series of SPE_{adj} for each predictor model is then regressed on a constant and the t-statistics of the constant are obtained from the estimations. We report the p-values of R^2_{OS} from the upper-tail t-statistic. Since the out-of-sample evaluation statistics in (2.13) and (2.14) do not follow specific assumptions, we mainly rely on those in our discussion.

To investigate the out-of-sample implications of each return prediction model for investors' risk-return profile, we next consider the utility gains for a mean-variance investor based on each return prediction model (Campbell and Thompson, 2008; Welch and Goyal, 2008; Rapach, Strauss and Zhou, 2009). An investor's optimal portfolio allocation[21] in the risky asset (the aggregate stock market portfolio) *w* in period *t* based on her forecasts of expected market return and market variance at *t* and her relative risk aversion γ is given by:

$$w_t = \left(\frac{1}{\gamma}\right)\left(\frac{\hat{r}_t^i}{\hat{\sigma}_t^2}\right) \tag{2.15}$$

where $\hat{\sigma}_t^2$ is the variance of market (risky asset) returns. We estimate market variance using the annualized 10-year rolling sum of squared market returns. To maintain comparability with other predictor variables, we do not account for investors' ambiguity attitudes in the optimal asset allocation problem. Our analysis considers the extracted ambiguity as a market timing factor given a constant risk aversion. Similar to Li, Ng and Swaminathan (2013), we report results in Table 2.7 based on γ = 3.[22] The resulting average utility of the representative investor managing an optimal portfolio *p* between the market (with weight *w*) and the risk-free asset is:

$$U_p = \mu_p - 0.5\gamma\sigma_p^2 \tag{2.16}$$

where μ_p and σ_p^2 are the sample mean and variance of portfolio p returns. We compute the investor's utility gain as the difference between utility based on portfolios with each predictor variable and the

[21] Following Campbell and Thompson (2008), we allow the allocation in the risky asset to be between 0 to 150%.

[22] The results are robust to alternative γ values.

historical average. This utility gain represents the certainty-equivalent excess return of a mean-variance investor in using a specific predictor variable (instead of using the historical mean model) in the asset allocation.

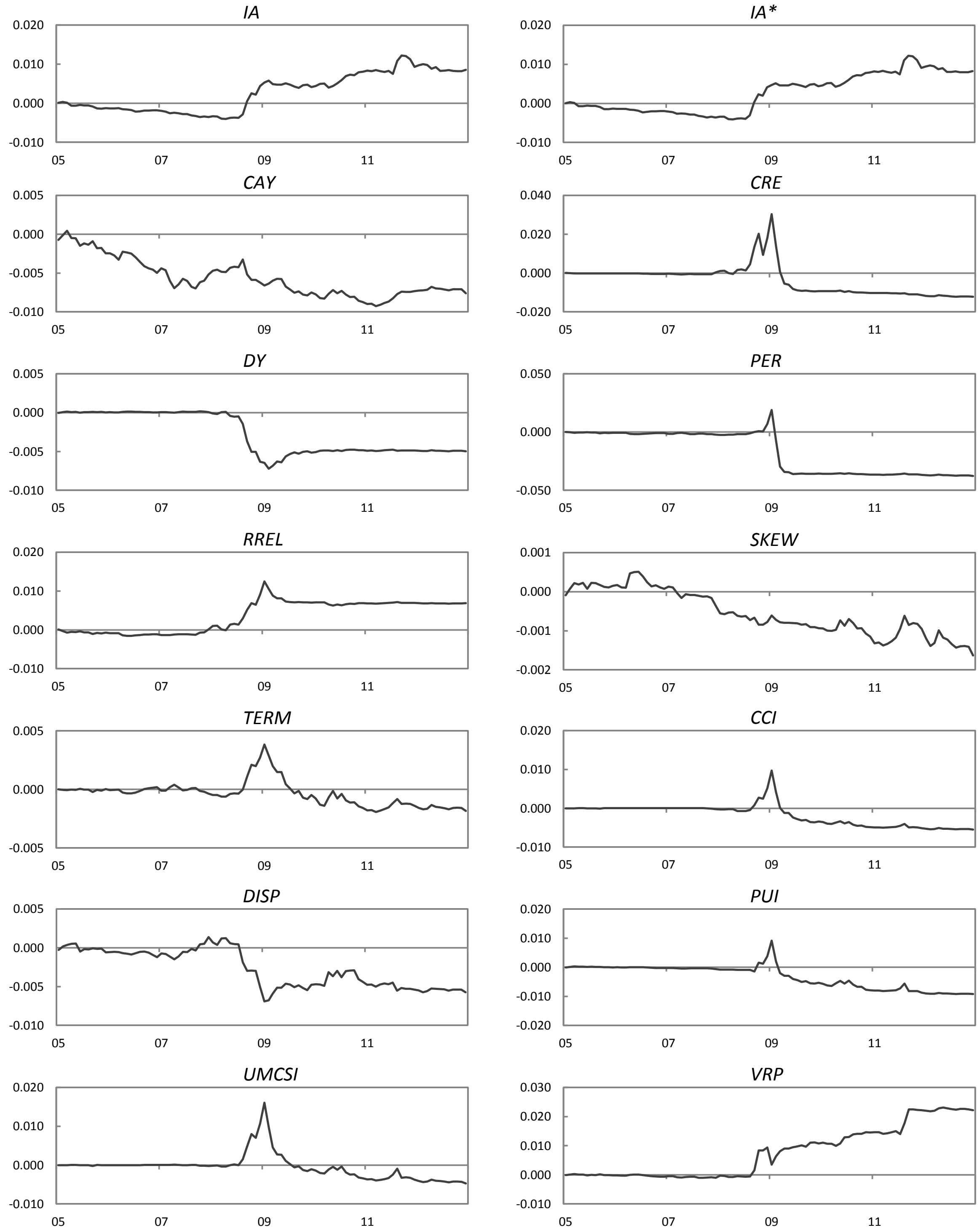
2.5.3.3. Out-of-sample Prediction Results

In producing reliable estimates for out-of-sample analysis, it is important to ensure the estimation period is not too short (e.g., Welch and Goyal, 2008); hence we use an estimation period containing at least 180 monthly observations. For one-month return prediction, the estimation period for the first forecast utilizes data from January 1990 to December 2004. Figure 2.3 shows the time-varying difference between the cumulative squared prediction error for the historical average benchmark and that of each predictor variable. The figure is based on one-month prediction (the forecasting period is January 2005 to December 2012). A reliable predictor variable should also show an upward-sloping plot indicating robust outperformance of the predictor variable compared to the historical average in out-of-sample prediction. Figure 2.3 shows that most standard predictors and ambiguity proxies (namely, DY, PER, CRE, TERM, CCI, DISP, PUI, and UMCSI) collapse during the 2007 to 2009 financial crisis. CAY and SKEW consistently underperform the historical average model. Only IA, VRP and RREL deliver forecasting advantages vs. the historical average.

Out-of-sample prediction results are reported in Table 2.7. Panels A and B report results based on rolling and recursive estimation periods, respectively. Panel A shows that at k = 1 and 6 only implied ambiguity IA (or IA^*), VRP and RREL show significant R^2_{OS}. A portfolio based on optimal allocation in the risky asset (market) according to these predictor variables generates an annualized return of more than 5% for a 1-month holding period (k = 1) and more than 4% for a 6-month holding period (k = 6). For a 12-month horizon (k = 12), IA (or IA*), RREL, VRP, DY and CAY are also significant. However, DY and VRP show negative utility gains. This suggests that while the out-of-sample forecasts of DY and VRP outperform the historical average at the 12-month horizon, they do not necessarily benefit the mean-variance investor. The risk-adjusted returns (given by the Sharpe ratio) generated by an optimal portfolio of DY and VRP are not superior to those of the historical market average. Results based on the recursive estimation period reported in Panel B are similar. Our out-of-sample prediction results are robust to the

Figure 2.3. Out-of-sample Relative Cumulative Squared Prediction Error (vs. Historical Average Benchmark)

Figure 2.3 shows the cumulative squared prediction error for the historical average benchmark model minus the cumulative squared prediction error for models involving each of the following predictors: option market implied ambiguity (IA), consumption-to-wealth ratio (CAY), credit spread (CRE), dividend yield (DY), price-to-earnings ratio (PER), stochastically detrended interest rate (RREL), CBOE's SKEW index (SKEW), term spread (TERM), consumer confidence index (CCI), Economic Policy Uncertainty Index (PUI), Dispersion (DISP) between Investors Intelligence Sentiment Indices – Bullish (IBULL), and Bearish (IBEAR), University of Michigan Consumer Sentiment Index (UMCSI), and variance risk premium (VRP). All y-axes are rescaled to 100 times. The out-of-sample forecasting period covers monthly observations from January 2005 to December 2012.



length of estimation period and parameter instability. The effectiveness of IA in out-of-sample prediction reaffirms its robust predictive ability shown in previous sections.

2.6. International Evidence

Given the strong evidence that ambiguity predicts excess stock market returns across various forecast horizons in the U.S., we next extend our scope globally examining eight other countries with VIX-type indices. Figure 2.4 shows the time-varying divergence in ambiguity beliefs in each of eight additional countries examined revealing differences in investors' perceived ambiguity across countries. Table 2.8 provides a descriptive summary for option-market ambiguity and excess market returns (Panel A), and correlation matrices in Panels B and C. Panel C shows that the average correlation among the excess market returns of Asian countries (i.e., Hong Kong and Japan) with European counterparts is lower than among European countries (0.61 vs. 0.83). The distinctive characteristics of the Asian markets highlight the importance of estimating a specific ambiguity measure per country.

Table 2.9 summarizes the predictive regressions findings for option-implied ambiguity (IA) in the eight countries examined.[23] In Belgium, France, Germany and Netherlands, IA predicts future aggregate market excess returns from 6-month to 36-month horizons. The predictive power of IA is highly significant with robust t-statistics. In the two Asian markets, Hong Kong and Japan, IA is significant for five out of seven horizons considered. In Switzerland and the United Kingdom, IA predicts in four and three out of seven horizons, respectively. Although predictability is not as strong as in the U.S., the predictive power of IA is quite impressive given the small sample size examined, relatively low liquidity and small scale of foreign option markets in comparison to the U.S. Yet the coefficient of IA in each foreign country is consistently negative (in line with CWZ's P2), suggesting that the negative relationship found between implied ambiguity and the equity premium is robust across countries and that ambiguity is priced in equity markets internationally.

[23] Due to similarity, we report the IA^* results in Supplementary Appendix Table SA2.2.

Table 2.7. Out-of-sample Prediction Results using Rolling and Recursive Estimation

Table 2.7 reports out-of-sample prediction results using forecasts from different traditional predictor variables, market ambiguity and alternative ambiguity proxies for k = 1, 6 and 12 months ahead according to equation (2.11). IA and IA* measure option market implied ambiguity. CAY is the consumption-wealth ratio. CRE is the credit spread. DY represents the aggregate dividend yield on the S&P500 index. PER is the price/earnings ratio. RREL is the stochastically detrended interest rate. SKEW denotes CBOE's SKEW index. TERM denotes the term spread between 10Y T-bond and 3M T-bill. CCI, DISP, PUI and UMCSI, denoting the Consumer Confidence Index, Dispersion (DISP) between Investors Intelligence Sentiment Indices – Bearish and Bullish, Policy Uncertainty Index, and University of Michigan's Consumer Sentiment Index respectively. VRP is the variance risk premium. MSPE is the mean squared prediction error, R^2_{OS} is the out-of-sample R^2 statistic of Campbell and Thompson (2008), p-value is based on the adjusted-MSPE statistic of Clark and West (2007); utility gain for a representative mean-variance investor according to equation (2.15) is presented in annualized percentage terms. The historical average is presented on the first row as benchmark. Panel A uses rolling estimation, and Panel B a recursive estimation approach.

Panel A. Out-of-sample Results for Rolling Estimation

		k=1					k=6					k=12				
	Variable	MSPE (x 10^3)	R^2_{OS} (%)	p-value	Annualized Return (%)	Utility Gain	MSPE (x 10^3)	R^2_{OS} (%)	p-value	Annualized Return (%)	Utility Gain	MSPE (x 10^3)	R^2_{OS} (%)	p-value	Annualized Return (%)	Utility Gain
	Historical Average	2.159	-	-	0.96		0.652	-	-	0.43		0.352	-	-	0.03	
	IA	2.070	4.13	0.02	6.21	4.76	0.643	1.36	0.07	4.20	3.63	0.347	1.51	0.06	2.15	2.01
	IA*	2.073	4.00	0.03	5.77	4.40	0.643	1.36	0.06	4.11	3.56	0.348	1.20	0.06	2.18	2.02
Traditional Predictor Variables	CAY	2.238	-3.66	-	-0.73	-1.74	0.663	-1.69	-	1.67	1.69	0.340	3.43	0.00	3.65	3.90
	CRE	2.287	-5.91	-	2.13	1.63	1.492	-128.94	-	3.26	3.58	0.759	-115.61	-	2.42	3.08
	DY	2.211	-2.40	-	-2.10	-3.92	0.654	-0.33	-	0.31	-2.86	0.331	5.93	0.02	2.62	-0.63
	PER	2.551	-18.16	-	2.26	1.25	1.685	-158.57	-	-1.15	-1.55	1.117	-217.18	-	-3.48	-5.46
	RREL	2.087	3.32	0.07	5.30	4.43	0.600	7.87	0.03	6.33	5.80	0.319	9.34	0.03	4.21	4.28
	SKEW	2.176	-0.79	-	-0.45	-1.31	0.678	-4.06	-	-0.45	-1.04	0.364	-3.29	-	-0.63	-1.07
	TERM	2.178	-0.88	-	1.13	0.34	0.680	-4.37	-	-1.09	-1.46	0.358	-1.68	-	-0.28	-0.70
Ambiguity Proxi	CCI	2.217	-2.66	-	-0.17	-0.88	0.784	-20.33	-	-3.00	-3.94	0.383	-8.78	-	-1.82	-2.94
	DISP	2.219	-2.76	-	-0.01	-3.09	0.665	-2.01	-	-0.01	-2.26	0.360	-2.18	-	-0.01	-1.47
	PUI	2.255	-4.45	-	-1.87	-2.69	0.708	-8.56	-	0.05	-0.15	0.404	-14.72	-	-1.08	-0.94
	UMCSI	2.208	-2.27	-	1.38	0.80	0.828	-27.04	-	-0.74	-0.54	0.455	-29.15	-	-3.61	-5.06
	VRP	1.928	10.70	0.02	7.83	5.52	0.602	7.65	0.00	4.36	1.90	0.349	0.72	0.09	0.58	-0.32

Panel B. Out-of-sample Results for Recursive Estimation

		k=1					k=6					k=12				
	Variable	MSPE (x 10^3)	R^2_{OS} (%)	p-value	Annualized Return (%)	Utility Gain	MSPE (x 10^3)	R^2_{OS} (%)	p-value	Annualized Return (%)	Utility Gain	MSPE (x 10^3)	R^2_{OS} (%)	p-value	Annualized Return (%)	Utility Gain
	Historical Average	2.159			0.96		0.652			0.43		0.352			0.03	
	IA	2.072	4.05	0.03	5.86	4.53	0.641	1.70	0.07	3.23	2.80	0.343	2.56	0.04	2.13	1.85
	IA*	2.072	4.04	0.03	5.61	4.30	0.640	1.87	0.05	3.53	3.06	0.344	2.28	0.04	2.29	1.97
Traditional Predictor Variables	CAY	2.241	-3.81	-	-0.61	-1.63	0.662	-1.59	-	1.69	1.73	0.340	3.55	0.00	3.59	3.86
	CRE	2.242	-3.82	-	2.26	1.69	1.177	-80.58	-	3.77	3.91	0.562	-59.66	-	1.88	2.37
	DY	2.169	-0.45	-	0.25	-1.04	0.637	2.32	0.09	1.17	-0.19	0.331	6.05	0.01	1.80	0.17
	PER	2.515	-16.48	-	3.83	2.73	1.602	-145.75	-	0.32	-0.05	1.086	-208.63	-	-2.43	-3.92
	RREL	2.100	2.74	0.05	5.32	4.30	0.606	7.02	0.01	6.23	5.58	0.313	10.98	0.00	4.29	4.07
	SKEW	2.174	-0.69	-	-0.51	-1.57	0.662	-1.54	-	0.51	-0.16	0.357	-1.51	-	-0.07	-0.66
	TERM	2.166	-0.33	-	1.05	0.08	0.663	-1.67	-	-0.09	-0.69	0.355	-0.88	-	0.04	-0.60
Ambiguity Proxi	CCI	2.199	-1.85	-	-0.47	-1.59	0.714	-9.59	-	-3.04	-4.60	0.366	-4.11	-	-0.79	-2.19
	DISP	2.210	-2.35	-	-0.79	-2.62	0.656	-0.61	-	-0.29	-1.46	0.354	-0.55	-	-0.26	-1.20
	PUI	2.236	-3.54	-	-2.86	-3.99	0.665	-1.97	-	0.20	-0.50	0.367	-4.39	-	-0.73	-0.96
	UMCSI	2.196	-1.71	-	-0.12	-0.97	0.760	-16.56	-	-3.77	-5.26	0.408	-15.81	-	-3.19	-5.07
	VRP	1.926	10.81	0.02	8.73	6.27	0.590	9.53	0.00	5.46	2.65	0.340	3.45	0.02	1.72	0.30

Figure 2.4. Option-market Implied Ambiguity in Eight Countries

Figure 2.4 shows the monthly estimates of our option-market implied ambiguity measure (IA) for Belgium (BEL20), France (CAC40), Germany (DAX30), Hong Kong (HSI), Japan (Nikkei225), Netherlands (AEX), Switzerland (SMI20), and United Kingdom (FTSE100). Shaded grey bands show (from left to right) the period of the "dot-com bubble", the 2007-2009 Great Financial Crisis, and the European debt crisis, respectively. The sample covers monthly observations from January 2000 to December 2012.

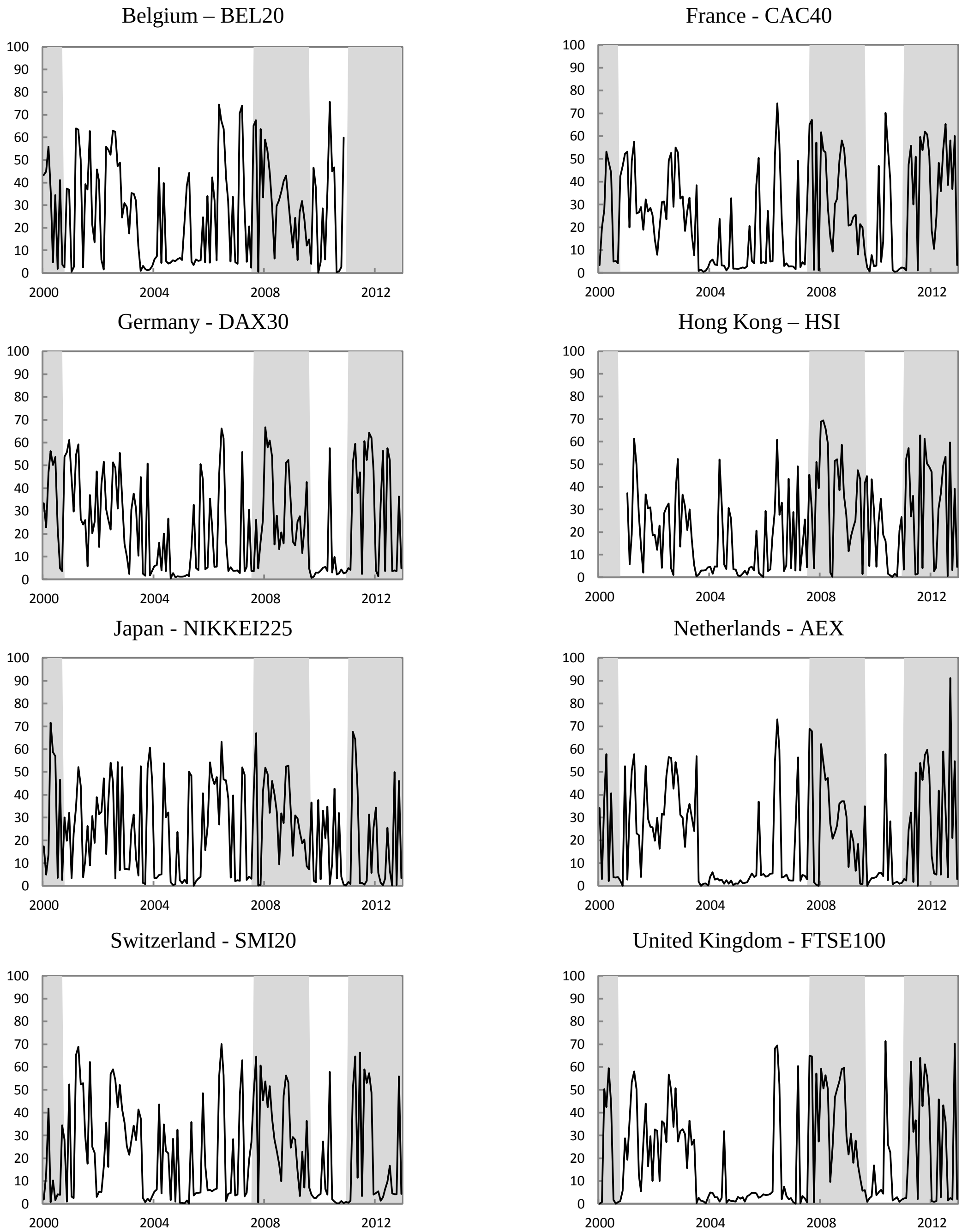


Table 2.8. Descriptive Statistics and Correlation Coefficients for Ambiguity and Excess Returns in Eight Countries

Table 2.8 presents descriptive statistics and correlation coefficients of international option-market implied ambiguity and excess returns in eight foreign countries with stock index option data. R_M-R_f is the excess market return calculated as the logarithmic return of the corresponding stock market index in excess of the logarithmic yield of 3 month government bond in each foreign country. IA and IA* are the option market ambiguity measures extracted from the corresponding volatility indices as defined in equations (2.7) and (2.8). All variables are denoted in annualized percentages. The sample covers monthly observations from January 2000 to December 2012 except Belgium and Hong Kong, which cover January 2000 to November 2010 and January 2001 to December 2012 respectively.

Panel A. Descriptive Statistics

Country	*Variable*	*Mean*	*Std. Dev.*	*Skewness*	*Kurtosis*	*AR(1)*	*Country*	*Variable*	*Mean*	*Std. Dev.*	*Skewness*	*Kurtosis*	*AR(1)*
	R_M-R_f	-5.26	68.51	-1.43	3.33	0.29		R_M-R_f	-4.61	71.61	-0.75	1.63	0.14
Belgium	*IA*	27.23	22.09	0.40	-1.03	0.33	*Japan*	*IA*	23.82	20.56	0.38	-1.21	0.20
	*IA**	41.06	14.45	0.72	-0.51	0.35		*IA**	39.04	13.09	0.61	-0.83	0.20
	R_M-R_f	-6.02	66.56	-0.61	0.67	0.13		R_M-R_f	-5.20	74.74	-1.00	2.15	0.11
France	*IA*	25.12	21.60	0.43	-1.20	0.48	*Netherlands*	*IA*	20.88	21.83	0.83	-0.45	0.35
	*IA**	39.84	13.95	0.66	-0.88	0.46		*IA**	37.28	14.07	1.16	0.59	0.32
	R_M-R_f	0.67	80.91	-0.94	2.77	0.08		R_M-R_f	-0.81	50.64	-0.72	0.62	0.25
Germany	*IA*	23.92	21.17	0.50	-1.23	0.42	*Switzerland*	*IA*	22.21	21.67	0.67	-1.01	0.35
	*IA**	39.15	13.64	0.70	-0.97	0.42		*IA**	38.21	13.96	0.86	-0.66	0.35
	R_M-R_f	2.02	78.55	-0.69	1.45	0.14		R_M-R_f	-0.02	0.53	-0.61	0.74	0.05
Hong Kong	*IA*	22.73	20.22	0.54	-0.95	0.28	*United Kingdom*	*IA*	21.13	22.15	0.74	-0.88	0.46
	*IA**	38.31	12.89	0.80	-0.47	0.29		*IA**	37.51	14.20	0.97	-0.39	0.43

Panel B. Correlation Coefficients for IA

	Belgium	*France*	*Germany*	*Hong Kong*	*Japan*	*Netherlands*	*Switzerland*	*United Kingdom*
Belgium	1.00							
France	0.58	1.00						
Germany	0.40	0.73	1.00					
Hong Kong	0.40	0.54	0.46	1.00				
Japan	0.27	0.30	0.36	0.33	1.00			
Netherlands	0.63	0.76	0.65	0.49	0.36	1.00		
Switzerland	0.59	0.61	0.57	0.41	0.28	0.66	1.00	
United Kingdom	0.63	0.77	0.68	0.56	0.33	0.76	0.70	1.00

Panel C. Correlation Coefficients for Excess Returns

	Belgium	*France*	*Germany*	*Hong Kong*	*Japan*	*Netherlands*	*Switzerland*	*United Kingdom*
Belgium	1.00							
France	0.81	1.00						
Germany	0.74	0.92	1.00					
Hong Kong	0.65	0.68	0.67	1.00				
Japan	0.48	0.60	0.57	0.63	1.00			
Netherlands	0.85	0.91	0.87	0.68	0.59	1.00		
Switzerland	0.78	0.82	0.77	0.60	0.53	0.81	1.00	
United Kingdom	0.82	0.87	0.81	0.70	0.59	0.86	0.79	1.00

Table 2.9. International Evidence on Predictive Regressions for Option-implied Ambiguity in Eight Countries

Table 2.9 presents predictive regression results for option market ambiguity IA in eight countries. Predictive regressions are specified according to equation (2.10). The sample covers monthly observations from January 2000 to December 2012 except Belgium and Hong Kong, which cover January 2000 to November 2010 and January 2001 to December 2012 respectively. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

k	*Cst*		*Belgium IA*		*Adj. R^2 (%)*	*k*	*Cst*		*France IA*		*Adj. R^2 (%)*
1	5.26	(0.68)	-0.32	(-1.38)	*0.36*	1	-0.15	(-0.02)	-0.21	(-0.95)	*-0.17*
6	6.73	(1.26)	-0.38	(-2.91) ***	*4.14*	6	0.30	(0.06)	-0.29	(-2.26) **	*3.19*
12	5.71	(1.34)	-0.36	(-3.36) ***	*6.27*	12	-3.53	(-0.76)	-0.16	(-1.70) *	*1.34*
18	5.26	(1.35)	-0.35	(-3.87) ***	*8.35*	18	-1.54	(-0.36)	-0.26	(-3.43) ***	*6.31*
24	1.84	(0.50)	-0.25	(-3.37) ***	*5.56*	24	-2.63	(-0.64)	-0.17	(-2.85) ***	*3.36*
30	1.26	(0.37)	-0.23	(-3.38) ***	*6.58*	30	-2.68	(-0.69)	-0.15	(-2.92) ***	*3.01*
36	0.71	(0.23)	-0.16	(-3.26) ***	*3.79*	36	-2.21	(-0.64)	-0.14	(-3.84) ***	*3.79*

k	*Cst*		*Germany IA*		*Adj. R^2 (%)*	*k*	*Cst*		*Hong Kong IA*		*Adj. R^2 (%)*
1	4.37	(0.52)	-0.14	(-0.54)	*-0.51*	1	8.84	(1.05)	-0.31	(-0.92)	*-0.08*
6	9.79	(1.78) *	-0.40	(-2.43) **	*4.60*	6	9.93	(1.77)	-0.32	(-1.86) *	*2.21*
12	7.54	(1.46)	-0.32	(-2.40) **	*5.23*	12	9.41	(1.87)	-0.26	(-1.69) *	*3.60*
18	6.60	(1.33)	-0.30	(-2.66) ***	*6.28*	18	7.21	(1.51)	-0.16	(-1.31)	*2.08*
24	6.98	(1.44)	-0.28	(-2.94) ***	*7.25*	24	8.15	(1.76)	-0.16	(-1.70) *	*3.55*
30	7.78	(1.72) *	-0.28	(-3.24) ***	*9.76*	30	9.44	(2.10)	-0.17	(-2.18) **	*8.71*
36	7.68	(1.88) *	-0.24	(-4.68) ***	*9.29*	36	8.40	(1.91)	-0.11	(-1.76) *	*5.74*

k	*Cst*		*Japan IA*		*Adj. R^2 (%)*	*k*	*Cst*		*Netherlands IA*		*Adj. R^2 (%)*
1	-0.23	(-0.03)	-0.17	(-0.69)	*-0.41*	1	-3.09	(-0.47)	-0.05	(-0.22)	*-0.62*
6	1.28	(0.23)	-0.27	(-2.39) **	*2.05*	6	2.54	(0.51)	-0.38	(-2.31) **	*4.45*
12	-1.83	(-0.35)	-0.12	(-1.40)	*0.38*	12	-1.17	(-0.25)	-0.23	(-1.80) *	*2.67*
18	-1.21	(-0.25)	-0.13	(-1.96) **	*1.21*	18	-0.52	(-0.12)	-0.29	(-2.67) ***	*6.68*
24	-1.14	(-0.26)	-0.10	(-2.02) **	*0.70*	24	-1.84	(-0.42)	-0.19	(-2.32) **	*3.62*
30	0.32	(0.08)	-0.14	(-2.79) ***	*2.64*	30	-2.25	(-0.54)	-0.15	(-2.32) **	*3.00*
36	0.40	(0.11)	-0.12	(-2.52) **	*2.40*	36	-2.45	(-0.65)	-0.11	(-2.54) **	*1.95*

k	*Cst*		*Switzerland IA*		*Adj. R^2 (%)*	*k*	*Cst*		*United Kingdom IA*		*Adj. R^2 (%)*
1	7.21	(1.44)	-0.30	(-1.65) *	*1.03*	1	0.05	(1.14)	-0.00	(-1.77) *	*1.09*
6	6.30	(1.65) *	-0.31	(-2.65) ***	*5.61*	6	0.04	(1.16)	-0.00	(-2.28) **	*6.02*
12	1.39	(0.42)	-0.12	(-1.50)	*1.08*	12	0.01	(0.35)	-0.00	(-1.44)	*2.61*
18	1.54	(0.53)	-0.15	(-2.18) **	*2.93*	18	0.01	(0.33)	-0.00	(-1.72) *	*3.48*
24	1.62	(0.58)	-0.14	(-2.31) **	*3.10*	24	0.00	(-0.10)	-0.00	(-0.88)	*0.07*
30	0.31	(0.11)	-0.07	(-1.46)	*0.32*	30	0.00	(-0.12)	-0.00	(-0.77)	*-0.25*
36	0.24	(0.10)	-0.04	(-1.15)	*-0.37*	36	0.00	(-0.18)	-0.00	(-0.32)	*-0.77*

2.7. Conclusion

We have examined the predictive power of market ambiguity for excess equity market returns or the equity premium in the U.S. and eight other countries. For the U.S., we extracted differences in ambiguity beliefs among heterogeneous investors using information contained in the VIX based on our ambiguity-adjusted OPM, and tested its forecasting power for S&P 500 index excess returns up to 36 months ahead. The predictive power of option market ambiguity is confirmed out-of-sample. Similar predictive regression results are obtained when ambiguity is extracted from traded option prices rather than backed out from the VIX. Our main results on the negative association between ambiguity and excess returns are also robust to an alternative measure of ambiguity, based on the difference between IBULL and IBEAR investor sentiments motivated by our Eq. (2.9), which does not rely on our A-OPM, VIX or option data. Globally, we extend our analysis to cover the equity markets of Belgium, France, Germany, Hong Kong, Japan, Netherlands, Switzerland and the UK, countries with major option trading activity but with a shorter option data series. We confirm that option-implied ambiguity is priced similarly in these international equity markets, raising the possibility of ambiguity contagion among different countries.

We have documented a robust and significant negative relationship between dispersion in ambiguity beliefs and the equity premium at the country and international levels, confirming empirically prior theoretical predictions by Cao, Wang and Zhang (2005) concerning the impact of ambiguity under limited market participation. Once ambiguity is properly controlled for, we confirm a positive risk-return trade-off as predicted by Merton’s (1973) ICAPM. The above results contribute to helping resolve the long-standing risk-return trade-off puzzle. Our overall findings underline the wider importance of accounting for ambiguity in asset pricing research.

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Appendix to Chapter 2

Table A2.1. Hodrick Reverse Regression Results for Option-market Implied Ambiguity (Extracted from VIX)

This table presents predictive regression results for market ambiguity (IA and IA*) using the reverse regressions according to Hodrick (1992) reorganization of the long-horizon regression:

$$r_{t+1} = \alpha + \beta \frac{\sum_{i=1}^{k} x_{t+1-i}}{k} + \varepsilon_t$$

where r_{t+1} is the one-step ahead market excess return, x_{t+1-i} is the predictor variable, and k is the prediction horizon. The sample covers monthly observations from January 2000 to December 2012. t-statistics are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Return Horizon (k)	Cst		IA		Adj. R^2 (%)	Return Horizon (k)	Cst		IA*		Adj. R^2 (%)
1	13.58	(3.72) ***	-0.46	(-2.41) **	*2.57*	1	31.37	(3.15) ***	-0.73	(-2.40) **	*2.61*
6	16.17	(3.17) ***	-0.63	(-2.46) **	*1.84*	6	42.49	(2.95) ***	-1.06	(-2.57) **	*2.04*
12	18.56	(3.29) ***	-0.75	(-2.51) **	*1.98*	12	49.36	(2.95) ***	-1.25	(-2.59) ***	*2.12*
18	18.83	(3.10) ***	-0.79	(-2.40) **	*1.81*	18	52.46	(2.81) ***	-1.35	(-2.51) **	*2.01*
24	17.66	(2.72) ***	-0.75	(-2.07) **	*1.29*	24	51.33	(2.51) **	-1.33	(-2.25) **	*1.58*
30	18.48	(2.63) ***	-0.78	(-1.97) **	*1.16*	30	55.04	(2.45) **	-1.43	(-2.20) **	*1.53*
36	18.56	(2.45) **	-0.79	(-1.84) *	*0.98*	36	55.74	(2.29) **	-1.45	(-2.06) **	*1.33*

Table A2.2. Hodrick Reverse Regression Results for Option-market Implied Ambiguity (Extracted from Option Prices)

This table presents predictive regression results for price-based market ambiguity (IA^{Price} and $IA*^{Price}$) using the reverse regressions according to Hodrick (1992) reorganization of the long-horizon regression:

$$r_{t+1} = \alpha + \beta \frac{\sum_{i=1}^{k} x_{t+1-i}}{k} + \varepsilon_t$$

where r_{t+1} is the one-step ahead market excess return, x_{t+1-i} is the predictor variable, and k is the prediction horizon. The sample covers monthly observations from January 2000 to December 2012. t-statistics are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Return Horizon (k)	Cst		IA^{Price}		Adj. R^2 (%)	Return Horizon (k)	Cst		$IA*^{Price}$		Adj. R^2 (%)
1	20.94	(2.65) ***	-0.58	(-1.98) **	*1.06*	1	43.83	(2.44) **	-0.95	(-2.11) **	*1.23*
6	30.85	(2.68) ***	-0.98	(-2.22) **	*1.43*	6	67.44	(2.49) **	-1.56	(-2.27) **	*1.52*
12	37.34	(2.76) ***	-1.22	(-2.32) **	*1.63*	12	80.36	(2.50) **	-1.87	(-2.30) **	*1.59*
18	40.36	(2.64) ***	-1.36	(-2.27) **	*1.58*	18	87.92	(2.40) **	-2.07	(-2.24) **	*1.53*
24	44.70	(2.64) ***	-1.55	(-2.32) **	*1.71*	24	98.77	(2.43) **	-2.36	(-2.29) **	*1.65*
30	54.40	(2.88) ***	-1.92	(-2.59) ***	*2.26*	30	121.21	(2.67) ***	-2.92	(-2.54) **	*2.16*
36	57.99	(2.80) ***	-2.07	(-2.54) **	*2.22*	36	130.58	(2.62) ***	-3.16	(-2.51) **	*2.15*

Table A2.3. Bivariate Regression Results with CAY and Each Predictor Variable

The table presents bivariate regression results involving CAY with each predictor variable, implied ambiguity and other alternative ambiguity proxies. IA and IA* measure option market implied ambiguity. CAY is the consumption-wealth ratio. CRE is the credit spread. DY represents the aggregate dividend yield on the S&P500 index. PER is the price/earnings ratio. RREL is the stochastically detrended interest rate. SKEW denotes CBOE's SKEW index. TERM denotes the term spread between 10Y T-bond and 3M T-bill. CCI, DISP, PUI and UMCSI, denoting the Consumer Confidence Index, Dispersion (DISP) between Investors Intelligence Sentiment Indices – Bearish and Bullish, Policy Uncertainty Index, and University of Michigan's Consumer Sentiment Index respectively. VRP is the variance risk premium. Predictive regressions are specified according to equation (2.10). The sample covers monthly observations from January 1990 to December 2012. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Panel A. Traditional Predictor Variables

k	*Cst*		*CAY*		*IA*		*Adj. R^2 (%)*
1	12.35	(3.36) ***	3.75	(2.26) **	-0.44	(-2.33) **	*3.48*
12	7.88	(2.94) ***	4.98	(3.43) ***	-0.19	(-1.78) *	*25.30*
24	6.61	(2.64) ***	5.49	(3.71) ***	-0.15	(-2.02) **	*48.19*
36	5.41	(2.24) **	5.29	(3.65) ***	-0.10	(-1.78) *	*62.35*

k	*Cst*		*CAY*		*CRE*		*Adj. R^2 (%)*
1	14.23	(1.26)	3.86	(2.29)	-9.00	(-0.74)	*1.25*
12	4.05	(0.58)	5.11	(3.51)	0.89	(0.12)	*20.92*
24	2.08	(0.38)	5.64	(3.84)	2.32	(0.43)	*44.38*
36	2.97	(0.62)	5.40	(3.77)	0.89	(0.21)	*59.57*

k	*Cst*		*CAY*		*DY*		*Adj. R^2 (%)*
1	0.77	(0.06)	3.56	(1.85) *	2.29	(0.38)	*0.78*
12	-2.66	(-0.24)	4.39	(2.47) **	3.71	(0.76)	*22.50*
24	-3.00	(-0.31)	4.92	(2.64) ***	3.58	(0.85)	*46.48*
36	-4.31	(-0.49)	4.61	(2.59) ***	3.99	(1.07)	*64.46*

k	*Cst*		*CAY*		*PER*		*Adj. R^2 (%)*
1	3.82	(0.61)	4.01	(2.43) **	0.07	(0.26)	*0.76*
12	4.46	(1.07)	5.11	(3.48) ***	0.02	(0.12)	*20.90*
24	2.38	(0.67)	5.66	(3.77) ***	0.07	(0.65)	*44.62*
36	2.39	(0.79)	5.44	(3.72) ***	0.05	(0.63)	*60.07*

k	*Cst*		*CAY*		*RREL*		*Adj. R^2 (%)*
1	7.11	(2.39) **	4.49	(2.72) ***	23.59	(2.49) **	*2.98*
12	6.37	(2.26) **	5.51	(3.64) ***	20.12	(2.24) **	*36.14*
24	4.79	(1.74) *	5.75	(3.70) ***	6.62	(1.16)	*46.74*
36	3.93	(1.46)	5.42	(3.61) ***	1.47	(0.39)	*59.67*

k	*Cst*		*CAY*		*SKEW*		*Adj. R^2 (%)*
1	31.94	(0.40)	3.76	(2.13) **	-0.23	(-0.34)	*0.76*
12	-87.59	(-1.71) *	5.87	(3.73) ***	0.80	(1.83) *	*25.66*
24	-31.22	(-1.00)	5.87	(3.79) ***	0.31	(1.17)	*44.98*
36	17.57	(0.89)	5.30	(3.58) ***	-0.12	(-0.74)	*59.70*

k	*Cst*		*CAY*		*TERM*		*Adj. R^2 (%)*
1	10.75	(1.88) *	4.34	(2.56) **	-2.86	(-1.11)	*1.11*
12	4.46	(0.84)	5.07	(3.37) ***	0.24	(0.10)	*20.90*
24	-0.41	(-0.08)	5.22	(3.48) ***	2.61	(1.34)	*48.97*
36	-1.02	(-0.20)	4.87	(3.35) ***	2.81	(1.59)	*67.77*

Panel B. Ambiguity Proxies

k	*Cst*		*CAY*		*CCI*		*Adj. R^2 (%)*
1	6.79	(0.53)	3.99	(2.42) **	-0.01	(-0.11)	*0.72*
12	8.52	(0.79)	5.08	(3.44) ***	-0.04	(-0.34)	*21.28*
24	14.23	(1.60)	5.48	(3.64) ***	-0.11	(-1.07)	*48.64*
36	15.09	(1.98) **	5.13	(3.48) ***	-0.12	(-1.34)	*67.45*

k	*Cst*		*CAY*		*DISP*		*Adj. R^2 (%)*
1	11.18	(1.54)	3.13	(1.70) *	-0.31	(-0.96)	*1.03*
12	2.99	(0.65)	5.39	(3.39) ***	0.10	(0.72)	*21.19*
24	4.54	(1.22)	5.57	(3.50) ***	-0.01	(-0.14)	*43.83*
36	3.94	(1.08)	5.38	(3.42) ***	-0.01	(-0.08)	*59.45*

k	*Cst*		*CAY*		*PUI*		*Adj. R^2 (%)*
1	7.19	(0.62)	3.99	(2.41) **	-0.02	(-0.14)	*0.73*
12	3.98	(0.55)	5.10	(3.48) ***	0.01	(0.14)	*20.90*
24	1.13	(0.16)	5.56	(3.71) ***	0.03	(0.56)	*44.30*
36	2.21	(0.32)	5.34	(3.65) ***	0.02	(0.33)	*59.61*

k	*Cst*		*CAY*		*UMCSI*		*Adj. R^2 (%)*
1	-2.76	(-0.10)	3.91	(2.32) **	0.10	(0.32)	*0.77*
12	4.92	(0.22)	5.10	(3.54) ***	0.00	(-0.00)	*20.87*
24	17.21	(0.97)	5.67	(3.90) ***	-0.15	(-0.72)	*45.82*
36	17.94	(1.17)	5.40	(3.79) ***	-0.16	(-0.89)	*62.74*

k	*Cst*		*CAY*		*VRP*		*Adj. R^2 (%)*
1	-5.16	(-1.00)	3.13	(1.84) *	0.59	(2.34) **	*5.92*
12	2.75	(0.87)	4.93	(3.35) ***	0.12	(1.62)	*22.83*
24	3.22	(1.08)	5.54	(3.71) ***	0.06	(1.11)	*44.62*
36	4.04	(1.40)	5.41	(3.72) ***	-0.01	(-0.27)	*59.50*

Supplementary Appendix to Chapter 2

Table SA2.1. Robustness Tests – Controlling for Subjective Investor Required Return and Risk-free Rate

The table presents the robustness regression results for market ambiguity IA and IA* controlling for subjective required rate of return (μ_{12m}) and risk-free rate (r_f). Predictive regressions according to equation (2.10). The sample covers monthly observations from January 1990 to December 2012. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

Panel A. Controlling for Subjective Required Rate of Return

k	*Cst*	*IA*	*μ_{12m}*	*Adj. R^2 (%)*	*k*	*Cst*	*IA**	*μ_{12m}*	*Adj. R^2 (%)*
1	12.88 (2.63) ***	-0.44 (-2.35) **	0.06 (0.23)	*2.24*	1	29.54 (2.75) ***	-0.69 (-2.32) **	0.08 (0.31)	*2.32*
6	10.08 (3.05) ***	-0.25 (-2.36) **	0.01 (0.05)	*3.52*	6	19.70 (3.62) ***	-0.40 (-2.38) **	0.02 (0.09)	*3.79*
12	9.31 (3.22) ***	-0.20 (-2.28) **	0.04 (0.23)	*4.91*	12	17.02 (3.83) ***	-0.32 (-2.28) **	0.05 (0.28)	*5.20*
18	9.89 (3.68) ***	-0.23 (-3.06) ***	-0.03 (-0.17)	*7.04*	18	19.16 (4.97) ***	-0.38 (-3.14) ***	-0.02 (-0.12)	*7.68*
24	9.72 (3.95) ***	-0.20 (-3.27) ***	-0.06 (-0.49)	*6.18*	24	18.16 (5.89) ***	-0.34 (-3.45) ***	-0.06 (-0.44)	*7.10*
30	9.40 (4.05) ***	-0.18 (-3.30) ***	-0.07 (-0.68)	*5.94*	30	17.09 (6.47) ***	-0.31 (-3.54) ***	-0.07 (-0.65)	*7.09*
36	9.30 (4.15) ***	-0.18 (-3.64) ***	-0.10 (-1.02)	*6.74*	36	16.60 (7.04) ***	-0.30 (-3.87) ***	-0.09 (-0.97)	*7.83*

Panel B. Controlling for Risk-free Rate

k	*Cst*	*IA*	*r_f*	*Adj. R^2 (%)*	*k*	*Cst*	*IA**	*r_f*	*Adj. R^2 (%)*
1	11.18 (1.77) *	-0.45 (-2.40) **	0.63 (0.45)	*2.29*	1	28.83 (2.63) ***	-0.73 (-2.39) **	0.65 (0.46)	*2.34*
6	8.85 (1.63)	-0.25 (-1.86) *	0.34 (0.26)	*3.62*	6	18.70 (2.55) **	-0.40 (-1.94) *	0.35 (0.27)	*3.89*
12	9.08 (1.81) *	-0.21 (-1.95) *	0.17 (0.15)	*4.83*	12	17.32 (2.62) ***	-0.34 (-2.02) **	0.19 (0.16)	*5.06*
18	10.56 (2.36) **	-0.22 (-2.45) **	-0.25 (-0.23)	*7.10*	18	19.56 (3.37) ***	-0.37 (-2.62) ***	-0.22 (-0.21)	*7.74*
24	11.75 (3.00) ***	-0.18 (-2.37) **	-0.67 (-0.73)	*6.81*	24	19.42 (3.95) ***	-0.31 (-2.62) ***	-0.65 (-0.70)	*7.72*
30	11.86 (3.40) ***	-0.16 (-2.32) **	-0.79 (-0.91)	*6.85*	30	18.70 (4.31) ***	-0.28 (-2.60) ***	-0.76 (-0.89)	*7.96*
36	11.53 (3.76) ***	-0.14 (-2.43) **	-0.77 (-0.97)	*6.76*	36	17.74 (4.86) ***	-0.25 (-2.71) ***	-0.75 (-0.95)	*7.87*

Table SA2.2. Additional International Evidence (with IA*)

The table presents predictive regression robustness results for option market ambiguity using IA* in eight countries. Predictive regressions are specified according to equation (2.10). The sample covers monthly observations from January 2000 to December 2012 except Belgium and Hong Kong, which cover January 2000 to November 2010 and January 2001 to December 2012 respectively. Robust t-statistics according to Hodrick (1992) adjustment 1B are reported in parentheses. The superscripts ***, **, and * indicate significance at 1%, 5%, and 10% levels, respectively.

k	*Cst*	*Belgium IA**	*Adj. R^2 (%)*	*k*	*Cst*	*France IA**	*Adj. R^2 (%)*
1	11.89 (0.82)	-0.37 (-1.11)	*-0.11*	1	5.57 (0.40)	-0.28 (-0.81)	*-0.31*
6	18.20 (2.10) **	-0.53 (-2.63) ***	*3.40*	6	9.33 (1.24)	-0.41 (-2.16) **	*2.55*
12	19.33 (3.27) ***	-0.57 (-3.52) ***	*6.73*	12	1.59 (0.25)	-0.23 (-1.69) *	*1.04*
18	19.07 (4.03) ***	-0.57 (-4.14) ***	*9.61*	18	7.81 (1.51)	-0.40 (-3.81) ***	*6.21*
24	11.91 (3.02) ***	-0.41 (-3.61) ***	*6.69*	24	4.25 (0.88)	-0.28 (-3.35) ***	*3.68*
30	10.08 (2.91) ***	-0.36 (-3.47) ***	*7.37*	30	3.38 (0.77)	-0.25 (-3.52) ***	*3.54*
36	7.14 (2.38) **	-0.26 (-3.45) ***	*4.60*	36	3.81 (0.96)	-0.24 (-4.61) ***	*4.62*

k	*Cst*	*Germany IA**	*Adj. R^2 (%)*	*k*	*Cst*	*Hong Kong IA**	*Adj. R^2 (%)*
1	8.04 (0.48)	-0.18 (-0.45)	*-0.56*	1	21.41 (1.09)	-0.51 (-0.96)	*0.00*
6	22.57 (2.55) **	-0.57 (-2.35) **	*3.83*	6	24.70 (2.46) **	-0.58 (-2.01) **	*3.09*
12	18.09 (2.52) **	-0.46 (-2.46) **	*4.56*	12	21.12 (2.40) **	-0.46 (-1.78) *	*4.70*
18	16.79 (2.65) ***	-0.44 (-2.86) ***	*5.69*	18	13.91 (1.98) **	-0.27 (-1.35)	*2.52*
24	17.40 (2.87) ***	-0.44 (-3.37) ***	*7.30*	24	15.11 (2.53) **	-0.28 (-1.83) *	*4.61*
30	18.55 (3.28) ***	-0.45 (-3.66) ***	*10.08*	30	16.45 (2.95) ***	-0.29 (-2.23) **	*9.91*
36	16.64 (3.36) ***	-0.38 (-5.23) ***	*9.52*	36	13.19 (2.50) **	-0.19 (-1.89) *	*7.00*

k	*Cst*	*Japan IA**	*Adj. R^2 (%)*	*k*	*Cst*	*Netherlands IA**	*Adj. R^2 (%)*
1	7.16 (0.45)	-0.29 (-0.75)	*-0.36*	1	-5.69 (-0.41)	0.04 (0.10)	*-0.64*
6	11.67 (1.45)	-0.43 (-2.48) **	*2.11*	6	15.09 (1.87) *	-0.55 (-2.33) **	*3.59*
12	2.94 (0.43)	-0.20 (-1.53)	*0.47*	12	7.22 (1.13)	-0.36 (-1.98) **	*2.46*
18	3.93 (0.68)	-0.21 (-2.10) **	*1.29*	18	10.83 (1.99) **	-0.47 (-3.15) ***	*6.98*
24	2.86 (0.59)	-0.17 (-2.18) **	*0.76*	24	5.92 (1.18)	-0.32 (-2.80) ***	*3.99*
30	5.94 (1.41)	-0.23 (-2.95) ***	*2.87*	30	4.47 (1.00)	-0.27 (-2.87) ***	*3.77*
36	5.43 (1.40)	-0.20 (-2.67) ***	*2.79*	36	2.59 (0.63)	-0.20 (-3.17) ***	*2.61*

k	*Cst*	*Switzerland IA**	*Adj. R^2 (%)*	*k*	*Cst*	*United Kingdom IA**	*Adj. R^2 (%)*
1	17.13 (1.59)	-0.43 (-1.53)	*0.81*	1	0.15 (1.50)	-0.00 (-1.59)	*0.74*
6	17.88 (2.68) ***	-0.48 (-2.71) ***	*5.64*	6	0.14 (2.38) **	-0.00 (-2.37) **	*5.88*
12	6.02 (1.33)	-0.19 (-1.62)	*1.11*	12	0.07 (1.31)	-0.00 (-1.52)	*2.64*
18	7.48 (2.17) **	-0.24 (-2.45) **	*3.30*	18	0.07 (1.69) *	-0.00 (-1.92) *	*3.99*
24	7.53 (2.32) **	-0.24 (-2.69) ***	*3.85*	24	0.02 (0.73)	-0.00 (-1.13)	*0.46*
30	3.50 (1.20)	-0.12 (-1.85) *	*0.80*	30	0.02 (0.62)	-0.00 (-1.07)	*0.17*
36	2.43 (0.96)	-0.08 (-1.67) *	*0.03*	36	0.01 (0.30)	-0.00 (-0.73)	*-0.47*

Chapter 3.

Option Market Ambiguity and Real Economic Activity

ABSTRACT

We document a negative relation between option-market ambiguity and real economic activity during 1990-2014. Our empirical analysis unveils a strong predictive link between market ambiguity and subsequent real economic activity covering production, employment, consumption and overall economic output. Corroborating prominent economic theories, our evidence indicates that ambiguity from the financial markets is associated with depressed production, lower consumption, higher unemployment and decreased aggregate economic output for up to eight quarters. Capturing divergence in beliefs, our ambiguity measure is able to predict economic activity beyond VIX, variance premium, credit spread and other established market predictors.

3.1. Introduction

Characterizing many economic phenomena, uncertainty refers to the inability to predict future outcomes (Nelson, 1961; Jurado, Ludvigson and Ng, 2015). A more general definition, as distinct from that of risk, is attributed to Knight (1921) and is illustrated by Ellsberg (1961) in his famous mind experiment. Knightian uncertainty or ambiguity refers to uncertain events where the probabilities of possible outcomes are unknown and cannot be estimated with confidence. Due to the inherent difficulty in inferring Knightian or “extreme” uncertainty, studies linking it to economic activity are rather scarce (Bloom, 2009; Bachmann, Elstner and Sims, 2013; Jurado, Ludvigson and Ng, 2015). This is the case despite more research being conducted recently on economic agents’ behavior and risk aversion characteristics in both real and financial markets (see, for example, Bloom, 2009; Christiano, Motto and Rostagno, 2010 and 2014; Drechsler and Yaron, 2011; Gourio, 2012; Bansal et. al., 2014; Caggiano, Castelnuovoa and Groshenny, 2014). Undoubtedly, changing investor sentiment creates more subjectivity and time-variability driving the risk and uncertainty tolerance of market participants, which in turn drives asset prices and contributes to financial system instability. In Bansal and Yaron (2004), time-variation in consumption uncertainty drives the equity risk premium. As such, and as a way of improving economic and financial prediction, many institutions have recently turned to sentiment indices or other proxies for ambiguity aversion and model uncertainty (e.g., see Coudert and Gex’s (2008) survey).

Economic uncertainty, while not directly or fully observable, remains a key driver in many economic theories (e.g., real options theory, precautionary saving, financial frictions, risk premia and ambiguity aversion, reviewed below) that connect agents’ behavior to subsequent real economic activity. Macroeconomic or monetary policy itself can also be linked to risk and ambiguity aversion (e.g., Bekaert, Hoerova and Lo Duca, 2013). Being able to infer the degree of market uncertainty and investor ambiguity aversion and assess its impact on real economic activity can therefore be key to understanding economic fluctuations and anticipating instability in the real economy. The present chapter extends research in this area examining the relationship between ambiguity, as inferred from the market through forward-looking option prices, and real economic activity in the U.S. over the last quarter century.

Aggregate investor expectations of the future level of uncertainty can be derived from the state of the economy itself and from the financial markets (e.g., Abdellaoui et al., 2011) as economic agents act according to their subjective uncertainty aversion or ambiguity preferences. Economists have extracted uncertainty or ambiguity information using different data proxies, such as forecast disagreements from surveys (Boero, Smith and Wallis, 2008; Bachmann, Elstner and Sims, 2013), coverage of 'uncertainty' (and other related keywords) in the press (Baker, Bloom and Davis, 2013), option market implied volatility or CBOE's volatility index (VIX)[1] (Bloom, 2009; Bekaert and Hoerova, 2014), and the variance risk premium or VRP (Bollerslev, Tauchen and Zhou, 2009; Zhou, 2009; Drechsler, 2013).

In this chapter we examine whether ambiguity, as inferred from the forward-looking options market, influences or anticipates subsequent real economic activity and whether it is a good predictor of changes in macroeconomic activity. Given the general inability to accurately predict fluctuations in economic activity (Jurado, Ludvigson and Ng, 2015), accounting for Knightian uncertainty might help improve predictive power and the quality of economic forecasts and policy interventions. Despite some mixed evidence that uncertainty proxies are associated with contemporaneous or subsequent economic activity fluctuations, so far no financial market-based measure offers comprehensive and robust predictive power for an extensive range of real economic activity indicators. Spanning the recent quarter century (from 1990 to 2014) of option market and macroeconomic data, our study documents a strong predictive relationship between option-market implied ambiguity (IA) and ex post fluctuations in real economic activity. Production, employment and consumption respond negatively to ambiguity as inferred from the financial option markets. These dynamics are consistent with those documented by economic proxies for uncertainty, such as the uncertainty measure proposed by Jurado, Ludvigson and Ng (2015) and disagreement indicators among economic forecasters. We extend this literature proposing a market-based indicator of ambiguity that comprehensively predicts real economic activity beyond other established financial market-based predictors. Option market

[1] While widely considered as a proxy for uncertainty, implied volatility such as the VIX is more a risk measure under Knight's stricter definition. As noted, Knightian/heightened uncertainty or ambiguity refers to situations in which agents are uncertain about the probability or distribution of possible outcomes. The 2007-2010 crisis was, for instance, characterized by Knightian uncertainty.

ambiguity robustly predicts the ex post performance of eight macroeconomic indicators and outperforms other prevalent financial market-based predictor variables in terms of reliability, consistency and robustness, including CBOE's volatility index (VIX), the variance risk premium (VRP), the credit spread (CS), and market excess returns (ER). Option market ambiguity also provides superior, comprehensive and robust predictive ability with regards to future real economic activity in multivariate settings relative to (the best of) other established predictor variables while controlling for risk aversion. Our findings suggest that option-market implied ambiguity provides important incremental information beyond that already contained in extant financial market predictors.

We particularly note three important contributions to the extant literature on the relationship between Knightian uncertainty and real economic activity. First, we document that option market investors' subjective ambiguity perceptions are important determinants of real economic activity. Second, we add to the evidence that option market ambiguity contains valuable information regarding macroeconomic uncertainty (Drechsler and Yaron, 2011; Drechsler, 2013). Given the basic nature of aggregate economic shocks and the inability to reliably predict, we show that forward-looking option market ambiguity is a more reliable proxy for heightened aggregate uncertainty and demonstrate its superior predictive power for a wide range of economic activity indicators. Finally, the empirical relationships we test corroborate the predictions of prominent economic theories on the negative association between time-varying uncertainty and real economic activity up to eight quarters forward.

3.2. Theory and Literature

Establishing a link between time-varying market ambiguity and ex post real economic activity requires an understanding of how heightened uncertainty (i.e., uncertainty beyond probabilistic risk or volatility) impacts real economic activity and verifying this through robust empirical evidence. A number of main economic theories provide predictions on this relationship: real options theory, precautionary saving, financial frictions, risk premia and ambiguity aversion. Under *real options theory* and the related bad news principle, economic agents follow a "wait and see" approach in guiding their future economic actions in the presence of uncertainty and irreversibility (see Dixit and Pindyck, 1994; Trigeorgis, 1996). Due to high adjustment costs to labor and capital, when uncertainty rises firms become more cautious and hence delay their investment and hiring, while individuals become more reserved with consumption spending, particularly on durables (Romer, 1990). These depressing effects on the economy are more pronounced under Knightian uncertainty as shown by Nishimura and Ozaki (2007) and Miao and Wang (2011). Resulting delays in investment, hiring and major consumption decisions might thus cause a slowdown in production, employment and consumption growth, leading to depressed economic activity. The depressing effect of uncertainty on investment has been confirmed by Rivoli and Salorio (1996) in foreign direct investment decisions, by Guiso and Parigi (1999) in explaining the negative effect of uncertainty on investments by Italian manufacturing firms, by Bloom, Bond and Van Reenen (2007) in describing firm-level investment dynamics, and by Bloom (2009) in explaining depressed hiring and investment following uncertainty shocks.

Precautionary saving theory prescribes that when facing increasing uncertainty, economic agents suppress costly activities concerning investment, production, consumption and hiring (Leland, 1968; Kimball, 1990; Guiso, Jappelli and Terlizzese, 1992; Bansal and Yaron, 2004). For example, one may have to save more during economic stress periods in case their job security is affected by adverse unanticipated circumstances. This shift of preferences by individuals and firms towards saving, rather than spending and investing, causes restraint in consumption, investment, hiring, and production growth. This also implies a negative relationship between uncertainty and overall real economic output.

Financial frictions theory argues that stricter financial constraints during times of financial stress and uncertainty (e.g., higher lending rates due to increased probability of default) produce adverse propagation effects onto real economic activity (Hall, 2010). This financial friction effect essentially slows down capital flows in the real economy, adversely affecting economic efficiency and productivity. Hall (2011) highlights the intricate linkage between uncertainty in financial markets and ex post real economic activity. The slowed capital flows in the real economy hinder investment and planned improvements in production, suggesting a depressed economic output following higher uncertainty in the economic system. Related to this, uncertainty raises the probability of default and the cost of finance and *risk premia* (Bansal and Yaron (2004); Liu and Miao (2015)). Moreover, when economic agents lack confidence and entertain worst-case beliefs, they fear and act as if the worst case scenario will occur, exhibiting *ambiguity aversion* (Ilut and Schneider, 2014). As the range of outcomes broadens under ambiguity, economic agents are more pessimistic reducing investment, hiring and consumption.

The above economic theories uniformly suggest that heightened economic uncertainty will likely have negative ramifications on investment, production, employment, consumption growth and overall output. In this chapter, we focus on the effect of Knightian-type uncertainty, particularly forward-looking ambiguity extracted from the option market, on subsequent real economic activity. Given the predictions made by the real options, precautionary saving, financial frictions, and risk premium and ambiguity aversion theories, we test the hypothesized negative effect of heightened uncertainty on production, employment, consumption growth and overall output empirically. An important perspective of this chapter is to corroborate the efficiency of option-market extracted ambiguity in predicting real economic activity.

Following decades of research in macroeconomics and finance on how financial market information is reflected in real economy performance indicators, it is widely accepted that financial market dynamics are associated with key fluctuations in economic activity. Fama (1981) documents a positive relationship between stock market returns and future economic activity, providing an explanation for the anomalous negative relationship between stock returns and inflation. Lee (1992) finds further supportive evidence using both interest rates and inflation in a VAR system. James, Koreisha and Partch (1985) find evidence that the stock market signals changes in real activity and the monetary base. Chen, Roll and Ross (1986) show that various macroeconomic risk factors are priced in stock market returns. Given that changes in expectations drive ex post economic activity (Keynes 1936), information on changes in financial market investor expectations is, not surprisingly, linked to fluctuations in economic uncertainty and real economic activity. Fama (1990) finds that stock returns, being a proxy for stock market investors' expectations, predict ex post industrial production (IP) growth in the U.S. from 1953 to 1987. Chen (1991) finds that credit spread (CS), proxying for default risk expectations, is correlated with economic output growth. Estrella and Hardouvelis (1991) find that the term spread (TS) predicts consumption and investment. Beaudry and Portier (2006) use stock price movements proxying for changes in investor expectations to explain business cycle fluctuations.

Motivated by the aforementioned evidence and the established informational efficiency of the financial options market, we extract ambiguity information from the market based on an ambiguity-extended option pricing model (A-OPM) and test its relevance and power as a proxy for uncertainty, beyond probabilistic risk, in predicting real economic activity in the U.S. during the recent quarter century. We extract economic agents' subjective ambiguity preferences (or degree of miscalibration) and divergence in beliefs from the Chicago Board Options Exchange (CBOE) volatility index (VIX) using our modified option pricing model (A-OPM) based on rank-dependent utility.[2] We verify the relevance of our market ambiguity measure by comparing it to a number of established macroeconomic uncertainty indicators from the extant economics literature. To investigate the empirical linkages between uncertainty and real economic activity, we employ a vector autoregressive (VAR) analysis of statistical causality between option-market implied ambiguity and real economic activity. Once we establish that uncertainty shocks are driven by ambiguity inferred from the option market, we test the predictive power of aggregate market-implied ambiguity (IA) on key economic indicators spanning production, employment, consumption, and overall economic performance. Controlling for risk aversion effects, we show that option-market ambiguity robustly predicts these real economic activity indicators for up to eight quarters (two years) with a consistent negative sign as predicted by the aforementioned economic theories. Our comprehensive findings contribute to growing evidence on the interaction between the financial markets and the real economy while revealing the informational efficiency and richness of option-market extracted ambiguity measures. More importantly, we provide empirical evidence on the linkages between option market ambiguity and macroeconomic uncertainty, showing that Knightian uncertainty expectations are subsumed in the pricing behavior of financial market investors.

[2] This A-OPM has been proposed by Chateauneuf, Kast and Lapied (1996) and applied to option pricing by Driouchi, Trigeorgis and Gao (2015), to implied volatility estimation by Driouchi, Trigeorgis and So (2016) and to real option analysis in environmental policy and corporate financing by Agliardi and Sereno (2011) and Agliardi et al. (2015).

The superior predictive ability of implied ambiguity (IA) for real economic activity is important to economic policy making as our measure can help enhance the predictability of economic activity fluctuations and anticipate potential shocks in economic output. Different from many existing economics-based uncertainty indicators that can only be estimated ex post, our financial market-based measure offers forward-looking information through the option pricing approach. This approach allows us to take advantage of the informational efficiency of the financial options market and obtain valuable information about agents' perceived level of ambiguity about the economy on a real-time basis. With respect to the economic implications, our empirical findings support key theory predictions that heightened uncertainty suppresses investment, production, employment, consumption and overall output. These findings suggest that option-implied ambiguity is a superior forward-looking proxy for aggregate uncertainty in the economy, contributing to the extant literature on the linkages between financial markets and the real economy (Fama, 1981; Chen, Roll and Ross, 1986; Estrella and Mishkin, 1998; McQueen and Roley, 1993; Liew and Vassalou, 2000; Beaudry and Portier, 2006). This is achieved through explicating the negative impact of heightened uncertainty on real economic activity.

3.3. Ambiguity Modeling and Empirical Setup

We infer economic agents' ambiguity preferences from the options market based on Choquet Brownian motion and rank-dependent utility proposed by Chateauneuf, Kast and Lapied (1996) and applied to option pricing by Driouchi, Trigeorgis and So (2016). The underlying asset return process is:

$$\frac{dS}{S} = (\mu + m\sigma)dt + s\sigma dz \quad (\forall m \in]-1,1[, \forall s \in]0,1]) \tag{3.1}$$

where S is the price of the underlying asset (the S&P 500 stock index or SPX) having mean drifts $\mu + m\sigma$ and standard deviations $s\sigma$ per unit time; m and s are the mean and standard deviation of a general Wiener process W following $dW = mdt + sdz$, with z being a standard Wiener process. Parameters $m\sigma$ and s entertain (multiple states of) uncertainty in the mean and variance of the process; these are functions of a capacity variable c, with $0 < c < 1$, summarizing the degree of investors' perceived ambiguity: c < 0.5 indicates investor ambiguity aversion, c = 0.5 risk/ambiguity neutrality, and c > 0.5 ambiguity-seeking attitudes. Eq. (3.1) implies that, due to ambiguity, there are multiple mean drifts and volatility states (scenarios), with mean drift(s) (of *dS/S*) of $\mu + m\sigma$ (set apart by m units of σ) and volatilities of sσ (with parameters m and s taking multiple values). Under this more general Brownian motion, the distance of c from the ambiguity neutrality value of 0.5 captures the amount of ambiguity perceived by the investor or decision maker[3] (Kast et al., 2014). Using the above Choquet (distorted) Brownian motion, the price of a European call option under ambiguity (A-OPM) takes the form (see Technical Appendix):

$$C_t^A = S_t e^{-\delta' T} N(d_1') - K e^{-r' T} N(d_2') \tag{3.2}$$

where

$$d_1' = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r' - \delta' + 0.5(s\sigma)^2\right)T}{s\sigma\sqrt{T}};\ d_2' = d_1' - (s\sigma)\sqrt{T} \tag{3.3}$$

$$r' = r + m\frac{[r - (\mu + m\sigma)]}{s^2\sigma};\ \delta' = \delta - \frac{(m + s^2\sigma - s\sigma)[(\mu + m\sigma) - r]}{s^2\sigma} \tag{3.4}$$

$$m = 2c - 1\ and\ s = \sqrt{4c(1-c)} \qquad (\forall c \in]0,1[) \tag{3.5}$$

In the A-OPM of Eq. (3.2), C_t^A is the price of a European call option under ambiguity at time *t*, S_t is the current price of the underlying asset (S&P500), K is the strike (exercise) price, σ is return volatility, δ is any form of 'dividend yield', and T is time to maturity. In Eqs. (3.1) to (3.4), m and s are subjective ambiguity parameters dependent on capacity function c (Eq. (3.4)). The above summarize uncertainty in model parameters and represent economic agents' model misspecification (miscalibration) under Knightian uncertainty (Sarin and Wakker, 1992; Hong and Karni, 1994; Ghirardato and Marinacci, 2002; Chateauneuf, Eichberger and Grant, 2007; Kast et al., 2014). Variables r' and δ' in Eq. (3.3) are the subjective discount rate and subjective dividend yield, respectively. When c = 0.5 (risk/ambiguity neutrality), m = 0 and s = 1, with Eq. (3.2) reducing to the Black-Scholes option pricing model (OPM) (adjusted for constant dividend yield δ). Similar to what has been shown by Gagliardini, Porchia and Trojani (2009) regarding the effect of ambiguity on bond

[3] While more popular in the literature for analysing agents' economic choice, the multiple-prior approach only accounts for uncertainty in the drift. We follow the more flexible Choquet and rank-dependent utility approach which allows for uncertainty in both drift and volatility.

premiums, miscalibration is reflected in adjusted discount rate r' and dividend yield δ' in our ambiguity-adjusted framework. This behavior can distort economic and financial fundamentals, affecting investment, production, hiring and consumption.

We use the VIX as an input[4] to our ambiguity-adjusted option pricing model (A-OPM) of Eqs. (3.2-3.4)[5] to extract ambiguity information from the options market.[6,7] Our extraction of ambiguity information from the VIX can be described intuitively as follows. We first input the VIX as a volatility measure into the Black-Scholes OPM for one-month options (T = 1 m) with strike (K) at the S&P level (at-the-money) to recover an equivalent market price (premium) of ATM options on the index. Using VIX as the source of primary information extraction is equivalent to inferring the price of ATM index options over a 23 year period. We then use our ambiguity-adjusted OPM of Eqs. (3.2 - 3.4) equated to this equivalent price of index options to extract the degree of investor ambiguity aversion (capacity measure c) on each day t. Since Black-Scholes OPM is nested in our A-OPM with c = 0.5 corresponding to risk/ambiguity neutrality, daily deviations of estimated c_t from 0.5 track investors' time-varying implied subjective ambiguity aversion (c < 0.5) or ambiguity seeking (c > 0.5) attitudes. The prediction ability of our ambiguity measure over OPM implied volatility or VIX arises from the explicit recognition of multiple mean ($\mu + m\sigma$) and volatility ($s\sigma$) scenarios in our A-OPM. IA in Eq. (3.7) thus captures ambiguity (Knightian uncertainty) attitudes, beyond risk, as it is the sum

[4] Known as the market fear gauge, VIX contains rich information regarding real economic activity. Bekaert, Hoerova and Lo Duca (2013) show strong co-movement between VIX and monetary policy measured by real interest rates. Bekaert and Hoerova (2014) find that VIX predicts industrial production growth for up to a year, while Zhang, Zhou and Zhu (2009) show VIX predicts cross-sectional credit default swap spreads.

[5] We also used VIX as the implied volatility of at-the-money (ATM) put options in inferring option market ambiguity. The specification of call or put does not affect our results and the informational content of the ambiguity measure inferred.

[6] Although VIX primarily measures risk (implied annualized standard deviation over the next month, interpreted as the % annual move in the S&P 500 index over a year with 67% probability) it has sometimes been used in the literature to partly extract information on uncertainty or ambiguity and as a barometer of investor sentiment (CBOE website). For example, Bloom (2009) finds that economic uncertainty measured by the VIX is associated with lower employment and output. Bekaert and Hoerova (2014) show that VIX^2 predicts industrial production growth (IP) up to a year.

[7] VIX initially (since its inception in 1990) was derived from at-the-money (ATM) options on the S&P100 index using the Black-Scholes OPM (still published by CBOE under VXO). After 2003 CBOE moved to a model-free estimation of near-term expected volatility using a weighted average of S&P500 (SPX) calls and puts with a wide range of strike prices (both at- and out-of-the money) with care to eliminate the volatility smile arising from the use of various strike prices (effectively being analogous to its previous estimation from ATM options using the OPM). A very high correlation between the VIX and VXO series since 1990 (ρ = 0.987) confirms that an equivalent option market price can be inferred by inverting the OPM using the VIX as input for ATM options.

of deviations from risk/ambiguity neutrality (c = 0.5). This represents the divergence in ambiguity beliefs or heterogeneity of ambiguity attitudes among representative ambiguity averse and ambiguity-loving investors in the market. Our results are unchanged if we use option prices, rather than VIX, as inputs in our extraction of the c measure.

By inverting Eq. (3.2) numerically and minimizing the absolute deviations between the theoretical model option price in Eq. (3.2) and the equivalent market price (for ATM options on S&P500) as implied by CBOE's implied volatility index (VIX) over a one-month maturity (T = 1 m), we extract investors' subjective ambiguity attitudes as follows:

$$IAA_t \equiv c_t^{AA} = \underset{c|0<c\leq 0.5}{\arg\min}\left\{\left|C_t^A(S_t, K, r, T, \sigma_t, \mu_t, c_t) - C_t^{Mkt}(S_t, K, r, T, VIX_t)\right|\right\} \tag{3.6}$$

$$IAS_t \equiv c_t^{AS} = \underset{c|0.5\leq c<1}{\arg\min}\left\{\left|C_t^A(S_t, K, r, T, \sigma_t, \mu_t, c_t) - C_t^{Mkt}(S_t, K, r, T, VIX_t)\right|\right\} \tag{3.7}$$

C_t^A above is the theoretical ambiguity-based call option price according to our A-OPM of Eq. (3.2), C_t^{Mkt} is the equivalent market option price (estimated by inverting the option price from VIX using the standard Black-Scholes OPM), S_t is the closing level of the S&P 500 index on day t, K is the strike price, r the risk-free rate, T the time to maturity (set at T = 1 m), σ_t is return volatility (estimated using RiskMetrics EWMA), c_t is the capacity (ambiguity degree) measure, μ_t is the subjective required rate of return (estimated as average return over the previous year), and VIX_t is the closing level of CBOE's VIX on day t.[8] This approach is analogous to Jiang and Tian (2005) in a curve-fitting exercise for computing model-free implied volatility, and to Cremers and Weinbaum (2010) in calculating put-call parity for return predictions. The above procedure allows inferring the equivalent market price of SPX options over a longer window (1990-2012) and extracting option-based ambiguity information. Our conclusions are generally unchanged if we use option prices, rather than VIX, for ambiguity extraction.

[8] The impact of dividend yield (δ) in the extraction process is negligible. In extracting the ambiguity attitudes, we set δ to 0 to avoid potential informational overlapping with dividend yield (DY), one of the alternative predictor variables used in our predictive regressions. For robustness, we also extracted ambiguity with the actual (non-zero) S&P dividend yield confirming the results are essentially the same (with correlation greater than 0.96). This is due to offsetting effects in the procedure as the errors from the two inverse operations (volatility-price conversion and price-ambiguity extraction) tend to cancel out. We present the results without dividend (DY) adjustment to further ensure that the information content picked up in the estimation of our ambiguity (IA) measure is not mixed with (or contaminated by) the information content from DY (itself claimed to be a good predictor of stock returns and certain economic indicators).

The resulting capacity variable (c_t) inferred from numerically solving the minimization problem of Eqs. (3.5) and (3.6) gives the degree of market investor implied ambiguity aversion (*IAA*, when $c <$ 0.5) or ambiguity seeking (*IAS*, when $c >$ 0.5) reflected in option prices. Once market investors' heterogeneous beliefs or time-varying ambiguity aversion attitudes are obtained, we estimate aggregate option market ambiguity on day t (IA_t) as the sum of deviations of each of the implied ambiguity or seeking beliefs (*IAA* and *IAS*) from neutrality (c = 0.5) as follows:

$$IA_t = (|IAS_t - 0.5| + |IAA_t - 0.5|) \tag{3.8}$$

The above is analogous to Abdellaoui et al. (2011) who estimate Knightian uncertainty through deviations from Bayesian expected-utility (rational behavior), and to Kast et al. (2014) who apply the Choquet framework to the Intertemporal CAPM (Merton, 1973). Eq. (3.8) and its deviations from ambiguity-neutrality enable us to capture the overall divergence in ambiguity beliefs among representative option investors. Other related measures of heterogeneity in beliefs relying on deviations from a norm include dispersion in analysts' forecasts and survey-based disagreement among professional forecasters (Anderson et al. 2009). Our ambiguity proxy is obtained directly from market-observed option pricing dynamics. It is also a related notion to the variance risk premium (VRP), being the difference between "risk-neutral" expected stock market variance (VIX^2) (corresponding to c = 0.5 in our A-OPM) and (actual or "physical") realized variance (RV) reflecting investor risk-aversion attitudes, but extended here to ambiguity-aversion involving c < 0.5 (aversion) or c > 0.5 (seeking). VRP has also been suggested to contain uncertainty or ambiguity information (e.g., Drechsler, 2013). However, although both IA and VRP rely on information from implied volatility (VIX or VIX^2) and realized variance (RV), allowing to infer investors' required premium, VRP simply relies on their difference whereas IA uses a different channel (distinct from VRP) capitalizing on our specific ambiguity-adjusted OPM (A-OPM) of Eq. (3.2) with parametric uncertainty in both drifts and volatilities (see Eq. (3.1)). Interpretations of VRP as an ambiguity proxy are, on the other hand, based on multiple-priors explanations (i.e., involving uncertainty in drift only) (e.g., Drechsler, 2013). Effectively, using VIX to infer a Black-Scholes option price equivalent and then using the ambiguity-based model (A-OPM) on that equivalent market price given the estimated realized variance and other inputs enables extracting more ambiguity-related information than the one contained in VIX or VRP. In light of this shared commonality in informational sources, we include VIX and VRP as benchmarks in our analysis and provide (in section 3.5.4.5) direct comparisons of their information content vs. IA. Our use of VIX as a source of ambiguity information extraction (as an alternative to directly using option prices) is due to (longer) data availability, coverage, quality and comparability to extant research.[9] In section 3.5.4., we confirm that our predictive regression findings are not due to using VIX as a source of ambiguity information extraction and that they generally hold if IA is inferred directly from SPX option prices. This holds even after controlling for market-induced risk aversion. To confirm the validity of option market ambiguity as a relevant financial market-based proxy for aggregate uncertainty, we compare it to other established uncertainty proxies from extant economics research. Validation results confirm IA is significantly correlated to seven out of eight macroeconomic uncertainty indicators, with expected signs.

To examine the behavior of a range of economic indicators in response to ambiguity shocks we employ a five-variable VAR system which, besides option market ambiguity (IA), includes industrial production growth (IP), total non-farm payroll (TNP), personal consumption expenditure (PCE), and the Chicago Fed National Activity Index (CFNAI). This VAR system considers the dynamics between option market ambiguity and changes in economic activity including production, employment, consumption and overall output. We use variance decomposition, Granger causality and impulse-response analysis for this purpose. After validating the predicted negative impact of ambiguity on real economic activity using the aforementioned methods, we investigate the informational efficiency of our option market extracted ambiguity measure in long horizon (up to eight quarter) economic predictions.

In ascertaining the relationship between option-market ambiguity and ex post economic activity, we consider an extensive set of economic activity indicators, two per sector. These indicators include:

[9] VIX represents the aggregate investor expectation of future volatility (based on realized option transactions). As such, VIX summarizes economic agents' expectation of future variability in the financial markets and the real economy. VIX further contains aggregate information from the most liquid portions of the volatility surface that is representative of the information set contained in the option markets. Finally, VIX data is readily and widely available providing information on economic agents' forward-looking expectations in real-time. VIX is a model-free option implied volatility measure which does not rely on a specific model such as the Black-Scholes OPM. The correlation between VIX and VXO, the former version of CBOE volatility index which relies on the Black-Scholes model with underlying the S&P100 index from 1990 to 2012 is 0.99. This extremely high correlation supports using the VIX as the at-the-money (ATM) implied volatility.

growth in industrial production (IP) and capacity utilization (CU) as measures of production activity; in total non-farm payroll (TNP) and in unemployment rate (UR) as measures of (un)employment; in personal consumption expenditure (PCE) and durable goods consumption (DG) as measures of consumption activity; and in real GDP per capita (GDPC) and the Chicago Fed National Activity Index (CFNAI) as measures of overall economic performance. The above eight indicators are used to test the theoretical predictions on the negative relationship between Knightian uncertainty and real economic activity. Since production drives demand for labor (employment), which in turn affects consumption decisions, we investigate the relationship between option-market ambiguity and real economic activity in the following order: production, employment, and consumption. After analysing the effect of market ambiguity on each of these categories of economic activity, we turn to the bigger picture concerning general economic output (based on real GDP growth per capita and the CFNAI indicators).

In benchmarking to other known financial market predictor variables, we compare implied ambiguity (IA) to the following: the credit spread (CS), dividend yield on the S&P 500 composite stock index (DY), earnings-to-price ratio (EP), market excess return (ER), term spread (TS), implied volatility of the S&P 500 index (VIX) and the variance risk premium (VRP).[10] Detailed specifications and references on these standard predictor variables are given in the next section and summarized in Table 3.1.

In the last part we employ standard long horizon predictive regressions (e.g., Fama, 1990; Schwert, 1990; Cochrane, 1991; Carroll, Fuhrer and Wilcox, 1994; Yang, 2011; Chen and Zhang, 2011) with various lags predicting ex post economic activity using our time-varying market implied ambiguity (IA) and other standard predictors. Our standard predictive regression takes the form:

$$y_{t+k}^{i} = \alpha + \beta x_t^j + \varepsilon_{t+k} \tag{3.9}$$

where y_{t+k}^{i} is ex post economic activity growth over k-months for economic indicator i, x_t^j is a 1 x h row vector of explanatory variables (excluding intercept), α is an h x 1 vector of intercepts, and β is

[10] The real risk-free interest rate was also considered as a possible predictor but was generally insignificant in univariate tests, except for CU, hence it was not included in subsequent analysis.

an h x 1 vector of slope coefficients. To address the overlapping issue arising from measurement of long-horizon growth (where $k > 1$) and for comparability, we closely follow extant literature (e.g. Cochrane, 1991; Estrella and Hardouvelis, 1991; De Lint and Stolin, 2003; Berardi and Torous, 2005; Berardi, 2009; Chen and Zhang, 2011; Yang, 2011; Allen, Bali and Tang, 2012; Bekaert and Hoerova, 2014) by considering robust t-statistics based on Newey and West (1987) standard errors.

3.4. Economic Activity Data and Variables Description

3.4.1. Dependent Variables

Production. In examining the predictive ability of market implied ambiguity (IA) concerning the growth of future production, we employ two indicators: growth in industrial production (IP) and growth in capacity utilization (CU). Monthly data on industrial production and capacity utilization from December 1989 to December 2014 are obtained from the Board of Governors of the Federal Reserve System. IP and CU are computed as the logarithmic change of the relevant indicator over a k-month horizon. Since industrial production values are denoted in real terms, no inflation adjustment is needed.

Employment. We use the unemployment rate growth (UR) and total non-farm payroll growth (TNP, net hiring) as indicators for employment activity. Monthly data of total non-farm payroll and unemployment rate from December 1989 to 2014 are obtained from the Bureau of Labor Statistics. UR and TNP are computed as the logarithmic change of the relevant indicator over a horizon of k months.

Consumption. For consumption indicators, we consider personal consumption expenditures growth (PCE) and personal consumption expenditures on durable goods consumption growth (DG). Monthly consumption data covering December 1989 to December 2014 are obtained from the Bureau of Economic Analysis (BEA). All values of the consumption indicators are divided by population and adjusted by the consumer price index (CPI) to obtain real consumption per capita. PCE and DG are the logarithmic change of the relevant per capita indicator in real terms over k month(s).

Overall Economic Output. In addition to the three key separate aspects of economic activity described above (namely production, employment and consumption), we also investigate the relationship of IA to overall economic activity. We consider real gross domestic product (GDP) per capita and the Chicago Fed National Activity Index (CFNAI) as overall economic indicators. Quarterly data of real GDP is collected from the BEA. Real GDP per capita growth (GDPC) is then computed as the logarithmic change of the relevant indicator over q quarter(s). Changes in aggregate economic output proxied by the CFNAI are computed as the average of the index over a k-month horizon.

3.4.2. Predictor Variables and Controls

We estimate option-market implied ambiguity (IA) based on Equations (3.4)-(3.6) using the closing level of the VIX index obtained from the Chicago Board Options Exchange (CBOE). Our option dataset relying on VIX covers the period from January 1990 to December 2012 when VIX data are available. Our results are robust to the use of VIX or option price data for extracting IA. We limit our dataset to a period up to 2012 to allow a 24-month window for the estimation of growth rates for the various economic indicators. To estimate IA, besides the VIX index closing levels, we estimate other input parameters needed for our calibration and option pricing models. We use the one-month USD LIBOR as the risk-free interest rate (r), the one-year geometric return on the S&P 500 index as a proxy for the subjective required return for S&P 500 investors (μ), and RiskMetrics EWMA volatility as the S&P return volatility measure (σ). Our results are robust to alternative input estimations.[11]

In addition to option-market implied ambiguity (IA), we consider a number of known financial market based predictors of economic activity for benchmarking. These include the aggregate dividend yield (DY) on the S&P 500 index (Yang, 2011), the term spread (TS) calculated as the difference between 10-year T-bond and 1-year T-bill yields (Harvey, 1988; Estrella and Hardouvelis, 1991; Plosser and Rouwenhorst, 1994; Rendu de Lint and Stolin, 2003; Estrella, 2005; Ang, Piazzesi and Wei, 2006; Chen and Zhang, 2011), the credit spread (CS) computed as the difference between Moody's BAA and AAA yield indices (Gilchrist, Yankov and Zakrajšek, 2009; Chen and Zhang, 2011), option implied volatility of the S&P 500 index as measured by the CBOE VIX (Bloom, 2009;

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[11] For robustness, we test the predictive power of IA using alternative inputs for the subjective required rate of return μ and volatility σ. As alternative proxies for μ we used 6, 12, 18, 24, 36, and 60 months recent historical annualized returns. As alternative volatility specifications (to using RiskMetrics EWMA), we used (i) a simple 22-day historical standard deviation of returns and (ii) out-of-sample GARCH(1,1) with three-year rolling estimation window. The predictive power of IA is not significantly affected by the choice of these inputs.

Bekaert and Hoerova, 2014), market excess return (ER) (Fama, 1981 and 1990; Barro, 1990; Schwert, 1990; Cochrane, 1991; Beaudry and Portier, 2006) of the S&P 500 index as measured by the monthly logarithmic return of S&P 500 index in excess of the logarithmic yield of 3-month treasury bills, the aggregate price-to-earnings ratio (EP) (Rapach, Strauss and Zhou, 2010) of S&P 500 index constituents, and the variance risk premium (VRP) (e.g., Zhou, 2009; Bekaert and Hoerova, 2014) as measured by the difference between S&P 500 index implied variance (VIX^2) and realized variance computed as the sum of squared returns using intra-day 5-mins index data. As our measure of option ambiguity is a market based predictor, we restrict comparison to analogous market based predictors to provide fair comparisons.

Aggregate DY and EP data are from Robert Shiller's website. US 10-year T-bill, 1-year T-bill yields, Moody's BAA yield index and Moody's AAA yield index data for computing the term and credit spreads (TS and CS) are from the Federal Reserve Bank of St. Louis's Federal Reserve Economic Data (FRED). S&P 500 index data for calculation of monthly excess returns are from Thomson Reuters Datastream. VRP data is obtained from Hao Zhou's website. A summary of all above variables is provided in Table 3.1.

Table 3.1. Descriptions of Variables, Data Series, and Data Sources

Table 3.1 describes the variables, data and data sources. All data series from websites of cited sources are downloaded in December 2014. All data series span a common sample period from January 1990 to December 2014 unless otherwise specified in the descriptions.

Panel A. Predictor Variables

Category	Abbreviation	Corresponding Indicator	Description	Source
Uncertainty Measures	IA	Options Market Ambiguity	Estimated by rank dependent option pricing model according to (7). End of month values.	-
	RV	S&P500 Realized Variance	Computed as the sum of squared returns using intra-day 5-min S&P500 index prices	Hao Zhou's website
	VIX	S&P500 Implied Volatility	S&P 500 option implied volatility based on the CBOE VIX index.	Chicago Board Options Exchange
	VRP	Variance Risk Premium	Variance risk premium defined as the difference between realized variance and implied variance of S&P 500 return.	Hao Zhou's website
Equity fundamentals	DY	Dividend Yield	Aggregate dividend yield of S&P 500 composite.	Robert Shiller's website
	EP	Earnings to Price Ratio	Reciprocal of aggregate price to earnings ratio of S&P 500 composite.	Robert Shiller's website
	ER	S&P500 Excess Return	S&P 500 index return in excess of 3-month treasury bond yield.	Thomson Datastream
Bond fundamentals	CS	Credit Spread (BAA yield - AAA yield)	Difference between Moody's BAA and AAA corporate bond yield.	Federal Reserve Bank of St. Louis FRED
	TS	Yield Curve (Term Spread, 10Y T-yield - 3M T-yield)	Difference between 10-year and 3-month U.S. treasury bond yield.	Federal Reserve Bank of St. Louis FRED

Panel B. Real Economic Activity Measures

Category	Abbreviation	Corresponding Indicator	Description	Source
Production	IP	Industrial Production Growth	Logorithmic change of k-month horizon industrial production index, measured as the real seasonally adjusted output for all facilities located in the United States manufacturing, mining, and electric, and gas utilities.	Board of Governors of the Federal Reserve System
	CU	Capacity Utilization Ratio Growth	Logorithmic change of k-month horizon capacity utilization measured as the percentage of resources used by corporations and factories to produce goods in manufacturing, mining, and electric and gas utilities for all facilities located in the United State.	Board of Governors of the Federal Reserve System
Employment	TNP	Total Non-farm Payroll Growth	Logorithmic change of k-month horizon total nonfarm payroll, measured as the seasonally adjusted number of U.S. workers in the economy that excludes proprietors, private household employees, unpaid volunteers, farm employees, and the unincorporated self-employed.	U.S. Bureau of Labor Statistics
	UR	Unemployment Rate Growth	Logorithmic change of k-month horizon unemployment rate, measured as the seasonally adjusted number of unemployed as a percentage of the labor force.	U.S. Bureau of Labor Statistics
Consumption	PCE	Personal Consumption Expenditure Growth (Real, per capita)	Logorithmic change of k-month horizon personal consumption expenditure per capita, measured as the seasonally adjusted per capita real value of goods and services purchased by U.S. residents.	U.S. Bureau of Economic Analysis
	DG	Durable Goods Expenditure Growth (Real, per capita)	Logorithmic change of k-month horizon durable goods expenditure per capita, measured as the seasonally adjusted per capita real value of durable goods purchased by U.S. residents. Durable goods is defined as tangible commodities that can be stored or inventoried and that have an average life of at least 3 years.	U.S. Bureau of Economic Analysis
Overall Output	GDPC	Real GDP per Capita Growth	Logorithmic change of q-month quarter real gross domestic product per capita. Sample: 1990Q1 to 2012Q4.	U.S. Bureau of Economic Analysis
	CFNAI	Chicago Fed National Activity Index	Sum of k-month Chicago Fed National Activity Index.	Federal Reserve Bank of Chicago

Panel C. Macroeconomic Uncertainty Proxies

Category	Abbreviation	Corresponding Indicator	Description	Source
Statistical based measures	CV_{CFNAI}	Conditional Variance of Chicago Fed National Activity Index	Conditional Variance of Chicago Fed National Activity Index estimated by GARCH(1,1)	-
	CV_{IP}	Conditional Variance of Industrial Production Growth	Conditional Variance of Industrial Production Growth estimated by GARCH(1,1)	-
	$MUNC_{BBC}$	Bali et al (2014) Macroeconomic Uncertainty	Macroeconomic Uncertainty measure according to Bali, Brown, and Caglayan (2014)	Turan Bali's website
	$MUNC_{JLN}$	Jurado et al.(2015) Macroeconomic Uncertainty	Macroeconomic Uncertainty measure with 1-, 3-, and 12-month forecasting horizons according to Jurado, Ludvigson and Ng (2015)	Sydney Ludvigson's website
Survey / media-coverage based measures	CCI	Consumer Confidence Index	Consumer Confidence Index	Federal Reserve Bank of St. Louis FRED
	PUI	Economic Policy Uncertainty Index	Economic Policy Uncertainty Index	http://www.policyuncertainty.com/
	SPF	Forecast Dispersion for Survey of Professional Forecaster	Forecast Dispersion for Survey of Professional Forecaster in forecasting GDP for current quarter, and 1-4 quarters ex post	Federal Reserve Bank of Philadelphia website
	UMCSI	University of Michigan Consumer Sentiment Index	University of Michigan Consumer Sentiment Index	Federal Reserve Bank of St. Louis FRED

3.5. Empirical Results

3.5.1. Summary Statistics

Figure 3.1 plots the time-varying levels of option-market implied ambiguity (IA) compared to option implied volatility (CBOE's VIX) and each of the eight economic activity indicators, with shaded areas representing NBER recessions. Graph 1 in Figure 3.1 reveals prolongedly inflated IA values during the more uncertain periods (recessions), including the 1990 recession, 1999 dot-com bubble, the 2008 financial crisis and the 2010 Eurozone debt crisis. IA is seen to be positively but loosely correlated with the VIX shown in Graph 2 and exhibits volatile fluctuations.[12] The IA plot in Graph 1 also has some resemblance to (and sometimes leads) the other graphs (shown in pairs) in Figure 3.1 depicting fluctuations in production (Panel B), employment (Panel C), consumption (Panel D), and overall economic output (Panel E).

Table 3.2 Panel A reports the descriptive statistics of our eight indicators of economic activity and the eight standard predictor variables concerning risk, uncertainty, equity and bond fundamentals (summarized in Table 3.1). All statistics for the predictor variables and indicators of economic activity are based on monthly observations, except for GDPC that is based on quarterly observations. Among the predictor variables, IA, ER and VRP show low levels of first-order autocorrelation (ranging from 0.07 to 0.36). Other predictor variables including DY, EP, CS, TS and VIX generally show very high first-order autocorrelations (ranging from 0.85 to 0.99 based on monthly observations). In light of the high persistence of these predictor variables, adjusted R^2 needs to be interpreted with care. The low first-order autocorrelation of our implied ambiguity measure IA largely mitigates the concern of spurious regressions when compared to highly auto-correlated predictor variables such as VIX, CS, EP, TS and DY. Despite the inference concerns regarding these highly persistent standard predictor variables, they are considered for benchmarking and comparability with extant research. Concerning the basic descriptive statistics of the economic activity indicators, their comparative

[12] IA is more volatile than the VIX as the latter captures mostly risk (volatility) whereas the former captures something akin to volatility of volatility (i.e., uncertainty in both rates of return and market volatility) and more extreme economic scenarios that are harder to specify with confidence. IA may also capture investor or consumer sentiment which may endogenously affect future macroeconomic outcome changes (such as consumption and employment growth), which IA is able to predict better than most standard predictors.

Figure 3.1. Market Ambiguity, S&P 500 Implied Volatility and Economic Activity Indicators

Figure 3.1 shows time-varying levels of option market ambiguity, S&P 500 implied volatility and (pairs of) economic activity indicators concerning production, employment, consumption, and overall economic output. All variables are sampled with monthly frequency except GDPC which is sampled with quarterly frequency. Shaded areas represent NBER recession periods based on quarterly dates. The sample period spans Jan 1990 to Dec 2014.

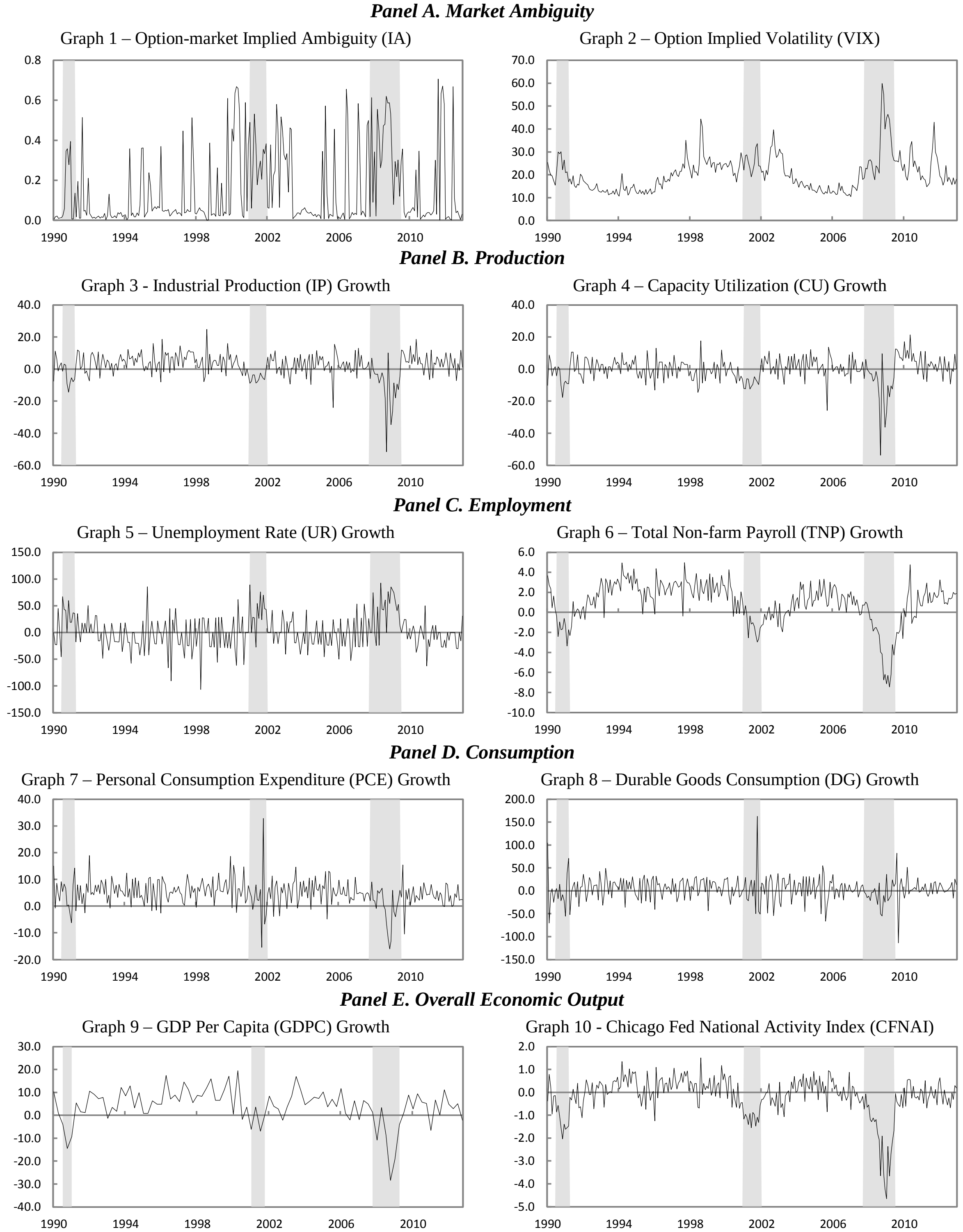


Table 3.2. Descriptive Statistics and Correlations

Table 3.2 reports the descriptive statistics and correlation matrices. IA is option market ambiguity. VIX is the CBOE volatility index. VRP is the variance risk premium calculated as the difference between implied variance and realized variance. DY is the dividend yield of the S&P 500 index. EP is the earnings to price ratio of the S&P 500 index. ER is the excess return of the S&P 500 index calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bill. CS is the credit spread between Moody's AAA and BAA bond yield indices. TS is the term spread between 10Y T-bond and 3M T-bill yields. IP and CU denote industrial production growth and capacity utilization ratio growth respectively. TNP and UR represent total non-farm payroll growth and unemployment rate growth respectively. PCE and DG denote personal consumption expenditure per capita growth and personal consumption expenditure on durable goods per capita growth respectively. GDPC denotes gross domestic product per capita growth. CFNAI is the Chicago Fed National Activity Index. All variables are reported in annualized percentage whenever possible. Descriptive statistics for predictor variables, production indicators, employment indicators, consumption indicators, and CFNAI are computed using monthly samples covering observations from 1990M01 to 2014M12. Descriptive statistics for GDPC are computed using quarterly data covering 1990Q1 to 2014Q4.

Panel A. Descriptive Statistics

	Predictor Variables								Real Economic Activity Measures							
	Uncertainty Measures			Equity Fundamentals			Bond Fundamentals		Production		Employment		Consumption		Overall Economic Activity	
	IA	*VIX*	*VRP*	*DY*	*EP*	*ER*	*CS*	*TS*	*IP*	*CU*	*TNP*	*UR*	*DG*	*PCE*	*GDPC*	*CFNAI*
Mean	0.15	20.43	18.47	2.10	4.67	6.03	0.97	1.88	2.02	-0.30	0.94	1.65	4.00	4.83	4.15	-0.17
Std. Dev.	0.20	7.77	20.35	0.66	1.40	52.51	0.42	1.16	7.98	8.00	2.10	31.67	26.52	5.16	7.59	0.86
Skewness	1.29	1.59	-2.48	0.66	-0.48	-0.77	3.06	-0.15	-1.74	-1.57	-1.22	0.29	0.36	-0.12	-1.28	-1.86
Kurtosis	0.29	4.04	35.17	-0.48	-0.10	1.57	12.23	-1.14	8.79	8.21	2.42	0.59	6.09	4.93	3.68	5.92
AR(1)	0.36	0.85	0.26	0.99	0.98	0.07	0.96	0.98	0.24	0.23	0.79	0.05	-0.26	-0.09	0.44	0.69

Panel B. Correlation Matrix for Monthly Sample

	IA	*VIX*	*VRP*	*DY*	*EP*	*ER*	*CS*	*TS*	*IP*	*CU*	*TNP*	*UR*	*DG*	*PCE*	*CFNAI*
IA	1.00														
VIX	0.37	1.00													
VRP	-0.20	0.31	1.00												
DY	-0.11	-0.04	-0.02	1.00											
EP	-0.16	-0.44	-0.20	0.26	1.00										
ER	-0.02	-0.39	-0.03	-0.02	0.06	1.00									
CS	0.30	0.61	0.04	0.29	-0.35	-0.13	1.00								
TS	-0.08	0.05	-0.03	0.35	-0.19	-0.04	0.27	1.00							
IP	-0.20	-0.25	-0.04	-0.13	0.20	0.02	-0.44	0.02	1.00						
CU	-0.18	-0.22	-0.06	-0.09	0.18	-0.01	-0.31	0.18	0.95	1.00					
TNP	-0.29	-0.51	-0.10	-0.24	0.43	0.13	-0.75	-0.24	0.53	0.40	1.00				
UR	0.17	0.28	0.05	0.15	-0.25	-0.09	0.37	0.04	-0.39	-0.35	-0.45	1.00			
DG	-0.10	-0.08	0.12	-0.07	-0.01	0.04	-0.13	0.00	0.10	0.07	0.15	0.00	1.00		
PCE	-0.19	-0.26	0.12	-0.15	0.05	0.11	-0.37	-0.08	0.24	0.20	0.32	-0.11	0.78	1.00	
CFNAI	-0.31	-0.49	-0.05	-0.28	0.31	0.12	-0.72	-0.08	0.82	0.74	0.82	-0.55	0.17	0.38	1.00

Panel C. Correlation Matrix for Quarterly Sample

	IA	*VIX*	*VRP*	*DY*	*EP*	*ER*	*CS*	*TS*	*GDPC*
IA	1.00								
VIX	0.28	1.00							
VRP	-0.14	0.72	1.00						
DY	-0.03	-0.04	0.03	1.00					
EP	-0.11	-0.42	-0.15	0.23	1.00				
ER	0.00	-0.24	-0.16	-0.05	0.00	1.00			
CS	0.39	0.56	0.15	0.28	-0.36	-0.05	1.00		
TS	-0.12	0.05	0.06	0.34	-0.20	-0.13	0.25	1.00	
GDPC	-0.38	-0.38	0.02	-0.37	0.15	0.14	-0.64	-0.06	1.00

annualized mean growth rates provide a good snapshot of aggregate shocks to the US economy during the last two decades. Real industrial production (mean IP of 2.02%) grew slower than overall per capita economic growth (mean GDPC of 4.15%), while real consumption per capita (mean PCE of 4.83%) grew in pace with overall per capita real economic growth.

Panel B of Table 3.2 reports the correlation matrix of the predictor variables and economic indicators based on monthly data. Correlations among contemporaneous predictor variables are generally low except for those between CS and VIX ($\rho = 0.61$) and EP and VIX ($\rho = -0.44$). To reduce the risk of incorrect and misleading inference due to multicollinearity, we avoid including highly-correlated predictor variables together in our multivariate regressions. Absolute values of correlations between contemporaneous predictor variables and monthly economic activity indicators are below 0.5, except for three pairs: CS and CFNAI (ρ = -0.72), TNP and CS (ρ = -0.75), and TNP and VIX (ρ = -0.51).

By comparison, the correlation matrix based on quarterly data in Panel C of Table 3.2 confirms that correlations among predictor variables are generally in line with those of the monthly sample (Panel B) except for VRP. When sampled with a quarterly frequency, the correlation between VRP and VIX, one of the sources of VRP's information extraction, increases from 0.31 to 0.72. This is likely due to the mean-reverting property of the realized volatility component of VRP. By contrast, despite also relying on VIX as a main information source, IA's correlation with VIX declines from 0.37 to 0.28 in quarterly data, suggesting that IA and VIX contain different sets of information for different sampling frequencies.

3.5.2. Validating Option Market Ambiguity as a Measure of Aggregate Uncertainty

Similar to Bali and Zhou (2016) and Bekaert and Hoerova (2016), we compare the correlations between IA and established economic uncertainty proxies to validate the suitability of IA as a financial market-based proxy for aggregate uncertainty. While comparing option market ambiguity to established macroeconomic uncertainty indicators, we bear in mind two important distinctions: 1) IA does not rely on ex post information, thus providing real-time predictive information to future real activity; 2) IA is a financial market-based measure that captures forward-looking information from options.

Table 3.3 reports correlations between IA and both statistical- and survey/media-based macroeconomic uncertainty proxies. For statistical-based uncertainty measures, following Bali and Zhou (forthcoming), we consider the conditional variance of the Chicago Fed National Activity Index (CV_{CFNAI}) and of industrial production growth (CV_{IP}) estimated using GARCH(1,1) models (Bollerslev (1986)); the macroeconomic uncertainty measure ($MUNC_{BBC}$) of Bali, Brown, and Caglayan (2014) based on Principle Component Analysis (PCA); and macroeconomic uncertainty ($MUNC_{JLN}^{1M}$ etc.) of Jurado, Ludvigson and Ng (2015) measured by a weighted conditional variance of financial and macroeconomic series with 1-, 3-, and 12-month forecasting horizons. For survey/media-based uncertainty measures, we consider the University of Michigan ConsumerSentiment Index (UMCSI); Consumer Confidence Index (CCI); disagreement among economic forecasters from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters ($SPF_{CQ}\ etc.$) with different forecasting horizons; and the Economic Policy Uncertainty Index (PUI, Baker, Bloom and Davis, 2013).

Table 3.3. Correlations with Macroeconomic Uncertainty Proxies

Table 3.3 reports correlations between option market ambiguity IA and other established macroeconomic uncertainty proxies. CV_{CFNAI} is the conditional variance of the Chicago Fed National Activity Index estimated by GARCH(1,1); CV_{IP} is the conditional variance of industrial production growth estimated by GARCH(1,1); $MUNC_{BBC}$ is the macroeconomic uncertainty measure according to Bali, Brown, and Caglayan (2014); $MUNC_{JLN}^{1M}$, $MUNC_{JLN}^{6M}$, and $MUNC_{JLN}^{12M}$ are the macroeconomic uncertainty measures according to Jurado, Ludvigson and Ng (2015) with 1-, 3-, and 12-month forecasting horizons respectively; CCI is the consumer confidence index; PUI is the economic policy uncertainty index; $SPF_{CQ}, SPF_{CQ}, SPF_{CQ}, SPF_{CQ}$, and SPF_{CQ} are dispersion of the Survey of Professional Forecasters in forecasting GDP for current quarter, and 1-4 quarters ex post respectively; and $UMCSI$ is the University of Michigan Consumer Sentiment Index. Correlations are computed using monthly samples covering observations from 1990M01 to 2014M12. p-values are shown in parenthesis. *, **, and *** represent significance at 90%, 95%, and 99% confidence levels respectively.

Panel A. Correlations with statistical based measures

	IA		IA
CV_{CFNAI}	0.189***	$MUNC_{JLN}^{1M}$	0.386***
	(0.002)		(0.000)
CV_{IP}	0.190***	$MUNC_{JLN}^{3M}$	0.393***
	(0.002)		(0.000)
$MUNC_{BBC}$	0.159**	$MUNC_{JLN}^{12M}$	0.405***
	(0.017)		(0.000)

Panel B. Correlations with survey / media-coverage based measures

	IA		IA
CCI	-0.026	SPF_{2Q}	0.263**
	(0.667)		(0.011)
PUI	0.181***	SPF_{3Q}	0.283***
	(0.003)		(0.006)
SPF_{CQ}	0.240**	SPF_{4Q}	0.356***
	(0.021)		(0.001)
SPF_{1Q}	0.268***	$UMCSI$	-0.155***
	(0.010)		(0.010)

From Panel A of Table 3.3, the correlations between IA and each of the statistical-based macroeconomic uncertainty measures are all positive and significant. Out the four statistical-based macro uncertainty indicators, Jurado, Luvigson and Ng's (2015) measures are reported to have relatively less noise (Bekaert and Hoerova, 2016) due to their use of forecasting errors with many economic series. The correlation between IA and $MUNC_{JLN}$ with different forecasting horizons ranges from 0.386 to 0.405 with expected positive signs. From Panel B of Table 3.3, IA is significantly correlated with all survey/media-coverage based uncertainty proxies, except for the consumer confidence index. Among this group of uncertainty indicators, IA achieves the highest correlation of

0.356 with SPF_{4Q} which measures the dispersion in forecasts among professional forecasters for GDP growth 4 quarters ahead. We also find our ambiguity measure to be more significantly positively correlated (ρ = 0.49) with a principal component combining statistical- and survey/media-based uncertainty information (i.e., $MUNC_{JLN}$ and SPF_{4Q}). The above and the results in Table 3.3 confirm the validity of IA as a proxy for aggregate uncertainty (and divergence in beliefs) in the economy. With a simple extraction methodology relying on forward-looking option data, IA captures a rich set of macroeconomic uncertainty information on a real-time basis.

3.5.3. Impact of Option Market Ambiguity on Real Economic Activity

To investigate the impact of option market ambiguity on various sectors of the real economy, we employ a five-variable VAR system using industrial production growth (IP), total non-farm payroll growth (TNP), personal consumption expenditure per capita growth (PCE), and changes in the Chicago Fed National Activity Index (CFNAI), in addition to implied ambiguity (IA). We specify the VAR system with a constant and five lags based on minimization of the Akaike information criterion:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{5,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_5 \end{bmatrix} + \begin{bmatrix} a_{1,1}^1 & a_{1,2}^1 & \cdots & a_{1,5}^1 \\ a_{2,1}^1 & a_{2,2}^1 & \cdots & a_{2,5}^1 \\ \vdots & \vdots & \ddots & \vdots \\ a_{5,1}^1 & a_{5,2}^1 & \cdots & a_{5,5}^1 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{5,t-1} \end{bmatrix} + \cdots + \begin{bmatrix} a_{1,1}^5 & a_{1,2}^5 & \cdots & a_{1,5}^5 \\ a_{2,1}^5 & a_{2,2}^5 & \cdots & a_{2,5}^5 \\ \vdots & \vdots & \ddots & \vdots \\ a_{5,1}^5 & a_{5,2}^5 & \cdots & a_{5,5}^5 \end{bmatrix} \begin{bmatrix} y_{1,t-5} \\ y_{2,t-5} \\ \vdots \\ y_{5,t-5} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{5,t} \end{bmatrix} \tag{3.10}$$

where $y_{1,t}$ to $y_{5,t}$ represent each of the five variables at time *t* including IP, TNP, PCE, CFNAI, and IA.

Table 3.4 summarizes our variance decomposition and Granger causality results from the above VAR system.[13] Panel A gives the percentage of 24-month forecast error variance explained by innovations (shocks) in each variable based on the VAR system, while Panel B reports the p-value from Granger causality analysis. Panel A of Table 3.4 indicates that market ambiguity IA is only minimally explained by the other four economy indicators considered in the system. Among the four economic indicators, TNP does best but only explains 3.86% of the 24-month forecast error in IA. By

[13] All the economic activity indicators used are log changes (e.g., industrial production growth rather than production level) so the dependent variables are not persistent. In the VAR system specification all the time series variables are stationary. Investors' Knightian uncertainty perceptions influence future macroeconomic activity by affecting the *changes* (growth) rather than the level itself. Since IA captures Knightian uncertainty and miscalibration, it represents a "top-up" part beyond VIX-related risk expectations. As an econometric analogue to IA and a market return predictor for up to 12 months (with low persistence), VRP captures only part of this miscalibration information.

contrast, IA seems to explain better the forecast error variance of all four economic indicators. IA explains 22.88% of CFNAI's and 25.22% of TNP's forecast error variance, though only 12.68% of IP's and 6.22% of PCE's. These results suggest ambiguity shocks are important in explaining the forecast variance in main economic indicators.

Table 3.4. Variance Decomposition and Granger Causality

Table 3.4 reports the variance decomposition and Granger causality results. Panel A reports the 24-month forecast error variance explained by innovations (shocks) in each of the variables. Panel B reports the p-value of Granger causality tests with null hypothesis of no Granger causality. IA is the option market ambiguity. IP denotes industrial production growth. TNP represents total non-farm payroll growth. PCE denotes personal consumption expenditure per capita growth. CFNAI is the Chicago Fed National Activity Index. The VAR system includes monthly sample covering observations from 1990M01 to 2014M12.

Panel A. Variance Decomposition

Dependent	Explained by Innovations in				
Variable	*IA*	*IP*	*PCE*	*TNP*	*CFNAI*
	(%)	(%)	(%)	(%)	(%)
IA	88.42	3.01	1.77	3.86	2.94
IP	12.68	67.07	4.90	4.90	10.45
PCE	6.22	6.10	79.81	3.94	3.93
TNP	25.22	20.13	4.08	38.10	12.47
CFNAI	22.88	37.55	6.38	10.73	22.45

Panel B. Granger Causality

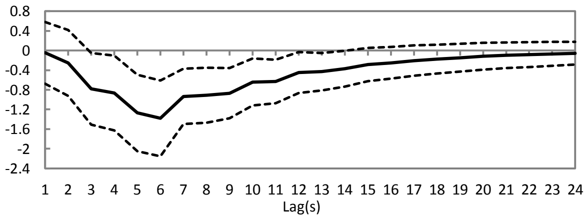
Dependent	Granger Caused by				
Variable	*IA*	*IP*	*PCE*	*TNP*	*CFNAI*
IA	-	0.69	0.12	0.93	0.75
IP	0.02	-	0.18	0.20	0.00
PCE	0.04	0.54	-	0.34	0.36
TNP	0.02	0.08	0.33	-	0.00
CFNAI	0.01	0.00	0.02	0.87	-

To further understand the impact of ambiguity (IA) shocks on real economic activity, Granger causality results are presented in Panel B of Table 3.4. IA explains in a Granger-causal sense all four economic indicators at the 95% confidence level (p-value < 0.05). In terms of causal relationships in the reverse direction, none of the four economic activity indicators Granger-causes IA in the system. We further perform impulse response analysis (shown in Figure 3.2) to guide as to the signs expected for long-horizon predictions (up to 24 months). Figure 3.2 suggests that industrial production IP, employment measured by TNP, and overall output measured by CFNAI respond negatively to shocks in option-market ambiguity (IA) throughout a majority of the 24-month lags considered. PCE generally also responds negatively to shocks in IA in the first two month lags. The above impulse-

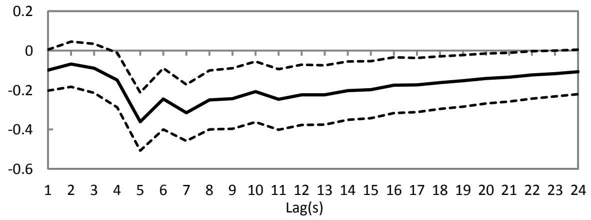
Figure 3.2. Responses of Select Economic Indicators to Shocks in Option Market Ambiguity (IA)

Figure 3.2 shows impulse responses of industrial production (IP) growth, total non-farm payroll (TNP) growth, personal consumption expenditure (PCE) growth, and the Chicago Fed National Activity Index (CFNAI) to shocks in IA. Dotted lines represent confidence bands at 90% level. The sample period spans Jan 1990 to Dec 2014.

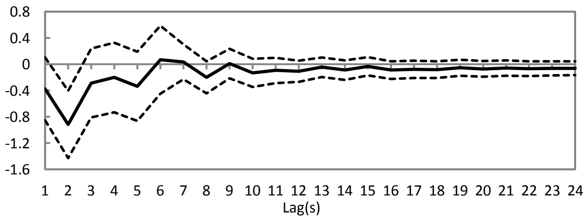
Panel A. Response of Industrial Production (IP) Growth to shock in IA



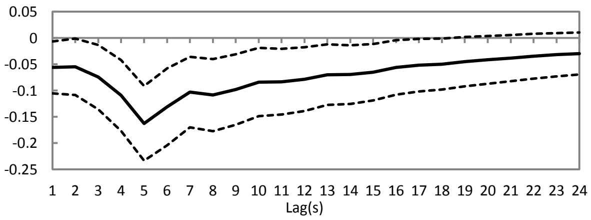
Panel B. Response of Total Non-farm Payroll (TNP) Growth to shock in IA



Panel C. Response of Personal Consumption Expenditure (PCE) Growth to shock in IA



Panel D. Response of Chicago Fed National Activity Index (CFNAI) to shock in IA



response analysis broadly confirms the findings obtained from the previous variance decomposition and Granger causality tests, suggesting that market ambiguity is a significant predictor of real economic activity. Overall, these findings highlight the important role of option market ambiguity in negatively impacting subsequent real economic activity and providing a solid foundation for long-horizon prediction of various economic sector indicators up to 24 months or 8 quarters. Given the negative impact of ambiguity shocks on economic activity corroborated by our Granger causality tests and impulse response analysis (in line with mentioned economic theories), we next turn to examining the informational efficiency of option market-implied ambiguity in long-horizon predictions of economic sector activity.

3.5.4. Predictive Performance of Market Ambiguity

The predictive ability of each of eight predictor variables (IA, VIX, VRP, DY, EP, ER, CS, TS) on the various economic sector indicators (two indicators each, in Panels A and B of Tables 3.5 - 3.8, related to production, employment, consumption, and overall economic output activity) are tested using standard predictive regressions with various horizons up to eight quarters (e.g., Fama, 1981; Fama, 1990; Schwert, 1990; Cochrane, 1991; Carroll, Fuhrer and Wilcox, 1994; Yang, 2011; Chen and Zhang, 2011). These are reported in Tables 3.5 – 3.8. The intercept, slope, Newey and West (1987)

Table 3.5. Predicting Production

Table 3.5 reports the predictive regression results for production activity. IA is the option market ambiguity. VIX is the implied volatility of the S&P 500 index based on the CBOE volatility index. VRP is variance risk premium, obtained as the difference between implied variance and realized variance of the S&P 500 index. DY is the dividend yield of the S&P 500 index. EP is the earnings to price ratio of the S&P 500 index. ER is the excess return of the S&P 500 index calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bill. CS is the credit spread between Moody's AAA and BAA bond yield indices. TS denotes the term spread between 10Y T-bond and 3M T-bill yields. The sample covers monthly observations from 1990M01 to 2014M12. Newey-West t-statistics with lags equal to the return horizon (in months) are reported in parentheses. *, **, and *** represent significance at 90%, 95%, and 99% confidence levels respectively.

Panel A. Predictive Regressions on Industrial Production (IP) Growth

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	3.69	3.65	3.43	3.37	3.29	3.20	3.11	2.99
	(8.85)	(7.99)	(6.65)	(6.23)	(5.71)	(5.24)	(4.67)	(4.15)
IA	-10.76***	-10.56***	-9.14**	-8.52***	-7.76***	-6.93***	-6.17***	-5.24***
	(-3.22)	(-2.69)	(-2.53)	(-2.80)	(-2.98)	(-3.18)	(-3.44)	(-3.44)
Adj. R^2 (%)	13.31	16.20	14.32	14.43	13.99	12.92	11.69	9.49
Cst	8.61	6.92	5.64	4.90	4.29	3.86	3.52	3.31
	(5.04)	(4.63)	(5.71)	(5.68)	(4.29)	(3.25)	(2.48)	(2.01)
VIX	-0.32***	-0.24***	-0.18***	-0.14***	-0.11**	-0.08*	-0.07	-0.06
	(-3.43)	(-2.70)	(-2.72)	(-2.65)	(-2.15)	(-1.79)	(-1.36)	(-1.03)
Adj. R^2 (%)	18.92	13.12	8.29	5.88	3.96	2.73	1.87	1.36
Cst	1.52	1.34	1.35	1.50	1.60	1.70	1.80	1.86
	(1.37)	(1.30)	(1.36)	(1.52)	(1.61)	(1.70)	(1.79)	(1.81)
VRP	0.03	0.04	0.04*	0.03*	0.03*	0.02	0.02	0.02
	(0.60)	(1.28)	(1.87)	(1.88)	(1.78)	(1.64)	(1.38)	(1.21)
Adj. R^2 (%)	0.63	1.85	2.22	1.67	1.52	1.30	0.91	0.84
Cst	4.65	3.21	2.09	1.25	0.46	-0.11	-0.44	-0.70
	(2.46)	(1.63)	(1.09)	(0.64)	(0.23)	(-0.06)	(-0.23)	(-0.37)
DY	-1.25	-0.56	-0.03	0.39	0.78	1.07	1.24	1.37*
	(-1.29)	(-0.58)	(-0.03)	(0.46)	(0.97)	(1.39)	(1.63)	(1.85)
Adj. R^2 (%)	1.72	0.17	-0.36	-0.02	1.30	3.24	5.16	7.34
Cst	-1.00	-0.01	0.79	1.29	1.48	1.55	1.64	1.68
	(-0.40)	(-0.00)	(0.33)	(0.61)	(0.81)	(0.99)	(1.16)	(1.23)
EP	0.65	0.44	0.26	0.16	0.13	0.13	0.11	0.11
	(1.41)	(0.97)	(0.66)	(0.45)	(0.41)	(0.41)	(0.36)	(0.32)
Adj. R^2 (%)	2.17	1.07	0.26	-0.08	-0.15	-0.14	-0.16	-0.15
Cst	1.81	1.84	1.88	1.93	1.99	2.05	2.09	2.12
	(3.30)	(2.72)	(2.51)	(2.48)	(2.52)	(2.57)	(2.61)	(2.65)
ER	0.04***	0.03***	0.03***	0.02***	0.02***	0.02***	0.01***	0.01***
	(3.23)	(2.97)	(2.91)	(3.06)	(3.25)	(3.23)	(3.27)	(3.32)
Adj. R^2 (%)	11.22	10.38	7.83	6.54	5.76	4.20	2.98	2.63
Cst	8.67	6.96	5.64	4.70	3.97	3.49	3.25	3.14
	(5.79)	(3.95)	(3.50)	(3.13)	(2.80)	(2.51)	(2.28)	(2.16)
CS	-6.84***	-5.09**	-3.72*	-2.73	-1.93	-1.39	-1.12	-0.99
	(-3.94)	(-2.41)	(-1.91)	(-1.58)	(-1.32)	(-1.14)	(-1.03)	(-0.96)
Adj. R^2 (%)	25.71	17.79	11.12	6.79	3.81	2.16	1.51	1.30
Cst	1.37	1.12	0.79	0.59	0.49	0.30	0.13	-0.02
	(1.82)	(1.16)	(0.69)	(0.45)	(0.34)	(0.20)	(0.08)	(-0.01)
TS	0.35	0.48	0.66	0.78*	0.86**	0.97**	1.08**	1.17**
	(0.92)	(1.09)	(1.45)	(1.76)	(1.96)	(2.15)	(2.23)	(2.31)
Adj. R^2 (%)	0.15	0.86	2.31	4.02	5.83	8.88	12.60	16.92

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	7.06	5.53	4.23	3.31	2.61	2.08	1.76	1.53
	(5.92)	(3.80)	(2.82)	(2.27)	(1.81)	(1.37)	(1.08)	(0.89)
IA	-6.21***	-7.27***	-6.59**	-6.55***	-6.26***	-5.62***	-4.86***	-3.80***
	(-3.11)	(-2.75)	(-2.46)	(-2.72)	(-2.89)	(-3.03)	(-3.18)	(-2.99)
CS	-6.16***	-4.27***	-3.12**	-2.19	-1.47	-1.14	-1.10	-1.22
	(-5.40)	(-2.98)	(-2.06)	(-1.50)	(-1.12)	(-0.97)	(-1.00)	(-1.15)
ER	0.03***	0.03***	0.02***	0.02***	0.02***	0.01***	0.01***	0.01***
	(4.62)	(4.31)	(3.96)	(4.05)	(4.47)	(4.47)	(4.55)	(4.54)
TS	0.91***	0.84***	0.91***	0.93***	0.94***	1.03**	1.14**	1.25**
	(3.23)	(2.66)	(2.64)	(2.70)	(2.60)	(2.55)	(2.50)	(2.53)
Adj. R^2 (%)	41.31	36.95	29.95	27.47	26.12	25.77	26.99	29.38

Panel B. Predictive Regressions on Capacity Utilization (CU) Growth

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	1.20	1.14	0.88	0.76	0.63	0.48	0.35	0.19
	(2.65)	(2.23)	(1.62)	(1.42)	(1.21)	(0.93)	(0.65)	(0.34)
IA	-9.67***	-9.24**	-7.55**	-6.55**	-5.47**	-4.29**	-3.29**	-2.15*
	(-3.06)	(-2.52)	(-2.29)	(-2.41)	(-2.40)	(-2.23)	(-2.08)	(-1.71)
Adj. R^2 (%)	10.86	12.75	10.20	9.18	7.73	5.72	3.95	1.86
Cst	5.62	3.72	2.23	1.27	0.48	-0.10	-0.53	-0.81
	(3.41)	(2.58)	(2.39)	(1.57)	(0.54)	(-0.10)	(-0.50)	(-0.70)
VIX	-0.29***	-0.20**	-0.12*	-0.07	-0.03	0.00	0.02	0.03
	(-3.19)	(-2.25)	(-1.86)	(-1.30)	(-0.60)	(-0.07)	(0.36)	(0.65)
Adj. R^2 (%)	15.48	8.96	4.04	1.58	0.12	-0.36	-0.15	0.44
Cst	-0.66	-0.88	-0.90	-0.76	-0.67	-0.57	-0.46	-0.41
	(-0.61)	(-0.90)	(-1.01)	(-0.90)	(-0.83)	(-0.74)	(-0.64)	(-0.59)
VRP	0.02	0.03	0.03	0.03	0.02	0.02	0.02	0.01
	(0.45)	(1.11)	(1.57)	(1.52)	(1.51)	(1.41)	(1.24)	(1.09)
Adj. R^2 (%)	0.19	1.37	1.94	1.50	1.47	1.24	0.82	0.75
Cst	1.33	-0.05	-1.11	-1.86	-2.52	-2.91	-3.02	-3.03
	(0.70)	(-0.03)	(-0.58)	(-0.96)	(-1.29)	(-1.50)	(-1.58)	(-1.64)
DY	-0.77	-0.11	0.40	0.77	1.10	1.30*	1.36*	1.38*
	(-0.80)	(-0.11)	(0.46)	(0.92)	(1.37)	(1.68)	(1.80)	(1.90)
Adj. R^2 (%)	0.44	-0.34	-0.03	1.13	3.37	6.00	8.06	9.97
Cst	-2.42	-1.05	0.07	0.83	1.22	1.42	1.58	1.65
	(-0.93)	(-0.38)	(0.03)	(0.32)	(0.51)	(0.64)	(0.75)	(0.80)
EP	0.46	0.17	-0.07	-0.23	-0.30	-0.34	-0.37	-0.38
	(0.94)	(0.33)	(-0.15)	(-0.51)	(-0.72)	(-0.84)	(-0.94)	(-0.97)
Adj. R^2 (%)	0.92	-0.15	-0.31	0.24	0.92	1.60	2.46	3.25
Cst	-0.48	-0.44	-0.40	-0.34	-0.28	-0.23	-0.19	-0.16
	(-0.86)	(-0.66)	(-0.55)	(-0.47)	(-0.39)	(-0.32)	(-0.28)	(-0.25)
ER	0.03***	0.03***	0.02**	0.02**	0.01**	0.01**	0.00*	0.00
	(2.94)	(2.60)	(2.42)	(2.38)	(2.41)	(2.07)	(1.67)	(1.31)
Adj. R^2 (%)	8.76	7.65	5.25	3.83	2.82	1.33	0.33	0.00
Cst	3.82	1.93	0.44	-0.64	-1.47	-1.98	-2.20	-2.23
	(2.16)	(1.00)	(0.28)	(-0.50)	(-1.38)	(-2.20)	(-2.60)	(-2.72)
CS	-4.23**	-2.29	-0.74	0.41	1.30	1.86*	2.10***	2.16***
	(-2.08)	(-0.98)	(-0.37)	(0.25)	(1.00)	(1.94)	(2.86)	(3.51)
Adj. R^2 (%)	9.76	3.42	0.11	-0.19	1.80	5.03	7.98	10.16
Cst	-2.87	-3.01	-3.21	-3.28	-3.25	-3.29	-3.32	-3.32
	(-3.93)	(-3.34)	(-3.13)	(-3.05)	(-3.08)	(-3.29)	(-3.62)	(-4.06)
TS	1.38***	1.45***	1.56***	1.61***	1.62***	1.65***	1.68***	1.69***
	(3.38)	(3.04)	(3.21)	(3.55)	(4.04)	(4.70)	(5.24)	(5.67)
Adj. R^2 (%)	7.64	11.01	15.53	20.03	24.59	31.48	39.28	47.82

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	1.65	-0.35	-1.80	-2.70	-3.37	-3.78	-4.04	-4.22
	(0.90)	(-0.21)	(-1.18)	(-1.68)	(-2.05)	(-2.35)	(-2.75)	(-3.33)
IA	-6.21**	-7.28**	-6.38*	-5.94**	-5.31**	-4.29**	-3.36**	-2.22**
	(-2.50)	(-2.23)	(-1.95)	(-2.07)	(-2.20)	(-2.21)	(-2.27)	(-2.02)
VIX	-0.18**	-0.08	-0.02	0.02	0.05	0.06	0.06	0.06
	(-2.19)	(-1.18)	(-0.42)	(0.28)	(0.77)	(1.00)	(1.26)	(1.54)
ER	0.02***	0.02***	0.02**	0.02**	0.02**	0.01**	0.01**	0.01*
	(3.06)	(3.04)	(2.54)	(2.35)	(2.44)	(2.14)	(1.96)	(1.92)
TS	1.39***	1.41***	1.51***	1.55***	1.55***	1.60***	1.63***	1.65***
	(4.04)	(3.49)	(3.58)	(3.95)	(4.51)	(5.22)	(5.81)	(6.41)
Adj. R^2 (%)	31.52	31.89	30.49	32.41	35.13	39.12	44.76	51.63

adjusted t-statistics, and the adjusted R^2 for each regression are reported for prediction horizons from one to eight quarters. Univariate regression results for each predictor variable are presented in Part 1. To further examine the collective information content provided by the set of significant predictor variables and assess whether option market ambiguity (IA) has unique and incremental information content compared to other good predictor variables, we additionally employ multivariate predictive regressions for each economic sector activity while paying careful attention to potential multicollinearity problems that may exist among highly-correlated predictor variables (shown in Part 2 at the bottom of each of Tables 3.5-3.8).

3.5.4.1. Predicting Production Activity

Results of our predictive regressions regarding production activity, including growth in industrial production (IP) and capacity utilization (CU), are reported in Table 3.5, Panels A and B.

Industrial Production (IP). Panel A of Table 3.5 shows that IA has a strong and robust predictive power for IP. IA significantly predicts IP for all prediction horizons from one to eight quarters at the 1% significance level. For all horizons the coefficient of IA is consistently negative as predicted, ranging from -10.76 to -5.24. This corroborates the theory prediction that increased ambiguity (as revealed in the options market) is associated with subsequent depressed real industrial production. The significant negative coefficient of IA is in line with the aforementioned economic theories predicting a negative relationship between aggregate ambiguity and economic output. Among other standard predictor variables, only market excess return (ER) contains comparable information content over all horizons. ER is positive and significant for all 8 quarters. When investors are optimistic about future economic output, market expectations tend to be priced in the stock market aggressively, manifested in a solid positive coefficient for excess market return (ER). These results are in line with previous studies (e.g., Fama, 1981; Fama, 1990; Schwert, 1990) and confirm that the predictive power of ER has not diminished in the post-1990s era. A few other known predictor variables also exhibit some predictive power for IP, but this is far weaker than what IA and ER offer. TS seems to predict IP for horizons longer than four quarters, while CS predicts only for short horizons (from one to three quarters). VIX, though providing the basis for information extraction for both IA and VRP, predicts industrial production up to six quarters only. VIX's (limited) predictive ability is in line with findings by Bloom (2009) and Bekaert and Hoerova (2014). Despite being widely viewed as a proxy for economic uncertainty and sometimes for Knightian uncertainty (Drechsler, 2013; Bekaert, Hoerova and Lo Duca, 2013; Miao, Wei and Zhou, 2014), VRP's predictive power for IP extends only to short (three- to five-quarter) horizons.

To further ensure that the information contained in IA is not due to an information overlap with other known predictor variables, we include it in multivariate regressions along with (the most significant of) other predictor variables. In Part 2 of Panel A in Table 3.5, we report multivariate regression results involving ER, CS and TS along with IA as predictors of IP. In these regressions, IA and ER still significantly predict IP for all horizons. According to robust t-statistics, ER is more significant for short horizons, yet IA contains a meaningful portion of information in predicting future economic output. Importantly, when VRP is included in the multivariate regressions it is rejected whenever IA is also included. The above implies that although VRP may contain some information on Knightian uncertainty as suggested by previous studies (Zhou, 2009; Drechsler, 2013; Miao, Wei and Zhou, 2014), this information content is not rich and incremental enough to remain significant in the presence of our implied ambiguity (IA) measure. This can be explained by the fact that VRP-related ambiguity information is mostly based on multiplier-priors arguments concerning uncertainty in the drift component of Eq. (3.1), while IA information relates to uncertainty in both drifts and volatilities (i.e., Choquet-based and rank-dependent utility arguments) specified in our ambiguity-adjusted OPM. Spreads CS and TS have complementary roles (as confirmed by a separate unreported bivariate regression) producing improved robust t-statistics.[14] The consistently negative and significant coefficient of IA in these multivariate regressions confirms the predictions that increased ambiguity in the financial markets is associated with depressed production.

Capacity Utilization (CU). Panel B of Table 3.5 shows that IA remains robust in predicting future CU for up to eight-quarter horizons. The slope of IA is again negative for all horizons confirming that higher ambiguity is associated with lower future capacity utilization. This corroborates the previous finding based on IP that ambiguity suppresses production. TS significantly predicts CU for all horizons up to eight quarters. Market excess return (ER) is no longer a significant predictor in the longer horizons. VIX, CS and DY show moderate predictive ability but are generally weaker and less consistent than IA, ER and TS. VIX only predicts up to three quarters, while VRP again does not

[14] Due to high correlation between VIX and CS, only one is included; VIX becomes insignificant for most horizons as confirmed by separate bivariate regressions (unreported) involving both VIX and CS.

seem to have predictive ability for capacity utilization. EP also does not predict CU (similar to IP). Overall, univariate predictive regression results confirm a strong and robust predictive power of IA with regards to production activity growth. They provide confirmatory empirical evidence concerning a negative lead-lag relationship between option market ambiguity and future production activity levels.

Part 2 of Panel B reports multivariate regression results for all significant predictors from part 1 of Panel B and examines the joint predictive power of VIX and IA, along with ER and TS. Based on robust t-statistics, the predictive power of IA holds up to eight-quarter horizons while TS improves in all horizons compared to the univariate regressions. ER also improves for all horizons when IA is included (unreported). VIX predicts only for the first quarter. Overall, IA remains significant in all horizons in predicting shocks in production activity measured by capacity utilization. As was the case with IP, IA contains a meaningful portion of information concerning future capacity utilization growth and remains a strong predictor in the presence of other known predictor variables.

Given that IA is the only variable that consistently predicts both production (IP and CU) indicators for all horizons, we conclude that option market ambiguity outperforms other known predictor variables of production activity in terms of consistency and reliability. These findings confirm that option market investors' subjective ambiguity preferences are a robust indicator of future production activity in the real economy and are in line with theoretical predictions linking increased aggregate uncertainty with depressed production activity. This holds even after controlling for market-induced risk aversion (i.e., consumption- and CAPM-based) in the regressions.

3.5.4.2. Predicting Employment

When greater uncertainty looms on the horizon, firms and economic agents tend to suppress their hiring plans waiting until they are more confident about what the future will bring (a real options hysteresis effect) or downscale investment due to precautionary saving motives. Recent studies (Bachmann, Elstner and Sims, 2012; Jurado, Ludivigson and Ng, 2013), using disagreement as proxy for uncertainty inferred from macroeconomic indicators, document a negative relationship between disagreement and labor market performance. We here examine the association between market ambiguity and ex post labor market activity using standard predictive regressions on unemployment

Table 3.6. Predicting Employment

Table 3.6 reports the predictive regression results for employment activity. IA is the option market ambiguity. VIX is the implied volatility of the S&P 500 index based on the CBOE volatility index. VRP is variance risk premium, obtained as the difference between implied variance and realized variance of the S&P 500 index. DY is the dividend yield of the S&P 500 index. EP is the earnings to price ratio of the S&P 500 index. ER is the excess return of the S&P 500 index calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bill. CS is the credit spread between Moody's AAA and BAA bond yield indices. TS denotes the term spread between 10Y T-bond and 3M T-bill yields. The sample covers monthly observations from 1990M01 to 2014M12. Newey-West t-statistics with lags equal to the return horizon (in months) are reported in parentheses. *, **, and *** represent significance at 90%, 95%, and 99% confidence levels respectively.

Panel A. Predictive Regressions on Unemployment Rate (UR) Growth

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	-3.24	-3.52	-3.58	-3.61	-3.61	-3.62	-3.40	-3.25
	(-2.00)	(-2.00)	(-1.84)	(-1.70)	(-1.60)	(-1.54)	(-1.35)	(-1.23)
IA	31.83***	33.20***	32.38***	31.22***	29.73***	28.26***	25.70***	23.47***
	(2.82)	(2.59)	(2.58)	(2.69)	(2.85)	(3.01)	(3.11)	(3.14)
Adj. R^2 (%)	9.09	13.31	14.30	14.69	14.69	14.49	12.93	11.59
Cst	-21.63	-18.70	-15.80	-13.52	-11.17	-9.50	-8.01	-6.49
	(-4.01)	(-3.81)	(-3.95)	(-3.54)	(-2.81)	(-2.19)	(-1.65)	(-1.20)
VIX	1.14***	0.99***	0.84***	0.72***	0.59***	0.50**	0.42**	0.34*
	(3.89)	(3.52)	(3.55)	(3.27)	(2.88)	(2.58)	(2.20)	(1.73)
Adj. R^2 (%)	18.77	18.94	15.28	12.28	9.11	6.99	5.21	3.49
Cst	1.67	2.24	2.19	2.19	1.91	1.65	1.33	1.23
	(0.45)	(0.59)	(0.58)	(0.56)	(0.48)	(0.40)	(0.32)	(0.30)
VRP	0.00	-0.04	-0.04	-0.05	-0.05	-0.05	-0.04	-0.05
	(-0.01)	(-0.31)	(-0.50)	(-0.73)	(-0.75)	(-0.75)	(-0.67)	(-0.77)
Adj. R^2 (%)	-0.36	-0.19	-0.08	0.13	0.13	0.15	0.04	0.17
Cst	-11.51	-8.61	-5.47	-2.59	0.03	2.15	3.75	5.24
	(-1.73)	(-1.12)	(-0.65)	(-0.29)	(0.00)	(0.23)	(0.39)	(0.56)
DY	6.26*	4.85	3.27	1.80	0.44	-0.68	-1.53	-2.33
	(1.94)	(1.32)	(0.85)	(0.45)	(0.11)	(-0.17)	(-0.38)	(-0.60)
Adj. R^2 (%)	3.80	2.95	1.33	0.20	-0.33	-0.27	0.17	0.97
Cst	21.07	16.75	13.32	10.84	8.95	7.43	6.16	5.31
	(2.82)	(2.04)	(1.62)	(1.38)	(1.21)	(1.06)	(0.92)	(0.82)
EP	-4.16***	-3.25**	-2.55*	-2.07	-1.71	-1.43	-1.20	-1.06
	(-2.92)	(-2.13)	(-1.70)	(-1.44)	(-1.24)	(-1.05)	(-0.87)	(-0.75)
Adj. R^2 (%)	7.87	6.32	4.28	3.00	2.18	1.59	1.12	0.88
Cst	2.16	2.12	1.89	1.63	1.36	1.08	0.84	0.62
	(1.10)	(0.88)	(0.69)	(0.55)	(0.44)	(0.34)	(0.26)	(0.19)
ER	-0.09**	-0.09**	-0.08***	-0.07***	-0.07***	-0.06***	-0.05***	-0.04***
	(-2.27)	(-2.50)	(-2.65)	(-2.76)	(-2.78)	(-2.87)	(-2.83)	(-3.05)
Adj. R^2 (%)	4.68	6.97	6.49	5.86	5.18	4.30	3.26	2.70
Cst	-21.70	-18.08	-14.73	-11.99	-9.61	-7.55	-5.81	-4.58
	(-5.32)	(-4.12)	(-3.38)	(-2.80)	(-2.29)	(-1.80)	(-1.33)	(-1.02)
CS	24.08***	20.27***	16.63***	13.59***	10.89***	8.53**	6.55**	5.09
	(5.53)	(4.18)	(3.48)	(3.02)	(2.72)	(2.43)	(1.97)	(1.59)
Adj. R^2 (%)	25.17	23.71	17.89	13.10	9.18	6.03	3.71	2.29
Cst	2.47	4.31	6.12	7.53	8.51	9.48	10.42	11.03
	(0.88)	(1.26)	(1.45)	(1.57)	(1.64)	(1.76)	(1.91)	(2.03)
TS	-0.44	-1.45	-2.51	-3.37*	-4.01**	-4.65***	-5.25***	-5.67***
	(-0.33)	(-0.97)	(-1.50)	(-1.93)	(-2.29)	(-2.71)	(-3.13)	(-3.43)
Adj. R^2 (%)	-0.30	0.56	2.74	5.81	9.31	13.82	19.18	24.24

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	-17.67	-13.25	-9.15	-5.81	-3.03	-0.56	1.65	3.25
	(-4.89)	(-3.53)	(-2.21)	(-1.39)	(-0.72)	(-0.13)	(0.36)	(0.70)
IA	15.67**	19.28**	20.16**	20.37**	20.17**	19.72***	17.75***	15.94***
	(2.00)	(2.14)	(2.20)	(2.36)	(2.56)	(2.77)	(2.82)	(2.79)
CS	22.76***	18.76***	15.59***	13.03***	10.74***	8.87***	7.62***	6.73**
	(6.88)	(5.48)	(4.42)	(3.76)	(3.33)	(3.08)	(2.74)	(2.46)
ER	-0.06***	-0.07***	-0.07***	-0.06***	-0.06***	-0.05***	-0.05***	-0.04***
	(-2.70)	(-3.92)	(-3.93)	(-4.18)	(-4.28)	(-4.59)	(-4.65)	(-5.19)
TS	-2.54**	-3.12***	-3.86***	-4.46***	-4.87***	-5.33***	-5.82***	-6.17***
	(-2.43)	(-2.93)	(-3.16)	(-3.35)	(-3.50)	(-3.78)	(-3.95)	(-4.03)
Adj. R^2 (%)	31.74	36.95	35.54	35.07	35.08	36.31	37.81	40.21

Panel B. Predictive Regressions on Total Non-farm Payroll (TNP) Growth

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	1.46	1.54	1.54	1.54	1.55	1.53	1.53	1.51
	(8.94)	(7.80)	(6.72)	(6.08)	(5.66)	(5.27)	(4.96)	(4.67)
IA	-3.49***	-3.94***	-3.89***	-3.84***	-3.76***	-3.58***	-3.44***	-3.25***
	(-3.17)	(-2.91)	(-2.71)	(-2.75)	(-2.85)	(-2.98)	(-3.16)	(-3.27)
Adj. R^2 (%)	12.09	16.78	17.33	17.96	18.45	17.91	17.76	16.94
Cst	3.85	3.67	3.45	3.23	2.99	2.78	2.61	2.46
	(7.47)	(6.49)	(6.69)	(6.62)	(6.07)	(5.32)	(4.51)	(3.83)
VIX	-0.14***	-0.13***	-0.12***	-0.11***	-0.10***	-0.09***	-0.08***	-0.07***
	(-4.96)	(-4.12)	(-4.02)	(-3.88)	(-3.60)	(-3.36)	(-3.06)	(-2.78)
Adj. R^2 (%)	32.78	31.09	27.56	23.99	20.27	17.08	14.59	12.58
Cst	0.99	0.92	0.91	0.91	0.90	0.92	0.93	0.94
	(2.44)	(2.12)	(2.04)	(1.97)	(1.89)	(1.90)	(1.89)	(1.87)
VRP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-0.18)	(0.04)	(0.13)	(0.22)	(0.38)	(0.39)	(0.44)	(0.47)
Adj. R^2 (%)	-0.26	-0.36	-0.33	-0.29	-0.17	-0.17	-0.13	-0.09
Cst	2.42	2.19	1.87	1.55	1.22	0.92	0.67	0.44
	(3.64)	(2.63)	(2.04)	(1.60)	(1.21)	(0.90)	(0.65)	(0.43)
DY	-0.71**	-0.60	-0.45	-0.29	-0.12	0.03	0.16	0.27
	(-2.04)	(-1.38)	(-0.95)	(-0.59)	(-0.25)	(0.06)	(0.33)	(0.59)
Adj. R^2 (%)	5.55	4.13	2.27	0.79	-0.14	-0.35	0.07	1.04
Cst	-1.65	-1.27	-0.95	-0.70	-0.50	-0.34	-0.18	-0.04
	(-2.00)	(-1.27)	(-0.91)	(-0.68)	(-0.51)	(-0.37)	(-0.21)	(-0.05)
EP	0.55***	0.47**	0.40**	0.35*	0.31*	0.28*	0.25	0.22
	(3.51)	(2.51)	(2.10)	(1.91)	(1.80)	(1.73)	(1.62)	(1.51)
Adj. R^2 (%)	15.52	12.16	9.37	7.57	6.33	5.47	4.61	3.85
Cst	0.88	0.87	0.88	0.90	0.92	0.94	0.96	0.97
	(4.40)	(3.29)	(2.86)	(2.66)	(2.55)	(2.50)	(2.49)	(2.49)
ER	0.01**	0.01**	0.01**	0.01***	0.01***	0.01***	0.01***	0.01***
	(2.17)	(2.12)	(2.39)	(2.63)	(2.71)	(2.84)	(2.84)	(3.01)
Adj. R^2 (%)	5.39	6.61	6.85	7.08	6.66	5.92	5.03	4.67
Cst	4.34	4.03	3.72	3.42	3.13	2.88	2.69	2.50
	(16.29)	(10.48)	(8.48)	(7.14)	(6.29)	(5.58)	(4.95)	(4.36)
CS	-3.52***	-3.20***	-2.87***	-2.54***	-2.23***	-1.96***	-1.74***	-1.54***
	(-12.56)	(-7.71)	(-6.17)	(-5.23)	(-4.81)	(-4.60)	(-4.30)	(-3.86)
Adj. R^2 (%)	59.63	53.07	45.20	37.51	30.86	25.38	21.53	17.89
Cst	1.43	1.24	1.05	0.87	0.72	0.56	0.42	0.29
	(6.14)	(3.77)	(2.53)	(1.79)	(1.30)	(0.94)	(0.66)	(0.44)
TS	-0.27**	-0.16	-0.06	0.04	0.13	0.22	0.31	0.38*
	(-2.01)	(-0.95)	(-0.29)	(0.20)	(0.62)	(1.02)	(1.40)	(1.73)
Adj. R^2 (%)	2.21	0.67	-0.23	-0.29	0.47	2.16	4.79	8.16

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	4.26	3.94	3.63	3.32	3.05	2.81	2.63	2.45
	(18.64)	(12.97)	(10.06)	(8.34)	(7.27)	(6.31)	(5.54)	(4.82)
IA	-1.38**	-2.10***	-2.28***	-2.46***	-2.59***	-2.58***	-2.58***	-2.51***
	(-2.35)	(-2.88)	(-2.77)	(-2.86)	(-2.97)	(-3.02)	(-3.18)	(-3.27)
CS	-3.25***	-2.81***	-2.46***	-2.10***	-1.77***	-1.51***	-1.30***	-1.11***
	(-14.73)	(-9.36)	(-6.87)	(-5.47)	(-4.73)	(-4.19)	(-3.68)	(-3.11)
ER	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(3.13)	(3.89)	(4.21)	(4.22)	(4.33)	(4.43)	(4.32)	(4.57)
Adj. R^2 (%)	63.06	60.10	53.70	47.74	42.32	37.04	33.29	29.68

rate (UR) growth in Panel A and on total non-farm payroll growth (net hiring) (TNP) in Panel B of Table 3.6.[15] Our empirical findings below are in line with studies suggesting a strong negative relationship between uncertainty and employment activity in the economy. Our innovation here is that we infer Knightian uncertainty and divergence in beliefs from the market and link it to real economic activity.

[15] For robustness we also examine the predictive power of market ambiguity for total jobless claim growth (JC). IA robustly predicts JC for all horizons considered. Results are available upon request.

Unemployment Rate (UR). We first examine the relationship between option market ambiguity and changes in unemployment rate in the economy. Predictive regression results of unemployment rate (UR) growth are summarized in Panel A of Table 3.6. IA predicts the unemployment growth rate from one- to eight-quarter horizons. The consistently positive coefficient of IA is in line with theories suggesting that uncertainty suppresses employment. This can be explained by either the wait-and-see (real option) behavior of hiring managers or the precautionary saving motive where companies save on costs to face uncertain periods. Market excess return (ER) and VIX are also efficient in predicting fluctuations in employment activities across horizons. This is in line with Bloom (2009) who uses the VIX to show that heightened uncertainty reduces employment and output. CS also predicts UR well (from one- to seven-quarter horizons). DY and EP exhibit limited, short-term predictive power (at one-quarter horizon and from one- to three-quarter horizons, respectively). Term spread (TS) predicts UR for longer horizons only (from four- to eight-quarter horizons). VRP fails to predict employment (UR).

In terms of multivariate predictive regressions (involving IA, ER, CS and TS) shown in Part 2 of Panel A in Table 3.6, the predictive power of IA essentially remains intact across horizons.[16] ER, CS and TS show improvements in significance for all horizons when IA is included. The overall findings from Panel A of Table 3.6 confirm that IA contains a meaningful and unique incremental set of information that helps explain ex post fluctuations in labor market activity as measured by growth in unemployment.

Total Non-farm Payroll (TNP). We next examine net hiring, another important indicator of employment activity in the real economy. Our results regarding total non-farm payroll (TNP) growth prediction are summarized in Panel B of Table 3.6. IA again significantly and robustly predicts ex post TNP for horizons up to eight quarters. That is, besides predicting unemployment rate changes, IA

[16] Similar to the IP multivariate regressions, due to high correlation between VIX and CS (with stronger predictive power being generally observed from CS), VIX is here omitted from the multivariate model. We have run separate sets of bivariate regressions involving IA and VIX which confirm IA predicts UR for all horizons in the presence of VIX.

Table 3.7. Predicting Consumption

Table 3.7 reports the predictive regression results for consumption activity. IA is the option market ambiguity. VIX is the implied volatility of the S&P 500 index based on the CBOE volatility index. VRP is variance risk premium, obtained as the difference between implied variance and realized variance of the S&P 500 index. DY is the dividend yield of the S&P 500 index. EP is the earnings to price ratio of the S&P 500 index. ER is the excess return of the S&P 500 index calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bill. CS is the credit spread between Moody's AAA and BAA bond yield indices. TS denotes the term spread between 10Y T-bond and 3M T-bill yields. The sample covers monthly observations from 1990M01 to 2014M12. Newey-West t-statistics with lags equal to the return horizon (in months) are reported in parentheses. *, **, and *** represent significance at 90%, 95%, and 99% confidence levels respectively.

Panel A. Predictive Regressions on Personal Consumption Expenditure (PCE) Growth

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	5.54	5.40	5.31	5.30	5.28	5.24	5.21	5.16
	(26.13)	(26.22)	(22.91)	(20.90)	(18.76)	(17.02)	(15.95)	(14.68)
IA	-4.75**	-3.92**	-3.47**	-3.40**	-3.20**	-2.97**	-2.79**	-2.40**
	(-2.50)	(-2.07)	(-1.98)	(-2.16)	(-2.18)	(-2.22)	(-2.33)	(-2.20)
Adj. R^2 (%)	9.69	10.37	10.16	11.12	11.11	10.49	10.04	8.04
Cst	7.74	6.83	6.39	6.18	5.96	5.68	5.50	5.26
	(10.15)	(10.98)	(13.71)	(12.99)	(12.17)	(10.58)	(9.62)	(8.23)
VIX	-0.14***	-0.10***	-0.08**	-0.07**	-0.06*	-0.04	-0.03	-0.02
	(-3.36)	(-2.66)	(-2.56)	(-2.19)	(-1.85)	(-1.40)	(-1.18)	(-0.79)
Adj. R^2 (%)	14.10	10.55	8.19	7.01	5.45	3.35	2.22	0.89
Cst	4.56	4.50	4.52	4.53	4.51	4.50	4.49	4.49
	(8.29)	(9.35)	(10.20)	(10.22)	(10.12)	(10.09)	(9.89)	(9.84)
VRP	0.01	0.02	0.01	0.01	0.01*	0.02**	0.02**	0.02**
	(0.62)	(1.16)	(1.43)	(1.57)	(1.83)	(2.00)	(2.17)	(2.16)
Adj. R^2 (%)	0.52	1.60	1.51	1.64	2.20	2.88	3.42	3.77
Cst	6.54	6.07	5.87	5.60	5.29	5.04	4.83	4.63
	(8.31)	(8.00)	(7.10)	(6.15)	(5.49)	(5.06)	(4.76)	(4.54)
DY	-0.82**	-0.61	-0.52	-0.39	-0.24	-0.12	-0.02	0.08
	(-1.99)	(-1.60)	(-1.34)	(-0.96)	(-0.59)	(-0.30)	(-0.05)	(0.20)
Adj. R^2 (%)	3.07	2.58	2.34	1.37	0.38	-0.15	-0.36	-0.27
Cst	4.52	4.70	4.97	5.12	5.28	5.46	5.62	5.68
	(4.29)	(4.21)	(4.17)	(4.22)	(4.44)	(4.78)	(5.18)	(5.55)
EP	0.06	0.02	-0.04	-0.07	-0.11	-0.15	-0.18	-0.19
	(0.32)	(0.11)	(-0.20)	(-0.36)	(-0.53)	(-0.74)	(-0.94)	(-1.05)
Adj. R^2 (%)	-0.28	-0.35	-0.30	-0.09	0.28	0.96	1.79	2.35
Cst	4.71	4.72	4.72	4.72	4.73	4.74	4.75	4.75
	(17.55)	(15.15)	(13.78)	(12.91)	(12.31)	(11.83)	(11.55)	(11.33)
ER	0.02***	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**
	(2.69)	(2.14)	(2.18)	(2.45)	(2.37)	(2.10)	(2.25)	(2.28)
Adj. R^2 (%)	8.25	6.74	5.05	5.93	5.09	3.48	3.44	2.99
Cst	7.76	7.36	7.07	6.80	6.55	6.35	6.18	6.03
	(11.07)	(10.78)	(11.60)	(12.87)	(14.03)	(13.50)	(12.50)	(11.47)
CS	-3.05***	-2.65***	-2.36***	-2.09***	-1.82***	-1.62***	-1.44***	-1.28***
	(-3.64)	(-3.18)	(-3.23)	(-3.44)	(-3.79)	(-3.84)	(-3.64)	(-3.38)
Adj. R^2 (%)	19.14	22.76	22.62	20.04	17.16	14.88	12.69	10.94
Cst	5.25	5.06	4.87	4.72	4.57	4.45	4.35	4.28
	(16.55)	(14.18)	(11.27)	(8.85)	(7.27)	(6.26)	(5.56)	(5.13)
TS	-0.23	-0.14	-0.05	0.03	0.11	0.18	0.23	0.27
	(-1.36)	(-0.76)	(-0.25)	(0.16)	(0.50)	(0.71)	(0.85)	(0.91)
Adj. R^2 (%)	0.49	0.13	-0.29	-0.33	0.15	1.01	2.19	3.37

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	7.53	7.21	6.96	6.69	6.46	6.29	6.11	5.97
	(14.45)	(13.11)	(12.86)	(14.44)	(15.65)	(14.61)	(13.36)	(12.08)
IA	-3.14**	-2.49**	-2.18*	-2.30**	-2.27**	-2.14**	-2.07**	-1.76**
	(-2.41)	(-2.00)	(-1.81)	(-2.09)	(-2.07)	(-2.11)	(-2.25)	(-2.07)
CS	-2.40***	-2.16***	-1.95***	-1.65***	-1.41***	-1.24***	-1.07***	-0.97***
	(-4.46)	(-3.45)	(-3.21)	(-3.34)	(-3.64)	(-3.40)	(-2.99)	(-2.65)
ER	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.00***	0.00***
	(3.49)	(2.97)	(3.12)	(3.33)	(3.19)	(2.79)	(3.12)	(3.06)
Adj. R^2 (%)	28.27	30.40	28.83	28.05	25.10	21.60	19.56	16.39

Panel B. Predictive Regressions on Durable Goods (DG) Consumption Growth

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	5.32	5.19	4.91	4.91	4.97	4.89	4.80	4.75
	(6.24)	(7.06)	(6.08)	(6.07)	(5.81)	(5.48)	(5.13)	(4.81)
IA	-9.86**	-8.40**	-6.26*	-5.85*	-5.92**	-5.08*	-4.21*	-3.67*
	(-2.07)	(-2.06)	(-1.85)	(-1.82)	(-1.98)	(-1.89)	(-1.85)	(-1.81)
Adj. R^2 (%)	2.56	4.76	3.88	4.06	5.20	4.35	3.35	2.76
Cst	6.96	5.93	5.01	5.06	4.80	3.96	3.84	3.63
	(3.54)	(4.35)	(4.00)	(3.60)	(3.10)	(2.20)	(1.91)	(1.61)
VIX	-0.15	-0.10	-0.05	-0.05	-0.04	0.01	0.02	0.03
	(-1.52)	(-1.36)	(-0.84)	(-0.77)	(-0.56)	(0.12)	(0.23)	(0.37)
Adj. R^2 (%)	0.76	0.77	0.09	0.18	-0.04	-0.35	-0.29	-0.09
Cst	3.41	3.11	3.44	3.46	3.49	3.53	3.59	3.66
	(3.42)	(3.30)	(3.50)	(3.32)	(3.18)	(3.13)	(3.06)	(3.03)
VRP	0.02	0.04**	0.03	0.03	0.03	0.03	0.03	0.03
	(0.63)	(2.04)	(1.29)	(1.39)	(1.53)	(1.46)	(1.50)	(1.42)
Adj. R^2 (%)	-0.21	1.08	0.53	0.88	1.28	1.62	1.69	1.69
Cst	7.57	6.16	5.45	4.64	3.79	3.11	2.63	2.17
	(2.55)	(2.66)	(2.22)	(1.82)	(1.47)	(1.20)	(1.02)	(0.85)
DY	-1.79	-1.08	-0.72	-0.30	0.13	0.48	0.72	0.96
	(-1.26)	(-0.95)	(-0.61)	(-0.26)	(0.12)	(0.44)	(0.70)	(0.99)
Adj. R^2 (%)	0.73	0.59	0.27	-0.23	-0.33	0.11	0.88	2.06
Cst	5.40	5.52	5.77	5.99	6.25	6.39	6.46	6.36
	(1.56)	(2.26)	(2.49)	(2.76)	(3.23)	(3.65)	(3.93)	(4.14)
EP	-0.34	-0.35	-0.39	-0.42	-0.47	-0.49	-0.49	-0.47
	(-0.50)	(-0.76)	(-0.89)	(-1.02)	(-1.22)	(-1.32)	(-1.31)	(-1.22)
Adj. R^2 (%)	-0.19	0.08	0.48	0.82	1.42	1.86	2.25	2.20
Cst	3.64	3.75	3.84	3.90	3.96	4.06	4.10	4.13
	(4.15)	(4.52)	(4.31)	(4.19)	(4.17)	(4.18)	(4.16)	(4.18)
ER	0.03**	0.02**	0.02**	0.02**	0.02***	0.01*	0.01**	0.01***
	(2.10)	(2.24)	(2.42)	(2.41)	(2.60)	(1.93)	(2.01)	(2.66)
Adj. R^2 (%)	1.47	2.52	2.07	2.62	2.87	0.79	0.81	1.35
Cst	7.63	7.31	7.03	6.66	6.14	5.79	5.61	5.45
	(3.24)	(3.79)	(4.08)	(4.13)	(3.92)	(3.58)	(3.38)	(3.23)
CS	-3.94	-3.52	-3.18*	-2.73*	-2.14	-1.73	-1.51	-1.30
	(-1.54)	(-1.63)	(-1.75)	(-1.74)	(-1.61)	(-1.42)	(-1.30)	(-1.19)
Adj. R^2 (%)	1.84	3.88	4.82	4.20	3.08	2.22	1.89	1.48
Cst	2.92	2.71	2.47	2.33	2.07	1.83	1.65	1.58
	(2.01)	(1.99)	(1.53)	(1.26)	(1.03)	(0.86)	(0.74)	(0.70)
TS	0.47	0.63	0.79	0.89	1.06	1.21	1.33*	1.38*
	(0.80)	(1.21)	(1.40)	(1.41)	(1.54)	(1.64)	(1.72)	(1.73)
Adj. R^2 (%)	-0.13	0.65	2.00	3.26	5.93	9.11	12.70	15.28

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	5.13	5.03	4.79	4.79	4.86	4.83	4.74	4.68
	(5.95)	(6.80)	(5.90)	(5.89)	(5.69)	(5.44)	(5.09)	(4.80)
IA	-9.71**	-8.28**	-6.17*	-5.76*	-5.83**	-5.03*	-4.16*	-3.62*
	(-2.13)	(-2.16)	(-1.93)	(-1.92)	(-2.09)	(-1.95)	(-1.93)	(-1.90)
ER	0.03**	0.02**	0.02***	0.02***	0.02***	0.01**	0.01**	0.01***
	(2.13)	(2.43)	(2.60)	(2.75)	(3.02)	(2.27)	(2.29)	(2.87)
Adj. R^2 (%)	3.96	7.16	5.85	6.56	7.93	5.07	4.09	4.03

is a robust predictor of net hiring. The IA coefficient is consistently negative for all prediction horizons, confirming that ambiguity shocks are associated with a decrease in net hiring. This negative relationship between uncertainty and labor activity corroborates previous studies and the aforementioned economic theories. In terms of the other predictor variables, it seems that information about future overall employment is subsumed in many predictor variables, including VIX, CS and ER. By contrast, DY and TS perform poorly as TNP predictors. The apparent inability of DY and TS variables to forecast future payroll growth is in line with Chen and Zhang (2011). Compared to Chen and Zhang (2011), CS in our study generally gives stronger predictive power primarily due to differences in sampling period and frequency (they employ quarterly data from 1952Q1 to 2009Q1). TNP has likely become more predictable using CS in the last two decades. As in the UR case, VRP fails to predict TNP.

Although TNP seems to be predictable by five out of the eight predictor variables, the information contained in those variables may be overlapping. To help analyse whether IA provides a meaningful and unique set of information concerning the prediction of total non-farm payroll growth, Part 2 of Panel B presents multivariate predictive regression results among the more significant variables (IA, CS and ER). Out of the five variables showing predictive power for TNP in Part 1, three are more significant and included in Part 2.[17] Given a stronger predictive signal offered by CS (than VIX and PE combined), CS is kept in the multivariate analysis. With IA, CS and ER jointly included in the multivariate model, IA still robustly predicts TNP with a negative coefficient for all horizons. This holds even after controlling for market-induced risk aversion in the regressions. Results confirm that IA contains a unique and meaningful portion of information that can predict future total non-farm payroll growth, while it complements the predictive power of CS and ER across all horizons. We next turn to consumption.

3.5.4.3. Predicting Consumption Activity

Consumer spending reflects the choice made by consumers during times of economic prosperity or hardship. In analysing the empirical linkage between option market ambiguity and ex post real consumption growth, we examine real personal consumption expenditure per capita growth (PCE) in Panel A, and real durable goods (DG) consumption expenditure per capita growth in Panel B of Table 3.7. All consumption figures are adjusted for both inflation and population to obtain per capita real consumption growth. The per capita measure of real consumption allows examining the lead-lag relationship between market ambiguity and subsequent personal consumption choices.

[17] In a separate three-variable regression (unreported) with VIX, CS and EP as the independent variables, we find the significance of VIX and EP to be weakened in the presence of CS. Due to high correlations between VIX and CS, separate bivariate regressions show that the predictive power of IA remains unchanged in the presence of either VIX or CS.

Personal Consumption Expenditure (PCE) Per Capita. Predictive regression results for PCE are summarized in Panel A of Table 3.7. Implied ambiguity (IA) robustly predicts PCE for all horizons. The IA coefficient is consistently negative, in line with aforementioned economic theories that predict higher uncertainty suppresses consumption. Investors in the options market respond early to anticipation of heightened uncertainty and its perceived impact on general households and their future consumption behavior. CS and ER also predict PCE for all horizons, whereas VIX and VRP are significant from one to five-quarter horizons and from six to eight-quarter horizons, respectively. DY performs poorly, only predicting PCE at the one-quarter horizon, while EP does not predict PCE at all. We do not observe any predictive ability concerning PCE from term spread (TS) as in early studies (Harvey, 1988; Estrella and Hardouvelis, 1991). It appears the predictive power afforded by TS may have diminished during the last two decades.[18] A separate set of regressions confirms the reduced predictive power afforded by TS in the post-1990s era (unreported).

In our multivariate predictive regression results summarized in Part 2 of Panel A in Table 3.7, IA, CS and ER reliably predict PCE jointly for all horizons, with adjusted R^2 ranging from 16.4% to 30.4%. Signs of slopes for all three predictor variables are consistent with our univariate results in Part 1, with the t-statistics improving for CS and ER in the presence of IA. The consistently negative coefficient of IA in predicting PCE validates various related economic theory predictions, including real options and precautionary saving motives that prescribe how increasing uncertainty reduces consumption.[19]

Real Durable Goods (DG) Consumption Per Capita. Predictive regression results for durable goods (DG) consumption growth are reported in Panel B of Table 3.7. DG is generally a good proxy for real personal consumption of durable goods. Durable goods, by definition, provide service flows for more than one period. Strictly speaking, period on period growth of consumption expenditure does not measure the exact consumption achieved during the period. Following standard literature (Yang,

[18] Our sample covers twenty-three years of more recent or post-1990 data, while previous studies mostly cover two decades of pre-1990 data (1953Q1-1987Q1 in Harvey 1988; 1955Q2-1988Q4 in Estrella and Hardouvelis 1991).

[19] Due to multicollinearity, VIX is here rejected by CS when included in the multivariate setup (verified by a separate bivariate regression, unreported). Despite the t-statistics of IA being slightly weaker in the multivariate regressions, we can still conclude that a very significant and meaningful portion of information contained in IA is not overlapped by the other two robust predictors (CS and ER).

2011), we assume consumption of durable goods to be proportional to net stock. Compared to PCE, DG is more difficult to predict. In Panel B, only IA and ER reliably predict DG for all horizons. The predictive power of IA is solid (with robust t-statistics varying from -1.81 to -2.07). The coefficient of IA is consistently negative across all horizons, in line with findings in previous subsections. VIX, VRP and all other known predictors do not predict DG well.

Part 2 of Panel B in Table 3.7 summarizes our multivariate predictive regression results based on only the two efficient predictors (IA and ER). In this multivariate setup, IA robustly predicts DG with improved t-statistics, consistent sign of slopes and similar slope values compared to the univariate results. The significance of IA and ER improves for all horizons in the presence of each other.[20] The above results suggest that IA and ER jointly capture incremental information about variations in future durable consumption growth while they contain distinct, complementary and non-overlapping information. Our overall findings confirm a clear negative relation between market ambiguity and subsequent consumption growth. This holds even after controlling for market-induced risk.

3.5.4.4. Predicting Overall Economic Output

In this section we turn our attention to the empirical linkage between market ambiguity and two key overall economic activity indicators, real GDP per capita growth (GDPC) in Panel A and changes in the Chicago Fed National Activity Index (CFNAI) in Panel B of Table 3.8. Using indicators in real terms (and using population growth in the case of GDPC) allows more fair examination of the relation between market ambiguity and ex post real economic growth.

Real GDP Per Capita (GDPC) Growth. Panel A of Table 3.8 reports the predictive regression results related to real GDP per capita growth. IA robustly predicts future real GDP per capita growth with a consistently negative coefficient and significant robust t-statistics at 98% confidence for all horizons. Adjusted R^2 ranges from 13.2% to 20.5% at an annual horizon. The predictive performance of IA for GDPC is strong across both short and long horizons with stable robust t-statistics. Results confirm theoretical predictions that Knightian uncertainty and ambiguity are associated with a decline

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[20] The joint adjusted R^2 of IA and ER almost equals the sum of the individual regression adjusted R^2s. For example, the 6-quarter horizon adjusted R^2s for IA and ER are 4.4% and 0.8%, respectively, while the multivariate regression adjusted R^2 is 5.1%.

Table 3.8. Predicting Overall Economic Output

Table 3.8 reports the predictive regression results for overall economic output. IA is the option market ambiguity. VIX is the implied volatility of the S&P 500 index based on the CBOE volatility index. VRP is variance risk premium, obtained as the difference between implied variance and realized variance of the S&P 500 index. DY is the dividend yield of the S&P 500 index. EP is the earnings to price ratio of the S&P 500 index. ER is the excess return of the S&P 500 index calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bill. CS is the credit spread between Moody's AAA and BAA bond yield indices. TS denotes the term spread between 10Y T-bond and 3M T-bill yields. The GDPC sample covers quarterly observations from 1990Q1 to 2014Q4. The CFNAI sample covers monthly observations from 1990M01 to 2014M12. Newey-West t-statistics with lags equal to the return horizon (in months) are reported in parentheses. *, **, and *** represent significance at 90%, 95%, and 99% confidence levels respectively.

Panel A. Predictive Regressions on GDP per capita (GDPC) growth

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	2.12	2.10	2.06	2.07	1.99	1.96	1.95	1.90
	(7.70)	(8.08)	(7.58)	(7.60)	(7.05)	(6.61)	(6.40)	(5.89)
IA	-5.27**	-5.06***	-4.69***	-4.50***	-3.86***	-3.48***	-3.29***	-2.86***
	(-2.43)	(-2.71)	(-2.71)	(-2.98)	(-2.71)	(-2.70)	(-2.97)	(-2.66)
Adj. R^2 (%)	14.12	18.47	18.77	20.51	17.43	16.00	15.98	13.19
Cst	3.00	2.81	2.49	2.26	2.05	1.89	1.72	1.63
	(3.18)	(3.42)	(3.64)	(3.68)	(3.57)	(3.33)	(2.78)	(2.35)
VIX	-0.08	-0.07	-0.05	-0.04	-0.03	-0.02	-0.01	-0.01
	(-1.52)	(-1.49)	(-1.32)	(-1.12)	(-0.90)	(-0.73)	(-0.43)	(-0.24)
Adj. R^2 (%)	4.90	5.43	3.36	2.01	0.76	-0.03	-0.71	-0.96
Cst	1.20	1.15	1.11	1.12	1.15	1.15	1.11	1.12
	(2.43)	(2.55)	(2.55)	(2.52)	(2.55)	(2.43)	(2.25)	(2.17)
VRP	0.01	0.01	0.01	0.02	0.01	0.02	0.02	0.02
	(0.42)	(0.70)	(1.09)	(1.23)	(1.16)	(1.38)	(1.49)	(1.50)
Adj. R^2 (%)	-0.81	-0.41	0.18	0.73	0.75	1.40	2.74	3.17
Cst	3.68	3.23	2.82	2.44	2.08	1.80	1.60	1.45
	(4.17)	(3.74)	(3.14)	(2.65)	(2.28)	(1.98)	(1.79)	(1.66)
DY	-1.10***	-0.89**	-0.68	-0.48	-0.30	-0.16	-0.06	0.02
	(-2.60)	(-2.09)	(-1.61)	(-1.18)	(-0.80)	(-0.46)	(-0.19)	(0.06)
Adj. R^2 (%)	7.24	6.42	4.11	2.02	0.32	-0.64	-1.04	-1.10
Cst	1.28	1.63	1.83	1.91	1.94	1.94	1.94	1.91
	(1.18)	(1.48)	(1.73)	(1.96)	(2.19)	(2.39)	(2.54)	(2.62)
EP	0.02	-0.06	-0.09	-0.10	-0.11	-0.10	-0.10	-0.09
	(0.09)	(-0.27)	(-0.49)	(-0.60)	(-0.70)	(-0.71)	(-0.69)	(-0.62)
Adj. R^2 (%)	-1.10	-0.98	-0.65	-0.44	-0.27	-0.23	-0.19	-0.28
Cst	1.29	1.29	1.32	1.37	1.39	1.42	1.44	1.47
	(4.80)	(4.34)	(4.07)	(4.04)	(4.04)	(4.02)	(4.00)	(3.99)
ER	0.01**	0.01**	0.01*	0.01*	0.01*	0.01	0.01	0.00
	(2.08)	(2.01)	(1.90)	(1.65)	(1.70)	(1.56)	(1.47)	(1.24)
Adj. R^2 (%)	6.56	10.30	8.99	6.37	5.72	3.86	2.65	1.12
Cst	4.17	3.66	3.33	3.00	2.75	2.52	2.42	2.39
	(7.56)	(6.37)	(5.52)	(5.24)	(4.97)	(4.55)	(4.22)	(4.07)
CS	-2.89***	-2.36***	-1.99***	-1.63***	-1.35**	-1.10**	-0.98**	-0.93**
	(-4.80)	(-3.60)	(-2.96)	(-2.65)	(-2.51)	(-2.34)	(-2.26)	(-2.27)
Adj. R^2 (%)	22.75	21.03	17.61	13.70	10.69	7.70	6.78	6.80
Cst	1.34	1.24	1.19	1.14	1.08	1.00	0.92	0.86
	(3.05)	(2.70)	(2.24)	(1.94)	(1.70)	(1.45)	(1.24)	(1.11)
TS	0.02	0.07	0.11	0.15	0.19	0.24	0.30	0.33
	(0.09)	(0.37)	(0.53)	(0.73)	(0.91)	(1.09)	(1.22)	(1.30)
Adj. R^2 (%)	-1.11	-0.97	-0.72	-0.20	0.61	2.07	4.16	6.38

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	3.95	3.42	3.11	2.81	2.58	2.37	2.29	2.29
	(9.35)	(9.99)	(8.36)	(7.50)	(6.58)	(5.56)	(4.86)	(4.46)
IA	-3.26*	-3.58***	-3.51***	-3.66***	-3.20***	-3.00***	-2.89***	-2.42**
	(-1.90)	(-2.97)	(-2.92)	(-3.23)	(-2.71)	(-2.62)	(-2.94)	(-2.53)
CS	-2.25***	-1.66***	-1.32***	-0.94**	-0.75**	-0.54	-0.45	-0.49
	(-4.94)	(-4.86)	(-3.42)	(-2.48)	(-2.08)	(-1.58)	(-1.34)	(-1.42)
ER	0.01**	0.01**	0.01**	0.01*	0.01*	0.01*	0.01*	0.00
	(2.42)	(2.45)	(2.22)	(1.85)	(1.87)	(1.73)	(1.65)	(1.34)
Adj. R^2 (%)	32.53	37.80	34.36	30.70	25.72	21.05	19.38	15.41

Panel B. Predictive Regressions on Chicago Fed National Activity Index (CFNAI)

Part 1 – Univariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	0.06	0.07	0.05	0.05	0.04	0.03	0.02	0.01
	(1.11)	(0.95)	(0.67)	(0.55)	(0.45)	(0.32)	(0.20)	(0.07)
IA	-1.55***	-1.57***	-1.48**	-1.40***	-1.32***	-1.21***	-1.12***	-1.00***
	(-3.14)	(-2.64)	(-2.54)	(-2.64)	(-2.73)	(-2.80)	(-2.93)	(-2.92)
Adj. R^2 (%)	15.27	17.30	16.82	16.70	16.45	15.33	14.21	12.40
Cst	0.98	0.80	0.64	0.53	0.43	0.34	0.27	0.21
	(4.08)	(3.42)	(3.54)	(3.35)	(2.60)	(1.89)	(1.31)	(0.90)
VIX	-0.06***	-0.05***	-0.04***	-0.03***	-0.03***	-0.02***	-0.02**	-0.02**
	(-4.24)	(-3.50)	(-3.59)	(-3.56)	(-3.19)	(-2.91)	(-2.54)	(-2.09)
Adj. R^2 (%)	32.13	25.12	19.21	15.63	12.23	9.53	7.51	5.85
Cst	-0.21	-0.23	-0.23	-0.22	-0.22	-0.21	-0.20	-0.19
	(-1.17)	(-1.32)	(-1.34)	(-1.28)	(-1.24)	(-1.17)	(-1.11)	(-1.07)
VRP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(0.23)	(0.55)	(0.76)	(0.84)	(0.96)	(0.97)	(0.94)	(0.94)
Adj. R^2 (%)	-0.15	0.35	0.44	0.42	0.57	0.52	0.46	0.51
Cst	0.49	0.33	0.18	0.05	-0.08	-0.18	-0.26	-0.32
	(1.85)	(1.09)	(0.57)	(0.15)	(-0.24)	(-0.55)	(-0.78)	(-0.99)
DY	-31.57**	-23.86	-16.73	-10.30	-3.93	1.42	5.27	8.55
	(-2.27)	(-1.54)	(-1.09)	(-0.68)	(-0.27)	(0.10)	(0.38)	(0.64)
Adj. R^2 (%)	6.97	4.26	2.14	0.68	-0.20	-0.34	0.01	0.70
Cst	-0.80	-0.63	-0.50	-0.40	-0.33	-0.28	-0.24	-0.20
	(-2.30)	(-1.67)	(-1.34)	(-1.15)	(-1.07)	(-1.01)	(-0.91)	(-0.83)
EP	0.13**	0.10	0.07	0.05	0.04	0.03	0.02	0.01
	(2.04)	(1.43)	(1.07)	(0.84)	(0.70)	(0.57)	(0.39)	(0.25)
Adj. R^2 (%)	5.53	3.18	1.58	0.72	0.30	0.05	-0.16	-0.27
Cst	-0.20	-0.20	-0.20	-0.19	-0.18	-0.17	-0.16	-0.16
	(-2.56)	(-1.94)	(-1.66)	(-1.50)	(-1.37)	(-1.26)	(-1.19)	(-1.14)
ER	0.00***	0.00**	0.00**	0.00***	0.00***	0.00***	0.00**	0.00***
	(2.62)	(2.35)	(2.43)	(2.63)	(2.65)	(2.61)	(2.58)	(2.65)
Adj. R^2 (%)	9.67	9.05	7.57	7.21	6.33	5.00	3.90	3.41
Cst	1.06	0.86	0.70	0.56	0.44	0.35	0.29	0.24
	(6.32)	(4.19)	(3.48)	(2.90)	(2.40)	(1.93)	(1.52)	(1.22)
CS	-1.27***	-1.07***	-0.90***	-0.75***	-0.62***	-0.52***	-0.45***	-0.40***
	(-6.64)	(-4.42)	(-3.81)	(-3.44)	(-3.29)	(-3.27)	(-3.15)	(-2.95)
Adj. R^2 (%)	48.96	38.26	29.48	22.75	17.30	13.35	10.84	9.14
Cst	-0.16	-0.22	-0.28	-0.33	-0.36	-0.41	-0.45	-0.48
	(-1.66)	(-1.64)	(-1.71)	(-1.74)	(-1.73)	(-1.78)	(-1.85)	(-1.93)
TS	-0.01	0.02	0.06	0.09	0.11	0.13*	0.16**	0.18**
	(-0.15)	(0.37)	(0.83)	(1.21)	(1.52)	(1.82)	(2.05)	(2.23)
Adj. R^2 (%)	-0.35	-0.22	0.54	1.86	3.60	6.35	9.91	14.03

Part 2 – Multivariate Regressions

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	1.00	0.81	0.66	0.52	0.41	0.33	0.27	0.22
	(8.32)	(5.44)	(4.18)	(3.38)	(2.75)	(2.12)	(1.63)	(1.27)
IA	-0.83***	-0.99***	-1.01***	-1.03***	-1.03***	-0.98***	-0.92***	-0.82***
	(-3.22)	(-2.88)	(-2.74)	(-2.82)	(-2.86)	(-2.85)	(-2.95)	(-2.96)
CS	-1.10***	-0.88***	-0.71***	-0.57***	-0.44***	-0.35***	-0.30**	-0.26**
	(-8.82)	(-5.61)	(-4.26)	(-3.51)	(-3.10)	(-2.78)	(-2.45)	(-2.18)
ER	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***
	(4.15)	(3.88)	(3.73)	(3.81)	(3.90)	(3.84)	(3.84)	(4.01)
Adj. R^2 (%)	57.85	49.42	40.79	35.16	30.22	25.45	21.94	18.90

in per capita real economic growth. In line with the findings by Ang, Piazzesi and Wei (2006), credit spread (CS) also robustly predicts GDPC for all horizons, being stronger for shorter horizons. Market excess return (ER), one of the best predictors for real activity so far, shows predictive power for short-to-intermediate horizons only. Dividend yield (DY) predicts GDPC with a negative coefficient for one to two-quarter horizons. VIX and VRP, along with EP and TS, show no predictive power at all. Previous studies have documented strong predictive power of the term spread (TS) for real economic output proxied by real GNP growth in the pre-1990s era (Harvey, 1988; Estrella and Hardouvelis, 1991). We do not observe predictive power by the term spread in our analysis. In additional analysis (unreported), we find the predictive ability of TS to largely disappear in the 1990s. Our results confirm the findings of Stock and Watson (2003) that the predictive power of TS has diminished after 1985.

In terms of multivariate predictive regression results, shown in Part 2 of Panel A in Table 3.8, IA continues to robustly predict GDPC for all horizons with a consistently negative coefficient. The predictive power of CS is weakened for longer horizons when IA is included, suggesting that IA subsumes most of the information contained in CS when predicting long-horizon real GDP per capita growth. In the multivariate setting, ER significantly predicts GDPC up to six quarters. Overall, for real GDP per capita growth prediction, IA is the best and only predictor that remains robust across all horizons in both univariate and multivariate regressions. [21]

Chicago Fed National Activity Index (CFNAI). Considering the robustness of overall economic output results, we examine the predictive power of IA and other predictors to ex post changes in the CFNAI. Predictive regression results for CFNAI are presented in Panel B of Table 3.8. Again, market ambiguity (IA) significantly predicts CFNAI for all horizons. The consistently negative coefficient of IA supports previous findings that heightened market uncertainty reduces overall economic activity. VIX, along with CS and ER, is a significant predictor of CFNAI across horizons, but VRP does not predict well. DY and EP predict only for the first quarter, while TS only for long horizons.

The multivariate regressions in Part 2 of Panel B include all robust predictors (IA, CS and ER; VIX here is excluded due to its high correlation with CS). IA still consistently predicts CFNAI (with slope coefficients significant at 99% confidence) across all horizons. Our overall results for the prediction of real economic per capita growth represented by GDPC and CFNAI suggest that IA is the only variable which robustly predicts both indicators across all horizons in both univariate and multivariate regression specifications. We conclude that IA contains unique information on future

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[21] In addition to GDP per capita growth, we ran a separate test (unreported) on the predictive power of market ambiguity for productivity growth (as proxied by real GDP growth per capita per annual hours work). This test confirms that market ambiguity is negatively associated with ex post productivity growth from two to eight-quarter horizons. This is in line with literature documenting declined productivity during uncertain periods (Ohanian, 2001; Kobayashi, 2006).

aggregate economic activity. Our results hold even after controlling for market-induced risk aversion (i.e., consumption- and CAPM-based) in the regressions. Overall, our chapter is the first to comprehensively document a robust negative relation between divergence in option market ambiguity beliefs and real economic activity.

3.5.4.5. Direct Comparisons with VIX, VRP, Credit Spread and Additional Robustness Results

In this section we perform additional robustness tests, specifically examining the comparative predictive performance of IA directly with other established financial proxies for uncertainty, namely the VIX, VRP and credit spread (CS), while controlling for ER, using the same set of predictors across all economic sectors (indicators). As a follow on to Bloom (2009) and Bekaert and Hoerova (2014) who use VIX as the main proxy for economic or measurable uncertainty, we first examine whether the predictive power of IA is robust when VIX is jointly included. By adding VIX (which is more strictly a measure of risk) and IA together in the predictive regressions we are able to assess the relative contributions of risk and ambiguity on economic activity. Subsequently we examine the comparative performance of IA vis-à-vis VRP (also related to VIX), another popular proxy for ambiguity; and finally we consider the inclusion of CS, a known predictor from the credit market (also highly correlated to VIX). Since VRP is also considered as a measure of risk aversion, its inclusion also serves as a control for risk aversion. As market excess returns were shown to be a solid predictor for most economic indicators, we control for ER in all our robustness regressions. We specify three trivariate regression setups for each economic indicator prediction. Our first setup includes IA, ER, and VIX; the second setup includes IA, ER, and VRP; and the third includes IA, ER, and CS. (These are in Supplementary Appendix, Tables A3.1 and A3.2.)

In all three regression setups considered, joint inclusion of VIX, VRP or CS with IA (while controlling for ER) does not reduce the predictive ability of IA. The results confirm the robust and superior predictive power of IA across the range of economic activities (spanning production, employment, consumption and overall output) even when VIX, VRP and CS are included in a direct "horse race" with IA while ER is controlled for. Results from the last trivariate specification, which combines information from three different markets, namely the stock market (ER), the bond or credit market (CS), and the option market (IA), seem to have the best overall predictive ability and highest adjusted R^2 among all robustness setups considered. This coincides with the best combination of predictors from the univariate specifications that entered the multivariate regressions in previous subsections (e.g., in part 2 of Table 3.8 for predicting overall economic activity). This confirms and justifies our previous use of a different set of predictors to predict different economic indicators in line with related literature (see e.g., Chen, Roll and Ross, 1986; Estrella and Hardouvelis, 1991; Gilchrist, Yankov and Zakrajšek, 2009; Chen and Zhang, 2011). The multivariate regression results presented in Part 2 of Tables 3.5 – 3.8 in the previous sections likely contain the strongest set of predictors for each different economic activity (although differing from economic indicator to indicator) without suffering from multicollinearity issues.

The direct comparisons of the predictive power of implied ambiguity (IA) with the VIX or VRP are particularly revealing. IA in general has a much better predictive power for long horizons (8 quarters or more). VIX (or VRP) is not well suited for long-term prediction of real economic activity. IA remains a significant predictor even when VIX (or VIX^2) is included. For example, in predicting capacity utilization (CU) growth (Table 3.5 Panel B), VIX is significant in univariate regressions up to 3 quarters but in multivariate regressions it is significant only for 1 quarter, while IA remains significant up to 8 quarters (or more). VIX tends not to be as good a predictor of economic activity in general (e.g., it predicts well for unemployment up to a year, but not so well for consumption or GDP per capita growth), whereas IA predicts well across all areas of economic activity. VRP is generally also a poor predictor of real economy activity across the board, with or without the inclusion of IA. IA in general dominates VIX or VRP (and their components) in predictive ability, especially over long horizons. In auxiliary Tobit and OLS regressions, we use an isolated and residuals-based IA (filtering potential commonality effects by regressing Eq. (3.7)'s IA on VIX, VRP and other predictors) and find that our prediction conclusions are maintained under the isolated IA^{iso} (residuals) specification. IA^{iso}-related results are available upon request.

For additional robustness, we also investigated the relationship between macroeconomic uncertainty, proxied by the uncertainty measures of Jurado, Ludvigson and Ng (2015), with ex post real activity. The signs of association between IA and economic activity are consistent with those of

$MUNC_{JLN}$ confirming that IA also captures information related to aggregate uncertainty in the economy. The advantage of our approach is that our IA measure is forward-looking and can be obtained in real-time from the options market for prediction purposes. For further robustness we also investigated the predictive power of IA when relative risk aversion (RRA) and time-varying risk aversion proxied by the Sharpe ratio (Merton, 1973; Anderson et al, 2009) are controlled for. The RRA measure is estimated with rolling 5-year data of market excess return and consumption growth using a standard consumption-based asset pricing approach (Campbell, 2003) with power utility. The Sharpe ratio is estimated with rolling 5 years data of ER and historical volatility of the market index. Our IA results hold when RRA and the Sharpe ratio are also controlled for in the predictive regressions (unreported).

Finally, for further robustness on methodology we extract option-market implied ambiguity (IA) directly from traded SPX option prices with short maturities for the period January 1990-December 2012 (instead of using VIX to infer an equivalent market price and then back out IA) using our A-OPM of Eq. (3.2) and confirm that the predictive ability of IA globally holds. That is, whether we extract IA from VIX or directly from traded SPX option prices, the superiority of IA in predicting an extensive range of economic activity indicators, in comparison to other established predictors including the VIX, VRP, CS or ER, is generally maintained.[22] All considered, our results show that IA is a robust and superior predictor of economic activity. Our overall findings underscore the need to consider the role of market-extracted ambiguity in economic policy and monitoring of real economic activity.

3.6. Conclusion

This chapter has examined linkages between option-market extracted ambiguity and macroeconomic real activity. We employ a market-based measure of Knightian uncertainty or ambiguity, inferred from financial options information, that allows us to investigate the propagation of ambiguity shocks

[22] Despite the excellent predictive power of IA when extracted from option prices, significance according to t-stat is less strong than when VIX is used partly due to relatively weaker data quality compared to CBOE's proprietary data. This may be an additional reason why VIX might be a more reliable domain of ambiguity information extraction for long horizon windows.

from the financial markets to the real economy. We further study the informational efficiency of implied ambiguity (IA) compared to other variables commonly used in predicting ex post real economic activity.

Extracted from the option markets, market ambiguity is informationally different from existing measures of risk and uncertainty, such as implied volatility (VIX), variance risk premium (VRP) or credit spread (CS). This chapter validates the relevance of option market implied ambiguity as a financial market-based measure of divergence in ambiguity beliefs and aggregate uncertainty in the economy. Using a 5-variable VAR system, market ambiguity (Granger) causes changes in economic sector indicators spanning production, employment, consumption and overall economic output. Variance decomposition analysis reveals significant portions of these indicators can be explained by ambiguity shocks inferred from the options market. Our analysis suggests a causal or lead-lag link between option market ambiguity and ex post economic activity extended over at least eight quarters.

Since important policy decisions rely on economic predictions of real economic activity, improving the ability to predict economic performance is a key task for policy making and monitoring the real economy. Incorporating investors' ambiguity perceptions as extracted from forward-looking option markets, consistent with an ambiguity-adjusted OPM, IA shows superior power in real economic activity predictions, outperforming the VIX, VRP, CS and other market-based predictors. Our results provide empirical support for the predictions of several key economic theories (including real options, precautionary saving, risk premium and financial frictions), which suggest that increasing uncertainty suppresses production, employment, consumption, and overall economic activity. The efficiency of option market extracted ambiguity in predicting a wide range of economic indicators might contribute to improved forecasts in monitoring the state of the economy and implementing robust macroeconomic policy decisions. Policy makers and economic forecasters can enrich and refine their models of economic activity prediction by incorporating market ambiguity information.

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Appendix to Chapter 3 - Comparisons with VIX, VRP and CS

Table A3.1. Predicting Production and Employment

Table A3.1 reports horse race comparisons of IA with VIX, with VRP, or with CS (while controlling for ER) in predictive regressions involving industrial production (IP) growth, capacity utilization (CU) growth, unemployment rate (UR) growth, and total non-farm payroll (TNP) growth. IA is the option-market implied ambiguity. VIX is the implied volatility of the S&P 500 index based on the CBOE volatility index. VRP is variance risk premium, obtained as the difference between implied variance and realized variance of the S&P 500 index. ER is the excess return of the S&P 500 index calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bill. CS is the credit spread between Moody's AAA and BAA bond yield indices. The sample covers monthly observations from 1990M01 to 2014M12. Newey-West t-statistics with lags equal to the return horizon (in months) are reported in parentheses. *, **, and *** represent significance at 90%, 95%, and 99% confidence levels respectively.

Panel A. Predicting Industrial Production (IP) Growth

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	6.77	5.05	4.00	3.39	2.87	2.65	2.51	2.40
	(4.43)	(4.44)	(4.91)	(3.63)	(2.48)	(1.93)	(1.58)	(1.32)
IA	-8.00***	-9.15***	-8.45***	-8.28***	-7.89***	-7.20***	-6.51***	-5.58***
	(-3.23)	(-2.83)	(-2.58)	(-2.78)	(-2.97)	(-3.08)	(-3.25)	(-3.21)
ER	0.03***	0.03***	0.02***	0.02***	0.02***	0.02***	0.01***	0.01***
	(3.52)	(3.74)	(3.21)	(3.14)	(3.33)	(3.09)	(2.89)	(2.71)
VIX	-0.18**	-0.09	-0.04	-0.01	0.02	0.02	0.03	0.03
	(-2.22)	(-1.47)	(-1.00)	(-0.21)	(0.33)	(0.45)	(0.47)	(0.41)
Adj. R^2(%)	28.22	27.18	21.83	20.38	19.24	16.76	14.45	11.94
Cst	3.21	2.99	2.78	2.86	2.86	2.84	2.84	2.73
	(0.65)	(0.45)	(0.51)	(0.57)	(0.65)	(0.73)	(0.83)	(0.92)
IA	-10.34***	-9.96***	-8.54***	-8.05***	-7.36***	-6.60***	-5.92***	-5.00***
	(-3.80)	(-3.02)	(-2.76)	(-3.01)	(-3.18)	(-3.36)	(-3.54)	(-3.42)
ER	0.04***	0.03***	0.03***	0.02***	0.02***	0.01***	0.01***	0.01***
	(3.99)	(4.21)	(4.24)	(4.41)	(4.78)	(4.86)	(4.82)	(4.67)
VRP	0.01	0.02	0.02**	0.02*	0.01	0.01	0.01	0.01
	(0.37)	(1.44)	(2.46)	(1.67)	(1.31)	(1.08)	(0.71)	(0.66)
Adj. R^2(%)	24.00	26.53	22.43	20.94	19.66	16.98	14.42	11.91
Cst	8.16	6.54	5.31	4.44	3.74	3.32	3.13	3.03
	(7.45)	(5.08)	(4.17)	(3.63)	(3.17)	(2.76)	(2.47)	(2.29)
IA	-7.16***	-8.15***	-7.53***	-7.52***	-7.25***	-6.70***	-6.04***	-5.10***
	(-3.58)	(-3.11)	(-2.82)	(-3.03)	(-3.12)	(-3.19)	(-3.37)	(-3.37)
ER	0.03***	0.03***	0.02***	0.02***	0.02***	0.01***	0.01***	0.01***
	(4.55)	(4.19)	(3.76)	(3.81)	(4.22)	(4.27)	(4.34)	(4.36)
CS	-5.37***	-3.54**	-2.34	-1.38	-0.65	-0.25	-0.11	-0.13
	(-4.47)	(-2.47)	(-1.61)	(-1.02)	(-0.55)	(-0.23)	(-0.11)	(-0.14)
Adj. R^2(%)	38.38	33.82	25.60	22.03	19.60	16.65	14.18	11.66

Panel B. Predicting Capacity Utilization (CU) Growth

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	4.03	2.07	0.79	-0.03	-0.71	-1.04	-1.25	-1.39
	(2.72)	(1.75)	(0.80)	(-0.03)	(-0.54)	(-0.74)	(-0.87)	(-0.95)
IA	-7.16***	-8.25***	-7.41**	-7.00**	-6.37**	-5.38**	-4.47**	-3.34**
	(-3.01)	(-2.63)	(-2.31)	(-2.38)	(-2.42)	(-2.32)	(-2.29)	(-2.03)
ER	0.02***	0.02***	0.02**	0.02**	0.02**	0.01*	0.01	0.01
	(2.90)	(2.73)	(2.21)	(2.00)	(1.99)	(1.68)	(1.40)	(1.25)
VIX	-0.16**	-0.06	0.00	0.04	0.07	0.08	0.08	0.08
	(-2.06)	(-0.89)	(-0.04)	(0.52)	(0.88)	(1.01)	(1.10)	(1.11)
Adj. R^2(%)	22.70	20.42	14.90	12.84	11.50	8.97	7.14	5.35
Cst	0.88	0.58	0.27	0.28	0.20	0.12	0.08	-0.08
	(0.68)	(0.47)	(0.47)	(0.47)	(0.48)	(0.51)	(0.56)	(0.60)
IA	-9.40***	-8.73***	-6.98**	-6.10**	-5.06**	-3.95**	-3.03*	-1.90
	(-3.66)	(-2.83)	(-2.44)	(-2.48)	(-2.36)	(-2.11)	(-1.87)	(-1.42)
ER	0.03***	0.03***	0.02***	0.02***	0.01***	0.01**	0.00*	0.00
	(3.41)	(3.37)	(3.16)	(2.96)	(3.00)	(2.52)	(1.91)	(1.47)
VRP	0.01	0.02	0.02	0.02	0.02	0.01	0.01	0.01
	(0.17)	(1.07)	(1.53)	(1.18)	(1.14)	(1.05)	(0.85)	(0.82)
Adj. R^2(%)	19.03	20.28	15.83	13.18	10.81	7.30	4.38	2.10
Cst	3.36	1.57	0.16	-0.86	-1.64	-2.09	-2.25	-2.28
	(2.39)	(1.06)	(0.13)	(-0.88)	(-2.09)	(-3.02)	(-3.24)	(-3.20)
IA	-7.79***	-8.66***	-7.87***	-7.56***	-6.99***	-6.07***	-5.12***	-3.91***
	(-3.69)	(-3.23)	(-2.89)	(-3.06)	(-3.07)	(-3.01)	(-3.08)	(-2.94)
ER	0.03***	0.03***	0.02***	0.02***	0.01***	0.01***	0.01**	0.01**
	(3.81)	(3.36)	(2.92)	(2.83)	(2.91)	(2.59)	(2.31)	(2.09)
CS	-2.70*	-0.69	0.67	1.72	2.49***	2.87***	2.92***	2.78***
	(-1.75)	(-0.43)	(0.49)	(1.47)	(2.71)	(4.14)	(5.24)	(5.79)
Adj. R^2(%)	22.73	20.10	15.25	15.31	17.26	18.17	18.44	17.32

Panel C. Predicting Unemployment Rate (UR) Growth

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	-18.86	-14.75	-11.70	-9.43	-7.07	-5.62	-4.56	-3.18
	(-3.64)	(-3.29)	(-3.11)	(-2.47)	(-1.68)	(-1.19)	(-0.86)	(-0.54)
IA	18.89**	23.61**	25.28**	25.96**	26.38***	26.11***	24.30***	23.07***
	(2.18)	(2.32)	(2.35)	(2.56)	(2.76)	(2.91)	(2.99)	(3.05)
ER	-0.04	-0.05**	-0.05**	-0.05***	-0.05***	-0.05***	-0.04***	-0.04***
	(-1.43)	(-2.41)	(-2.57)	(-2.66)	(-2.75)	(-2.77)	(-2.62)	(-2.75)
VIX	0.87***	0.64***	0.47**	0.34**	0.21	0.13	0.08	0.01
	(3.17)	(2.73)	(2.54)	(1.98)	(1.21)	(0.70)	(0.41)	(0.06)
Adj. R^2(%)	21.38	25.46	23.68	22.01	20.16	18.59	15.82	13.80
Cst	-3.87	-3.38	-3.31	-3.10	-3.17	-3.21	-3.14	-2.83
	(2.56)	(2.14)	(2.13)	(2.41)	(2.63)	(2.82)	(3.12)	(3.33)
IA	32.56***	33.19***	32.24***	30.82***	29.39***	27.95***	25.51***	23.12***
	(3.27)	(2.95)	(2.87)	(2.94)	(3.10)	(3.25)	(3.29)	(3.28)
ER	-0.08**	-0.09***	-0.08***	-0.07***	-0.06***	-0.06***	-0.05***	-0.04***
	(-2.39)	(-2.94)	(-3.20)	(-3.49)	(-3.56)	(-3.82)	(-3.77)	(-4.33)
VRP	0.05	0.02	0.01	0.00	0.00	0.00	0.00	-0.01
	(0.50)	(0.29)	(0.27)	(-0.01)	(0.00)	(-0.01)	(0.06)	(-0.12)
Adj. R^2(%)	13.53	19.74	20.21	19.96	19.29	18.23	15.67	13.81
Cst	-20.72	-17.01	-13.78	-11.16	-8.89	-6.95	-5.33	-4.17
	(-6.18)	(-5.15)	(-4.00)	(-3.30)	(-2.64)	(-2.04)	(-1.45)	(-1.08)
IA	18.32**	22.54**	24.18***	25.01***	25.25***	25.26***	23.81***	22.37***
	(2.32)	(2.54)	(2.61)	(2.77)	(2.95)	(3.06)	(3.16)	(3.23)
ER	-0.06***	-0.07***	-0.07***	-0.06***	-0.06***	-0.05***	-0.05***	-0.04***
	(-2.69)	(-3.71)	(-3.49)	(-3.61)	(-3.56)	(-3.80)	(-3.64)	(-4.02)
CS	20.56***	16.05***	12.25***	9.17***	6.51**	4.25	2.57	1.38
	(6.30)	(4.94)	(3.70)	(2.88)	(2.24)	(1.59)	(0.96)	(0.51)
Adj. R^2(%)	30.06	33.31	29.13	25.49	22.38	19.66	16.24	13.97

Panel D. Predicting Total Non-farm Payroll (TNP) Growth

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	3.70	3.39	3.12	2.85	2.60	2.40	2.24	2.10
	(6.76)	(5.91)	(5.84)	(5.63)	(4.97)	(4.22)	(3.58)	(3.02)
IA	-1.68**	-2.41**	-2.57**	-2.74**	-2.86***	-2.83***	-2.82***	-2.73***
	(-2.09)	(-2.41)	(-2.30)	(-2.43)	(-2.61)	(-2.75)	(-2.98)	(-3.11)
ER	0.00	0.00	0.00	0.00**	0.00**	0.00**	0.00**	0.00**
	(0.63)	(1.24)	(1.60)	(1.96)	(2.10)	(2.15)	(2.12)	(2.21)
VIX	-0.12***	-0.10***	-0.09***	-0.07***	-0.06**	-0.05*	-0.04	-0.03
	(-3.95)	(-3.18)	(-2.96)	(-2.69)	(-2.27)	(-1.89)	(-1.56)	(-1.25)
Adj. R^2(%)	34.78	36.32	34.12	32.17	29.93	27.19	25.15	23.22
Cst	1.61	1.62	1.59	1.58	1.55	1.55	1.53	1.51
	(0.29)	(0.26)	(0.25)	(0.28)	(0.31)	(0.34)	(0.37)	(0.40)
IA	-3.64***	-4.03***	-3.96***	-3.89***	-3.78***	-3.60***	-3.46***	-3.26***
	(-3.75)	(-3.38)	(-3.08)	(-3.08)	(-3.15)	(-3.24)	(-3.39)	(-3.47)
ER	0.01**	0.01**	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(2.21)	(2.35)	(2.76)	(3.15)	(3.39)	(3.62)	(3.71)	(4.05)
VRP	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00
	(-0.69)	(-0.59)	(-0.65)	(-0.61)	(-0.43)	(-0.42)	(-0.35)	(-0.28)
Adj. R^2(%)	17.88	23.26	23.91	24.67	24.61	23.35	22.31	21.11
Cst	4.26	3.94	3.63	3.32	3.05	2.81	2.63	2.45
	(18.64)	(12.97)	(10.06)	(8.34)	(7.27)	(6.31)	(5.54)	(4.82)
IA	-1.38**	-2.10***	-2.28***	-2.46***	-2.59***	-2.58***	-2.58***	-2.51***
	(-2.35)	(-2.88)	(-2.77)	(-2.86)	(-2.97)	(-3.02)	(-3.18)	(-3.27)
ER	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(3.13)	(3.89)	(4.21)	(4.22)	(4.33)	(4.43)	(4.32)	(4.57)
CS	-3.25***	-2.81***	-2.46***	-2.10***	-1.77***	-1.51***	-1.30***	-1.11***
	(-14.73)	(-9.36)	(-6.87)	(-5.47)	(-4.73)	(-4.19)	(-3.68)	(-3.11)
Adj. R^2(%)	63.06	60.10	53.70	47.74	42.32	37.04	33.29	29.68

Table A3.2. Predicting Consumption and Overall Output

Table A3.2 reports comparisons of IA with VIX, with VRP, or with CS (while controlling for ER) in predictive regressions involving personal consumption expenditure (PCE) growth, durable goods (DG) consumption growth, real GDP per capita (GDPC) growth, and the Chicago Fed National Activity Index (CFNAI). IA is the option-market implied ambiguity. VIX is the implied volatility of the S&P 500 index based on the CBOE volatility index. VRP is variance risk premium, obtained as the difference between implied variance and realized variance of the S&P 500 index. ER is the excess return of the S&P 500 index calculated as the logarithmic return of S&P 500 in excess of the logarithmic yield of 3M T-bill. CS is the credit spread between Moody's AAA and BAA bond yield indices. The sample covers monthly observations from 1990M01 to 2014M12. Newey-West t-statistics with lags equal to the return horizon (in months) are reported in parentheses. *, **, and *** represent significance at 90%, 95%, and 99% confidence levels respectively.

Panel A. Personal Consumption Expenditure (PCE) Growth

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	6.93	6.18	5.86	5.59	5.41	5.19	5.01	4.79
	(12.34)	(12.56)	(13.11)	(11.44)	(10.18)	(8.97)	(8.05)	(6.78)
IA	-3.51**	-3.19*	-2.95*	-3.08**	-3.02**	-2.94**	-2.88**	-2.63**
	(-2.24)	(-1.93)	(-1.81)	(-2.12)	(-2.16)	(-2.25)	(-2.41)	(-2.34)
ER	0.01***	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**
	(2.85)	(2.15)	(2.05)	(2.46)	(2.28)	(2.12)	(2.34)	(2.29)
VIX	-0.08***	-0.05*	-0.03	-0.02	-0.01	0.00	0.01	0.02
	(-2.71)	(-1.78)	(-1.27)	(-0.69)	(-0.34)	(0.00)	(0.29)	(0.54)
Adj. R^2(%)	20.69	18.26	15.72	16.89	15.77	13.45	13.09	11.07
Cst	5.30	5.10	5.07	5.06	5.01	4.96	4.91	4.84
	(0.31)	(0.23)	(0.23)	(0.26)	(0.29)	(0.32)	(0.34)	(0.37)
IA	-4.54***	-3.64**	-3.24**	-3.18**	-2.96**	-2.70**	-2.50**	-2.11**
	(-2.91)	(-2.28)	(-2.08)	(-2.26)	(-2.25)	(-2.24)	(-2.35)	(-2.15)
ER	0.02***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(3.29)	(2.79)	(2.98)	(3.41)	(3.37)	(3.23)	(3.64)	(3.66)
VRP	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	(0.44)	(1.23)	(1.30)	(1.16)	(1.27)	(1.33)	(1.47)	(1.46)
Adj. R^2(%)	17.51	17.26	15.32	17.18	16.68	14.97	14.91	12.93
Cst	7.53	7.21	6.96	6.69	6.46	6.29	6.11	5.97
	(14.45)	(13.11)	(12.86)	(14.44)	(15.65)	(14.61)	(13.36)	(12.08)
IA	-3.14**	-2.49**	-2.18*	-2.30**	-2.27**	-2.14**	-2.07**	-1.76**
	(-2.41)	(-2.00)	(-1.81)	(-2.09)	(-2.07)	(-2.11)	(-2.25)	(-2.07)
ER	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.00***	0.00***
	(3.49)	(2.97)	(3.12)	(3.33)	(3.19)	(2.79)	(3.12)	(3.06)
CS	-2.40***	-2.16***	-1.95***	-1.65***	-1.41***	-1.24***	-1.07***	-0.97***
	(-4.46)	(-3.45)	(-3.21)	(-3.34)	(-3.64)	(-3.40)	(-2.99)	(-2.65)
Adj. R^2(%)	28.27	30.40	28.83	28.05	25.10	21.60	19.56	16.39

Panel B. Predicting Durable Consumption (DG) Growth

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	4.87	4.08	3.50	3.57	3.30	2.83	2.78	2.46
	(2.30)	(3.02)	(2.36)	(2.29)	(1.92)	(1.43)	(1.25)	(1.00)
IA	-9.91**	-9.02**	-7.17**	-6.70**	-7.04**	-6.57**	-5.67**	-5.34**
	(-2.03)	(-2.19)	(-1.98)	(-2.05)	(-2.27)	(-2.18)	(-2.15)	(-2.26)
ER	0.03*	0.03**	0.02**	0.02***	0.02***	0.02**	0.01**	0.02***
	(1.94)	(2.44)	(2.57)	(2.73)	(2.84)	(2.32)	(2.29)	(2.76)
VIX	0.01	0.05	0.07	0.07	0.08	0.11	0.11	0.12
	(0.13)	(0.83)	(1.03)	(1.00)	(1.26)	(1.45)	(1.29)	(1.35)
Adj. R^2(%)	3.61	7.04	6.10	6.86	8.88	7.16	6.41	7.51
Cst	5.01	4.38	4.40	4.33	4.38	4.32	4.22	4.17
	(1.13)	(0.79)	(0.88)	(0.90)	(1.00)	(1.09)	(1.16)	(1.27)
IA	-9.59**	-7.65**	-5.79*	-5.31*	-5.37*	-4.53*	-3.66*	-3.12
	(-2.00)	(-1.99)	(-1.81)	(-1.77)	(-1.93)	(-1.76)	(-1.70)	(-1.56)
ER	0.03**	0.02***	0.02***	0.02***	0.02***	0.01**	0.01**	0.01***
	(2.15)	(2.62)	(2.64)	(3.02)	(3.20)	(2.44)	(2.38)	(2.77)
VRP	0.01	0.03	0.02	0.02	0.02	0.02	0.02	0.02
	(0.14)	(1.51)	(0.76)	(0.92)	(1.01)	(1.00)	(1.10)	(1.02)
Adj. R^2(%)	3.61	7.50	5.86	6.81	8.39	5.78	4.96	5.01
Cst	7.25	7.01	6.81	6.42	5.92	5.71	5.52	5.32
	(3.29)	(4.14)	(4.16)	(4.14)	(3.98)	(3.66)	(3.43)	(3.27)
IA	-8.18*	-6.85*	-4.70*	-4.58	-5.07*	-4.39*	-3.59*	-3.16*
	(-1.86)	(-1.91)	(-1.65)	(-1.62)	(-1.85)	(-1.72)	(-1.68)	(-1.67)
ER	0.03*	0.02**	0.01**	0.02***	0.02***	0.01**	0.01**	0.01***
	(1.83)	(2.48)	(2.48)	(2.59)	(3.03)	(2.14)	(2.23)	(3.06)
CS	-2.41	-2.26	-2.30	-1.86	-1.21	-1.00	-0.90	-0.72
	(-1.07)	(-1.22)	(-1.43)	(-1.31)	(-1.01)	(-0.84)	(-0.77)	(-0.67)
Adj. R^2(%)	4.35	8.40	7.95	8.12	8.58	5.51	4.45	4.19

Panel C. Predicting GDP Per Capita (GDPC) Growth

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	2.55	2.33	2.06	1.92	1.73	1.63	1.51	1.47
	(3.32)	(3.78)	(4.28)	(4.58)	(4.03)	(3.49)	(2.70)	(2.24)
IA	-4.98***	-4.88***	-4.66***	-4.56***	-4.00***	-3.66***	-3.54***	-3.11***
	(-2.84)	(-3.54)	(-3.41)	(-3.53)	(-3.03)	(-2.90)	(-3.07)	(-2.72)
ER	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**	0.00*
	(2.17)	(2.32)	(2.32)	(2.06)	(2.19)	(2.06)	(2.05)	(1.76)
VIX	-0.03	-0.02	0.00	0.00	0.01	0.01	0.02	0.02
	(-0.70)	(-0.56)	(-0.15)	(0.23)	(0.54)	(0.75)	(0.91)	(0.81)
Adj. R^2(%)	20.66	28.69	27.36	26.47	22.84	19.71	19.08	14.76
Cst	1.89	1.79	1.71	1.72	1.66	1.61	1.55	1.52
	(0.41)	(0.33)	(0.29)	(0.29)	(0.30)	(0.34)	(0.38)	(0.43)
IA	-5.20***	-4.95***	-4.54***	-4.34***	-3.72***	-3.32***	-3.10***	-2.67***
	(-2.76)	(-3.19)	(-3.16)	(-3.38)	(-2.98)	(-2.91)	(-3.22)	(-2.80)
ER	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**	0.00**
	(2.30)	(2.46)	(2.47)	(2.24)	(2.38)	(2.30)	(2.37)	(2.10)
VRP	0.01	0.01	0.01	0.01	0.01	0.01*	0.02*	0.02
	(0.45)	(0.92)	(1.42)	(1.45)	(1.42)	(1.66)	(1.69)	(1.55)
Adj. R^2(%)	20.28	28.98	28.45	27.83	24.09	21.21	21.04	16.92
Cst	3.95	3.42	3.11	2.81	2.58	2.37	2.29	2.29
	(9.35)	(9.99)	(8.36)	(7.50)	(6.58)	(5.56)	(4.86)	(4.46)
IA	-3.26*	-3.58***	-3.51***	-3.66***	-3.20***	-3.00***	-2.89***	-2.42**
	(-1.90)	(-2.97)	(-2.92)	(-3.23)	(-2.71)	(-2.62)	(-2.94)	(-2.53)
ER	0.01**	0.01**	0.01**	0.01*	0.01*	0.01*	0.01*	0.00
	(2.42)	(2.45)	(2.22)	(1.85)	(1.87)	(1.73)	(1.65)	(1.34)
CS	-2.25***	-1.66***	-1.32***	-0.94**	-0.75**	-0.54	-0.45	-0.49
	(-4.94)	(-4.86)	(-3.42)	(-2.48)	(-2.08)	(-1.58)	(-1.34)	(-1.42)
Adj. R^2(%)	32.53	37.80	34.36	30.70	25.72	21.05	19.38	15.41

Panel D. Chicago Fed National Activity Index (CFNAI)

	Prediction Horizon (Quarters)							
	1	2	3	4	5	6	7	8
Cst	0.82	0.61	0.46	0.35	0.25	0.18	0.13	0.08
	(3.69)	(3.08)	(2.88)	(2.26)	(1.45)	(0.92)	(0.57)	(0.30)
IA	-0.93***	-1.11**	-1.13**	-1.14**	-1.13***	-1.07***	-1.01***	-0.92***
	(-2.74)	(-2.46)	(-2.34)	(-2.46)	(-2.60)	(-2.65)	(-2.76)	(-2.76)
ER	0.00**	0.00**	0.00**	0.00**	0.00***	0.00**	0.00**	0.00**
	(2.26)	(2.49)	(2.40)	(2.56)	(2.60)	(2.44)	(2.31)	(2.28)
VIX	-0.04***	-0.03***	-0.02***	-0.02**	-0.01	-0.01	-0.01	0.00
	(-3.47)	(-2.90)	(-2.80)	(-2.36)	(-1.63)	(-1.14)	(-0.77)	(-0.48)
Adj. R^2(%)	37.48	33.52	28.59	26.28	23.83	20.74	18.16	15.60
Cst	0.05	0.03	0.02	0.01	0.01	0.00	-0.01	-0.02
	(0.11)	(0.09)	(0.09)	(0.10)	(0.11)	(0.12)	(0.13)	(0.14)
IA	-1.55***	-1.54***	-1.45***	-1.38***	-1.29***	-1.19***	-1.09***	-0.97***
	(-3.83)	(-3.08)	(-2.87)	(-2.94)	(-3.00)	(-3.05)	(-3.15)	(-3.10)
ER	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***
	(2.95)	(2.90)	(3.12)	(3.47)	(3.62)	(3.65)	(3.69)	(3.92)
VRP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(-0.17)	(0.11)	(0.25)	(0.25)	(0.36)	(0.32)	(0.28)	(0.35)
Adj. R^2(%)	24.33	25.68	23.78	23.30	22.22	19.81	17.62	15.37
Cst	1.00	0.81	0.66	0.52	0.41	0.33	0.27	0.22
	(8.32)	(5.44)	(4.18)	(3.38)	(2.75)	(2.12)	(1.63)	(1.27)
IA	-0.83***	-0.99***	-1.01***	-1.03***	-1.03***	-0.98***	-0.92***	-0.82***
	(-3.22)	(-2.88)	(-2.74)	(-2.82)	(-2.86)	(-2.85)	(-2.95)	(-2.96)
ER	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***	0.00***
	(4.15)	(3.88)	(3.73)	(3.81)	(3.90)	(3.84)	(3.84)	(4.01)
CS	-1.10***	-0.88***	-0.71***	-0.57***	-0.44***	-0.35***	-0.30**	-0.26**
	(-8.82)	(-5.61)	(-4.26)	(-3.51)	(-3.10)	(-2.78)	(-2.45)	(-2.18)
Adj. R^2(%)	57.85	49.42	40.79	35.16	30.22	25.45	21.94	18.90

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Chapter 4.

Accounting for Ambiguity Aversion in GARCH Volatility Models

ABSTRACT

Distinguishing between risk and uncertainty, this chapter proposes a volatility forecasting framework that incorporates asymmetric ambiguity shocks in the (exponential) GARCH-M conditional volatility process. Spanning 23 years of daily data and considering the differential role of ambiguity attitudes in the gain and loss domains, our models capture a rich set of information and provide more accurate volatility forecasts both in-sample and out-of-sample when compared to the unambiguous or risk-based counterparts. Volatility-timing trading strategies confirm the economic significance of our proposed methodology and indicate that an annualized excess return of 4.09% over the benchmark could be earned from 1995 to 2012.

4.1. Introduction

Pioneered by Engle (1982), the autoregressive conditional heteroskedasticity (ARCH) volatility modeling approach has revolutionized the way we predict volatility and allowed us to understand time-varying volatility under different assumptions. Engle's seminal approach in modeling conditional volatility has been extended to the generalized ARCH (GARCH, Bollerslev, 1986), exponential ARCH (E(G)ARCH, Nelson 1991), and GJR-(G)ARCH models (Glosten et al., 1993). In recent years, an important stream of research concerned with the notion of ambiguity, as uncertainty beyond probabilistic risk, has emerged to highlight the relevance and significance of model uncertainty in asset pricing, volatility prediction, policy evaluation, and decision making (e.g., Manski, 2000; Cao et al., 2005; Brock et al., 2007; Agliardi and Agliardi, 2009; Easley and O'Hara, 2009; Kast et al., 2014). The concept of uncertainty and its distinction from risk was highlighted almost a century ago by Knight (1921), was further conceptualized by Keynes (1921, 1937), and corroborated by Ellsberg (1961) in his famous thought experiments on individual decision making under ambiguity. Although ambiguity has been widely recognized and theorized by researchers in the context of financial markets (see for e.g., Gilboa and Schmeidler, 1989; Chateauneuf et al., 1996; Cao et al., 2005; Handel et al., 2013), empirical work linking model uncertainty to volatility modeling is still scarce (e.g., Buraschi and Jiltsov, 2006; Anderson et al., 2009; Fan and Mancini, 2009; Driouchi et al., 2016). This is due to the inherent difficulties in quantifying ambiguity empirically.

Motivated by the need to quantify ambiguity and assess the potential implications of model uncertainty in volatility prediction, this chapter investigates the empirical relation between ambiguity attitudes and risk in financial markets and highlights the value of incorporating ambiguity in GARCH volatility forecasts. In a recent paper, Driouchi et al. (2016) study the lead-lag relationship between ambiguity implied by option prices and realized volatility around the subprime crisis in a standard historical variance setting, and demonstrate that forward-looking ambiguity can be important in volatility prediction especially in uncertain times (i.e., 2006-2008). In their effort to estimate the impact of uncertainty on expected returns, Anderson et al. (2009) also examined the effect of uncertainty on conditional volatility as a robustness check. However, their study only focuses on quarterly data, as limited by the availability of professional forecasters' survey data, and does not provide information on how uncertainty affects conditional volatility in higher frequency settings (e.g., daily). Also related, Fan and Mancini (2009) show how accounting for learning and model misspecification in option pricing can minimize empirical pricing errors and improve volatility prediction. They validate their approach using option pricing data for the 2002-2004 period. No study has highlighted the role of ambiguity aversion, as a decision-theoretic construct, in volatility forecasting over an extensive time window in- and out-of-sample and using a large dataset of option prices in the context of GARCH volatility modeling.

We fill this gap in research by examining the relationship between investors' attitudes to ambiguity, as inferred from market traded option prices (and also the CBOE's VIX), and conditional volatility over the 1990-2012 period in the context of the GARCH volatility framework. More specifically, we extract investors' attitudes to ambiguity from S&P 500 index options using a modified option pricing formula under ambiguity and account for ambiguity innovations in our GARCH volatility forecasts. This approach allows us to capture and quantify the ambiguity attitudes of sophisticated options investors/traders on a real-time basis. Our chapter differs from that of Driouchi et al. (2016) in that we explicitly incorporate ambiguity innovations in the GARCH methodology, control for downside and upside markets (i.e., gains vs. losses), and assess economic significance and forecasting accuracy in- and out-of-sample over the entire 1990-2012 period. Not concerned with the GARCH apparatus, Driouchi et al. (2016) are focused on the subprime crisis and the incremental information content of ambiguity implied from put option prices over 2006-2008 in a standard historical variance setting. By allowing asymmetric uncertainty shocks in the GARCH conditional volatility process for calls/puts, gains and losses, we show that option market ambiguity attitudes (OMAA) are quantitatively important in determining the subsequent level of conditional volatility. Our analysis reveals a strong relationship between OMAA and ex post conditional volatility over a quarter century of daily data. Ambiguity aversion is positively associated with ex post conditional volatility in the gain domain, while negatively associated with ex post conditional volatility in the loss domain. This special S-shape relationship has been prominent in the behavioral economics, decision theory and psychology literatures regarding agents' decision making behavior. We unveil it in the context of financial markets and GARCH volatility forecasting. Our results are robust to a range of forecasting tests (e.g., in-sample, out-of-sample, and economic significance) and various modeling specifications.

Back in the 1990s, many studies suggested that the relationship between an individual's decision and her ambiguity attitude may not be explained by a simple linear relationship, especially when considering emotional sensitivities to gains and losses (Thaler et. Al, 1997; Tversky and Kahneman, 1986; Thaler and Johnson, 1990; Low, 2004). For example, Viscusi and Chesson (1999) explained how an individual's ambiguity attitude may shift from ambiguity aversion to ambiguity seeking (and vice versa) under the fear and hope effects. Their work underlines the differential role of ambiguity attitudes in the gain and loss domains. They suggest that in the gain domain, subjects are more ambiguity averse for high probabilities of gains but become more ambiguity seeking for low probabilities of gains. On the other hand in the loss domain, subjects are more ambiguity seeking for high probabilities of loss and more ambiguity averse for low probabilities of loss. A similar shift in ambiguity preference is also documented by Ho, Keller and Keltyka (2002) and Chkravarty and Roy (2009). Kelsey et al. (2011) also point out that under Knightian uncertainty or ambiguity investors react differently to past winners and losers, and explain how momentum profitability relates to such an asymmetry. In line with this pattern of attitudes, we posit that the different impacts of positive and negative returns on investors' ambiguity and sentimental attributes might explain the difficulties researchers have faced in modeling the interaction between market risk and ambiguity empirically. This is especially important in our analysis of the relationship between volatility and ambiguity, as the sought-after association might change from time to time depending on the level of gain/loss on the investment.

Inspired by the observed pattern of shifting ambiguity attitudes in decision making experiments, this chapter analyses the relationship between volatility and ambiguity by taking into account the gains/losses of market investors, and examines the extent to which adopting an ambiguity-based approach to modeling volatility can contribute to improving the accuracy and practical relevance of GARCH volatility forecasts. Given the behavioral observations of Viscusi and Chesson (1999) and that behavioral biases from ambiguity neutrality create instability, we expect ambiguity aversion (seeking) to contribute to upward revisions in conditional volatility in the gain (loss) domain.

Our proposed GARCH volatility modeling methodology incorporates the asymmetric impact of option market ambiguity attitudes on ex post conditional volatility and shows that ambiguity or model uncertainty, as proxied by OMAA, can improve the forecasting accuracy of GARCH volatility models using daily data that spans 23 years from 1990 to 2012. The inclusion of OMAA yields significant improvements in the in-sample root mean squared error (RMSE) for up to 7.8% versus the standard benchmark. We also assess the out-of-sample forecasting ability of our ambiguity-based volatility models when compared to their unambiguous and risk-based benchmark counterparts. Analyses based on different estimation windows, forecasting windows, sampling frequencies of intraday realized volatility, and four different loss functions, consistently confirm that OMAA is statistically significant in improving the accuracy of both in-sample and out-of-sample volatility forecasts. As a robustness test, we also examine the economic significance of considering OMAA in GARCH volatility modeling by comparing portfolio returns generated from two simple volatility timing trading strategies based on our out-of-sample volatility forecasts under ambiguity. For the out-of-sample estimation window 1995-2012, an annualized 4.09% return can be earned in excess of that generated by the unambiguous or risk-based forecasts. We contribute to the literature by presenting robust and extensive empirical evidence on the importance of miscalibration and the efficiency of option market information in GARCH volatility forecasting.

4.2. Empirical Framework

To investigate the role of ambiguity attitudes in the formation of subsequent conditional variance of excess returns, we employ the GARCH-in-mean model and account for ambiguity and option implied variance as exogenous variables in the variance equation. For robustness, we examine the same linkage under an exponential version of GARCH to ensure the positivity of forecasted variance. We expand our study out-of-sample with different estimation windows in order to verify if the relationship observed from the in-sample estimation is stable and consistent in out-of-sample settings, and whether the discovered relationship provides economic value to volatility forecasting practice.

4.2.1 *Inferring Ambiguity Attitudes from the Option Market*

To assess the relationship between investors' ambiguity attitudes and ex post conditional variance, daily estimations of ambiguity attitudes are crucial. To this aim, we employ the rank dependent utility framework under ambiguity proposed by Chateauneuf et al. (1996) and extended to option pricing by Driouchi et al. (2015). Underlying the expanded option pricing framework is a modified version of geometric Brownian motion that allows subjective attitudes towards Knightian uncertainty to come into play. Related to the uncertain expected utility of Gul and Pesendorfer (2014), this type of Brownian motion has been validated by Kast and Lapied (2010) and Kast et al. (2014) in a number of decision theoretic contributions. The ambiguity-based Brownian motion is, thus, specified as follows:

$$\frac{dS}{S} = (\mu + m\sigma)dt + s\sigma dz \;\; (\forall\, m \in \,]-1,1[\,, \forall\, s \in]0,1]\,) \tag{4.1}$$

$$dW = mdt + sdz \tag{4.2}$$

where *S* is the price of the underlying asset, *m* and *s* are the mean and standard deviation of a general Wiener process *W*, *z* is a standard Wiener process. Agent's model misspecification under ambiguity is summarized by a capacity variable *c* which determines *m* and *s*, where 0 < c < 1 (Kast et al., 2014). Driouchi et al. (2015) employ this set of Brownian motions to derive an ambiguity-adjusted model for European exchange option pricing under Knightian uncertainty. As a special case of that model, the price of a European call[1] option with fixed strike *K* under ambiguity takes the following modified Black-Scholes form:[2]

$$\begin{aligned} P_t^C = {} & S_0 e^{-\delta' T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r' - \delta' + 0.5(s\sigma)^2)T}{s\sigma\sqrt{T}}\right) \\ & - Ke^{-r'T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r' - \delta' - 0.5(s\sigma)^2)T}{s\sigma\sqrt{T}}\right) \end{aligned} \tag{4.3}$$

where:

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[1] As robustness, we also obtained OMAA using VIX-implied model prices with comparable results. The alternative procedure consists of 1) taking VIX as the implied volatility of at-the-money options, 2) converting implied volatility into option prices using the standard Black-Scholes model, and 3) extracting OMAA from the VIX-implied option prices using Eq. (4.3). Results are in line with the OMAA from traded option prices presented herein. The additional VIX-based OMAA results are available from the authors.

[2] Technical details of the model are included in the supplementary appendix.

$$r' = r + m\frac{[r - (\mu + m\sigma)]}{s^2\sigma};$$
$$\delta' = \delta - \frac{(m + s^2\sigma - s\sigma)[(\mu + m\sigma) - r]}{s^2\sigma} \tag{4.4}$$

$$m = 2c - 1 \; and \; s = \sqrt{4c(1-c)} \quad (\forall\, c \in]0,1[) \tag{4.5}$$

where P_t^C is the rank dependent utility or ambiguity-adjusted option price for 0 < c < 1, K is the strike price of the option, r is the risk-free rate, T is the time to maturity in years, μ is the subjective expected return, σ is the volatility measure, c is the capacity variable proxying for miscalibration, r' is the subjective discount rate and δ' is the subjective dividend yield.

To obtain an estimate of investors' ambiguity attitudes, we invert Eq. (4.3) numerically by minimizing the absolute deviations between the model price and market price:

$$OMAA_t \equiv c_t^* = \underset{c|0<c<1}{\arg\min}\left[\left|P_t^C(S_t, K, r, T, \sigma_t, \mu_t, \delta_t, c_t) - P_t^{Mkt}\right|\right] \tag{4.6}$$

where P_t^{Mkt} is the market traded SPX option price, S_t is the closing level of S&P 500 index on day t, σ_t is the index volatility, c_t is the time-varying ambiguity measure, μ_t is the rate of return on the index, and δ_t is the dividend yield of S&P 500 portfolio. Investors' ambiguity attitudes are summarized by the capacity variable c_t through the minimization of the absolute error function in (4.6). The resulting capacity variable c_t from (4.6) is, therefore, our proxy for option market ambiguity attitudes (OMAA). In the extraction process, we take the 1-month LIBOR rate as the risk-free rate, the trailing twelve months dividend yield of the S&P 500 index as dividend yield, RiskMetrics EWMA volatility[3] (JP Morgan 1996) as the volatility input (σ_t), and 12-month historical returns as a proxy for the subjective rate of return[4] (μ_t). The choice of inputs in the extraction process

[3] We have also used out-of-sample GARCH(1,1) with a three-year rolling estimation window and a simple 22-day standard deviation of returns as alternative measures of expected volatility. The use of alternative volatility measures does not change our overall conclusions. Our results are not crucially affected by the choice of volatility measure.

[4] The choice of proxy for the subjective rate of return relates to investors' memory about past returns. Barberis, Huang and Santos (2001) emphasize the importance of investors' memory in determining their required returns. They show that investors tend have a short memory when recalling past gains and losses. We employ the 12-month past returns as our proxy for the subjective rate of return as this time frame fits well with the short memory assumption plus it gives us a reliable sample size of 252 trading days. As a robustness check, we also extracted OMAA using shorter (6 months) and longer (up to 3 years) periods of returns as proxies for μ_t and found the conclusions still hold.

is in line with the relevant literature (Barberis, Huang and Santos, 2001; Gonzalez-Riveria, Lee and Mishra, 2004; Harris and Nguyen, 2013).

4.2.2 GARCH-in-mean Estimations

To begin with our specifications of econometric model, we assume the following mean equation for all the ARCH class models used in this chapter:

$$r_t = \alpha_0 + \alpha_1 h_t + \varepsilon_t \tag{4.7}$$

where r_t is the daily logarithmic return of the S&P 500 index in excess of the logarithmic yield of 3-month treasury bill, h_t is the conditional variance at time t, which is estimated dynamically in the variance equation, and ε_t represents the unexpected excess return at time t. Rearranging terms, we get:

$$\varepsilon_t = r_t - (\alpha_0 + \alpha_1 h_t) \tag{4.8}$$

In order to compare and assess the information content of ambiguity attitudes in determining ex post conditional variance, we first estimate a benchmark vanilla GARCH model without the innovation from investors' ambiguity attitudes. We define the variance equation of the benchmark Model 1.1 as:

Model 1.1:

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} \tag{4.9}$$

To assess the information content of the variance model with ambiguity attitudes under different gains and losses, we specify two variance Models 1.2 and 1.3 as follows:

Model 1.2:

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 D_{t-1|r_{t-1}>0} OMAA_{t-1} \tag{4.10}$$

Model 1.3:

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 \left(1 - D_{t-1|r_{t-1}>0}\right) OMAA_{t-1} \tag{4.11}$$

where $D_{t-1|r_{t-1}>0}$ is a dummy variable that takes a value of 1 when the excess return is positive and 0 otherwise, and $OMAA_{t-1}$ is the ambiguity attitude measure. The dummy variables in Models 1.2 and 1.3 aim to capture the asymmetric effects of ambiguity attitude and ex post conditional variance in the gain (Model 1.2) and loss (Model 1.3) domains in with ambiguity theory predictions. Since the inferred ambiguity attitude is based on option market information and to ensure the additional information content provided by ambiguity innovation is not due to informational overlaps with implied variance (despite the low correlation between OMAA and IV), we specify three additional models with option implied variance[5] as one of the exogenous variables:

Model 1.4:

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 IV_{t-1} \tag{4.12}$$

where IV_{t-1} is the daily implied variance of the S&P 500 index. Model 1.4 represents the vanilla GARCH specification with implied variance and without the innovation from ambiguity attitudes. To differentiate between gains and losses, we again consider the following two models with ambiguity:

Model 1.5:

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 IV_{t-1} + \beta_4 D_{t-1|r_{t-1}>0} OMAA_{t-1} \tag{4.13}$$

Model 1.6:

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} + \beta_3 IV_{t-1} + \beta_4 \left(1 - D_{t-1|r_{t-1}>0}\right) OMAA_{t-1} \tag{4.14}$$

4.2.3 EGARCH-in-mean Estimations

The introduction of exogenous variables in ARCH class models may sometimes suffer from yielding negative conditional variances. As a robustness test and to ensure positivity in our out-of-sample volatility forecasts, we follow Engle (1982) and Nelson (1991), and consider equivalents of Models 1.1 to 1.6 in exponential form:

Model 2.1:

$$\log(h_t) = \beta_0 + \beta_1 \left|\varepsilon_{t-1}/\sqrt{h_{t-1}}\right| + \beta_2 \log(h_{t-1}) \tag{4.15}$$

Model 2.2:

$$\log(h_t) = \beta_0 + \beta_1 \left|\varepsilon_{t-1}/\sqrt{h_{t-1}}\right| + \beta_2 \log(h_{t-1}) + \beta_3 D_{t-1|r_{t-1}>0} OMAA_{t-1} \tag{4.16}$$

[5] We have also investigated the role of volatility of volatility (as measured by CBOE VVIX index) in volatility modeling with a reduced sample period from 2007 to 2012 (since VVIX data is only available from 2007). In general VVIX is not significant under both GARCH-M and EGARCH-M estimations, while OMAA remains significant in the presence of VVIX.

Model 2.3:

$$\log(h_t) = \beta_0 + \beta_1 \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| + \beta_2 \log(h_{t-1}) + \beta_3 \left(1 - D_{t-1|r_{t-1}>0}\right) OMAA_{t-1} \tag{4.17}$$

Model 2.4:

$$\log(h_t) = \beta_0 + \beta_1 \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| + \beta_2 \log(h_{t-1}) + \beta_3 \log(IV_{t-1}) \tag{4.18}$$

Model 2.5:

$$\log(h_t) = \beta_0 + \beta_1 \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| + \beta_2 \log(h_{t-1}) + \beta_3 \log(IV_{t-1}) + \beta_4 D_{t-1|r_{t-1}>0} OMAA_{t-1} \tag{4.19}$$

Model 2.6:

$$\log(h_t) = \beta_0 + \beta_1 \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| + \beta_2 \log(h_{t-1}) + \beta_3 \log(IV_{t-1}) + \beta_4 \left(1 - D_{t-1|r_{t-1}>0}\right) OMAA_{t-1} \tag{4.20}$$

Models 2.1 to 2.6 are the exponential versions of Models 1.1 to 1.6. Model 2.1 is the vanilla EGARCH without ambiguity attitudes, and acts as a benchmark for Models 2.2 and 2.3 which consider OMAA under gains and losses respectively. In Models 2.4 to 2.6, we also take into account the impact of implied variance on subsequent conditional variance and investigate if the information from OMAA innovations is still significant (Models 2.5 and 2.6).

4.2.4 Estimation, Inference and Diagnostic Analysis

We estimate the models described in the previous section by maximizing their log-likelihood functions. Inference of variables is based on robust t-statistics as described in Bollerslev and Woodridge (1992). In addition to the estimated coefficients and robust t-statistics, the likelihood ratio and its chi-squared test statistics are also reported to judge the significance of the added OMAA parameter. In-sample error statistics based on four loss functions (described in the next section) are also reported to evaluate the in-sample forecasting performance of each model. For diagnostic analysis, we examine whether the inclusion of option market ambiguity attitudes reduces the skewness and excess kurtosis of standardized residuals from the mean equations. Jarque-Bera test statistics are also reported to compare the normality of standardized residuals from each model.

4.2.5 Out-of-sample Forecasting

We further test the consistency of the relationship between option market ambiguity attitude and conditional variance, and the ability of improving variance forecasts by assessing the out-of-sample forecasting accuracy of the models. To ensure the positivity of conditional variance forecasts out-of-sample, we carry out out-of-sample forecasting based on Models 2.1 to 2.6. To compare the variance forecasts across models, an estimated variance benchmark is needed. This benchmark relates to a one-step ahead realized volatility computed using intra-day returns. Due to the high degree of noise in using daily squared returns as the variance benchmark (Anderson et al. 2001; Anderson et al. 2003), we employ model-free realized variance computed from intraday 1-minute return data. As evidence suggests that the realized variance computed from 1-minute return data can be noisy due to intraday market microstructure issues, we also compute realized variance with rolling 5-minute and 10-minute squared return on 1-minute grid (e.g., Stroud and Johannes, 2014). This approach follows Christoffersen et al. (2014) by minimizing the noise while preserving all the information subsumed in 1-minute returns.

Following Chou (2005), Wei et al. (2010), and Hou and Suardi (2011), we consider the following four loss functions (LF) to evaluate the performance of each of the variance models (2.1 to 2.6).

LF 1: *Root mean squared error (RMSE)*

$$RMSE_M = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(EV_{t+1} - FV_{M,t+1}\right)^2} \tag{4.21}$$

LF 2: *Mean absolute error (MAE)*

$$MAE_M = \frac{1}{T}\sum_{t=1}^{T}\left|EV_{t+1} - FV_{M,t+1}\right| \tag{4.22}$$

LF 3: *Logarithmic loss function (LL)*

$$LL_M = \frac{1}{T}\sum_{t=1}^{T}\left[\ln\left(\frac{EV_{t+1}}{FV_{M,t+1}}\right)\right]^2 \tag{4.23}$$

LF 4: Loss implied by Gaussian likelihood (QLIKE)

$$QLIKE_M = \frac{1}{T}\sum_{t=1}^{T}\left[\ln\left(FV_{M,t+1}\right) + \frac{EV_{t+1}}{FV_{M,t+1}}\right] \tag{4.24}$$

where EV_{t+1} represents the estimated variance benchmark as measured by our intraday variance proxies at time t+1, $FV_{M,t+1}$ represents the forecasted variance from model M in which M represents each of the Models 2.1 to 2.6. While RMSE is the most commonly used loss function in variance forecasting, MAE and QLIKE have been shown to be more robust (Fan et al., 2008; Wei et al., 2010; Hou and Suardi, 2011). More importantly, QLIKE measures relative forecasting error and is known to penalize more heavily forecasts that underestimate the benchmark estimated variance. In this chapter, we mainly rely on QLIKE in drawing our conclusions regarding the accuracy of our ambiguity-based GARCH volatility forecasts. QLIKE is indeed more preferable for risk management and investment purposes. From a risk management standpoint, underestimating volatility may cost more than overestimating it by the same amount. From an investment point of view, relative forecasting errors are more relevant than absolute forecasting errors as returns are computed relative to an investment cost. We also implement the test for superior predictive ability (SPA)[6] using the bootstrapping method proposed by Hansen (2005) as a reality check for data snooping issues.

4.3. Data and Variables

4.3.1 Option Data

We employ a dataset of European S&P 500 index options for the period of 2 Jan 1990 to 31 Dec 2012. To ensure the liquidity of option contracts included, we follow the option contract selection method of the Chicago Board Options Exchange (CBOE) in computing the VIX (Chicago Board Options Exchange, 2014). To confirm that the information extracted from traded option prices is not biased

[6] A similar diagnostic technique is the reality check (RC) for data snooping of White (2000). Hansen and Lunde (2005) compared 330 GARCH-based models and found RC to be less powerful and fails to detect inferior models. Herein, we rely on the SPA test for out-sample model evaluation.

towards certain option specification, we include both call and put options[7] with different moneyness including out-of-the-money (OTM), at-the-money (ATM), and in-the-money (ITM). Similar to the CBOE, we include both near-term options and next-term options in our sample. The average day-to-maturity (DTM) of our near-term and next-term option samples are 19.04 days and 46.59 days respectively. Our sample contains 947,314 contract-days with an average moneyness (S/K) of 1.06, and an average DTM of 31.92 days. Given the large scale of the dataset, we report basic summary statistics with option data sorted by DTM, VIX level, and the degree of OMAA. This helps us

Table 4.1. Summary Statistics for Option Data:

Panels A, B, and C report the basic descriptive statistics sorted by day-to-maturity (DTM), level of VIX, and level of OMAA respectively. The data period covers 2 Jan 1990 to 31 Dec 2012.

Panel A. By Day-to-maturity (DTM)

	DTM < 14	*14 ≤ DTM < 30*	*30 ≤ DTM < 45*	*45 ≤ DTM < 60*	*60 ≤ DTM*	*All*
Number of Contract-days	159,278	292,004	248,574	180,336	67,122	947,314
Average Price	42.520	58.191	67.021	68.536	68.286	60.558
Average Implied Volatility	0.316	0.279	0.260	0.237	0.228	0.269
Average Bid-ask Spread	1.758	1.792	1.889	1.813	1.805	1.817
Average Bid-ask Spread / Price	4.14%	3.08%	2.82%	2.64%	2.64%	3.00%

Panel B. By level of VIX

	VIX < 15	*15 ≤ VIX < 20*	*20 ≤ VIX < 25*	*25 ≤ VIX < 30*	*30 ≤ VIX*	*All*
Number of Contract-days	194,076	271,666	232,506	113,472	135,594	947,314
Average Price	37.288	54.706	67.175	73.103	83.740	60.558
Average Implied Volatility	0.153	0.221	0.274	0.323	0.475	0.269
Average Bid-ask Spread	1.232	1.728	1.883	1.976	2.583	1.817
Average Bid-ask Spread / Price	3.30%	3.16%	2.80%	2.70%	3.08%	3.00%

Panel C. By level of OMAA

	OMAA < 0.25	*0.25 ≤ OMAA < 0.5*	*0.5 ≤ OMAA < 0.75*	*0.75 ≤ OMAA*	*All*
Number of Contract-days	194	259,936	680,484	6,700	947,314
Average Price	59.275	60.905	60.012	102.545	60.558
Average Implied Volatility	0.309	0.280	0.262	0.551	0.269
Average Bid-ask Spread	2.988	1.815	1.798	3.793	1.817
Average Bid-ask Spread / Price	5.04%	2.98%	3.00%	3.70%	3.00%

understand relationships among implied volatility, ambiguity attitudes, and option contract specification. Table 4.1 summarizes the key characteristics of the option data we use including the number of option prices / contract-days, average price, average Black Scholes implied volatility, average bid-ask spread, and average bid-ask spread to average price ratio. From Panel C of Table 4.1,

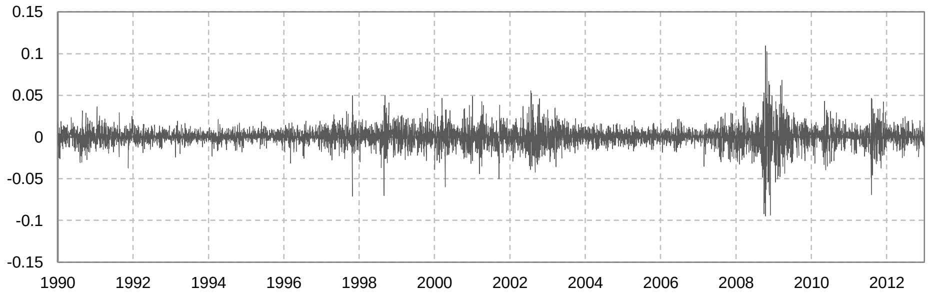
[7] According to the CBOE selection methodology, call and put options have matched strikes meaning there are equal number of call and put option contracts selected at any given moment. As a robustness check, we also extracted OMAA from only calls and then only puts. Our results are robust regardless of using OMAA from call options, put options prices or an average of both.

we can observe that when investors are extremely ambiguity seeking (0.75 ≤ OMAA) or very ambiguity averse (OMAA < 0.25), the average bid-ask spread to price ratio tends to be higher. Taking the percentage bid-ask spread as a proxy for illiquidity, our data confirms a positive association between market illiquidity and ambiguity in line with extant research on market microstructure (Routledge and Zin, 2009).

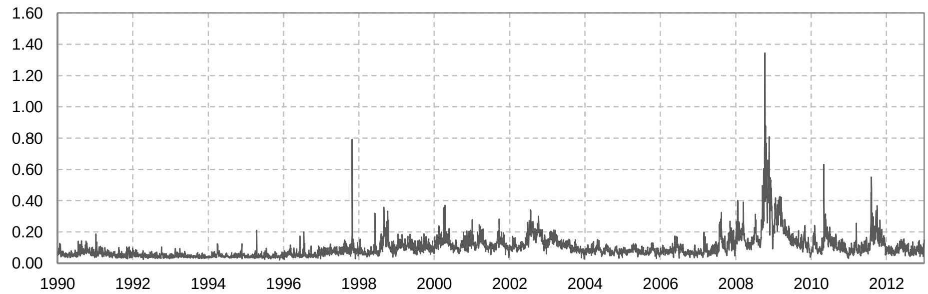
Figure 4.1. S&P 500 Daily Returns, Realized Volatility, VIX, and OMAA.

Graph A plots the daily S&P 500 returns. Graph B plots the daily realized volatility from an average RV estimator with rolling 5-minute squared return on 1-minute grid. Realized volatility in Graph B is shown in annualized standard deviation terms. Graph C plots the daily VIX index. Graph D plots the daily ambiguity attitudes from S&P 500 options. All plots span 2 Jan 1990 to 31 Dec 2012.

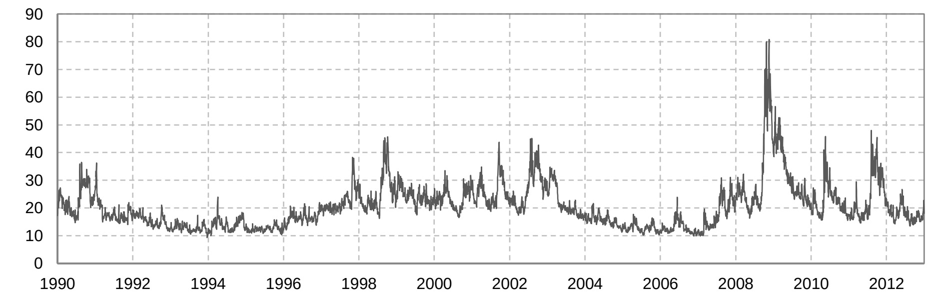
Graph A. Daily S&P 500 Returns



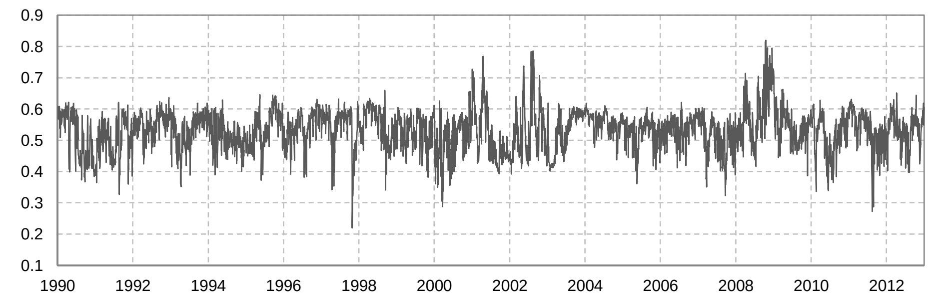
Graph B. Daily Realized Volatility from Average RV Estimator (5-min)



Graph C. Daily Level of VIX



Graph D. Daily Ambiguity Attitudes from S&P 500 Options (OMAA)



4.3.2 Stock Market Data

Daily closing levels of the S&P 500 index, daily 1-month USD LIBOR, and the daily trailing twelve month dividend yield of the S&P 500 index were obtained from Thomson Datastream. Data of U.S. 3-month Treasury bill rate was obtained from the Federal Reserve Bank of St. Louis.

To assess the out-of-sample forecasting performance of our models, intraday high frequency return data was used to compute the realized variance based on different sampling grids. This intraday dataset consists of 2.2 million data points from 2 January 1990 to 31 Dec 2012. We construct three realized variance measures as the estimated variance used in our out-of-sample forecasting performance evaluations. Realized variance based on 1-minute returns is simply the sum of squared 1-minute returns during the day. As noted in the literature (Marterns and van Dijk, 2007; Bandi et al., 2008; Christoffersen et al., 2014), the 1-minute realized variance can be noisy due to market microstructure effects, we also compute 5-minute and 10-minute realized variance with rolling grids as a result. For realized variance based on 5-minute returns, we start computing realized variance as the sum of squared 5-minute returns from the first price on 1-minute grid. Once finished with the rolling approach from the first minute of the day, we compute the realized variance starting from the second minute price of the day. We repeat these steps until we have 5 realized variance estimates in a day and take the sample average to obtain the 5-minute realized variance.

The daily returns on the S&P 500 index, realized variance (measured as squared realized volatility) from the average RV estimator based on 5-minute intraday returns, daily level of VIX, and option market ambiguity attitudes are plotted in Figure 1. The Great Financial Crisis of 2007-2008 dominates the picture in Graph A. The realized variance plot in Graph B is characterized by two episodes of high volatility: a mini-crash[8] in Oct 1997 and the crisis of 2007-2008. A prolonged low volatility era from 2003 to 2006 is also evident in Graph B. From Graph D, we can see that 2007-2008 is dominated by ambiguity seeking behavior (where OMAA>0.5). OMAA recorded its maximum value of 0.8191 on 21 Oct 2008 and minimum value of 0.2196 on 28 Oct 1997, the day right after the mini-crash. While OMAA has a daily average of 0.5312 over the entire 23-year sample period, ambiguity seeking

[8] The Mini-crash refers to the stock market crash of October 27, 1997 that is believed to have been caused by the economic crisis in Asia.

Table 4.2. Descriptive Statistics and Correlation Matrix

Panel A reports the basic descriptive statistics for excess returns (ER), implied variance (IV), options market ambiguity attitudes (OMAA), and realized variances (RV) using different estimation grids. Panel B reports the correlation matrix. The data period covers 2 Jan 1990 to 31 Dec 2012.

Panel A. Descriptive Statistics

	ER	*IV*	*OMAA*	RV_{1min}	RV_{5min}	RV_{10min}
Mean	0.0249	0.0484	0.5312	0.0165	0.0158	0.0153
Median	0.0970	0.0356	0.5394	0.0069	0.0066	0.0064
SD	2.9549	0.0002	0.0644	0.0003	0.0002	0.0002
Skewness	-0.2193	4.7023	-0.1036	35.3165	36.4704	34.6836
Kurtosis	11.4454	37.5037	4.0978	1760.7280	1858.6330	1709.8930
AR(1)	-0.0572	0.9705	0.8253	0.3165	0.3025	0.3214

Panel B. Correlation Matrix

	ER	*IV*	*OMAA*	RV_{1min}	RV_{5min}	RV_{10min}
ER	1.0000					
IV	-0.1331	1.0000				
OMAA	0.1041	0.1536	1.0000			
RV_{1min}	-0.0184	0.5114	0.1315	1.0000		
RV_{5min}	-0.0156	0.5017	0.1289	0.9995	1.0000	
RV_{10min}	-0.0187	0.5163	0.1320	0.9994	0.9996	1.0000

behavior is most prominent in the crisis year of 2008. Daily average OMAA in 2008 is 0.5870, the highest level in the 23 years covered.

Table 4.2 reports the descriptive statistics and correlation matrix for excess returns, implied variance, option market ambiguity attitudes, and the three realized variances specified above. In Panel A of Table 4.2, we observe that the long-run mean excess return of the market is close to zero. Excess returns are negatively skewed and have excess kurtosis. On the other hand, average OMAA is at 0.5312, just above the ambiguity neutral threshold (0.5). This implies investors in the S&P 500 index option market are on average moderately ambiguity seeking[9].

4.4. GARCH Volatility Forecasting and the Role of Ambiguity Attitudes

This section presents our empirical results examining the relationship between ambiguity attitudes and ex post conditional variance in the gain/loss domains. As noted, we assess the informational efficiency of OMAA by comparing our models with ambiguity to benchmark models (without ambiguity) under GARCH-in-mean and also via the exponential GARCH-in-mean to ensure positivity of the conditional variance.

[9] OMAA is moderately correlated (correlation = 0.26) to the United States Valuation Index developed by the Yale School of Management, indicating investors' ambiguity seeking is positively related to their confidence / aggressiveness in valuing stocks.

4.4.1 GARCH-in-mean Estimation and In-sample Forecasting

Table 4.3 presents our estimates for Models 1.1 to 1.6. Model 1.1 is the vanilla GARCH model (without ambiguity) used as benchmark for comparison with Models 1.2 and 1.3 (with ambiguity), while Model 1.4 serves as the benchmark model involving IV as an exogenous variable for comparison with Models 1.5 and 1.6 containing OMAA innovations. From Models 1.1 to 1.3, it is clear that ambiguity innovations provide additional prediction information and have different effects on conditional variance for gains and losses. From Model 1.2, innovations from ambiguity in the gain domain result in downward revisions of the conditional variance. Turning to the loss specification (Model 1.3), innovations from ambiguity generate positive revisions of the conditional variance. In both cases, judging by the reported likelihood ratios, ambiguity attitude information is significant in explaining ex post variations in conditional variance. Without controlling for the effect of implied variance in the model, ambiguity attitudes matter more to the revisions of conditional variance in the domain of losses. In general, from Models 1.2 and 1.3, we learn that increased ambiguity seeking is associated with upward revisions in the conditional variance in the loss domain while increased ambiguity aversion is associated with upward revisions of the conditional variance in the gain domain. This is directly in line with the qualitative prescriptions of the hope and fear effects.

In Models 1.4 to 1.6, we consider implied variance as one of the explanatory variables in the variance equations. When compared to Models 1.1-1.3, Models 1.4 to 1.6 show improved fit, as measured by the log likelihood, when implied variance is included in the variance process. In Model 1.5, OMAA is significant and once again negatively related to conditional variance in the gain domain. The LR statistic confirms that the inclusion of OMAA is meaningful and significant. The negative relationship between OMAA and conditional variance in the gain domain is improved by the inclusion of implied variance. This confirms the result obtained in Model 1.2. Option market ambiguity attitudes are robustly correlated with ex post conditional variance in the gain domain. Turning to Model 1.6, the OMAA coefficient in the loss domain is positive and significant with an

Table 4.3. GARCH-In-Mean Estimates of the Daily Ambiguity-Volatility Relation

Panel A reports the estimated parameters of Models 1.1 to 1.6 based on maximum likelihood estimation. Robust t-statistics reported in parentheses are computed based on Bollerslev and Wooldridge (1992). Panel B reports the in-sample error statistics. Model 1.1 is the benchmark model for Models 1.2 and 1.3. Model 1.4 is the benchmark model for Models 1.5 and 1.6. We estimate the models using daily close-to-close returns of S&P 500 index, implied variance and option market ambiguity attitudes for the period 2 Jan 1990 to 31 Dec 2012. ***,**, and * indicate 99%, 95%, and 90% confidence levels respectively.

Panel A. Estimation Results

	Model 1.1	*Model 1.2*	*Model 1.3*	*Model 1.4*	*Model 1.5*	*Model 1.6*
α_0	1.68E-04	5.72E-05	2.19E-05	4.62E-06	-1.84E-05	-1.39E-05
	(1.073)	(0.358)	(0.136)	(0.028)	(-0.112)	(-0.085)
α_1	2.746 *	3.231 **	3.304 **	0.668	0.981	0.937
	(1.712)	(1.967)	(2.009)	(0.339)	(0.498)	(0.478)
β_0	1.04E-06 ***	3.05E-06 ***	-1.29E-06 **	-1.16E-05 ***	-4.03E-07	-1.20E-05 ***
	(3.927)	(3.056)	(-1.981)	(-3.331)	(-0.093)	(-4.991)
β_1	0.076 ***	0.075 ***	0.071 ***	-0.017	0.001	0.000
	(8.345)	(8.273)	(8.274)	(-0.682)	(0.052)	(0.019)
β_2	0.917 ***	0.914 ***	0.919 ***	-0.014	0.314 **	0.310 **
	(109.713)	(104.733)	(111.268)	(-0.072)	(2.355)	(2.280)
$\beta_{OMAA,Gain}$	-	-6.22E-06 **	-	-	-2.11E-05 ***	-
	-	(-2.152)	-	-	(-3.280)	-
$\beta_{OMAA,Loss}$	-	-	9.86E-06 ***	-	-	2.14E-05 ***
	-	-	(3.503)	-	-	(3.191)
β_{IV}	-	-	-	0.717 ***	0.466 ***	0.471 ***
	-	-	-	(5.537)	(4.706)	(4.657)
Log Likelihood	18,752.46	18,760.97	**18,774.31**	18,855.27	18,868.95	**18,869.76**
LR	-	17.02	43.7	-	27.36	28.98
$P(LR) > \chi^2$	-	3.70E-05	3.83E-11	-	1.69E-07	7.31E-08

Panel B. In-sample Error Statistics

(x1000)	*Model 1.1*	*Model 1.2*	*Model 1.3*	*Model 1.4*	*Model 1.5*	*Model 1.6*
RMSE	0.1734	**0.1684**	0.1687	0.1411	**0.1398**	0.1399
MAE	0.0859	0.0843	**0.0839**	0.0751	**0.0743**	0.0744
LL	1.7821	1.7866	**1.7763**	1.7741	**1.7664**	1.7665
QLIKE	0.5542	0.5543	**0.5523**	0.5470	0.5448	**0.5447**

improved robust t-statistic of 3.191. The log-likelihood of Model 1.6 reveals a better explanatory power for OMAA in the loss domain (than under gains) when implied variance is controlled for. Considering all models in Table 4.3 collectively, our study unveils the important role of ambiguity attitudes in the conditional variance process. Judging by the log likelihood output, Model 1.6 is the most successful in terms of in-sample forecasting accuracy and efficiency[10]. Our results overall suggest that option market ambiguity attitudes capture important information regarding future

[10] For robustness, we also included a GJR term for asymmetric shocks and found OMAA remains significant in the variance equations. With the inclusion of asymmetric shocks in Models 1.5(1.6), OMAA remains significant and negatively (positively) associated with ex post variance. The coefficients of OMAA are generally more significant than those of GJR. The GJR term is insignificant under EGARCH, and thus not suitable for out-of-sample comparison.

evolutions of conditional volatility and are efficient in improving the accuracy of GARCH volatility forecasts in practice.

Turning to the in-sample error statistics in Panel B of Table 4.3, for models without implied variance as an exogenous variable (Models 1.1 to 1.3), the RMSE indicates that the model based on gains (Model 1.2) generates the best volatility forecasts, while MAE, LL and QLIKE point to Model 1.3 as the best model overall. As LL and QLIKE penalize underestimated volatility forecasts, this implies that considering OMAA in the loss domain minimizes the underestimation of forecasted volatility. Considering all error statistics at the same time suggests that accounting for OMAA improves the accuracy of the in-sample volatility forecasts. For Models 1.4 to 1.6, with implied variance as an exogenous variable, the error statistics of Models 1.5 and 1.6 beat those of Model 1.4. The in-sample error statistics support the model estimation results and suggest that option market ambiguity attitude is quantitatively important in determining ex post conditional volatility. This sheds light on how to improve the accuracy of GARCH volatility forecasts by considering investors' attitudes to uncertainty in prediction exercises.

Given the non-normality property of US stocks return data, a good variance model should also reduce, or ideally remove, the negative skewness and excess kurtosis in the residuals (Campbell and Hentschel, 1992). To further understand the importance of ambiguity aversion in variance modeling, we carry out additional diagnostic tests to check the impact of ambiguity attitudes on conditional variance forecasting while considering skewness and kurtosis dynamics. Table 4.4 reports diagnostic tests on the standardized residuals from the mean equations of Models 1.1 to 1.6. Table 4.4 shows

Table 4.4. GARCH-In-Mean Diagnostic Tests of the Daily Ambiguity-Volatility Relation

The table reports the diagnostic tests of Models 1.1 to 1.6. Skewness and excess kurtosis are the estimated skewness and excess kurtosis of the standardized residuals from the mean equation. Model 1.1 is the benchmark model for Models 1.2 and 1.3. Model 1.4 is the benchmark model for Models 1.5 and 1.6. We estimate the models using daily close-to-close returns of S&P 500 index, implied variance and option market ambiguity attitudes over the period 2 Jan 1990 to 31 Dec 2012.

	Model 1.1	*Model 1.2*	*Model 1.3*	*Model 1.4*	*Model 1.5*	*Model 1.6*
Skewness	-0.405	-0.400	-0.378	-0.379	-0.353	-0.346
	(-12.572)	(-12.423)	(-11.735)	(-11.774)	(-10.972)	(-10.750)
Excess Kurtosis	1.765	1.645	1.528	1.602	1.332	1.294
	(27.415)	(25.553)	(23.741)	(24.887)	(20.692)	(20.101)
Jarque-Bera	909.625	807.304	701.350	757.988	548.531	519.630

that skewness and excess kurtosis levels are reduced in the models containing the OMAA measure. Although the t-statistics still suggest that skewness and excess kurtosis cannot be ruled out, reductions in skewness and excess kurtosis confirm the findings in Table 4.3 that with the inclusion of ambiguity attitudes in the variance processes, better model fits can be obtained. Jarque-Bera statistics that summarize information from skewness and excess kurtosis dynamics further confirm that the residuals from the models with ambiguity are much closer to normality.

The success of Models 1.2, 1.3, 1.5 and 1.6 renews our understanding of the economic value of model uncertainty and option market ambiguity attitudes for conditional variance estimations. Results from this section lead to the following conclusions on the impact of OMAA on conditional variance:

1. The relationship between conditional variance and option market ambiguity attitude is negative and statistically significant in the gain domain, meaning increases in ambiguity aversion is related to increases in conditional variance.
2. The relationship between conditional variance and option market ambiguity attitude is positive and statistically significant in the loss domain, meaning increases in ambiguity seeking is related to increases in conditional variance.
3. In general the inclusion of OMAA allows better fits of empirical data and that conclusions 1-2 hold after controlling for option implied variance or risk-based option information.

The next section turns to the exponential versions of Models 1.1 to 1.6 for robustness and also as a prelude to the out-of-sample forecasting analysis (covered in Section 4.4.3).

4.4.2 Exponential GARCH-in-mean Estimation and In-sample Forecasting

Table 4.5 reports the estimation results for Models 2.1 to 2.6. Model 2.1 is the benchmark model without ambiguity or implied variance. Comparing Models 2.1 and 2.2, OMAA towards gains exhibits the same negative relationship with conditional variance as in the previous section. The robust t-statistic of OMAA in the gain domain is -4.52, which suggests that OMAA is more significant in Model 2.2 than in Model 1.2. Model 2.3 shows a positive and statistically significant relationship between OMAA and conditional variance in the loss domain, confirming our results from the previous section. In general, the OMAA measure is statistically significant for both gains and losses, with the likelihood ratio being significant at the 99.99% confidence level.

Turning to the specifications with controls for implied variance, Models 2.4 to 2.6 are showing similar conclusions as those in Models 1.4 to 1.6. From Model 2.5, OMAA towards gains exhibits a negative relationship with conditional variance but with a slightly weaker robust t-statistics when compared to Model 2.2. On the other hand, in Model 2.6, OMAA towards losses also shows

Table 4.5. Exponential GARCH-In-Mean Estimates of the Daily Ambiguity-Volatility Relation

Panel A reports the estimated parameters of Models 2.1 to 2.6 based on maximum likelihood estimation. Robust t-statistics reported in parentheses are computed based on Bollerslev and Wooldridge (1992). Panel B reports the in-sample error statistics. Model 2.1 is the benchmark model for Models 2.2 and 2.3. Model 2.4 is the benchmark model for Models 2.5 and 2.6. We estimate the models using daily close-to-close returns of S&P 500 index, implied variance and option market ambiguity attitudes over the period 2 Jan 1990 to 31 Dec 2012. ***,**, and * indicate 99%, 95%, and 90% confidence levels respectively.

Panel A. Estimation Results

	Model 2.1	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
α_0	1.42E-04	-5.33E-05	-6.39E-05	1.95E-07	-1.69E-05	-1.67E-05
	(0.863)	(-0.312)	(-0.401)	(0.001)	(-0.113)	(-0.113)
α_1	2.791	2.686	1.921	0.550	0.891	0.888
	(1.550)	(1.420)	(1.144)	(0.328)	(0.539)	(0.545)
β_0	-2.40E-01 ***	-2.15E-01 ***	-3.09E-01 ***	1.65E+00 ***	1.20E+00 ***	8.99E-01 ***
	(-7.837)	(-6.650)	(-9.382)	(4.077)	(3.574)	(2.678)
β_1	0.165 ***	0.161 ***	0.148 ***	-0.111 **	-0.063	-0.054
	(9.047)	(8.997)	(8.993)	(-2.354)	(-1.475)	(-1.303)
β_2	0.988 ***	0.985 ***	0.986 ***	-0.086	0.267 *	0.312 **
	(365.602)	(326.034)	(414.242)	(-0.534)	(1.953)	(2.436)
$\beta_{OMAA,Gain}$	-	-1.76E-01 ***	-	-	-4.07E-01 ***	-
	-	(-4.520)	-	-	(-4.176)	-
$\beta_{OMAA,Loss}$	-	-	2.73E-01 ***	-	-	4.44E-01 ***
	-	-	(6.776)	-	-	(4.390)
β_{IV}	-	-	-	1.327 ***	0.894 ***	0.839 ***
	-	-	-	(6.763)	(5.118)	(5.130)
Log Likelihood	18,745.50	18,768.21	**18,794.72**	18,878.90	18,888.76	**18,890.31**
LR	-	45.42	98.44	-	19.72	22.82
$P(LR) > \chi^2$	-	1.59E-11	3.35E-23	-	8.97E-06	1.78E-06

Panel B. In-sample Error Statistics

(x1000)	*Model 2.1*	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
RMSE	0.1542	**0.1422**	0.1720	0.1569	**0.1553**	0.1583
MAE	0.0824	**0.0795**	0.0839	**0.0818**	0.0820	0.0823
LL	1.7951	1.7939	**1.7688**	1.7513	1.7422	**1.7397**
QLIKE	0.5588	0.5563	**0.5506**	0.5408	0.5383	**0.5377**

a strong positive relationship with ex post conditional variance. OMAA is significant under gains and losses. In general, the significance of OMAA is improved when the models are specified in exponential form (when compared to robust t-statistics in Table 4.3).

Turning to the in-sample error statistics for Models 2.1 to 2.6 in Panel B of Table 4.5, similar conclusions can be drawn: the inclusion of option market ambiguity attitudes does improve the accuracy of GARCH volatility forecasts. When we consider the in-sample error statistics of Models 2.4 to 2.6, RMSE and MAE tend to give mixed suggestions as to which model gives the most accurate forecasts. In light of that, and as noted earlier, we rely on QLIKE, which has proved to be more reliable in assessing the quality of forecasts (e.g., Kumar, 2015), for our model comparison. The in-sample QLIKE for Model 2.6 gives 0.5377 x 10^3, the smallest error in all six models (Table 4.5). The results from Table 4.5 confirm our findings and highlight the suitability of using exponential models in forecasting exercises.

Table 4.6. Exponential GARCH-In-Mean Diagnostic Tests of the Daily Ambiguity-Volatility Relation

The table reports the diagnostic tests of Models 2.1 to 2.6. Skewness and excess kurtosis are the estimated skewness and excess kurtosis of the standardized residuals from the mean equation. Model 2.1 is the benchmark model for Models 2.2 and 2.3. Model 2.4 is the benchmark model for Models 2.5 and 2.6. We estimate the models using daily close-to-close returns of S&P 500 index, implied variance and option market ambiguity attitudes over the period 2 Jan 1990 to 31 Dec 2012.

	Model 2.1	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
Skewness	-0.395	-0.404	-0.359	-0.371	-0.364	-0.354
	(-12.278)	(-12.567)	(-11.148)	(-11.531)	(-11.320)	(-11.009)
Excess Kurtosis	1.779	1.662	1.443	1.553	1.445	1.391
	(27.641)	(25.826)	(22.420)	(24.118)	(22.444)	(21.608)
Jarque-Bera	914.788	824.921	626.936	714.645	631.888	588.128

In line with Table 4.4, we present the diagnostic test results for Models 2.1 to 2.6 in Table 4.6. Skewness and excess kurtosis of the standardized residuals from the mean equations are in general reduced in all models involving OMAA except for skewness in Model 2.2. This suggests that models with OMAA tend to better account for the non-normality of S&P 500 returns. Standardized residuals from Model 2.6 have the least skewness and least excess kurtosis out of the six models. The magnitude of skewness is reduced from -0.395 in benchmark Model 2.1 to -0.354 in Model 2.6. Excess kurtosis decreases from 1.779 in Model 2.1 to 1.391 in Model 2.6. Jarque-Bera statistics in Table 4.6 also suggest that the standardized residuals from the models with OMAA innovations are closer to normality.

The in-sample forecasting success of option market ambiguity helps us understand the asymmetric property of OMAA in affecting revisions in conditional variance. Our analysis is the first empirical study to reveal this special relationship in financial markets data in the context of the GARCH framework. The approach of asymmetric modeling of conditional variance with ambiguity information provides insightful and statistically significant explanatory power beyond what traditional variance models offer. Despite that, the economic value of including OMAA in variance forecasting might still largely depend on the model and parameter stability. In the next section, we examine the out-of-sample forecasting ability of our ambiguity-based models to further corroborate our conclusions.

4.4.3 Out-of-sample Volatility Forecasting

While the in-the-sample analysis in the previous sections allowed us to understand the empirical implications of option market ambiguity attitudes for GARCH volatility modeling, the validity of the improved variance models as an economically valuable tool for market statisticians also depends on the out-of-sample forecasting performance. Herein, we examine the out-of-sample forecasting performance of Models 2.1 to 2.6 over a period of 20 years from 1993 to 2012. To ensure the stability of model parameters, we employ at least 3 years of data in the estimation. For each model, we use both 3-year and 5-year rolling estimation windows for parameters' estimation. Conditional variance forecasts are then compared to realized variance with different intra-day estimation grids (1, 5 and 10 min grids as mentioned in Section 4.3). We employ four loss functions / error statistics with respect to the measured variance for the evaluation of the forecasting ability of each model. In addition to RMSE and MAE, the most popular error statistics used in out-of-sample forecasting evaluation, we once again take into account LL, which measures the logarithmic forecasting accuracy, and QLIKE which measures the relative forecasting accuracy and penalizes underestimated variance forecasts. As noted, due to the important properties of QLIKE in risk management and investment, our conclusions mainly rely on the QLIKE statistics. As discussed, we also compare the various models using Hansen (2005)'s superior predictive ability (SPA) test.

Table 4.7. Out-of-sample Forecasting

The table reports error statistics from out-of-sample forecasting with 3 and 5-year rolling estimation windows. RMSE, MAE, LL and QLIKE are root mean squared error, mean absolute error, logarithmic loss function and loss implied by Gaussian likelihood respectively. Loss functions are specified according to (4.21)-(4.24). Hansen (2005) SPA test p-values are reported in parentheses. Model 2.1 is the benchmark model for Models 2.2 and 2.3. Model 2.4 is the benchmark model for Models 2.5 and 2.6. Bolded figures are the minimum errors in each set of models. We estimate the models using daily close-to-close returns of S&P 500 index, implied variance and option market ambiguity attitudes over the period 2 Jan 1990 to 31 Dec 2012. ***,**, and * indicate 99%, 95%, and 90% confidence levels respectively.

Panel A. Error Statistics Based On 3-Year Rolling Estimation Window

	Model 2.1	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
RMSE						
RV_{1min}	0.2743	**0.2542**	0.3122	0.3691	0.3403	**0.3328** **
		(0.150)	(1.000)		(0.158)	(0.095)
RV_{5min}	0.1882	**0.1568**	0.2438	0.3144	0.2791	**0.2697** **
		(0.154)	(1.000)		(0.197)	(0.092)
RV_{10min}	0.2617	**0.2399**	0.3049	0.3650	0.3341	**0.3262** **
		(0.129)	(1.000)		(0.171)	(0.098)
MAE						
RV_{1min}	0.0906	**0.0829** *	0.0958	0.1116	0.1066	**0.1048** **
		(0.100)	(1.000)		(0.164)	(0.098)
RV_{5min}	0.0897	**0.0820**	0.0952	0.1111	0.1061	**0.1043** **
		(0.105)	(1.000)		(0.163)	(0.094)
RV_{10min}	0.0931	**0.0854**	0.0987	0.1147	0.1096	**0.1079** **
		(0.112)	(1.000)		(0.160)	(0.080)
LL						
RV_{1min}	1.3087	**1.2462** ***	1.2810	1.2577	1.2398 **	**1.2336** ***
		(0.029)	(0.228)		(0.082)	(0.015)
RV_{5min}	1.4154	**1.3502** ***	1.3882	1.3642	1.3452 **	**1.3386** ***
		(0.033)	(0.243)		(0.072)	(0.016)
RV_{10min}	1.4923	**1.4251** ***	1.4639	1.4404	1.4203 **	**1.4135** ***
		(0.028)	(0.222)		(0.067)	(0.012)
QLIKE						
RV_{1min}	0.4628	0.4457 ***	**0.4366** **	0.4289	0.4223 **	**0.4219** **
		(0.043)	(0.068)		(0.085)	(0.053)
RV_{5min}	0.4685	**0.4495** ***	0.4555	0.4454	0.4401 **	**0.4392** ***
		(0.044)	(0.101)		(0.083)	(0.031)
RV_{10min}	0.5091	0.4907 ***	**0.4839** **	0.4762	0.4691 **	**0.4686** ***
		(0.042)	(0.070)		(0.071)	(0.025)

Panel B. Error Statistics Based On 5-Year Rolling Estimation Window

	Model 2.1	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
RMSE						
RV_{1min}	0.2871	**0.2695**	0.3053	0.3491	0.3281	**0.3236**
		(0.143)	(1.000)		(0.182)	(0.136)
RV_{5min}	0.1956	**0.1680**	0.2234	0.2818	0.2544	**0.2484**
		(0.116)	(1.000)		(0.182)	(0.130)
RV_{10min}	0.2732	**0.2540**	0.2948	0.3425	0.3197	**0.3150**
		(0.140)	(1.000)		(0.177)	(0.135)
MAE						
RV_{1min}	0.0971	**0.0897** **	0.0988	0.1125	0.1077	**0.1066** **
		(0.090)	(1.000)		(0.135)	(0.094)
RV_{5min}	0.0960	**0.0886** **	0.0979	0.1118	0.1069	**0.1058** *
		(0.093)	(1.000)		(0.142)	(0.100)
RV_{10min}	0.0997	**0.0922** **	0.1016	0.1158	0.1108	**0.1097** **
		(0.091)	(1.000)		(0.151)	(0.099)
LL						
RV_{1min}	1.2100	**1.1402** ***	1.1553 ***	1.1614	1.1420 ***	**1.1386** ***
		(0.006)	(0.017)		(0.027)	(0.006)
RV_{5min}	1.2981	**1.2252** ***	1.2410 ***	1.2499	1.2290 ***	**1.2257** ***
		(0.003)	(0.012)		(0.020)	(0.002)
RV_{10min}	1.3675	**1.2926** ***	1.3085 ***	1.3197	1.2977 ***	**1.2943** ***
		(0.005)	(0.007)		(0.024)	(0.003)
QLIKE						
RV_{1min}	0.4221	**0.4037** ***	0.4038 ***	0.4026	**0.3968** ***	0.3979 ***
		(0.012)	(0.005)		(0.027)	(0.020)
RV_{5min}	0.4325	**0.4117** ***	0.4152 ***	0.4121	**0.4061** ***	0.4065 ***
		(0.008)	(0.008)		(0.015)	(0.014)
RV_{10min}	0.4629	**0.4433** ***	0.4439 ***	0.4434	**0.4369** ***	0.4379 ***
		(0.009)	(0.009)		(0.016)	(0.010)

Table 4.7 reports the error statistics from the out-of-sample forecasting analysis. Panel A reports the loss functions from forecasts based on a 3-year (750 trading days) rolling estimation window. Panel B reports the loss functions from forecasting models based on a 5-year (1250 trading days)

rolling estimation window. Hansen's SPA test bootstrapped p-values are reported in parentheses. Since we have 5792 daily observations from Jan 1990 to Dec 2012, loss functions in Panel A and Panel B are based on 5042 and 4542 days out-of-sample forecasting windows respectively.

The first three columns of Table 4.7 report out-of-sample forecasting errors for models (2.1 to 2.3) that do not involve implied variance as an exogenous variable, while the last three columns report the errors based on models (2.4 to 2.6) with implied variance in the variance process. The smallest out-of-sample forecasting errors are reported in bold in each sub-group for easier comparison. From Panel A of Table 4.7, comparing loss functions in Models 2.1-2.3 points to Model 2.2 as the best out-of-sample forecasting model with the lowest errors in a majority of loss functions and RV computation grid. Model 2.3 dominates QLIKE statistics and appears to be the best for risk management purposes. According to the LL and QLIKE criteria, models with option market ambiguity attitudes consistently have smaller out-of-sample forecasting errors when compared to the benchmark Model 2.1. SPA test p-values suggest that Model 2.2 is the best model when data snooping bias is accounted for. Turning to Models 2.4 to 2.6, which take into account the impact of ex ante implied variance, models (2.5 and 2.6) containing OMAA innovations produce better variance forecasts when compared to the benchmark Model 2.4 without ambiguity. All four error statistics unequivocally point to Model 2.6 as the best model to forecast conditional volatility. SPA test p-values also clearly point to Model 2.6 as the best forecasting model overall. Interestingly but unsurprisingly when we compare out-of-sample forecasting errors under RMSE and MAE, the inclusion of implied variance actually increases the absolute value of forecasting errors. Alternatively when we assess models using LL and QLIKE statistics, the inclusion of implied variance does help avoid the under-estimation of conditional variance problem. From Panel A of Table 4.7, it is clear that the inclusion of OMAA in the variance models significantly improves the accuracy of forecasts for different realized variance specifications and considering various loss functions.

Turning to Panel B of Table 4.7, comparing the error statistics of RMSE and MAE against those presented in Panel A, it appears that the forecasting errors are in general smaller under the 5-year estimation window. LL and QLIKE error statistics based on 5-year estimation windows tend to be smaller than those presented in Panel A. This suggests that volatility forecasts based on the 5-year estimation window are less underestimated and, thus, are more suitable for risk management matters. In Panel B, among the first group of models that do not include implied variance, all four loss functions and SPA p-values unequivocally indicate that Model 2.2 generates the most accurate volatility forecasts for the 5-year rolling estimation window. This reaffirms the findings from Panel A of Table 4.7. Error statistics based on RMSE, MAE, and LL for Models 2.4 to 2.6 indicate that Model 2.6, which considers OMAA for losses, produces the most accurate volatility forecasts, while QLIKE points to Model 2.5. This suggests that volatility models that account for investors' ambiguity attitudes in the gain domain may be more suitable for risk management purposes. One explanation for this finding is that in the loss domain, the market is dominated by ambiguity seeking investors who will underestimate their risk exposure. Considering all four loss functions and the out-of-sample analysis, we can conclude that OMAA is important in producing more accurate GARCH volatility forecasts, and thus is quantitatively important and valuable for volatility forecasting practice.

The striking out-of-sample results highlight the efficiency of option market information and ambiguity attitudes in forecasting volatility. To further appreciate the economic significance of option market ambiguity attitudes in financial markets and volatility modeling, the next section studies the potential economic gains made from trading strategies based on our out-of-sample GARCH volatility forecasts from Models 2.1 to 2.6.

4.4.4 Economic Significance Analysis

We extend the analysis to verify if investors can benefit from more accurate variance forecasts and generate excess profits based on the proposed forecasting models (Fleming et al., 2003; Marquering and Verbeek, 2004; Boguth et al., 2011). Similar to Han (2006) and Driesprong et al. (2008), we implement a daily-rebalancing volatility timing strategy based on the level of variance forecasts relative to the historical average realized variance. We consider a risk-seeking investor[11] who invests in the S&P 500 portfolio whenever the next day variance forecast is exceeding the one-month average

[11] According to Merton (1973), risk is positively related to expected returns. Since we consider the case of one-asset (S&P 500 index) volatility timing strategy, it is logical to assume investors who would follow this strategy to be risk-seeking individuals.

Table 4.8. Economic Significance Analysis

The table reports the trading statistics for the long-only and long/short market-timing trading strategy based on variance forecasts from each model. The trading window covers 2 Jan 1995 to 31 Dec 2012.

Panel A. Economic Gain Based on Long-only Volatility-timing Trading Strategy

FULL PERIOD	*Model 2.1*	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
Cumulative Return	158.43%	263.50%	252.94%	271.71%	392.30%	325.23%
Annualized Return	5.43%	7.46%	7.28%	7.59%	9.29%	8.40%
Portfolio Volatilty	19.59%	19.71%	19.49%	19.77%	19.62%	19.60%
Return/risk Reward Ratio	0.28	0.38	0.37	0.38	0.47	0.43
Superiority to Benchmark Strategy (Percentage Points)	-	105.07	94.51	-	120.59	53.52

1995 - 2004	*Model 2.1*	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
Cumulative Return	143.65%	138.65%	152.71%	153.67%	174.98%	178.93%
Annualized Return	9.36%	9.13%	9.76%	9.80%	10.70%	10.86%
Portfolio Volatilty	18.14%	18.07%	17.99%	18.12%	18.06%	18.02%
Return/risk Reward Ratio	0.52	0.51	0.54	0.54	0.59	0.60
Superiority to Benchmark Strategy (Percentage Points)	-	-5.01	9.06	-	21.31	25.26

2005 - 2012	*Model 2.1*	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
Cumulative Return	6.07%	52.32%	39.56%	46.43%	78.94%	52.04%
Annualized Return	0.74%	5.41%	4.26%	4.89%	7.56%	5.38%
Portfolio Volatilty	21.25%	21.59%	21.22%	21.65%	21.40%	21.41%
Return/risk Reward Ratio	0.03	0.25	0.20	0.23	0.35	0.25
Superiority to Benchmark Strategy (Percentage Points)	-	46.25	33.49	-	32.51	5.61

Panel B. Economic Gain Based on Long/Short Volatility-timing Trading Strategy

FULL PERIOD	*Model 2.1*	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
Cumulative Return	105.31%	310.06%	281.13%	330.95%	647.63%	459.17%
Annualized Return	4.09%	8.18%	7.74%	8.48%	11.87%	10.07%
Portfolio Volatilty	20.09%	20.08%	20.08%	20.08%	20.07%	20.07%
Return/risk Reward Ratio	0.20	0.41	0.39	0.42	0.59	0.50
Superiority to Benchmark Strategy (Percentage Points)	-	204.75	175.82	-	316.68	128.22

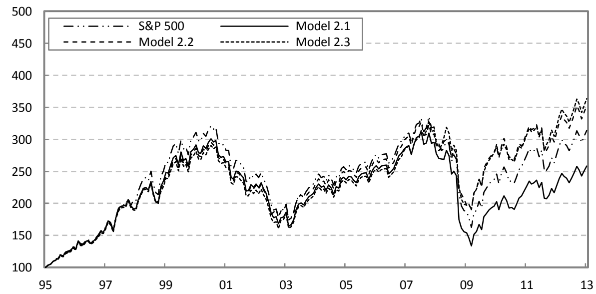
1995-2004	*Model 2.1*	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
Cumulative Return	127.73%	118.23%	144.71%	147.70%	191.37%	200.14%
Annualized Return	8.62%	8.16%	9.41%	9.54%	11.34%	11.68%
Portfolio Volatilty	18.25%	18.25%	18.24%	18.24%	18.24%	18.24%
Return/risk Reward Ratio	0.47	0.45	0.52	0.52	0.62	0.64
Superiority to Benchmark Strategy (Percentage Points)	-	-9.50	16.98	-	43.66	52.43

2005-2012	*Model 2.1*	*Model 2.2*	*Model 2.3*	*Model 2.4*	*Model 2.5*	*Model 2.6*
Cumulative Return	-9.39%	90.93%	57.71%	79.12%	168.44%	93.39%
Annualized Return	-1.23%	8.43%	5.87%	7.57%	13.16%	8.61%
Portfolio Volatilty	22.16%	22.15%	22.16%	22.15%	22.14%	22.15%
Return/risk Reward Ratio	-0.06	0.38	0.26	0.34	0.59	0.39
Superiority to Benchmark Strategy (Percentage Points)	-	100.32	67.10	-	89.32	14.27

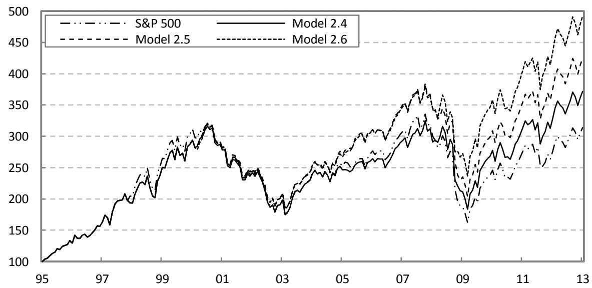
Figure 4.2. Portfolio Value for Economic Significance Analysis.

Graph A and B plot the daily portfolio values for the long-only market-timing strategy. Graph C and D plot the daily portfolio values for the long/short market-timing strategy. All portfolio values are computed net of a transaction fee of 0.1% each way. All plots span 2 Jan 1995 to 31 Dec2012.

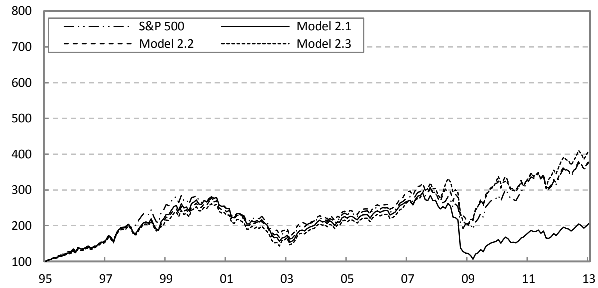
Graph A. Portfolio Value for Long-only Strategy Model 2.1-2.3



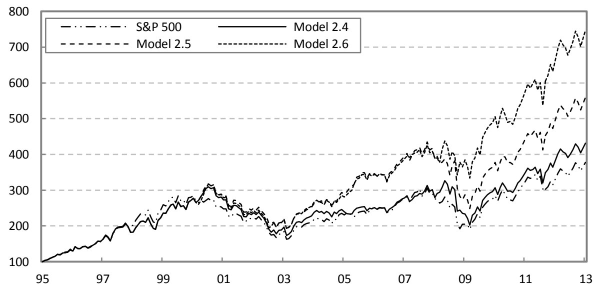
Graph B. Portfolio Value for Long-only Strategy Model 2.4-2.6



Graph C. Portfolio Value for Long/short Strategy Model 2.1-2.3



Graph D. Portfolio Value for Long/short Strategy Model 2.4-2.6



of realized variance. At the end of each trading day, (s)he will decide whether to buy (or sell if the position was already established) the portfolio. With this strategy simply consisting of "buy" and "sell" timing without accounting for portfolio reallocation, we assess the economic significance of OMAA variance models in market timing. To allow for a more flexible trading strategy, we also consider an alternative situation in which the investor can go short when the variance forecast is lower than the historical average. In the economic significance analysis, we use out-of-sample forecasts from each model based on the 5-year rolling estimation window[12]. The trading window in which the strategies are implemented therefore covers the 1995-2012 period. Following Driesprong et al. (2008), we assume a transaction cost[13] of 0.1% each way for "buy" and "sell".

[12] These results also hold when we use a 3-year rolling estimation window.

[13] Research on the economic significance of volatility timing trading strategies adopts various transaction costs assumptions. Bhardwaj and Brooks (1992), and Balduzzi and Lynch (1999) suggest 0.5% for individual equity trading; Driesprong et al. (2008) suggest that 0.1% is more reasonable for futures trading on commodities; and Fleming et al. (2003) suggest 0.01% for futures trading on equities. Investors can easily invest in index tracking ETFs, such as SPDR SPY which has a correlation of 99% with S&P500 index, and incur very low transaction costs. For example the largest electronic brokerage house in the U.S. charges only $0.005 per share for SPY transactions, which amounts to a transaction cost of 0.002% (as of Sept 2016). Although investors can trade the index cheaply by various ETF and index futures, we employ a more conservative rate of 0.1% each way as suggested by Driesprong et al. (2008).

Figure 4.2 plots the portfolio values based on the strategies described above. Graph A and B show the portfolio values for long-only while Graphs C and D illustrate the portfolio values for long/short volatility timing strategies.

Table 4.8 reports the portfolio performance of the volatility timing strategy under each forecasting model. The benchmark for Models 2.2 and 2.3 is Model 2.1, while the benchmark for Models 2.5 and 2.6 is Model 2.4. From Panel A, under the long-only strategy over the full period, ambiguity-based volatility Models 2.2 and 2.3 generate 105.07 and 94.51 percentage points return in excess of the benchmark (i.e., Model 2.1 without ambiguity innovations). Turning to Model 2.4 and its variants which incorporate information from implied variance, ambiguity-based volatility Models 2.5 and 2.6 outperform the benchmark (Model 2.4 without ambiguity) by 120.59 and 53.52 percentage points over the whole period. All strategies based on models with ambiguity innovations generate significantly higher returns than the benchmark models. This suggests that, when used as part of a volatility timing strategy, the inclusion of investors' ambiguity attitudes in volatility forecasting exercises can help generate superior returns. Turning to the sub-period results in Panel A, the superior returns generated by the volatility-timing strategies involving OMAA generally hold during (and after) the financial crisis of 2008 and the collapse of Lehman Brothers. While these strategies tend to generate higher returns than the benchmark (except for Model 2.2) in 1995-2004, their returns are even more impressive in the 2005-2012 follow-up period. Model 2.5, which considers option market ambiguity attitudes in the gain domain together with implied variance, produces a striking return of 392.3% during 2005-2012. This is equivalent to 120.59 percentage points in excess of the benchmark Model 2.4.

To further appreciate the importance of option market ambiguity attitudes in volatility-timing investment strategies, we consider a more flexible case which allows the investors to go short when the forecasted volatility is below the historical average level. Since the long-short strategies ensure the investor to have a continuous exposure to the S&P 500 index (either short or long at a given time), volatilities of the portfolio values are identical to those of the S&P 500 index[14]. In Panel B of Table 4.8, we can observe that the long-short strategy relying on GARCH volatility forecasts from our ambiguity Model 2.5 generates an astonishing 647.63% return over our entire sample period of 1995-2012 compared to only 330.95% for the benchmark (Model 2.4). The annualized return from a strategy based on Model 2.2 (8.18%) is exactly double than that of the benchmark Model 2.1 without ambiguity (4.09%). Considering the full period performance of the various strategies in Panel B, ambiguity-based GARCH volatility models produce superior returns than the benchmarks. In 1995-2012, all ambiguity-based strategies generated between 128.22 and 316.68 percentage points in excess of the corresponding benchmark or unambiguous models. Similar to the long-only strategy, our models perform equally well (and outperform benchmarks) during more uncertain periods.

The superior performance of our ambiguity-based strategies highlights the economic value of option market ambiguity attitudes in GARCH volatility forecasting, risk management, and market timing. The economic significance analysis confirms that the proposed volatility forecasting models not only provide more accurate volatility forecasts both in-sample and out-of-sample, but are also important in market-timing especially during periods of economic uncertainty.

4.5. Conclusion

We propose an ambiguity-based GARCH-in-mean volatility forecasting framework which capitalizes on the asymmetric and well documented role of market investors' ambiguity attitudes in the gain and loss domains. Our empirical results, based on a comprehensive dataset that spans 23 years of option pricing data and containing 5792 daily observations, confirm a significant relationship between ambiguity attitudes and ex post conditional volatility, and suggest that ambiguity and model uncertainty, as inferred from the option market, are quantitatively important in determining ex post conditional variance and in improving GARCH forecasts.

[14] Slight discrepancies in portfolio volatilities as shown in Panel B are due to the transaction costs. In the case of a perfect market without transaction costs, the portfolio volatility based on long-short strategy in all models will be identical.

We evaluate the practical relevance of our ambiguity-based volatility forecasting models by examining their out-of-sample forecasting accuracy. Out-of-sample forecasting analysis using the SPA test, different estimation windows, comparison windows, estimated volatility benchmarks, and an extensive range of loss functions unequivocally documents robust and consistent improvements in the accuracy of GARCH volatility forecasts when investors' ambiguity attitudes and implied variance are controlled for. In addition to the out-of-sample analysis, we also examine the economic value of including option market ambiguity attitudes in volatility forecasting practice by comparing portfolio returns based on two simple volatility-timing trading strategies with transaction costs. Economic significance results confirm the findings of the in-sample and out-of-sample estimations, and show that an annualized return of 4.09% in excess of the benchmark portfolio return can be earned using the volatility forecasts from our proposed models.

Given the consistent findings from the in-sample estimations, out-of-sample forecasting, and economic significance analyses, option market ambiguity attitudes can be considered a statistically significant and quantitatively important factor in GARCH volatility forecasting. This chapter validates the hypothesis of efficiency of options markets in informing future volatility fluctuations and serves as a foundation for research on the role of ambiguity aversion and model uncertainty in other risk management areas such as value at risk analysis and stress testing. We leave this for future research.

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Chapter 5.

Ambiguity Attitudes, the Variance Premium and International Stock Market Volatility

ABSTRACT

This chapter estimates a behavioral/subjective volatility scaling factor, which captures investors' ambiguity attitudes in the market, and adjusts the variance risk premium and its two components for model uncertainty to test the incremental role of ambiguity information in financial and economic prediction. Assessing the predictive power of the resulting ambiguity-adjusted measures in predicting market returns, economic activity and financial instability, our results document improved informational content over the standard variance premium and its benchmark market variance components. Analysis of international volatility effects further suggests that investors can be better informed about spillovers if ambiguity aversion is incorporated in volatility estimations.

5.1. Introduction

Option market information has long been praised for its informational efficiency in predicting a variety of important financial and economic variables. Cremers and Weinbaum (2010) document the return predicting ability of call and put option implied volatility deviations. Relying on VIX as a proxy for uncertainty, Bloom (2009) shows that non-Knightian uncertainty shocks can be associated with ex post depressed economic activity including aggregate output and employment. Bali and Murray (2013) shows that risk-neutral skewness predicts the cross section of equity option portfolio returns.

The CBOE volatility index VIX has long been referred to as the “fear gauge” of the equity market (Whaley 2000). Computed using a model-free approach, VIX represents the risk-neutral expectation of future volatility of S&P 500 index returns. As it captures the expected variability in returns for a leading equity index, a long list of papers have used VIX as a proxy for market and economic uncertainty (Bloom, 2009; Chung and Chuwonganant, 2014). Ang, Hodrick, Xing and Zhang (2006) show that changes in aggregate volatility, as proxied by changes in VIX, are priced in cross-sectional returns. Bloom (2009) analyzes in a vector autoregressive model the relationship between VIX and economic activity to find that VIX has a potentially depressing effect on ex post economic activity. Bekaert, Hoerova and Lo Duca (2013) document a strong lead-lag relationship between components of VIX and changes in monetary policy. All the empirical evidence points to the richness of information embedded in VIX. Despite this, no study has been devoted to interpreting or estimating investors’ fear gauge and its association with the variance premium from a model uncertainty perspective (i.e., ambiguity or uncertainty beyond probabilistic risk).

Over the last decade, as another option-based indicator related to VIX, the variance risk premium (VRP, Carr and Wu, 2009) has found prominence in the economics and finance literatures. Computed as the difference between implied variance (IV, squared VIX) and an estimated conditional variance (RV) of the underlying market returns, VRP has been shown to predict short term market excess return robustly (Bollerslev, Tauchen and Zhou, 2009; Bollerslev, Marrone, Xu and Zhou, 2014). However, the measurement of VRP is not without limitations (Bekaert and Hoerova, 2014; Barras and

Malkhozov, 2016). In a recent econometric study that identifies the best VRP specification, Bekaert and Hoerova (2014) show how their variance risk premium and its market variance components are able to predict stock market returns, volatility, economic activity and financial instability. While other attempts have also been made to improve variance risk premium estimations, efforts are largely focused on capturing a more accurate conditional variance, which in essence reduces the improvement exercise to a volatility forecasting issue. Herein, we examine the VRP predictive ability and its related estimation problems from a model uncertainty point of view. This chapter investigates whether, and the extent to which, the information content of variance risk premium can be augmented by incorporating investors' ambiguity aversion in its two variance components (i.e., IV and RV).

While it is important to address the problem of ambiguity attitudes in the literature (Epstein and Chen, 2002; Hatchondo, 2008; Dimmock et. al, 2016), ambiguity is still not a main-stream issue in forecasting and economic prediction. This is explained by the inherent difficulties in estimating investors' ambiguity attitudes from financial market data. While limited evidence has attempted to interpret VRP using ambiguity theories (Miao et al., 2012; Dreschler, 2013), an increasing stream of research suggests that VRP's predictive power is primarily driven or dominated by investors' time-varying risk aversion (Rosenberg and Engle, 2002; Bakshi and Madan, 2006; Bekaert, Hoerova and Duca, 2013). Although research on variance premium estimation has been on an upward trend, no study has shown explicitly whether ambiguity and model uncertainty information can be incorporated into VRP (and its two variance components) as a test of informational content. This study fills this gap by explicitly estimating an ambiguity-adjusted VRP with information of investors' ambiguity attitudes, i.e., the behavioral adjustment factor, extracted from option prices. On the one hand, if VRP already captures ambiguity aversion, any attempt to incorporate ambiguity information into its components would fail because of information overlaps. On the other hand, if accounting for ambiguity aversion improves VRP predictive ability, it will indicate that VRP might not capture ambiguity aversion effectively after all. It may also suggest or confirm that risk aversion dominates VRP dynamics (Bekaert and Hoerova, 2016). Our main hypothesis is that accounting for investors' ambiguity attitudes should increase the informational content of VRP and its variance components when used for the purpose of economic and financial prediction. As an additional contribution, we extend the verification of this hypothesis to international markets and the role of volatility spillovers.

Motivated by the forward-looking nature of option markets, we extract investors' ambiguity attitudes using an ambiguity-adjusted option pricing model (A-OPM) based on Choquet utility, compute the subjective adjustment factor (SAF) for market variance, and compare the informational content of the subjectively adjusted variance measures to those of their "ambiguity-free" or risk-based counterparts. For ease of comparison, we follow the prediction sequence of Bekaert and Hoerova (2014). We also use alternative proxies for ambiguity, which are not option-based, for robustness and validation. After controlling for ambiguity aversion, we find that the subjectively adjusted variance premium (SVRP) outperforms the vanilla or ambiguity-free variance premium in predicting the future excess returns of the S&P 500 index. We also examine whether the ambiguity-adjusted implied (SIV) and realized variances (SRV) provide additional information content in predicting economic activity and financial instability. Our results document that both ambiguity-adjusted implied and realized variances gain extra predictive power in forecasting economic activity proxied by industrial production growth and financial instability proxied by the Kansas City Financial Stress Index (KCFSI). Our results are robust to alternative specifications of SAF based on ambiguity proxies that do not solely rely on option market information, such as the Economic Policy Uncertainty Index (PUI), Investors Intelligence Indices – Bearish (BEAR) and Bullish (BULL), University of Michigan Consumer Sentiment Index (UMCSI), and Consumer Confidence Index (CCI).

We further investigate cross border volatility dynamics and find that investors can be better informed about spillovers across markets with option trading activity, and that international stock market volatility predictions can be enhanced if ambiguity information is explicitly accounted for. This chapter contributes to the extant literature by incorporating ambiguity attitudes information into the variance premium, implied variance and realized variance, and documenting an improved predictive ability to (international) market excess returns, economic activity and financial instability. Our analysis also contributes to the volatility spillovers literature by showing that ambiguity aversion can be important in explaining international risk dynamics.

5.2. Modeling Framework and Empirical Methodology

To adjust the variance premium and its two volatility components for ambiguity, we need ex ante estimations of investors' ambiguity attitudes. To preserve the real-time characteristics of the variance premium, we employ a contingent claim approach in which forward-looking ambiguity expectations and level of miscalibration are estimated from traded option market prices. For this purpose, we employ the rank dependent utility framework first proposed by Chateauneuf et al (1996) and later extended to European type option pricing by Driouchi et al. (2016). Under this framework, a modified set of Brownian motions that allows ambiguity attitudes to enter the valuation process via non-additive Choquet capacities (see e.g., Eichberger and Kelsey, 2014), instead of standard Bayesian additive probabilities, is used. Validated by Kast and Lapied (2010), Kast et al. (2014) and Agliardi et al. (2016) to be dynamically consistent, the ambiguity-adjusted Brownian motion or set of Brownian motions is specified as follows:

$$\frac{dS}{S} = (\mu + m\sigma)dt + s\sigma dz \ \ (\forall m \in]-1,1[\,, \forall s \in]0,1]\,) \tag{5.1}$$

$$dW = mdt + sdz \tag{5.2}$$

where S is the spot price of the underlying asset, μ is the drift, σ is the volatility term, m and s are the mean and standard deviation of a standard Wiener process W, z is a standard Wiener process. Examining the level of ambiguity surrounding the subprime crisis and its daily effects on S&P volatility, Driouchi et al (2016) develop an ambiguity-adjusted Black-Scholes option pricing model using the above Brownian motion framework. The price of a European call option is specified as follows:

$$\begin{aligned} P_t^C = S_0 e^{-\delta' T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r' - \delta' + 0.5(s\sigma)^2)T}{s\sigma\sqrt{T}}\right) \\ - Ke^{-r'T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r' - \delta' - 0.5(s\sigma)^2)T}{s\sigma\sqrt{T}}\right) \end{aligned} \tag{5.3}$$

where:

$$\begin{aligned} r' = r + m\frac{[r - (\mu + m\sigma)]}{s^2\sigma}; \\ \delta' = \delta - \frac{(m + s^2\sigma - s\sigma)[(\mu + m\sigma) - r]}{s^2\sigma} \end{aligned} \tag{5.4}$$

$$m = 2c - 1 \; and \; s = \sqrt{4c(1-c)} \quad (\forall \; c \; \in]0,1[) \tag{5.5}$$

P_t^C is the ambiguity-adjusted price of the call, K is the strike price, r is the risk-free rate, δ is the dividend yield, σ is the volatility measure, c is the capacity variable summarizing investors' miscalibration, μ is the subjective required rate of return, r' and δ' are the subjective discount rate and subjective dividend yield respectively. Under this approach, agents' model misspecification under ambiguity is summarized by the capacity variable c, which in turn determines behavioral factors m and s. If investors are ambiguity averse, c will be less than 0.5 (Driouchi et al., 2016; Agliardi et al., 2016). When there is no ambiguity in the economic environment or market, c = 0.5, the pricing equation reduces to the classical Black Scholes model. From (5.1), s represents the subjective adjustment factor to the standard deviation in the Brownian motion. Before obtaining s from option prices, we estimate investors' ambiguity attitudes (OMAA) by inverting (5.3) numerically and minimizing the absolute deviations between model and market price[1]:

$$OMAA_t \equiv c_t^* = \underset{c|0<c<1}{\arg\min}\left[\left|P_t^C(S_t, K, r, T, \sigma_t, \mu_t, \delta_t, c_t) - P_t^{Mkt}\right|\right] \tag{5.6}$$

where P_t^{Mkt} is the observed market price of the S&P 500 option. The resulting capacity variable c_t is then transformed into the scaling variance factor or the subjective adjustment factor SAF (see (5.5)): [2]

$$SAF_t \equiv \; s^2 = 4c(1-c) \tag{5.7}$$

where SAF_t is the subjective adjustment factor for variance at time t. In the estimation of SAF_t, we use the 1-month USD LIBOR as risk-free rate, trailing 12-month aggregated dividend yield of the S&P 500 index constituents as the expected dividend yield, RiskMetrics EWMA volatility (JP Morgan, 1996) as the volatility input, and 12-month historical returns as the subjective required rate of return[3].

[1] A similar procedure is followed in Driouchi, Trigeorgis and So (2016) and So, Driouchi and Trigerogis (2016).

[2] For robustness, we used alternative proxies for ambiguity (i.e., PUI, BEAR, BULL, UMCSI and CCI) in our SAF estimations to find comparable and consistent results. The SAF based on the Investors Intelligence Bearish Index (SAF_{BEAR}) shows the most consistent improvements (reported in Table A5.2). Although not as good as our option-based ambiguity-adjusted measures, the predictive ability of the SAF_{BEAR}-adjusted VRP and its components is generally higher than that of alternative proxies. In a related study exploring the relationships among the conditional volatility of Investors Intelligence Bull/Bear ratio, stock market returns and volatility, Escobari and Jafarinejad (2016) achieve improved explanatory power when investor sentiment is accounted for.

[3] The choice of subjective required rate of return relates to investors' memory of past returns. Barberis, Huang and Santos (2001) highlight the importance of how far back an investor's mind stretches when determining her required return, and that investors generally have a short

The variance risk premium and its two variance components – implied variance and realized variance – have been shown to predict a variety of important economic and financial variables. This includes equity market returns, economic activity indicators, and financial sector instability indicators. To investigate if the SAF provides any additional informational content to those predictions, we compare the information content of VRP, and its two variance components, with and without ambiguity adjustment. For comparability with extant literature (e.g. see Fama and French, 1989; Lewellen, 2004; Bollerslev, Tauchen and Zhou, 2009; Li, Ng and Swaminathan, 2013) we employ the following standard long-horizon prediction regression:

$$y_{t+k}^{i} = \alpha + \beta x_t^j + \varepsilon_{t+k} \tag{5.8}$$

where y_{t+k}^{i} is the ex post excess return, industrial production growth and Kansas City Financial Stress Index over k-months horizons, x_t^j is a 1 x h row vector of explanatory variables (excluding intercept), α is an h x 1 vector of intercepts, and β is an h x 1 vector of slope coefficients. As noted in Ang and Bekaert (2009), Hodrick (1992) standard error provides the best inference for small samples, we report the robust t-statistics based on Hodrick (1992) standard error in regressions involving excess returns and industrial production growth. Since the computation of Hodrick (1992) spectral density relies on an artificially-created overlapping setup in the dependent variable, the computation is not suitable for the prediction of the Kansas City Financial Stress Index. For the prediction of KCFSI, we follow Bekaert and Hoerova (2014) by using Newey West with lags equal to max [3, 2*horizon].

5.3. Data and Variable

5.3.1 Option Data

To obtain daily investors' ambiguity attitudes and compute the subjective adjustment factor (SAF), we employ a large dataset of S&P 500 European-type options from 1990 to 2012. To ensure that we infer investors' ambiguity attitudes based on the most liquid option contracts, we follow the option contract selection procedure of the CBOE in computing VIX (Chicago Board Options Exchange,

memory. We choose the 12-month return as it reflects short memory features plus gives a reasonable sample size (typically 252 trading days). We have also used different historical returns from 6 month to 5 years and volatility measures including simple historical standard deviation and out-of-sample GARCH-M(1,1), and found the results hold.

Table 5.1. Summary Statistics of the Option Dataset

This table reports general summary statistics in Panel A and descriptive statistics of the option dataset sorted by day-to-maturity (DTM), level of VIX, and the level of OMAA in Panel B, C, and D respectively. The option dataset covers Jan 1990 to Dec 2012.

Panel A. General Summary Statistics	
Total Number of Option Prices	947,314
Average Moneyness	1.060
Average DTM	31.921

Panel B. By Day-to-maturity (DTM)	DTM < 14	14 ≤ DTM < 30	30 ≤ DTM < 45	45 ≤ DTM < 60	60 ≤ DTM	All
Number of Contract-days	159,278	292,004	248,574	180,336	67,122	947,314
Average Price	42.520	58.191	67.021	68.536	68.286	60.558
Average Implied Volatility	0.316	0.279	0.260	0.237	0.228	0.269
Average Bid-ask Spread	1.758	1.792	1.889	1.813	1.805	1.817
Average Bid-ask Spread / Price	4.14%	3.08%	2.82%	2.64%	2.64%	3.00%

Panel C. By level of VIX	VIX < 15	15 ≤ VIX < 20	20 ≤ VIX < 25	25 ≤ VIX < 30	30 ≤ VIX	All
Number of Contract-days	194,076	271,666	232,506	113,472	135,594	947,314
Average Price	37.288	54.706	67.175	73.103	83.740	60.558
Average Implied Volatility	0.153	0.221	0.274	0.323	0.475	0.269
Average Bid-ask Spread	1.232	1.728	1.883	1.976	2.583	1.817
Average Bid-ask Spread / Price	3.30%	3.16%	2.80%	2.70%	3.08%	3.00%

Panel D. By level of OMAA	OMAA < 0.25	0.25 ≤ OMAA < 0.5	0.5 ≤ OMAA < 0.75	0.75 ≤ OMAA	All
Number of Contract-days	194	259,936	680,484	6,700	947,314
Average Price	59.275	60.905	60.012	102.545	60.558
Average Implied Volatility	0.309	0.280	0.262	0.551	0.269
Average Bid-ask Spread	2.988	1.815	1.798	3.793	1.817
Average Bid-ask Spread / Price	5.04%	2.98%	3.00%	3.70%	3.00%

2014). We include both call and put options with various moneyness levels covering out-of-the-money (OTM), at-the-money (ATM), and in-the-money (ITM). In line with the VIX computational approach, we consider both near-term and next-term options. The aggregate option market ambiguity attitude measure is then taken as the average of OMAA across different maturities. Table 5.1 reports the descriptive statistics for our option data. From Panel A of Table 5.1, our option dataset covers nearly one million contract-days with an average moneyness (S/K) of 1.06 and an average day-to-maturity of 31.92 days. We also report summary statistics according to day-to-maturity, level of VIX, and level of OMAA for a clear description of the sample characteristics. Panels B and C show that our dataset characteristics are consistent with those of standard option datasets used in extant research. Fundamental characteristics of option prices can be noted. A decreasing implied volatility across increasing days-to-maturity is consistent with the observed volatility term structure. Interestingly, Panel D of Table 5.1 documents an increased bid-ask spread when ambiguity is high (i.e., when OMAA is away from neutrality 0.5). This is consistent with existing literature relating depressed liquidity and limited participation to increased ambiguity (Epstein and Schneider, 2007; Cao, Wang and Zhang, 2005; Ui, 2011).

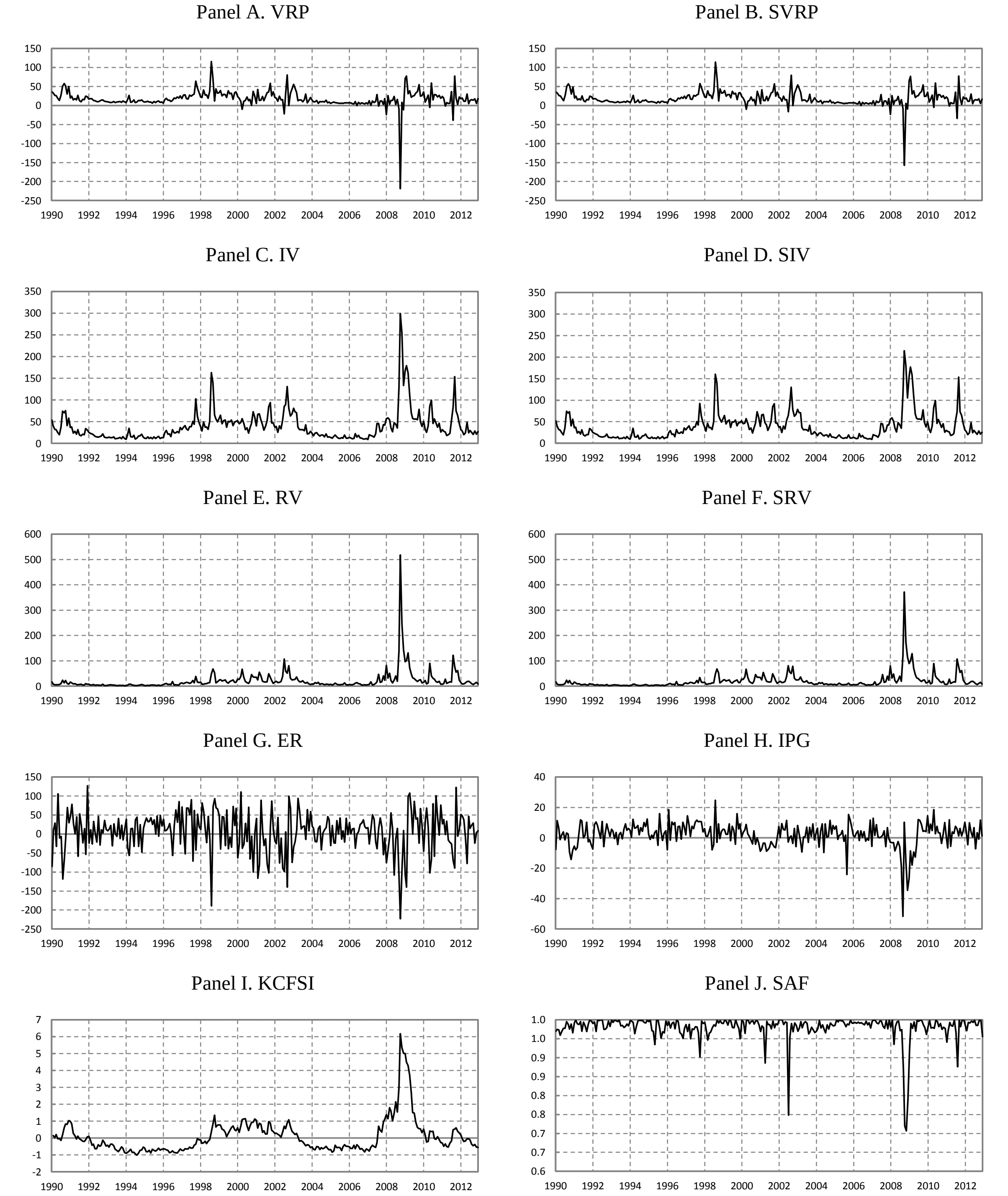
5.3.2 Other Data

Besides our option data, we obtain the daily closing level of S&P 500 index, daily 1-month USD LIBOR, and trailing 12-month dividend of the S&P 500 portfolio from Thomson Datastream. For the international volatility spillover analysis, we employ implied volatility indices data from seven countries with option trading activity obtained from Thomson Datastream. Our dataset spans the period from 1990 to 2014.

We consider the predictive power of the improved VRP and its variance components to market excess returns, industrial production growth, and the Kansas City Financial Stress Index (KCFSI). Excess return is computed as the monthly annualized logarithmic return of the S&P 500 index in excess of the 3-month treasury yield. Industrial production growth is computed as the annualized logarithmic change of industrial production. In market excess returns prediction, we also control for other established predictors of returns including the consumption wealth ratio (CAY) measured by a linear combination of consumption, labor income and asset holdings (Lettau and Ludvigson, 2001), credit spread (CS) measured as the difference between Moody’s BAA and AAA yield indices, dividend yield (DY) of S&P 500 portfolio, price-to-earnings ratio (PER) of S&P 500 portfolio, stochastically detrended short rate (RREL), and term spread (TS) measured as the difference between 10-year treasury bond and 3-month treasury bill yields. The KCFSI, industrial production, Moody’s BAA and AAA yield indices, and treasury yields used to compute TS were obtained from the website of the Federal Reserve Bank of St. Louis. CAY data is obtained from Martin Lettau’s website. VRP data is obtained from Hao Zhou’s website. Monthly data on S&P 500 dividend yield and price-earnings ratios (PER) were obtained from Robert Shiller’s website. S&P 500 data was obtained from Thomson Datastream. Table A5.1 in the Appendix provides descriptions of the data series, variables and data sources used. Figure 5.1 plots the VRP and its two components together with their subjectively adjusted counterparts. Table 5.2 reports the descriptive statistics for all predictor variables and the target variables predicted. In Panel A of Table 5.2, market excess returns are on average positive meaning the US equity market in general delivers positive risk-adjusted returns to investors. Predictor variables used are generally highly autocorrelated except VRP (AR(1)=0.26) and SVRP (AR(1)=0.30).

Figure 5.1. Predicting Excess Returns

This figure plots the key variables used in this study, including variance premium (VRP), ambiguity-adjusted variance premium (SVRP), implied variance (IV), ambiguity-adjusted implied variance (SIV), realized variance (RV), ambiguity-adjusted realized variance (SRV), industrial production growth (IPG), S&P 500 excess returns (ER), Kansas City Financial Stress Index (KCFSI), and Subjective Adjustment Factor (SAF). The sample covers monthly observations from Jan 1990 to Dec 2012.



Turning to Panel B of Table 5.2, contemporary market excess returns are weakly correlated with predictor variables except IV (ρ=-0.42), RV (ρ=-0.38) and their subjectively adjusted counterparts SIV (ρ=-0.41) and SRV (ρ=-0.38).

Table 5.2. Summary Statistics of Variables

This table reports the summary statistics and correlation matrix of the variables considered. ER is the excess return of S&P 500 index computed as the logarithmic return of S&P500 index in excess of the logarithmic yield of 3-month treasury yield. IPG is the logarithmic industrial production growth. KCFSI is the Kansas City Financial Stress Index. OMAA is option market ambiguity attitudes computed according to eq. 6. SAF is the subjective adjustment factor computed according to eq. 7. VRP and SVRP are variance premium and ambiguity-adjusted variance premium respectively. IV and SIV are implied variance and ambiguity- adjusted implied variance respectively. RV and SRV are realized variance and ambiguity- adjusted realized variance respectively. DY is the dividend yield of S&P 500 index portfolio. PER is the price-to-earnings ratio of S&P 500 index portfolio. CS is the credit spread computed as the difference between Moody's BAA and AAA bond yield indices. TS is the term spread computed as the difference between 10-year and 3-month treasury yields. CAY is consumption wealth ratio. RREL is stochastically detrended short rate. Variables are denoted in annualized percentages whenever suitable. The sample period covers end-of-month observations from Jan 1990 to Dec 2012.

Panel A. Descriptive Statistics

				Uncertainty measures		Variance Premium		Implied Variance		Realized Variance		Equity Fundamental		Bond Fundamental		Other Predictors	
	ER	IPG	KCFSI	OMAA	SAF	VRP	SVRP	IV	SIV	RV	SRV	DY	PER	CS	TS	CAY	RREL
Mean	6.03	2.02	0.09	0.53	0.98	18.01	17.94	39.79	38.31	21.78	20.37	2.10	25.15	-23.05	1.88	0.27	-0.08
Std.Dev.	52.51	7.98	1.04	0.06	0.03	21.89	19.30	35.61	31.19	39.94	31.19	0.66	16.00	15.99	1.16	1.57	0.33
Skewness	-0.77	-1.74	2.85	0.02	-5.50	-3.88	-1.71	3.34	2.52	8.14	6.42	0.66	4.18	-4.06	-0.15	0.04	-0.39
Kurtosis	1.57	8.79	11.20	1.76	36.06	50.78	26.83	16.31	8.36	90.10	61.41	-0.48	20.07	19.25	-1.14	-0.99	0.10
Max	126.91	24.71	6.18	0.77	1.00	115.85	113.87	298.90	214.98	517.46	372.17	3.88	123.73	-10.33	3.76	3.10	0.68
Min	-222.77	-51.59	-1.01	0.32	0.71	-218.56	-157.20	9.05	9.02	1.73	1.70	1.11	13.50	-120.83	-0.53	-3.19	-1.02
AR(1)	0.07	0.24	0.95	0.35	0.56	0.26	0.30	0.80	0.80	0.65	0.68	0.99	0.97	0.97	0.98	0.95	0.83

Panel B. Correlation Matrix

	ER	IPG	KCFSI	OMAA	SAF	VRP	SVRP	IV	SIV	RV	SRV	DY	PER	CS	TS	CAY	RREL
ER	1.00																
IPG	0.02	1.00															
KCFSI	-0.31	-0.53	1.00														
OMAA	0.03	-0.16	0.16	1.00													
SAF	0.23	0.24	-0.49	-0.48	1.00												
VRP	0.00	-0.01	-0.01	-0.26	0.39	1.00											
SVRP	-0.05	0.00	0.04	-0.26	0.35	0.99	1.00										
IV	-0.42	-0.25	0.81	0.13	-0.56	0.10	0.19	1.00									
SIV	-0.41	-0.24	0.79	0.06	-0.44	0.22	0.31	0.99	1.00								
RV	-0.38	-0.22	0.73	0.26	-0.71	-0.46	-0.38	0.84	0.76	1.00							
SRV	-0.38	-0.24	0.77	0.21	-0.66	-0.39	-0.31	0.87	0.81	0.99	1.00						
DY	-0.02	-0.13	0.09	0.02	-0.11	-0.05	-0.04	0.04	0.01	0.06	0.04	1.00					
PER	0.03	-0.23	0.49	0.02	-0.06	0.19	0.22	0.40	0.44	0.25	0.30	0.04	1.00				
CS	-0.04	0.22	-0.49	-0.02	0.06	-0.19	-0.22	-0.40	-0.44	-0.25	-0.30	0.00	-1.00	1.00			
TS	-0.04	0.02	0.03	0.10	-0.11	-0.06	-0.05	0.06	0.05	0.09	0.08	0.35	0.22	-0.21	1.00		
CAY	0.04	-0.01	-0.05	-0.02	-0.12	0.12	0.14	0.10	0.09	0.02	0.01	0.45	-0.03	0.05	0.17	1.00	
RREL	0.10	0.27	-0.43	-0.04	0.20	-0.07	-0.09	-0.31	-0.31	-0.24	-0.25	-0.20	-0.31	0.30	-0.36	-0.10	1.00

5.4. Financial and Economic Predictability Findings

To assess the informational content of our ambiguity-adjusted measures of variance risk premium and the two market variance components, we revisit the predictive power evidence documented in the literature. Similar to Bekaert and Hoerova (2014), we consider three aspects or dimensions of uncertainty by predicting stock market excess returns, economic activity proxied by industrial production growth, and financial instability proxied by the Kansas City Financial Stress Index. In the additional results section, we examine how the SAF affects international volatility prediction dynamics and its role in related spillovers. We find that the predictive power of SVRP is more significant than VRP in predicting market excess returns, and SIV (SRV) is more significant in predicting industrial production growth than IV (RV). In addition, we also find improved information content to financial instability by SRV when compared to RV. Analysis of market volatility across seven countries (with option trading activity) suggests that accounting for model uncertainty and ambiguity in international volatility modeling can improve the detection of spillover effects.

5.4.1 Predicting Equity Market Excess Returns

Table 5.3 reports the predictive regression results according to equation (8). From Panel A of Table 5.3, VRP shows strong predictive power to market excess return from 1- to 12-month horizons with the adjusted R^2 ranging from 2.91% to 11.89%. The most significant prediction horizon is at 3-month. This predictive pattern is generally in line with the findings reported in the literature (Bolleslev et. al, 2009; Bolleslev et. al, 2014). Turning to the SVRP results, a significant improvement in informational content is observed. Robust t-statistics of the estimated coefficient of SVRP and the adjusted R^2 improve in every single predictive horizon. Robust t-statistics of SVRP range from 1.65 in 24-month horizon to 3.88 in 3-month horizon. The general predictive pattern of VRP is preserved with the strongest predictive power at

Table 5.3. Predicting Excess Returns

This table reports the excess returns prediction results. VRP and SVRP are variance premium and ambiguity-adjusted variance premium respectively. IV and SIV are implied variance and ambiguity-adjusted implied variance respectively. RV and SRV are realized variance and ambiguity-adjusted realized variance respectively. DY is the dividend yield of S&P 500 index portfolio. PER is the price-to-earnings ratio of S&P 500 index portfolio. CS is the credit spread computed as the difference between Moody's BAA and AAA bond yield indices. TS is the term spread computed as the difference between 10-year and 3-month treasury yields. CAY is consumption wealth ratio. RREL is stochastically detrended short rate. Sample covers 1990M01 to 2012M12. Hodrick (1992) t-statistics are reported in squared brackets. *,**, and *** denote significance at 90%, 95%, and 99% confidence levels respectively.

Horizon	**1**	**3**	**6**	**12**	**18**	**24**	**1**	**3**	**6**	**12**	**18**	**24**
Panel A. Regressions with (ambiguity-adjusted) variance premium												
Constant	-3.94	-2.69	0.70	3.91	4.48	4.71	-5.68	-4.32	-0.46	3.26	3.94	4.28
	[-0.77]	[-0.69]	[0.19]	[1.22]	[1.48]	[1.60]	[-1.06]	[-1.08]	[-0.12]	[1.00]	[1.28]	[1.43]
VRP	0.58**	0.50***	0.31***	0.14*	0.10	0.08						
	[2.43]	[3.65]	[3.03]	[1.92]	[1.56]	[1.54]						
SVRP							0.68***	0.60***	0.38***	0.18**	0.13*	0.11*
							[2.62]	[3.88]	[3.32]	[2.07]	[1.71]	[1.65]
Adj R^2 (%)	5.61	11.89	8.02	2.91	1.73	1.52	5.99	12.98	9.18	3.64	2.39	2.03

Horizon	**1**	**3**	**6**	**12**	**18**	**24**	**1**	**3**	**6**	**12**	**18**	**24**
Panel B. Regressions with (ambiguity-adjusted) variance premium and other predictors												
Constant	-18.49	-19.66	-18.80	-15.02	-12.01	-10.55	-20.18	-21.23	-19.90	-15.62	-12.47	-10.82
	[-1.06]	[-1.27]	[-1.41]	[-1.24]	[-1.01]	[-0.97]	[-1.16]	[-1.38]	[-1.49]	[-1.29]	[-1.05]	[-0.99]
CAY	1.89	2.34	3.27*	4.45***	4.65***	4.84***	1.58	2.07	3.08*	4.35**	4.59***	4.80***
	[0.87]	[1.23]	[1.79]	[2.61]	[2.72]	[2.84]	[0.72]	[1.08]	[1.68]	[2.55]	[2.68]	[2.82]
CS	-7.13	-5.09	1.89	7.30	3.61	2.43	-8.01	-5.78	1.50	7.11	3.56	2.34
	[-0.48]	[-0.35]	[0.16]	[0.94]	[0.63]	[0.49]	[-0.54]	[-0.40]	[0.13]	[0.92]	[0.62]	[0.47]
DY	9.32	9.10	7.07	4.30	3.18	2.61	9.70	9.43	7.28	4.42	3.25	2.66
	[1.51]	[1.56]	[1.28]	[0.90]	[0.73]	[0.69]	[1.56]	[1.62]	[1.31]	[0.92]	[0.74]	[0.70]
PER	0.19	0.26	0.18	0.00	0.04	0.03	0.17	0.24	0.17	-0.01	0.04	0.03
	[0.65]	[1.02]	[0.84]	[-0.00]	[0.30]	[0.29]	[0.60]	[0.96]	[0.78]	[-0.04]	[0.26]	[0.27]
RREL	27.02**	28.03***	29.30***	28.32***	20.25**	13.56**	27.56***	28.52***	29.64***	28.50***	20.39**	13.65**
	[2.58]	[2.97]	[3.13]	[3.02]	[2.44]	[2.14]	[2.62]	[3.02]	[3.16]	[3.03]	[2.45]	[2.15]
TS	-0.62	-0.97	-0.12	1.89	2.91	3.32	-0.48	-0.84	-0.03	1.95	2.95	3.34
	[-0.21]	[-0.34]	[-0.04]	[0.77]	[1.32]	[1.56]	[-0.16]	[-0.29]	[-0.01]	[0.79]	[1.33]	[1.57]
VRP	0.57**	0.48***	0.30***	0.15**	0.09*	0.07						
	[2.28]	[3.52]	[3.21]	[2.37]	[1.72]	[1.61]						
SVRP							0.69**	0.58***	0.37***	0.19**	0.12*	0.09
							[2.48]	[3.78]	[3.41]	[2.42]	[1.82]	[1.61]
Adj R^2 (%)	8.07	23.06	29.32	46.31	51.50	59.30	8.61	24.35	30.47	46.95	51.94	59.51

3-month horizon. More importantly, the predictive power of SVRP is present from 1- to 24-month horizons, extending the predictive horizon of the original VRP from 12- to 24-month. This confirms the informational relevance of ambiguity attitudes and model uncertainty in estimating the variance risk premium[4].

While the univariate results show promising evidence about the improvement provided by the SAF and the extracted ambiguity information from option prices, it is also important to consider other variables at the same time. Evidence has shown that the equity risk premium is driven by more than one source of information (Ang and Bekaert, 2007; Menzly, Santos and Veronesi, 2004), making it important to control for other known predictors of future excess returns. By considering other known predictor variables, we can also verify if the information embedded in SAF is overlapped with other market-based quantities. Panel B considers a multivariate specification that includes other known predictors of excess returns such as the consumption wealth ratio (CAY), credit spread (CS), dividend yield (DY), price-to-earnings ratio (PER), stochastically detrended short rate (RREL), and term spread (TS). In Panel B, VRP's significance improves from 6- to 24-month horizons with the inclusion of other predictor variables. The most significant prediction horizon is achieved at 3-month with a robust t-statistics of 3.52. Overall the predictive pattern is in line with Bolleslev et. al's (2009) findings regarding the enhancing effect of PER. RREL is significant in predicting future excess returns in all horizons while CAY is significant from 6- to 24-month horizons. Turning to the regressions involving SVRP and other predictors, the estimated t-statistics and adjusted R^2 improve in all horizons when compared to the multivariate specification with VRP. The most significant prediction is still at 3-month with robust t-statistics of 3.78, equivalent to a 7% improvement in t-statistics. The coefficients and significance of other predictor variables are very similar to those of the standard VRP regressions. These results confirm that the additional informational content harbored in SVRP is unique and not due to information overlap with other known predictors. The informational content of SAF seems to

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[4] We have also examined the out-of-sample forecasting power of SVRP versus VRP. Following Campbell and Thompson (2008), Welch and Goyal (2008), and Rapach, Strauss and Zhou (2009), we compared the out-of-sample R^2, utility gain for investors' asset allocation and annualized returns from optimal portfolios using out-of-sample return forecasts by SVRP and VRP. Our ambiguity-adjusted VRP delivers higher out-of-sample R^2, utility gain and annualized returns when compared to VRP forecasts. Results are available from the authors upon request.

be distinct from other predictor variables and VRP itself. Our finding confirms the improvement brought by the inclusion of ambiguity aversion from option prices and calls for the consideration of ambiguity corrections in financial and economic prediction.[5] As mentioned above, these findings are robust to alternative proxies for ambiguity, which do not rely solely on option information. Having demonstrated the improvement to VRP with a simple SAF correction, we now turn to its two variance components. As (S)IV and (S)RV do not normally predict market excess returns, we investigate the improvement in information content when predicting real economic activity and financial instability. This is in line with Bekaert and Hoerova (2014) who analyze the improvements brought by better estimations of the conditional variance. Our analysis is different in that it tackles the problem of (mis)estimation of market variance from a model uncertainty point of view.

Table 5.4. Predicting Economic Activity

This table reports the industrial production growth prediction results. VRP and SVRP are variance premium and ambiguity-adjusted variance premium respectively. IV and SIV are implied variance and ambiguity-adjusted implied variance respectively. RV and SRV are realized variance and ambiguity-adjusted realized variance respectively. Sample covers 1990M01 to 2012M12. Hodrick (1992) t-statistics are reported in squared brackets. *,**, and *** denote significance at 90%, 95%, and 99% confidence levels respectively.

Horizon	1	3	6	12	18	24	30	36	1	3	6	12	18	24	30	36
Panel A. Regressions with (ambiguity-adjusted) variance premium and realized variance																
Constant	4.49 ***	4.30 ***	3.40 ***	2.55 ***	2.23 ***	2.11 ***	2.06 ***	2.07 ***	4.36 ***	4.26 ***	3.34 ***	2.56 ***	2.26 ***	2.14 ***	2.09 ***	2.10 ***
	[4.39]	[5.84]	[5.34]	[4.60]	[4.10]	[3.81]	[3.68]	[3.75]	[4.77]	[5.89]	[5.29]	[4.61]	[4.14]	[3.82]	[3.69]	[3.75]
VRP	-0.04	-0.03	-0.01	0.01	0.01	0.01	0.01	0.01								
	[-1.20]	[-1.20]	[-0.32]	[0.67]	[1.02]	[0.98]	[0.96]	[0.70]								
RV	-0.08 **	-0.08 ***	-0.06 ***	-0.03 **	-0.02 *	-0.01	-0.01	-0.01								
	[-2.48]	[-2.68]	[-2.92]	[-2.43]	[-1.84]	[-1.47]	[-1.28]	[-1.28]								
SVRP									-0.03	-0.01	0.01	0.02	0.02	0.01	0.01	0.01
									[-0.78]	[-0.49]	[0.49]	[1.23]	[1.42]	[1.26]	[1.18]	[0.89]
SRV									-0.09 ***	-0.10 ***	-0.07 ***	-0.04 ***	-0.03 **	-0.02 *	-0.01 *	-0.01 *
									[-2.65]	[-2.76]	[-3.02]	[-2.65]	[-2.20]	[-1.86]	[-1.70]	[-1.74]
Adj R^2 (%)	11.46	27.40	20.17	8.28	4.08	2.24	1.61	0.95	10.88	26.90	20.42	9.64	5.46	3.23	2.40	1.67

Horizon	1	3	6	12	18	24	30	36	1	3	6	12	18	24	30	36
Panel B. Regressions with (ambiguity-adjusted) implied variance																
Constant	5.02 ***	5.14 ***	4.24 ***	3.15 ***	2.69 ***	2.46 ***	2.36 ***	2.30 ***	5.04 ***	5.16 ***	4.21 ***	3.17 ***	2.74 ***	2.50 ***	2.38 ***	2.34 ***
	[4.54]	[4.97]	[5.80]	[5.96]	[5.56]	[5.13]	[4.95]	[4.89]	[4.93]	[5.37]	[5.85]	[5.86]	[5.44]	[4.95]	[4.74]	[4.71]
IV	-0.07 **	-0.08 ***	-0.06 ***	-0.03 **	-0.02 *	-0.01	-0.01	-0.01								
	[-2.50]	[-2.75]	[-2.91]	[-2.36]	[-1.74]	[-1.37]	[-1.16]	[-1.16]								
SIV									-0.08 ***	-0.08 ***	-0.06 ***	-0.03 **	-0.02 *	-0.01	-0.01	-0.01
									[-2.73]	[-2.99]	[-2.96]	[-2.36]	[-1.78]	[-1.38]	[-1.13]	[-1.17]
Adj R^2 (%)	10.90	23.51	15.07	4.90	1.78	0.70	0.30	0.19	9.04	19.71	12.02	4.14	1.66	0.66	0.27	0.24

5.4.2 Predicting Real Economic Activity

To understand better the dynamics between SAF, VRP and its two variance components, we examine the economic activity predicting ability of VRP and its two variance components vis-à-vis their subjectively adjusted counterparts. Table 5.4 reports the predictive regression results for (S)VRP and (S)RV in Panel A, and (S)IV in Panel B. In Panel A of Table 5.4, stock market returns variance RV

[5] We have also examined if SVRP provide improved predictive power to market excess returns in six other countries including France, Germany, Japan, Netherlands, Switzerland, and United Kingdom. Except for Japan, in which VRP does not predict ex post excess return, SVRP shows improved predictive power in all countries. Results are available from the authors upon request.

robustly predicts economic activity, proxied by industrial production growth, from 1- to 18- month horizons. The coefficient of RV is consistently negative suggesting that heightened volatility is associated with depressed ex post economic output. For the prediction horizons considered, the robust t-statistics of RV range from -1.28 to -2.92. VRP does not show any predictive ability towards industrial production growth when RV is controlled for. This is in line with the findings of Bekaert and Hoerova (2014). Turning to the set of results on the right hand side of Panel A, SAF clearly improves the predictive ability of RV in all horizons considered. SRV robustly predicts IPG from 1- to 36-month horizons with improved robust t-statistics from -1.70 to -3.02. The coefficient of SRV is consistently negative. This is in line with studies suggesting that market volatility is negatively associated with ex post economic activity (Bloom, 2009; Caggiano, Castelnuovo and Groshenny, 2014). This impressive improvement echoes the findings in the last section regarding the importance of considering ambiguity and model uncertainty in the estimation of market variance.

Panel B reports the predictive regression results of IV and SIV. The predictive performance of IV is similar to that of RV. IV robustly predicts ex post IPG from 1- to 18-month horizons with robust t-statistics from -1.74 to -2.91. In the last specification with SIV as the sole independent variable to predict IPG, SIV robustly predicts IPG with negative coefficients. Robust t-statistics of SIV, ranging from -1.78 to -2.99 for 1- to 18-month horizons, are generally improved in short- and medium-term horizons compared to IV. Results in Table 5.4 confirm that in addition to more efficiently predicting market excess return, ambiguity-adjusted measures of market volatility harbor additional informational content towards the prediction of economic activity. Our results hold when alternative proxies for ambiguity such as BEAR, BULL, UMCSI, and CCI are used instead of the option-based scaling factor.

5.4.3 Predicting Financial Instability

This section investigates whether ambiguity-adjusted market variance (SRV and SIV) and variance premium (SVRP) are also able to improve the predictive ability of standard VRP and variance (IV and RV) when it comes to financial instability. While there are several financial instability indicators available, finding one that spans our entire sample period is not easy. We use the Kansas City

Financial Stress Index (KCFSI) as our target indicator because it covers data from 1990.[6] KCFSI is a principle component of 11 indicators including treasury yield, interest rate swap spread, idiosyncratic volatility of bank share prices, cross-sectional dispersion of bank stock returns, and VIX. We regress the level of KCFSI 1 to 24 months ahead on our VRP, IV, RV variables, and their subjectively adjusted counterparts. One important point to note is that given that KCFSI contains direct information from VIX, regressions involving IV can be biased by the autocorrelation of VIX. Any adjustment to VIX with a stochastic adjustment factor will therefore diminish its forecasting accuracy vis-à-vis the KCFSI especially for shorter prediction horizons. Therefore in analyzing the predictive power of IV and its ambiguity-adjusted counterpart SIV, we focus on longer prediction horizons (i.e., >12-months) when the autocorrelation problem is far less pronounced.

Table 5.5. Predicting Financial Instability

This table reports the Kansas City Financial Street Index prediction results. VRP and SVRP are variance premium and ambiguity-adjusted variance premium respectively. IV and SIV are implied variance and ambiguity-adjusted implied variance respectively. RV and SRV are realized variance and ambiguity-adjusted realized variance respectively. Sample covers 1990M01 to 2012M12. Newey and West (1987) t-statistics with lag equals to max[3,2*horizon] are reported in squared brackets. *,**, and *** denote significance at 90%, 95%, and 99% confidence levels respectively.

Horizon	**1**	**3**	**6**	**12**	**18**	**24**	**1**	**3**	**6**	**12**	**18**	**24**
Panel A. Regressions with (ambiguity-adjusted) variance premium												
Constant	0.10	0.16	0.16	0.06	0.07	0.11	0.04	0.13	0.15	0.05	0.07	0.13
	[0.36]	[0.58]	[0.56]	[0.19]	[0.22]	[0.35]	[0.16]	[0.43]	[0.47]	[0.16]	[0.22]	[0.36]
VRP	0.00	0.00	0.00	0.00	0.00	0.00						
	[-0.05]	[-0.45]	[-0.64]	[0.15]	[0.00]	[-0.42]						
SVRP							0.00	0.00	0.00	0.00	0.00	0.00
							[0.19]	[-0.22]	[-0.42]	[0.19]	[-0.03]	[-0.42]
Adj R^2 (%)	-0.35	0.33	0.42	-0.35	-0.39	-0.18	-0.18	-0.20	0.01	-0.33	-0.39	-0.12
Horizon	**1**	**3**	**6**	**12**	**18**	**24**	**1**	**3**	**6**	**12**	**18**	**24**
Panel B. Regressions with (ambiguity-adjusted) implied variance												
Constant	-0.85 ***	-0.67 ***	-0.41 **	-0.16	-0.02	0.13	-0.92 ***	-0.71 ***	-0.44 ***	-0.20	-0.05	0.14
	[-11.98]	[-6.83]	[-2.50]	[-0.52]	[-0.06]	[0.30]	[-9.10]	[-5.86]	[-2.70]	[-0.66]	[-0.12]	[0.29]
IV	0.02 ***	0.02 ***	0.01 ***	0.01 *	0.00	0.00						
	[10.84]	[8.22]	[6.42]	[1.95]	[0.63]	[-0.30]						
SIV							0.03 ***	0.02 ***	0.01 ***	0.01 **	0.00	0.00
							[7.69]	[5.49]	[4.89]	[2.13]	[0.69]	[-0.28]
Adj R^2 (%)	65.30	41.82	17.49	3.55	0.25	-0.18	62.72	38.58	16.27	4.13	0.44	-0.17
Horizon	**1**	**3**	**6**	**12**	**18**	**24**	**1**	**3**	**6**	**12**	**18**	**24**
Panel C. Regressions with (ambiguity-adjusted) realized variance												
Constant	-0.32 ***	-0.27 **	-0.16	-0.02	0.03	0.08	-0.43 ***	-0.35 ***	-0.22	-0.06	0.01	0.08
	[-3.42]	[-2.31]	[-0.92]	[-0.08]	[0.10]	[0.27]	[-4.85]	[-3.42]	[-1.36]	[-0.24]	[0.02]	[0.26]
RV	0.02 ***	0.02 ***	0.01 ***	0.00 *	0.00	0.00						
	[4.76]	[7.45]	[7.04]	[1.91]	[0.81]	[-0.19]						
SRV							0.03 ***	0.02 ***	0.01 ***	0.01 **	0.00	0.00
							[6.10]	[10.12]	[7.43]	[2.02]	[0.93]	[-0.15]
Adj R^2 (%)	52.70	38.62	17.70	2.41	0.11	-0.37	58.51	41.74	19.47	3.54	0.46	-0.38

[6] We have also used CISS, a composite index of financial stress for Europe, and the Macroeconomic Uncertainty Index developed by Bali, Brown and Caglayan (2014), which is a principal component of eight financial and economic based risk factors, as alternative proxies and found similar results.

Our predictive regression results are presented in Table 5.5. In Panel A, VRP is not significant at all in predicting ex post financial stress. This is consistent with Bekaert and Hoerova (2014). In Panel B, IV is efficient in predicting financial instability from 1- to 13-month horizons. As noted, any adjustment to IV would weaken predictive ability as measured by the robust t-statistics due to the inclusion of VIX in the estimation of the KCFSI. In comparing the predictive power of IV and SIV, we focus on longer predictive horizons. Indeed from the 9-month horizon onwards, the predictive ability of SIV exceeds that of IV. Ambiguity-adjusted IV extends the prediction horizon of implied variance to 14-months. Similar findings can be observed in Panel C. RV predicts KCFSI from 1- to 14-month horizons with robust t-statistics ranging from 1.71 to 4.76. On the other hand, SRV robustly improves the predictive ability to 1- to 15-month with robust t-statistics ranging from 1.81 to 6.10. These improvements in information content once again confirm the importance of controlling for model uncertainty in prediction exercises. These findings are robust to alternative ambiguity proxies (e.g., BULL, BEAR and UMCSI) that do not rely on option market information. The improved predictive power of SIV and SRV to financial instability suggests that accounting for ambiguity attitudes information may be key to more accurate predictions or anticipations of systemic financial shocks (see e.g., Fernholz, 2015 on FX interventions in times of crisis).

5.4.4 Additional Results: Predicting International Volatility

The evidence presented in the last three sections confirms the relevance of ambiguity attitudes and the SAF in economic and financial prediction. This section investigates if the informational improvement in volatility estimations extends to international markets. We consider major global equity markets including the Netherlands (AEX index), France (CAC index), Germany (DAX index), the United Kingdom (FTSE index), Japan (NIKKEI index), Switzerland (SMI index), and the United States (SPX Index). Due to data availability, we extract the SAF from the publicly available VIX-equivalent volatility indices in those countries[7]. While volatility spillover effects are well documented in the literature (see for example, Hamao, Masulis and Ng, 1990; King and Wadhwani, 1990; Lin, Engle and

[7] The selection of these seven countries is due to data availability and liquidity considerations. Similar analyses focusing on these countries include Bollerslev, Marrone, Xu and Zhou (2014).

Ito, 1994; Fleming, Kirby and Ostdiek, 1998; Bekaert and Wu, 2000; Baele, 2005), the potential role played by ambiguity aversion has not been examined empirically. It is, therefore, important to verify whether international volatility prediction can be improved with a simple adjustment for ambiguity. To compare the efficiency of capturing volatility spillovers internationally, we estimate a system of equations that allows for multiple channels of information propagation. A similar methodology has been adopted by Degryse and Ongena (2001) in analyzing the association between bank relationships and firm profitability, and by Buch, Koch and Koetter (2013) in analyzing the relationship between banks' internationalization and risk. We first specify the following system of regression equations to understand how foreign realized variance affects local realized variance:

Model 1

$$\begin{aligned}
\Delta RV_t^{AEX} &= \alpha_1 + \beta_1 \Delta RV_{t-1}^{CAC} + \beta_2 \Delta RV_{t-1}^{DAX} + \beta_3 \Delta RV_{t-1}^{FTSE} + \beta_4 \Delta RV_{t-1}^{NIKKEI} + \beta_5 \Delta RV_{t-1}^{SMI} + \beta_6 \Delta RV_{t-1}^{SPX} \\
\Delta RV_t^{CAC} &= \alpha_2 + \beta_7 \Delta RV_{t-1}^{AEX} + \beta_2 \Delta RV_{t-1}^{DAX} + \beta_3 \Delta RV_{t-1}^{FTSE} + \beta_4 \Delta RV_{t-1}^{NIKKEI} + \beta_5 \Delta RV_{t-1}^{SMI} + \beta_6 \Delta RV_{t-1}^{SPX} \\
\Delta RV_t^{DAX} &= \alpha_3 + \beta_7 \Delta RV_{t-1}^{AEX} + \beta_1 \Delta RV_{t-1}^{CAC} + \beta_3 \Delta RV_{t-1}^{FTSE} + \beta_4 \Delta RV_{t-1}^{NIKKEI} + \beta_5 \Delta RV_{t-1}^{SMI} + \beta_6 \Delta RV_{t-1}^{SPX} \\
\Delta RV_t^{FTSE} &= \alpha_4 + \beta_7 \Delta RV_{t-1}^{AEX} + \beta_1 \Delta RV_{t-1}^{CAC} + \beta_2 \Delta RV_{t-1}^{DAX} + \beta_4 \Delta RV_{t-1}^{NIKKEI} + \beta_5 \Delta RV_{t-1}^{SMI} + \beta_6 \Delta RV_{t-1}^{SPX} \qquad (5.9) \\
\Delta RV_t^{NIKKEI} &= \alpha_5 + \beta_7 \Delta RV_{t-1}^{AEX} + \beta_1 \Delta RV_{t-1}^{CAC} + \beta_2 \Delta RV_{t-1}^{DAX} + \beta_3 \Delta RV_{t-1}^{FTSE} + \beta_5 \Delta RV_{t-1}^{SMI} + \beta_6 \Delta RV_{t-1}^{SPX} \\
\Delta RV_t^{SMI} &= \alpha_6 + \beta_7 \Delta RV_{t-1}^{AEX} + \beta_1 \Delta RV_{t-1}^{CAC} + \beta_2 \Delta RV_{t-1}^{DAX} + \beta_3 \Delta RV_{t-1}^{FTSE} + \beta_4 \Delta RV_{t-1}^{NIKKEI} + \beta_6 \Delta RV_{t-1}^{SPX} \\
\Delta RV_t^{SPX} &= \alpha_7 + \beta_7 \Delta RV_{t-1}^{AEX} + \beta_1 \Delta RV_{t-1}^{CAC} + \beta_2 \Delta RV_{t-1}^{DAX} + \beta_3 \Delta RV_{t-1}^{FTSE} + \beta_4 \Delta RV_{t-1}^{NIKKEI} + \beta_5 \Delta RV_{t-1}^{SMI}
\end{aligned}$$

where ΔRV_t^i represents the first difference of realized variance of index i return at time t, α denotes the constants, and β denotes the coefficients of the lagged foreign market realized variance. It is worth noting that we constrain the coefficient of one independent variable to be equal across all seven equations. This allows us to understand the joint impact of any specific market variance on the variance of other markets. The setting also facilitates easier comparison between models using "ambiguity-free" and ambiguity-adjusted variance. For comparison with (9), we also specify the following ambiguity-adjusted realized variance system:

Model 2

$$\begin{aligned}
\Delta RV_t^{AEX} &= \alpha_1 + \beta_1 \Delta SRV_{t-1}^{CAC} + \beta_2 \Delta SRV_{t-1}^{DAX} + \beta_3 \Delta SRV_{t-1}^{FTSE} + \beta_4 \Delta SRV_{t-1}^{NIKKEI} + \beta_5 \Delta SRV_{t-1}^{SMI} + \beta_6 \Delta SRV_{t-1}^{SPX} \\
\Delta RV_t^{CAC} &= \alpha_2 + \beta_7 \Delta SRV_{t-1}^{AEX} + \beta_2 \Delta SRV_{t-1}^{DAX} + \beta_3 \Delta SRV_{t-1}^{FTSE} + \beta_4 \Delta SRV_{t-1}^{NIKKEI} + \beta_5 \Delta SRV_{t-1}^{SMI} + \beta_6 \Delta SRV_{t-1}^{SPX} \\
\Delta RV_t^{DAX} &= \alpha_3 + \beta_7 \Delta SRV_{t-1}^{AEX} + \beta_1 \Delta SRV_{t-1}^{CAC} + \beta_3 \Delta SRV_{t-1}^{FTSE} + \beta_4 \Delta SRV_{t-1}^{NIKKEI} + \beta_5 \Delta SRV_{t-1}^{SMI} + \beta_6 \Delta SRV_{t-1}^{SPX} \\
\Delta RV_t^{FTSE} &= \alpha_4 + \beta_7 \Delta SRV_{t-1}^{AEX} + \beta_1 \Delta SRV_{t-1}^{CAC} + \beta_2 \Delta SRV_{t-1}^{DAX} + \beta_4 \Delta SRV_{t-1}^{NIKKEI} + \beta_5 \Delta SRV_{t-1}^{SMI} + \beta_6 \Delta SRV_{t-1}^{SPX} \quad (5.10) \\
\Delta RV_t^{NIKKEI} &= \alpha_5 + \beta_7 \Delta SRV_{t-1}^{AEX} + \beta_1 \Delta SRV_{t-1}^{CAC} + \beta_2 \Delta SRV_{t-1}^{DAX} + \beta_3 \Delta SRV_{t-1}^{FTSE} + \beta_5 \Delta SRV_{t-1}^{SMI} + \beta_6 \Delta SRV_{t-1}^{SPX} \\
\Delta RV_t^{SMI} &= \alpha_6 + \beta_7 \Delta SRV_{t-1}^{AEX} + \beta_1 \Delta SRV_{t-1}^{CAC} + \beta_2 \Delta SRV_{t-1}^{DAX} + \beta_3 \Delta SRV_{t-1}^{FTSE} + \beta_4 \Delta SRV_{t-1}^{NIKKEI} + \beta_6 \Delta SRV_{t-1}^{SPX} \\
\Delta RV_t^{SPX} &= \alpha_7 + \beta_7 \Delta SRV_{t-1}^{AEX} + \beta_1 \Delta SRV_{t-1}^{CAC} + \beta_2 \Delta SRV_{t-1}^{DAX} + \beta_3 \Delta SRV_{t-1}^{FTSE} + \beta_4 \Delta SRV_{t-1}^{NIKKEI} + \beta_5 \Delta SRV_{t-1}^{SMI}
\end{aligned}$$

where ΔSRV_t^i is the first difference of ambiguity-adjusted realized variance for index i at time t.

We also investigate similar relationships among global VRPs according to the following system:

Model 3

$$\begin{aligned}
VRP_t^{AEX} &= \alpha_1 + \beta_1 VRP_{t-1}^{CAC} + \beta_2 VRP_{t-1}^{DAX} + \beta_3 VRP_{t-1}^{FTSE} + \beta_4 VRP_{t-1}^{NIKKEI} + \beta_5 VRP_{t-1}^{SMI} + \beta_6 VRP_{t-1}^{SPX} \\
VRP_t^{CAC} &= \alpha_2 + \beta_7 VRP_{t-1}^{AEX} + \beta_2 VRP_{t-1}^{DAX} + \beta_3 VRP_{t-1}^{FTSE} + \beta_4 VRP_{t-1}^{NIKKEI} + \beta_5 VRP_{t-1}^{SMI} + \beta_6 VRP_{t-1}^{SPX} \\
VRP_t^{DAX} &= \alpha_3 + \beta_7 VRP_{t-1}^{AEX} + \beta_1 VRP_{t-1}^{CAC} + \beta_3 VRP_{t-1}^{FTSE} + \beta_4 VRP_{t-1}^{NIKKEI} + \beta_5 VRP_{t-1}^{SMI} + \beta_6 VRP_{t-1}^{SPX} \\
VRP_t^{FTSE} &= \alpha_4 + \beta_7 VRP_{t-1}^{AEX} + \beta_1 VRP_{t-1}^{CAC} + \beta_2 VRP_{t-1}^{DAX} + \beta_4 VRP_{t-1}^{NIKKEI} + \beta_5 VRP_{t-1}^{SMI} + \beta_6 VRP_{t-1}^{SPX} \qquad (5.11) \\
VRP_t^{NIKKEI} &= \alpha_5 + \beta_7 VRP_{t-1}^{AEX} + \beta_1 VRP_{t-1}^{CAC} + \beta_2 VRP_{t-1}^{DAX} + \beta_3 VRP_{t-1}^{FTSE} + \beta_5 VRP_{t-1}^{SMI} + \beta_6 VRP_{t-1}^{SPX} \\
VRP_t^{SMI} &= \alpha_6 + \beta_7 VRP_{t-1}^{AEX} + \beta_1 VRP_{t-1}^{CAC} + \beta_2 VRP_{t-1}^{DAX} + \beta_3 VRP_{t-1}^{FTSE} + \beta_4 VRP_{t-1}^{NIKKEI} + \beta_6 VRP_{t-1}^{SPX} \\
VRP_t^{SPX} &= \alpha_7 + \beta_7 VRP_{t-1}^{AEX} + \beta_1 VRP_{t-1}^{CAC} + \beta_2 VRP_{t-1}^{DAX} + \beta_3 VRP_{t-1}^{FTSE} + \beta_4 VRP_{t-1}^{NIKKEI} + \beta_5 VRP_{t-1}^{SMI}
\end{aligned}$$

where VRP_t^i is the VRP of index i at time t. The ambiguity-equivalent system is specified as follows:

Model 4

$$\begin{aligned}
VRP_t^{AEX} &= \alpha_1 + \beta_1 SVRP_{t-1}^{CAC} + \beta_2 SVRP_{t-1}^{DAX} + \beta_3 SVRP_{t-1}^{FTSE} + \beta_4 SVRP_{t-1}^{NIKKEI} + \beta_5 SVRP_{t-1}^{SMI} + \beta_6 SVRP_{t-1}^{SPX} \\
VRP_t^{CAC} &= \alpha_2 + \beta_7 SVRP_{t-1}^{AEX} + \beta_2 SVRP_{t-1}^{DAX} + \beta_3 SVRP_{t-1}^{FTSE} + \beta_4 SVRP_{t-1}^{NIKKEI} + \beta_5 SVRP_{t-1}^{SMI} + \beta_6 SVRP_{t-1}^{SPX} \\
VRP_t^{DAX} &= \alpha_3 + \beta_7 SVRP_{t-1}^{AEX} + \beta_1 SVRP_{t-1}^{CAC} + \beta_3 SVRP_{t-1}^{FTSE} + \beta_4 SVRP_{t-1}^{NIKKEI} + \beta_5 SVRP_{t-1}^{SMI} + \beta_6 SVRP_{t-1}^{SPX} \\
VRP_t^{FTSE} &= \alpha_4 + \beta_7 SVRP_{t-1}^{AEX} + \beta_1 SVRP_{t-1}^{CAC} + \beta_2 SVRP_{t-1}^{DAX} + \beta_4 SVRP_{t-1}^{NIKKEI} + \beta_5 SVRP_{t-1}^{SMI} + \beta_6 SVRP_{t-1}^{SPX} \qquad (5.12) \\
VRP_t^{NIKKEI} &= \alpha_5 + \beta_7 SVRP_{t-1}^{AEX} + \beta_1 SVRP_{t-1}^{CAC} + \beta_2 SVRP_{t-1}^{DAX} + \beta_3 SVRP_{t-1}^{FTSE} + \beta_5 SVRP_{t-1}^{SMI} + \beta_6 SVRP_{t-1}^{SPX} \\
VRP_t^{SMI} &= \alpha_6 + \beta_7 SVRP_{t-1}^{AEX} + \beta_1 SVRP_{t-1}^{CAC} + \beta_2 SVRP_{t-1}^{DAX} + \beta_3 SVRP_{t-1}^{FTSE} + \beta_4 SVRP_{t-1}^{NIKKEI} + \beta_6 SVRP_{t-1}^{SPX} \\
VRP_t^{SPX} &= \alpha_7 + \beta_7 SVRP_{t-1}^{AEX} + \beta_1 SVRP_{t-1}^{CAC} + \beta_2 SVRP_{t-1}^{DAX} + \beta_3 SVRP_{t-1}^{FTSE} + \beta_4 SVRP_{t-1}^{NIKKEI} + \beta_5 SVRP_{t-1}^{SMI}
\end{aligned}$$

where $SVRP_t^i$ is the SVRP of index i at time t.

We estimate the regression systems using joint OLS and report the results in Table 5.6. We only report coefficients but not the constants for ease of presentation. From Table 5.6, the realized variances of DAX, NIKKEI, and SPX are significant as sources of international market volatility. Interestingly the strongest spillover signal comes from NIKKEI with t-statistics of -4.55. Turning to Model 2 which uses the SRVs of foreign markets as determinants of volatility spillovers, some clear improvements are observed. The t-statistics of AEX, CAC, FTSE, NIKKEI, SMI, and SPX SRVs all improved documenting six statistically significant sources of spillovers instead of three in the case without ambiguity (Model 1). With improvements in six out of seven countries considered, the average adjusted R^2 from the seven individual regression equations increases from 3.81% in Model 1 to 4.05% in Model 2. This confirms that correcting for model uncertainty and option-based ambiguity attitudes in volatility estimations can better explain volatility spillovers internationally.

Table 5.6. Additional Results – Predicting International Volatility

This table reports the system regressions results according to eq. (5.9), (5.10), (5.11), and (5.12). VRP and SVRP are variance premium and ambiguity-adjusted variance premium respectively. RV and SRV are realized variance and ambiguity-adjusted realized variance respectively. Sample covers 1990M01 to 2012M12. t-statistics are reported in parentheses. *,**, and *** denote significance at 90%, 95%, and 99% confidence levels respectively.

			Model 1		Model 2		Model 3		Model 4	
Index	**Country**	**Coefficient**	**RV**		**SRV**		**VRP**		**SVRP**	
AEX	*Netherlands*	β_7	0.023		-0.189	*	0.213	***	0.272	***
			(0.338)		(-1.704)		(4.119)		(4.465)	
CAC	*France*	β_1	-0.034		0.166	*	-0.283	***	-0.289	***
			(-0.565)		(1.788)		(-5.128)		(-4.548)	
DAX	*Germany*	β_2	-0.123	***	-0.055		-0.054		0.020	
			(-2.619)		(-0.795)		(-1.100)		(0.326)	
FTSE	*United Kingdom*	β_3	0.047		0.291	***	0.498	***	0.421	***
			(0.539)		(2.812)		(6.792)		(5.262)	
NIKKEI	*Japan*	β_4	-0.305	***	-0.753	***	-0.040		0.059	*
			(-4.548)		(-7.601)		(-1.551)		(1.692)	
SMI	*Switzerland*	β_5	-0.102		-0.293	**	0.270	***	0.295	***
			(-1.133)		(-2.112)		(3.229)		(2.771)	
SPX	*United States*	β_6	0.156	***	0.313	***	-0.101	**	-0.294	***
			(3.154)		(4.055)		(-2.039)		(-4.384)	
Average Adj. R^2 (%)			3.81		4.05		17.46		24.01	

Turning to Models 3 and 4, we investigate variance risk premia spillovers among countries and verify whether investors can be better informed about international variance risk dynamics through the VRP lens. In Model 3, five out of seven countries' variance premia contribute significantly to local country variance premia without accounting for SAF. Turning to Model 4 which incorporates SAF adjustments into VRP, six out of seven countries are once again significant in explaining domestic variance premia in the system. Improvements in t-statistics can be particularly observed for AEX, NIKKEI, and SPX. The overall average adjusted R^2 is also improved from 17.46% in Model 3 to 24.01% in Model 4. These findings suggest that investors and policy makers can be better informed about international market volatility dynamics if ambiguity about volatility is explicitly accounted for. This also confirms the improved informational content of our ambiguity-adjusted variance and variance premium measures, and suggests that uncertainty beyond risk can also contribute to spillovers.

5.5. Conclusion

We examine the information content of the variance premium, implied variance and realized variance from the perspective of ambiguity theory. We extract option market ambiguity attitudes from traded option prices on the S&P 500 index and compute a subjective adjustment factor following Choquet and rank-dependent utility. The predictive power and informational content of the resulting ambiguity-adjusted variance premium and its two variance components are then tested against their standard (risk-based) counterparts in predicting stock market excess returns, economic activity, financial instability, and international volatility.

Our results reveal clear improvements in the predictive power of the variance premium for stock market excess returns. The improved predictive power provided by the subjective and behavioral adjustment factor could still be observed when other known predictor variables of market excess returns are controlled for. Our results suggest that the additional information content of the ambiguity-adjusted variance premium is unique and not due to information overlaps with other known predictors. Our findings also suggest that the standard risk-based VRP may not capture ambiguity aversion information effectively. This also validates our hypothesis that accounting for investors' ambiguity attitudes improves the market returns predicting ability of the variance premium. We also document a robust improvement to implied and realized variance in predicting economic activity as proxied by industrial production growth, and to realized variance in predicting financial instability in the United States. Our results hold for alternative ambiguity proxies that are not inferred from option prices namely PUI, BULL, BEAR, UMCSI and CCI. We further investigate the volatility spillover effect among seven major equity markets and find that investors can be better informed about this effect if ambiguity aversion information is embedded in international volatility spillover detection. Our findings contribute to the rapidly growing literature on model uncertainty and the (mis-)estimation of the variance premium.

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Appendix to Chapter 5

Table A5.1. Descriptions of Variables, Data Series, and Data Sources

This table describes the variables, data series and data sources. Data series from websites are downloaded in April 2016. All data series span a common sample period from Jan 1990 to Dec 2012.

Panel A. Predictor Variables

Category	Abbreviation	Corresponding Indicator	Description	Source
Uncertainty Measures	OMAA	Option Market Ambiguity Attitudes	Estimated by rank dependent option pricing model according to eq. (6). End of month values.	-
	SAF	Subjective Adjustment Factor	Computed as 4*OMAA*(1-OMAA)according to eq. (7). End of month values.	-
Variance Premium	VRP	Variance Risk Premium	Variance risk premium defined as the difference between realized variance and implied variance of S&P 500 return, end of month values.	Hao Zhou's website
	SVRP	Ambiguity-adjusted Variance Risk Premium	Computed as SAF*VRP, end of month values.	-
Option Implied Variance	IV	S&P500 Implied Variance	S&P 500 option implied variance computed as VIX^2, end of month values.	CBOE website
	SIV	Ambiguity-adjusted Implied Variance	Computed as SAF*IV, end of month values.	-
Realized Variance	RV	S&P 500 Realized Variance	Sum of squared 5-min intraday returns of S&P 500 index, end of month values	Hao Zhou's website
	SRV	Ambiguity-adjusted Realized Variance	Computed as SAF*RV, end of month values.	-
Equity fundamentals	DY	Dividend Yield	Aggregate dividend yield of S&P 500 composite.	Robert Shiller's website
	PER	Price-to-earnings Ratio	Reciprocal of aggregate price to earnings ratio of S&P 500 composite.	Robert Shiller's website
Bond fundamentals	CS	Credit Spread (BAA yield - AAA yield)	Difference between Moody's BAA and AAA corporate bond yield.	Federal Reserve Bank of St. Louis FRED
	TS	Yield Curve (Term Spread, 10Y T-yield - 3M T-yield)	Difference between 10-year and 3-month U.S. treasury bond yield.	Federal Reserve Bank of St. Louis FRED
Other Predictors	CAY	Consumption Wealth Ratio	Consumption Wealth Ratio by Lettau and Ludvigson 2004	Martin Lettau's website
	RREL	Stochastically Detrended Short Rate	Computed as the difference between one-month treasury bill rate and its 12-month moving average	-

Panel B. Prediction Targets

Category	Abbreviation	Corresponding Indicator	Description	Source
Equity Market Returns	ER	S&P500 Excess Return	S&P 500 index return in excess of 3-month treasury bond yield.	Thomson Datastream
Economic Activity	IPG	Industrial Production Growth	Logorithmic change of k-month horizon industrial production index, measured as the real seasonally adjusted output for all facilities located in the United States manufacturing, mining, and electric, and gas utilities.	Board of Governors of the Federal Reserve System
Finanical Instability	KCFSI	Kansas City Financial Stress Index	Principal component of 11 variables.	Federal Reserve Bank of Kansas City

Table A5.2. Results with Alternative Ambiguity Proxy

This table reports the prediction results of adjusted VRP, IV, and RV using an alternative proxy for ambiguity. The alternative proxy used is Investors Intelligence Bearish Index (BEAR). Ambiguity attitudes (AA_{BEAR}) are proxied by the scaled Bearish Index computed as:

$$AA_{BEAR,t} = \left(BEAR_t - MIN(BEAR\ over\ [0,t])\right) / \left(MAX(BEAR\ over\ [0,t]) - MIN(BEAR\ over\ [0,t])\right)$$

$$SAF_{BEAR,t} = 4 \cdot \left(1 - AA_{BEAR,t}\right) \cdot AA_{BEAR,t}$$

VRP is the variance premium. IV is implied variance. RV is realized variance. DY is the dividend yield of S&P 500 index portfolio. PER is the price-to-earnings ratio of S&P 500 index portfolio. CS is the credit spread computed as the difference between Moody's BAA and AAA bond yield indices. TS is the term spread computed as the difference between 10-year and 3-month treasury yields. CAY is consumption wealth ratio. RREL is stochastically detrended short rate. Sample covers 1990M01 to 2012M12. Hodrick (1992) t-statistics are reported in squared brackets for regressions on market excess returns and industrial production growth. Newey and West (1987) t-statistics with lag equals to max[3,2*horizon] are reported in squared brackets for regressions on Kansas City Financial Stress Index. *,**, and *** denote significance at 90%, 95%, and 99% confidence levels respectively.

Horizon	**1**	**3**	**6**	**12**	**18**	**24**
Panel A. Predicting Excess Returns with adjusted variance premium and other predictors						
Constant	-19.68	-20.39	-19.21	-15.20	-12.15	-10.66
	[-1.12]	[-1.32]	[-1.44]	[-1.26]	[-1.02]	[-0.98]
CAY	158.94	211.52	312.98 *	438.48 ***	461.17 ***	480.68 ***
	[0.72]	[1.10]	[1.71]	[2.58]	[2.69]	[2.83]
CS	-7.12	-5.22	1.79	7.22	3.60	2.42
	[-0.48]	[-0.36]	[0.15]	[0.93]	[0.63]	[0.49]
DY	9.69	9.40	7.25	4.40	3.24	2.65
	[1.56]	[1.61]	[1.31]	[0.92]	[0.74]	[0.70]
PER	0.17	0.24	0.17	0.00	0.04	0.03
	[0.57]	[0.96]	[0.80]	[-0.02]	[0.28]	[0.28]
RREL	27.45 ***	28.32 ***	29.46 ***	28.40 ***	20.31 **	13.61 **
	[2.61]	[3.00]	[3.15]	[3.03]	[2.45]	[2.15]
TS	-0.27	-0.71	0.04	1.97	2.96	3.36
	[-0.09]	[-0.25]	[0.01]	[0.80]	[1.33]	[1.57]
SAF_{BEAR}*VRP	0.65 **	0.54 ***	0.33 ***	0.17 **	0.11 *	0.08 *
	[2.36]	[3.59]	[3.36]	[2.47]	[1.80]	[1.67]
Adj R^2 (%)	8.53	23.46	29.51	46.35	51.56	59.36

Horizon	**1**	**3**	**6**	**12**	**18**	**24**
Panel B. Predicting Industrial Production Growth with adjusted variance premium and realized variance						
Constant	4.51 ***	4.25 ***	3.37 ***	2.55 ***	2.22 ***	2.10 ***
	[4.87]	[6.05]	[5.53]	[4.77]	[4.21]	[3.89]
SAF_{BEAR}*VRP	-0.05	-0.03	0.00	0.01	0.01	0.01
	[-1.28]	[-1.05]	[-0.19]	[0.80]	[1.22]	[1.15]
SAF_{BEAR}*RV	-0.08 ***	-0.09 ***	-0.06 ***	-0.03 **	-0.02 *	-0.01
	[-2.65]	[-2.76]	[-3.00]	[-2.52]	[-1.95]	[-1.57]
Adj R^2 (%)	11.99	27.79	20.54	8.72	4.45	2.49

Horizon	**1**	**3**	**6**	**12**	**18**	**24**
Panel C. Predicting Industrial Production Growth with adjusted implied variance						
Constant	5.01 ***	5.05 ***	4.18 ***	3.13 ***	2.67 ***	2.44 ***
	[4.93]	[5.26]	[5.98]	[6.07]	[5.61]	[5.15]
SAF_{BEAR}*IV	-0.08 ***	-0.08 ***	-0.06 ***	-0.03 **	-0.02 *	-0.01
	[-2.71]	[-2.88]	[-2.99]	[-2.40]	[-1.76]	[-1.36]
Adj R^2 (%)	11.36	23.35	14.90	4.94	1.73	0.64

Horizon	**1**	**3**	**6**	**12**	**18**	**24**
Panel D. Predicting Financial Instability with adjusted realized variance						
Constant	-0.34 ***	-0.28 **	-0.16	-0.02	0.03	0.08
	[-3.75]	[-2.51]	[-0.98]	[-0.10]	[0.09]	[0.27]
SAF_{BEAR}*RV	0.02 ***	0.02 ***	0.01 ***	0.00 **	0.00	0.00
	[5.17]	[8.38]	[7.31]	[1.97]	[0.85]	[-0.18]
Adj R^2 (%)	54.53	39.61	18.05	2.65	0.21	-0.37

Chapter 6.
Conclusions

This research focuses on obtaining a forward looking measure of ambiguity inferred from traded option prices and examining the informational efficiency of such a measure in several important research areas including market returns prediction, economic activity prediction, volatility forecasting, and the estimation of variance premium and market variance. The study enriches our knowledge of the informational dynamics among option market ambiguity, the financial market, and the real economy, and calls for the consideration of Knightian uncertainty principles in finance and economics topics in which uncertainty beyond risk has largely been ignored. This research also shows that the concept of ambiguity aversion, which seems complex but yet represents a natural predisposition of human beings, is highly relevant in determining expected returns in the equity market, producing more accurate economic forecasts, improving the accuracy of GARCH-based volatility forecasts, estimating variance premium, and detecting international volatility spillovers.

Findings in Chapter 2 contribute to solving the puzzle of a negative risk-return tradeoff generally reported in the literature and show the richness of information embedded in our measure of option market ambiguity. Chapter 3 reveals the importance of ambiguity in determining the level of economic activity in a macro setting. Being the only variable to robustly predict all of the economic activity indicators considered, option market ambiguity serves as the most powerful piece of evidence supporting various theories about the negative relationship between uncertainty and economic activity. Chapter 4 contributes to the volatility forecasting literature by showing the importance of option market ambiguity and confirming the role of the hope and fear effect in volatility formation. Chapter 5 studies the (mis)estimation of variance premium and its market variance components from an ambiguity point of view and highlights an improved information content when ambiguity is explicitly accounted for. This contributes to our understanding of the role and nature of the variance premium as a popular, but imperfect, proxy for Knightian uncertainty, and further confirms the informational efficiency of ambiguity as inferred from traded option prices. In addition to providing a better understanding of the empirical relationships among risk, uncertainty, economic activity and financial market returns, the results derived from this study can also contribute to improving investment management, risk management, and asset pricing practices.

Technical Appendix - Option Pricing under Ambiguity (Based on Driouchi, Trigeorgis and So (2016) and Driouchi, Trigeorgis and Gao (2015))

Let B be the price of a riskless bond with instantaneous rate of return *r* such that:

$$\frac{dB}{B} = rdt \qquad (A1)$$

Let *O* be the price of a contingent-claim (e.g., a European call or put option on the S&P index) which depends only on *S* and time *t*, $\mathrm{O}(S, t)$. From Ito's lemma and Eq. (1), the dynamics of option price *O* can be written ($\forall\, m \in]-1{,}1[\ , \forall\, s \in]0{,}1]$) as:

$$dO(S,t) = \frac{\partial O}{\partial t}dt + \frac{\partial O}{\partial S}[(\mu + m\sigma)Sdt + (s\sigma)SdZ] + \frac{1}{2}\frac{d^2O}{dS^2}([(s\sigma)SdZ]^2 + [(\mu + m\sigma)Sdt]^2 + [(s\sigma)SdZ \times (\mu + m\sigma)Sdt]) \qquad (A2)$$

This simplifies to:

$$dO(S,t) = \left[\frac{\partial O}{\partial t} + \frac{\partial O}{\partial S}(\mu + m\sigma)S + S^2\frac{1}{2}\frac{d^2O}{dS^2}(s\sigma)^2\right]dt + \frac{\partial O}{\partial S}(s\sigma)SdZ \qquad (A3)$$

Using Eq. (1), the level(s) of marginal utility in the economy ξ under Choquet ambiguity is:

$$\frac{d\xi}{\xi} = [mg(\xi,S) + f(\xi,S)]dt + sg(\xi,S)dz \qquad (A4)$$

This results from standard economic dynamics $\frac{d\xi}{\xi} = f(\xi,S)dt + g(\xi,S)dW$ (see Harrison and Kreps, 1979) and the characteristics of *W* in the Choquet ambiguity universe. Functions g and *f* help derive the pricing kernel under uncertainty. Thus:

$$\begin{aligned} d(\xi B) &= \xi(rBdt) + B\big[\big(mg(\xi,S) + f(\xi,S)\big)\xi dt + sg(\xi,S)\xi dz\big] \\ &= \xi B\big[\big(r + mg(\xi,S) + f(\xi,S)\big)dt + sg(\xi,S)\xi dz\big] \end{aligned} \qquad (A5)$$

Applying martingale theory, the drift (dt) term is set to zero. This implies:

$$r + mg(\xi,S) + f(\xi,S) = 0 \ \text{ or } \ f(\xi,S) = -r - mg(\xi,S) \qquad (A6)$$

Following a similar procedure for *S*:

$$d(\xi S) = \xi dS + Sd\xi + d < \xi, S > \tag{A7}$$

$$d(\xi S) = \xi S[(\mu + m\sigma)dt + s\sigma dz] + S\xi\{[mg(\xi, S) - r - mg(\xi, S)]dt + sg(\xi, S)dz\}$$
$$+ s^2\sigma\xi S g(\xi, S)dt$$

$$= \xi S[(\mu + m\sigma) - r + s^2\sigma g(\xi, S)]dt + S\xi[s\sigma + sg(\xi, S)]dz \tag{A8}$$

Setting the drift term to zero, we obtain the ambiguity-adjusted Sharpe ratio $g(\xi, S)$:

$$(\mu + m\sigma) - r + s^2\sigma g(\xi, S) = 0 \text{ or } g(\xi, S) = \frac{[r - (\mu + m\sigma)]}{s^2\sigma} \tag{A9}$$

The market pricing kernel follows Harrison and Kreps (1979) dynamics but, due to market incompleteness, multiple marginal utility levels and Knightian uncertainty, *f* and *g* are not unique as they are affected by investors' ambiguity parameters *m* and *s*. This means that Choquet ambiguity impacts the fundamental component of the market pricing kernel (via parameters *m* and *s*) but not the purely sentimental element (see Cochrane, 2001; Shefrin, 2005). Relaxing this general (market incompleteness) assumption reduces to the perfect replication or risk-neutral case of Black-Scholes (1973) OPM. Using the results from Eqs. (A6) and (A9):

$$\frac{d\xi}{\xi} = f(\xi, S)dt + g(\xi, S)dz$$

$$= -r - m\left\{\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right\}dt + \left(\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right)dz \tag{A10}$$

Consider the value of a call or put option *O* written on underlying stock index *S* (with dividend yield δ).

$$d(\xi O) = \xi dO + Od\xi + d < \xi, O >$$

$$= \xi\left\{\left[\frac{\partial O}{\partial t} + \frac{\partial O}{\partial S}(\mu - \delta + m\sigma)S + S^2\frac{1}{2}\frac{d^2O}{dS^2}(s\sigma)^2\right]dt + \frac{\partial O}{\partial S}(s\sigma)Sdz\right\}$$

$$+ \xi O\left[-r - m\left\{\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right\}dt + \left(\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right)dz\right]$$

$$+ \xi\left[\left(\frac{[r - (\mu + m\sigma)]}{s^2\sigma}\right)\frac{\partial O}{\partial S}(s\sigma)Sdt\right] \tag{A11}$$

Setting the drift (dt) term of the option to zero results in the fundamental equation for pricing derivatives or contingent-claims $O_{c/p}$:

$$\xi\left[\frac{\partial O}{\partial t}+\frac{\partial O}{\partial S}(\mu-\delta+m\sigma)S+S^2\frac{1}{2}\frac{d^2O}{dS^2}(s\sigma)^2\right]dt+\xi O\left[-r-m\left\{\frac{[r-(\mu+m\sigma)]}{s^2\sigma}\right\}dt\right]$$
$$+\xi\left[\left(\frac{[r-(\mu+m\sigma)]}{s^2\sigma}\right)\frac{\partial O}{\partial S}(s\sigma)Sdt\right]=0 \qquad (A12)$$

Solving Eq. (A12) for European options written on S leads to:

$$P_t^C=S_te^{-\delta' T}N\left(\frac{\ln\left(\frac{S_0}{K}\right)+\left(r'-\delta'+0.5(s\sigma)^2\right)T}{s\sigma\sqrt{T}}\right)$$
$$-Ke^{-r'T}N\left(\frac{\ln\left(\frac{S_0}{K}\right)+\left(r'-\delta'-0.5(s\sigma)^2\right)T}{s\sigma\sqrt{T}}\right) \qquad (A13)$$

where:

$$r'=r+m\frac{[r-(\mu+m\sigma)]}{s^2\sigma},\delta'=\delta-\frac{(m+s^2\sigma-s\sigma)[(\mu+m\sigma)-r]}{s^2\sigma},$$
$$m=2c-1\ and\ s=\sqrt{4c(1-c)}\quad (\forall\ c\ \in]0,1[)$$

References for the Technical Appendix:

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