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Event-Triggered Positive Filter Design for Positive Polynomial Fuzzy Systems via Premise Integration and Membership-Function-Dependent Methods

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***Abstract*—This paper focuses on the design of the positive fuzzy filter with event-triggered mechanism for positive polynomial fuzzy-model-based (PPFMB) system. The main objective is to design a positive and stable fuzzy filter which makes the L_1 performance index of the estimated output signal as good as possible with limited networked bandwidth resources. Since the positive constraints exist in the positive systems, while the disturbance signal of the output signal and the transmission delay in the event-triggered mechanism are taken into account in this research, the augmented system is rebuilt by introducing the weight coefficient of disturbance signal, and a novel linear copositive Lyapunov function (LCLF) is proposed. In order to adapt to the analysis framework based on linear Lyapunov function, a novel linear event-triggered condition is proposed. Furthermore, to handle the mismatched premise variables caused by the event-triggered mechanism, the premise variable associated with event-triggered instants are integrated into the interval of continuous premise variable with the help of the novel event-triggered condition, which means that the original multi-dimensional type-1 membership functions (MFs) are transformed into single-dimensional interval type-2 MFs. Then, an interval type-2 membership-function-dependent (IT2MFD) method is employed to introduce MFs information into the resultant conditions, so that more optimized L_1 performance index is obtained. Finally, an example with simulation results is given to verify the effectiveness of all the fuzzy filter design strategies proposed in this paper for optimizing performance index.**

***Index Terms*—positive polynomial fuzzy-model-based (PPFMB) system, novel linear copositive Lyapunov function (LCLF), positive fuzzy filter, event-triggered, interval type-2 membership function dependent (IT2MFD) method**

I. Introduction

Among the practical dynamic systems, there is a class of systems whose system variables and outputs have non-negative characteristics, such as drug metabolism systems [1], population systems [2], network systems [3] and DC-DC power converters [4]. In view of the non-negative characteristic of dynamic variables of the above systems, positive systems [5] whose states are constrained in positive orthant instead of the whole space exhibit advantages over general systems in modeling such systems. However, the non-negative restriction of the positive systems makes the research encounter challenge. For example, in the control field, the positive system states needs to be not only stable but also positive [6], especially for the design of positive observer [7]–[9] and positive filter [10], [11], both the systems states and estimated states or estimated signal need to be guaranteed to be stable and positive.

Referring to the research of the nonlinear positive systems, in addition to the challenge mentioned above, another thorny problem is how to deal with the nonlinear characteristics. One feasible method is to represent the nonlinear positive systems with T-S fuzzy model [12], which can transform a nonlinear system into a series of linear systems weighted by membership functions (MFs), then the existing research results on linear positive systems can be used as references. In the past decade, nonlinear positive systems are further represented by the polynomial fuzzy model [13] that not only inherits the merit of the T-S fuzzy model mentioned above but also enhances the modeling capability. For the positive polynomial-fuzzy-model-based (PPFMB) systems, various synthesis problems have been researched, for example, state feedback control [14], static output feedback control [15], observer-based control [16], static output feedback tracking control [17], impulsive control [18], and so on. Up to now, there still exist many open problems to the synthesis of PPFMB systems.

Filters are the effective tool for estimating signal that are disturbed by noises. In particular, ℓ_1/L_1 filtering [19] and H_∞ filtering [20] have good signal processing performance in cases where the statistics of the external noise are not exactly known. Considering the positive properties of the positive systems, L_1-norm can describe the size of the states or output signals in more detail than H_∞-norm. In recent years, some enlightening research results on positive ℓ_1/L_1 filtering have emerged. In reference [21], a linear programming approach was proposed to design linear ℓ_1 filter, this approach was then extended to the design of positive T-S fuzzy filter [22]. In order to perform the stability analysis and positivity analysis for filter error systems under the linear programming approach frame, an augmentation approach with auxiliary variable was introduced in [23]. However, to the best of our knowledge, positive filter design for PPFMB systems has not been addressed, and the design approach need to be further studied.

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In the past decades, networked control systems (NCSs) have been widely used in various fields due to its merits, such as flexible connections between components, low power consumption and low maintenance costs [24]. However, bandwidth limitation is an inevitable problem, a variety of network-induced problems will appear in the case of a large amount of data to be transmitted, such as packet dropouts [25], poor real-time performance and measurements degradation. In response to above problems, some effective coping strategies have been reported in some literatures. For example, literature [26] treated measurements degradation as a probability problem, and proposed a promising chance-constrained method to optimize the control performance. In addition, setting up a time-triggered [27], two-bit-triggered [28] or event-triggered mechanism [29]–[31] between the sensors and the controllers or filters also is a very effective strategy to save bandwidth.

For the event-triggered mechanism, the form and parameters of the event-triggered condition directly affect the bandwidth saving effectiveness and the difficulty of comprehensive analysis. Since the analytical framework of the positive systems is different from that of general systems, the traditional quadratic-form event-triggered conditions cannot be used for positive systems under the linear-copositive-Lyapunov-function-based analytical framework [32]–[34]. Literatures [35] and [36] proposed linear-form event-triggered conditions to adapt to the analysis of positive systems, especially the latter one has an advantage in breaking the monotonic limit of the output signal, but this method results in conservatism and analysis complexity. Although literature [34], [37] proposed a 1-norm event-triggered conditions which have no restrictions on output nor does it need to introduce predefined variable, but communication delay was not considered in this literature, which prompts us to study the event-triggered mechanism under the consideration of transmission delay.

When the event-triggered mechanism is applied to T-S or polynomial fuzzy systems, the premise variable of filter is the variable that is transmitted when the event is triggered, which is different from the continuous premise variable of the plant. This premise variable mismatch problem prevents the use of PDC method to reduce the conservatism of the analysis results. In order to deal with mismatched premise variables and thus reduce conservatism, some pioneering methods have been reported in the literatures, such as partition method [38], [39], non-PDC approach [40], [41] and new asynchronous premise reconstruction approach [42]. However, partition method only works for discrete time systems; non-PDC approach either require limited conditions [41] or treat mismatched premise variables as two completely unrelated variables [40], which all introduce conservatism; new asynchronous premise reconstruction approach introduces the upper and lower bounds information of the proportion of asynchronous MFs to reduce conservatism, but the resultant conditions still do not depend on the MFs information, which means that the resultant conditions are still conservative. Therefore, it is an open problem to deal with the mismatch of premise variable brought by event-triggered mechanisms, which motivates our current research.

In this paper, positive filter design for PPFMB systems under the event-triggered mechanism is investigated, several challenges appear in the current research: (i) How to construct the augmented system when the communication delay and the output signal disturbed by noise are considered, so that the positivity of the plant and the filter can be guaranteed? (ii) How to design the event-triggered condition and Lyapunov function, so that the stability and L_1 performance can be guaranteed? (iii) How to handle the mismatched premise variables caused by event-triggered mechanism, so that the conservatism is reduced. The solutions to the above challenges are presented as follows, they are the contributions of this paper:

1) This is the first attempt to design event-triggered positive fuzzy filter for the PPFMB systems, taking into account disturbance of the output signal and the transmission delay in the event-triggered mechanism. To facilitate the positive analysis, a novel augmented system is constructed by introducing a weight coefficient which allows the flexible relationship between the disturbance signals at different time.
2) Since the system own positive characteristics, and disturbance of the output signal and transmission delay are considered at the same time, the quadratic Lyapunov functions and the existing linear copositive Lyapunov functions (LCLFs) are not suitable for the present research, so a novel LCLF is proposed. In addition, a novel linear event-triggered condition is proposed, which facilitates to perform the positive analysis and stability analysis by using the lower and upper bounds of the augmented system, respectively.
3) To handle the mismatched premise variables caused by event-triggered mechanism, the relationship between different premise variables is deeply explored, and mismatched premise variables are integrated into the same variable in interval form with the help of the event-triggered condition, then the interval type-2 membership function dependent (IT2MFD) analysis method [43] is used to introduce the premise variables and MFs information into stability conditions and positivity conditions, so that the L_1 performance of the filter can be improved.

The organization of this paper is as follows. In Section II, the notations, the PPFMB plant model and filter with novel linear event-triggered condition are described. In Section III, an augmented system with their upper and lower bounds are constructed, and the stability analysis and positivity analysis are performed by using the proposed novel LCLF. In Section IV, the mismatched premise variables are integrated, and IT2MFD method is utilized to relax the resultant conditions. In Section V, an example is used to illustrate the effectiveness of proposed positive fuzzy filter design strategies for PPFMB system. In Section VI, a conclusion is drawn.

II. Preliminary

A. Notation

The following notations are used within this paper. $\underline{p}$ represents $\{1, 2, \ldots, p\}$. A polynomial $f(\mathbf{x}(t))$ is an SOS if there exist polynomials $f_1(\mathbf{x}(t)), f_2(\mathbf{x}(t)), \ldots, f_m(\mathbf{x}(t))$ such that $f(\mathbf{x}(t)) = \sum_{i=1}^{m} f_i^2(\mathbf{x}(t))$, where $f_i(\mathbf{x}(t))$ is a polynomial and

m is a positive integer. It is clear that $f(\mathbf{x}(t))$ being an SOS naturally implies $f(\mathbf{x}(t)) \geq 0$ for all $\mathbf{x}(t)$. The expressions of $\mathbf{A} \prec 0$ and $\mathbf{A} \succ 0$, mean that all elements of $\mathbf{A}$ are negative and positive, respectively; $\mathbf{A} < 0$ and $\mathbf{A} > 0$ mean that $\mathbf{A}$ is negative definite and positive definite, respectively. $\mathbf{A}^{(\alpha,\beta)}$ is the α^{th} row, β^{th} column element of $\mathbf{A}$. $\mathbf{A}^{(\alpha,:)}$ or $\mathbf{A}^{(:,\beta)}$ is a vector denoting the α^{th} row of $\mathbf{A}$, or β^{th} column of $\mathbf{A}$. Matrix $\mathbf{Q}$ is called Metzler matrix [5], if its off diagonal elements are all nonnegative. For a vector $\mathbf{x}(t)$, $\|\mathbf{x}(t)\|_{L_1} = \sum_{i=1}^{n} |x_i(t)|$, with $x_i(t)$ being the i^{th} element of $\mathbf{x}(t)$.

B. Positive Polynomial Fuzzy Plant Model

The following polynomial fuzzy model with p fuzzy rules is adopted to represented the nonlinear system, the i-th rule is of the following format:

Rule i : IF $f_1(\mathbf{y}(t))$ is $\mathbf{M}_1^i$ AND $\cdots$ AND $f_\phi(\mathbf{y}(t))$ is $\mathbf{M}_\phi^i$

$$\text{THEN} \quad \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{w}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{w}(t) \\ \mathbf{z}(t) = \mathbf{E}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{F}_i(\mathbf{x}(t))\mathbf{w}(t) \end{cases}$$

where $\mathbf{x}(t) \in \Re^n$, $\mathbf{w}(t) \in \Re^g$, $\mathbf{y}(t) \in \Re^q$ and $\mathbf{z}(t) \in \Re^m$ are the state vector, disturbance signal, measurement output and the signal to be estimated, respectively; n, g, q and m are their dimensions. $f_\phi(\mathbf{y}(t))$ is the premise variable and $\mathbf{M}_\phi^i$ is a fuzzy set corresponding to premise variable $f_\phi(\mathbf{y}(t))$ in rule i, $i \in \underline{p}$, $\vartheta \in \psi$, and ψ is a positive integer; $\mathbf{A}_i(\mathbf{x}(t)) \in \Re^{n\times n}$, $\mathbf{B}_i(\mathbf{x}(t)) \in \Re^{n\times g}$, $\mathbf{C} \in \Re^{q\times n}$, $\mathbf{D} \in \Re^{q\times g}$, $\mathbf{E}_i(\mathbf{x}(t)) \in \Re^{m\times n}$ and $\mathbf{F}_i(\mathbf{x}(t)) \in \Re^{m\times g}$ are the known system matrices.

Based on the above fuzzy rules, the dynamics of the nonlinear system is defined as follows:

$$\begin{cases} \dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{y}(t))[\mathbf{A}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{w}(t)] \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{w}(t) \\ \mathbf{z}(t) = \sum_{i=1}^{p} w_i(\mathbf{y}(t))[\mathbf{E}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{F}_i(\mathbf{x}(t))\mathbf{w}(t)] \end{cases} \tag{1}$$

where $w_i(\mathbf{y}(t))$ is the normalized grade of membership, $w_i(\mathbf{y}(t)) \geq 0$, $\sum_{i=1}^{p} w_i(\mathbf{y}(t)) = 1$, and $w_i(\mathbf{y}(t)) = (\prod_{\eta=1}^{\phi} \mu_{M_\eta^i}(f_\eta(\mathbf{y}(t))))/(\sum_{k=1}^{p}\prod_{\eta=1}^{\phi} \mu_{M_\eta^k}(f_\eta(\mathbf{y}(t))))$; $\mu_{M_\eta^i}(f_\eta(\mathbf{y}(t)))$ is the grade of membership corresponding to the fuzzy term M_η^i.

Definition 1: [22] The polynomial fuzzy system (1) is said to be positive only if for every initial conditions $\mathbf{x}(0) \succ 0$ and $\mathbf{w}(0) \succ 0$, its state variables $\mathbf{x}(t)$, measurement output $\mathbf{y}(t)$ and estimated signal $\mathbf{z}(t)$ are all nonnegative for all $t \geq 0$.

Lemma 1: A polynomial fuzzy system (1) is guaranteed to be positive if $\sum_{i=1}^{p} w_i(\mathbf{y}(t))\mathbf{A}_i(\mathbf{x}(t))$ is Metzler matrix, and $\sum_{i=1}^{p} w_i(\mathbf{y}(t))\mathbf{B}_i(\mathbf{x}(t)) \succ 0$, $\mathbf{C} \succ 0$, $\mathbf{D} \succ 0$, $\sum_{i=1}^{p} w_i(\mathbf{y}(t))\mathbf{E}_i(\mathbf{x}(t)) \succ 0$, $\sum_{i=1}^{p} w_i(\mathbf{y}(t))\mathbf{F}_i(\mathbf{x}(t)) \succ 0$.

Proof 1: The proof of Lemma 1 is given in supplementary file due to page limit.

C. Event-Triggered Positive Polynomial Fuzzy Filter

To reduce the influence of disturbance signals on the system output signals, the positive polynomial fuzzy filter is design for the PPFMB system (1). Also, to avoid the waste of communication bandwidth and improve the communication efficiency, the event-triggered mechanism is introduced. The event trigger moments are denoted as t_0h, t_1h, t_kh, ... Considering that the transmission delay exist in the network communication, the communication delay of the k-th transfer is denoted as τ_k with satisfying $\tau_k \in [0, \bar{\tau})$, where $\bar{\tau} = \max\{\tau_k\}$, then the signals $\mathbf{y}(t_0h)$, $\mathbf{y}(t_1h)$, $\mathbf{y}(t_2h)$, ... will arrive at the filter side at the moments $t_0h + \tau_0$, $t_1h + \tau_1$, $t_2h + \tau_2$, ..., respectively.

To facilitate the analysis, the idea of event-triggered interval partitioning and transmission delay processing method [44] are applied in this paper, the time interval between two adjacent signals arriving at the filter is divided into some subintervals as following:

$$[t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) = \cup_{s_k=0}^{\varsigma_k} \Im_{s_k} \tag{2}$$

where

$$\begin{aligned} &\Im_0 = [t_kh + \tau_k, t_kh + h + \bar{\tau}); \\ &\Im_{s_k} = [t_kh + s_kh + \bar{\tau}, t_kh + (s_k+1)h + \bar{\tau}),\ s_k \in \underline{\varsigma_k - 1}; \\ &\Im_{\varsigma_k} = [t_kh + \varsigma_kh + \bar{\tau}, t_{k+1}h + \tau_{k+1}),\ \varsigma_k = t_{k+1} - t_k - 1. \end{aligned}$$

Define a time-varying function $\tau(t)$ to represent the time difference between continuous time and sampling instants,

$$\tau(t) = \begin{cases} t - t_kh,\ t \in \Im_0 \\ t - t_kh - s_kh,\ t \in \Im_{s_k}, s_k \in \underline{\varsigma_k - 1} \\ t - t_kh - \varsigma_kh,\ t \in \Im_{\varsigma_k} \end{cases} \tag{3}$$

where

$$0 \leq \tau(t) \leq \tau_M = \bar{\tau} + h \tag{4}$$

Then the following equation can be obtained:

$$\mathbf{y}(t_kh) = \mathbf{y}(t - \tau(t)) + \Delta\mathbf{y}(t_kh). \tag{5}$$

where

$$\Delta\mathbf{y}(t_kh) = \begin{cases} \mathbf{y}(t_kh) - \mathbf{y}(t_kh),\ t \in \Im_0 \\ \mathbf{y}(t_kh) - \mathbf{y}(t_kh + s_kh),\ t \in \Im_{s_k}, s_k \in \underline{\varsigma_k - 1} \\ \mathbf{y}(t_kh) - \mathbf{y}(t_kh + \varsigma_kh),\ t \in \Im_{\varsigma_k} \end{cases} \tag{6}$$

Considering the positive of the measurement outputs, the event-triggered condition is not necessary to be designed as quadratic-form, so we propose a linear-form event-triggered condition as follows:

$$-\theta_1\mathbf{y}(t_kh) \preceq \Delta\mathbf{y}(t_kh) \preceq \theta_2\mathbf{y}(t_kh), \tag{7}$$

where θ_1 and θ_2 are the predefined event-triggered coefficients. Based on the equation (5), the above event-triggered condition is equivalent to the following conditions:

$$-\frac{\theta_1}{1+\theta_1}\mathbf{y}(t - \tau(t)) \preceq \Delta\mathbf{y}(t_kh) \preceq \frac{\theta_2}{1-\theta_2}\mathbf{y}(t - \tau(t)). \tag{8}$$

Combining the above event-triggered mechanisms, the event-triggered positive polynomial fuzzy filter with c rules is designed, the j-th rule of filter is as follows:

Rule j : IF $g_1(\mathbf{y}(t))$ is $\mathbf{N}_1^j$ AND $\cdots$ AND $g_\psi(\mathbf{y}(t))$ is $\mathbf{N}_\psi^j$

$$\text{THEN} \quad \begin{cases} \dot{\mathbf{x}}_f(t) = \mathbf{A}_{fj}(\mathbf{x}_f(t))\mathbf{x}_f(t) + \mathbf{B}_{fj}(\mathbf{x}_f(t))\mathbf{y}(t_k h) \\ \mathbf{z}_f(t) = \mathbf{C}_{fj}(\mathbf{x}_f(t))\mathbf{x}_f(t) + \mathbf{D}_{fj}(\mathbf{x}_f(t))\mathbf{y}(t_k h) \end{cases}$$

where $\mathbf{x}_f(t) \in \Re^n$ and $\mathbf{z}_f \in \Re^m$ are the filter state and estimated output. $g_\vartheta(\mathbf{y}(t))$ is the premise variable and N_ϑ^j is an IT2 fuzzy set corresponding to its premise variable in rule j, $j \in \underline{c}$, $\vartheta \in \underline{\psi}$, and ψ is a positive integer. $\mathbf{A}_{fj}(\mathbf{x}_f(t))$, $\mathbf{B}_{fj}(\mathbf{x}_f(t))$, $\mathbf{C}_{fj}(\mathbf{x}_f(t))$ and $\mathbf{D}_{fj}(\mathbf{x}_f(t))$ are the positive polynomial fuzzy filter gain matrices to be determined.

Based on the above fuzzy rules, the event-triggered positive polynomial fuzzy filter is represented as follows:

$$\begin{cases} \dot{\mathbf{x}}_f(t) = \sum_{j=1}^{c} m_j(\mathbf{y}(t_k h)) \times \\ \qquad [\mathbf{A}_{fj}(\mathbf{x}_f(t))\mathbf{x}_f(t) + \mathbf{B}_{fj}(\mathbf{x}_f(t))\mathbf{y}(t_k h)] \\ \mathbf{z}_f(t) = \sum_{j=1}^{c} m_j(\mathbf{y}(t_k h)) \times \\ \qquad [\mathbf{C}_{fj}(\mathbf{x}_f(t))\mathbf{x}_f(t) + \mathbf{D}_{fj}(\mathbf{x}_f(t))\mathbf{y}(t_k h)] \end{cases} \tag{9}$$

III. Positivity and Stability Analysis

A. Augmented Event-Triggered Positive Polynomial Fuzzy Filter Error System

Denote $\Delta\mathbf{x}(t) = \mathbf{x}_f(t) - \mathbf{x}(t)$ and $\Delta\mathbf{z}(t) = \mathbf{z}_f(t) - \mathbf{z}(t)$, from (1), (5) and (9), the augmented event-triggered positive polynomial fuzzy filter error system is obtained as follows:

$$\begin{cases} \dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}(t)) m_j(\mathbf{y}(t_k h)) \times \\ \qquad [\mathbf{A}_i(\mathbf{x}(t))\mathbf{x}(t) + \mathbf{B}_i(\mathbf{x}(t))\mathbf{w}(t)] \\ \dot{\Delta\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}(t)) m_j(\mathbf{y}(t_k h)) \times \\ \qquad [(\mathbf{A}_{fj}(\mathbf{x}_f(t)) - \mathbf{A}_i(\mathbf{x}(t)))\mathbf{x}(t) \\ \qquad + \mathbf{A}_{fj}(\mathbf{x}_f(t))\Delta\mathbf{x}(t) + \mathbf{B}_{fj}(\mathbf{x}_f(t))\mathbf{y}(t-\tau(t)) \\ \qquad + \mathbf{B}_{fj}(\mathbf{x}_f(t))\Delta\mathbf{y}(t_k h)) - \mathbf{B}_i(\mathbf{x}(t))\mathbf{w}(t)] \\ \Delta\mathbf{z}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}(t)) m_j(\mathbf{y}(t_k h)) \times \\ \qquad [(\mathbf{C}_{fj}(\mathbf{x}_f(t)) - \mathbf{E}_i(\mathbf{x}(t)))\mathbf{x}(t) \\ \qquad + \mathbf{C}_{fj}(\mathbf{x}_f(t))\Delta\mathbf{x}(t) + \mathbf{D}_{fj}(\mathbf{x}_f(t))\mathbf{y}(t-\tau(t)) \\ \qquad + \mathbf{D}_{fj}(\mathbf{x}_f(t))\Delta\mathbf{y}(t_k h)) - \mathbf{F}_i(\mathbf{x}(t))\mathbf{w}(t)] \end{cases} \tag{10}$$

In the following analysis, for simplicity, the time t is dropped for the simulation without ambiguity, e.g., $\mathbf{x}(t)$, $\mathbf{z}(t)$, $\mathbf{z}_f(t)$ and $\mathbf{w}(t)$ are denoted by $\mathbf{x}$, $\mathbf{z}$, $\mathbf{z}_f$ and $\mathbf{w}$, respectively; $\mathbf{x}(t-\tau(t))$, $\mathbf{w}(t-\tau(t))$ and $\mathbf{y}(t-\tau(t))$ are denoted by $\mathbf{x}_\tau$, $\mathbf{w}_\tau$ and $\mathbf{y}_\tau$, respectively; $\mathbf{y}(t_k h)$ is denoted by $\mathbf{y}_{t_k}$.

According to event-triggered condition (8), the upper bounded and lower bounded of the term $\Delta\mathbf{y}_{t_k}$ in system (10) are $\frac{\theta_2}{1-\theta_2}\mathbf{y}_\tau$ and $-\frac{\theta_1}{1+\theta_1}\mathbf{y}_\tau$, respectively, then the upper bounded system and the lower bounded system of the augmented system (10) can be obtained when the term $\Delta\mathbf{y}_{t_k}$ is replaced by its boundary.

Then, by replacing the term $\Delta\mathbf{y}_{t_k}$ in system (10) with its upper bound $\frac{\theta_2}{1-\theta_2}\mathbf{y}_\tau$ and lower bound $-\frac{\theta_1}{1+\theta_1}\mathbf{y}_\tau$, where $\mathbf{y}_\tau = \mathbf{C}\mathbf{x}_\tau + \mathbf{D}\mathbf{w}_\tau$, the upper bound and lower bound of the augmented filter error system (10) can be obtained as follows, respectively:

$$\begin{cases} \dot{\mathbf{x}} = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k})[\mathbf{A}_i(\mathbf{x})\mathbf{x} + \mathbf{B}_i(\mathbf{x})\mathbf{w}] \\ \dot{\Delta\mathbf{x}} \prec \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [(\mathbf{A}_{fj}(\mathbf{x}_f) - \mathbf{A}_i(\mathbf{x}))\mathbf{x} + \mathbf{A}_{fj}(\mathbf{x}_f)\Delta\mathbf{x} \\ \qquad + (1 + \frac{\theta_2}{1-\theta_2})\mathbf{B}_{fj}(\mathbf{x}_f)(\mathbf{C}\mathbf{x}_\tau + \mathbf{D}\mathbf{w}_\tau) \\ \qquad - \mathbf{B}_i(\mathbf{x}(t))\mathbf{w}] \\ \Delta\mathbf{z} \prec \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [(\mathbf{C}_{fj}(\mathbf{x}_f) - \mathbf{E}_i(\mathbf{x}))\mathbf{x} + \mathbf{C}_{fj}(\mathbf{x}_f)\Delta\mathbf{x} \\ \qquad + (1 + \frac{\theta_2}{1-\theta_2})\mathbf{D}_{fj}(\mathbf{x}_f)(\mathbf{C}\mathbf{x}_\tau + \mathbf{D}\mathbf{w}_\tau) \\ \qquad - \mathbf{F}_i(\mathbf{x})\mathbf{w}] \end{cases} \tag{11}$$

$$\begin{cases} \dot{\mathbf{x}} = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k})[\mathbf{A}_i(\mathbf{x})\mathbf{x} + \mathbf{B}_i(\mathbf{x})\mathbf{w}] \\ \dot{\Delta\mathbf{x}} \succ \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [(\mathbf{A}_{fj}(\mathbf{x}_f) - \mathbf{A}_i(\mathbf{x}))\mathbf{x} + \mathbf{A}_{fj}(\mathbf{x}_f)\Delta\mathbf{x} \\ \qquad + (1 - \frac{\theta_1}{1+\theta_1})\mathbf{B}_{fj}(\mathbf{x}_f)(\mathbf{C}\mathbf{x}_\tau + \mathbf{D}\mathbf{w}_\tau) \\ \qquad - \mathbf{B}_i(\mathbf{x})\mathbf{w}] \\ \Delta\mathbf{z} \succ \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [(\mathbf{C}_{fj}(\mathbf{x}_f) - \mathbf{E}_i(\mathbf{x}))\mathbf{x} + \mathbf{C}_{fj}(\mathbf{x}_f)\Delta\mathbf{x} \\ \qquad + (1 - \frac{\theta_1}{1+\theta_1})\mathbf{D}_{fj}(\mathbf{x}_f)(\mathbf{C}\mathbf{x}_\tau + \mathbf{D}\mathbf{w}_\tau) \\ \qquad - \mathbf{F}_i(\mathbf{x})\mathbf{w}] \end{cases} \tag{12}$$

For a positive system (1), the matrices $\sum_{i=1}^{p} w_i(\mathbf{y})\mathbf{B}_i(\mathbf{x}) \succ 0$ and $\sum_{i=1}^{p} w_i(\mathbf{y})\mathbf{F}_i(\mathbf{x}) \succ 0$, whose positiveness is not conducive to deriving the positivity conditions for the upper and lower bounded augmented filter error systems (11) and (12), so the systems (11) and (12) are equivalently transformed by introducing a variable $\Delta\mathbf{w}$ which are defined as $\Delta\mathbf{w} = \frac{1}{\sigma}\mathbf{w}_\tau - \mathbf{w}$, where σ is a weight coefficient such that $\Delta\mathbf{w} \succ 0$ hold. Meanwhile, denoting $\boldsymbol{\eta} = [\mathbf{x}\ \Delta\mathbf{x}]$ and $\boldsymbol{\varphi} = [\mathbf{w}\ \Delta\mathbf{w}]$, the equivalent upper bounded and lower bounded augmented filter error systems are as follows:

$$\begin{cases} \dot{\boldsymbol{\eta}} \prec \hat{\dot{\boldsymbol{\eta}}} = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [\tilde{\mathbf{A}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\eta} + \hat{\tilde{\mathbf{A}}}_{\tau j}(\mathbf{x}_f)\mathbf{x}_\tau + \hat{\tilde{\mathbf{B}}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\varphi}] \\ \Delta\mathbf{z} \prec \hat{\Delta\mathbf{z}} = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [\hat{\tilde{\mathbf{C}}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\eta} + \hat{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f)\mathbf{x}_\tau + \hat{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\varphi}] \end{cases} \tag{13}$$

where

$$\tilde{\mathbf{A}}_{ij}(\mathbf{x},\mathbf{x}_f) = \begin{bmatrix} \mathbf{A}_i(\mathbf{x}) & 0 \\ \mathbf{A}_{fj}(\mathbf{x}_f) - \mathbf{A}_i(\mathbf{x}) & \mathbf{A}_{fj}(\mathbf{x}_f) \end{bmatrix} \tag{14}$$

$$\hat{\tilde{\mathbf{A}}}_{\tau j}(\mathbf{x}_f) = \begin{bmatrix} 0 \\ (1+\frac{\theta_2}{1-\theta_2})\mathbf{B}_{fj}(\mathbf{x}_f)\mathbf{C} \end{bmatrix} \tag{15}$$

$$\hat{\tilde{\mathbf{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) = \begin{bmatrix} \mathbf{B}_i(\mathbf{x}) & 0 \\ \hat{\mathbf{B}}_j(\mathbf{x}_f) - \mathbf{B}_i(\mathbf{x}) & \hat{\mathbf{B}}_j(\mathbf{x}_f) \end{bmatrix} \tag{16}$$

$$\hat{\mathbf{B}}_j(\mathbf{x}_f) = \sigma(1+\frac{\theta_2}{1-\theta_2})\mathbf{B}_{fj}(\mathbf{x}_f)\mathbf{D} \tag{17}$$

$$\tilde{\mathbf{C}}_{ij}(\mathbf{x},\mathbf{x}_f) = [\mathbf{C}_{fj}(\mathbf{x}_f) - \mathbf{E}_i(\mathbf{x}) \quad \mathbf{C}_{fj}(\mathbf{x}_f)] \tag{18}$$

$$\hat{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f) = \left[(1+\frac{\theta_2}{1-\theta_2})\mathbf{D}_{fj}(\mathbf{x}_f)\mathbf{C}\right] \tag{19}$$

$$\hat{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f) = \left[\hat{\mathbf{D}}_j(\mathbf{x}_f) - \mathbf{F}_i(\mathbf{x}) \quad \hat{\mathbf{D}}_j(\mathbf{x}_f)\right] \tag{20}$$

$$\hat{\mathbf{D}}_j(\mathbf{x}_f) = \sigma(1+\frac{\theta_2}{1-\theta_2})\mathbf{D}_{fj}(\mathbf{x}_f)\mathbf{D} \tag{21}$$

$$\begin{cases} \dot{\boldsymbol{\eta}} \succ \check{\dot{\boldsymbol{\eta}}} = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [\tilde{\mathbf{A}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\eta} + \check{\tilde{\mathbf{A}}}_{\tau j}(\mathbf{x}_f)\mathbf{x}_\tau + \check{\tilde{\mathbf{B}}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\varphi}] \\ \Delta\mathbf{z} \succ \check{\Delta\mathbf{z}} = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \times \\ \qquad [\tilde{\mathbf{C}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\eta} + \check{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f)\mathbf{x}_\tau + \check{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f)\boldsymbol{\varphi}] \end{cases} \tag{22}$$

where $\tilde{\mathbf{A}}_{ij}(\mathbf{x},\mathbf{x}_f)$ and $\tilde{\mathbf{C}}_{ij}(\mathbf{x},\mathbf{x}_f)$ are defined in (14) and (18), respectively

$$\check{\tilde{\mathbf{A}}}_{\tau j}(\mathbf{x}_f) = \begin{bmatrix} 0 \\ (1-\frac{\theta_1}{1+\theta_1})\mathbf{B}_{fj}(\mathbf{x}_f)\mathbf{C} \end{bmatrix} \tag{23}$$

$$\check{\tilde{\mathbf{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) = \begin{bmatrix} \mathbf{B}_i(\mathbf{x}) & 0 \\ \check{\mathbf{B}}_j(\mathbf{x}_f) - \mathbf{B}_i(\mathbf{x}) & \check{\mathbf{B}}_j(\mathbf{x}_f) \end{bmatrix} \tag{24}$$

$$\check{\mathbf{B}}_j(\mathbf{x}_f) = \sigma(1-\frac{\theta_1}{1+\theta_1})\mathbf{B}_{fj}(\mathbf{x}_f)\mathbf{D} \tag{25}$$

$$\check{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f) = \left[(1-\frac{\theta_1}{1+\theta_1})\mathbf{D}_{fj}(\mathbf{x}_f)\mathbf{C}\right] \tag{26}$$

$$\check{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f) = [\check{\mathbf{D}}_j(\mathbf{x}_f) - \mathbf{F}_i(\mathbf{x}) \quad \check{\mathbf{D}}_j(\mathbf{x}_f)] \tag{27}$$

$$\check{\mathbf{D}}_j(\mathbf{x}_f) = \sigma(1-\frac{\theta_1}{1+\theta_1})\mathbf{D}_{fj}(\mathbf{x}_f)\mathbf{D} \tag{28}$$

In this paper, the objective is to design a positive and stable filter which makes system states and the estimated output signals positive and stable with best possible L_1 performance. It should be noted that the positivity and stability conditions used to ensure the positivity and stability of the system $[\mathbf{x} \ \mathbf{z}]$ and filter $[\mathbf{x}_f \ \mathbf{z}_f]$ cannot be directly derived. From another point of view, it can be founded from the definitions $\Delta\mathbf{x} = \mathbf{x}_f - \mathbf{x}$ and $\Delta\mathbf{z} = \mathbf{z}_f - \mathbf{z}$ that the positivity and stability of $[\mathbf{x} \ \mathbf{z}]$ and $[\mathbf{x}_f \ \mathbf{z}_f]$ can be guaranteed by the positivity and stability of $[\mathbf{x}_f \ \Delta\mathbf{x}]$ and $[\mathbf{z}_f \ \Delta\mathbf{z}]$. So the positivity analysis and stability analysis will be performed for $[\mathbf{x}_f \ \Delta\mathbf{x}]$ and $[\mathbf{z}_f \ \Delta\mathbf{z}]$ in the following.

Remark 1: In reference [36], a predefined variable exists in the event-triggered condition, which breaks the constrain that the output signals must change monotonically but introduces conservatism; In reference [37], although above issue was avoided, the network communication delay which is the common phenomenon in practical engineering was not considered. In this paper, a novel event-triggered condition (7) is proposed that does not impose any additional restrictions, also the upper bounded augmented filter error system (13) and lower bounded augmented filter error system (22) are established to facilitate the stability analysis and positivity analysis of the event-triggered filter error system with transmission delay.

B. Stability Analysis

In order to obtained the positive polynomial fuzzy filter matrices, the stability analysis of upper bounded augmented filter error system (13) is performed, then the obtained stability conditions can guarantee the stability of the augmented filter error system (10). To facilitate the stability analysis, the following novel LCLF is proposed:

$$V = \boldsymbol{\lambda}^T\boldsymbol{\eta} + \int_{t-\tau}^{t} \boldsymbol{\delta}_1^T\mathbf{C}\mathbf{x}(s_1)d_{s_1} + \int_{t-\tau}^{t} \boldsymbol{\delta}_2^T\mathbf{D}\mathbf{w}(s_2)d_{s_2} \tag{29}$$

where $\boldsymbol{\lambda}^T = [\boldsymbol{\lambda}_1^T \quad \boldsymbol{\lambda}_2^T]$, $\boldsymbol{\lambda}_1 \in \Re^{n\times 1}$, $\boldsymbol{\lambda}_2 \in \Re^{n\times 1}$; $\boldsymbol{\delta}_1 \in \Re^{m\times 1}$, $\boldsymbol{\delta}_2 \in \Re^{m\times 1}$, m is the dimension of output $\mathbf{y}$.

Remark 2: In this paper, the output signal disturbed by noise, and communication delay in the event-triggered mechanism is considered. Neither the normal LCLF $V = \boldsymbol{\lambda}^T\boldsymbol{\eta}$ nor the LCLF in reference [23] can reflect above characteristics of the system, resulting in the failure to derive the stability conditions of the system, so novel LCLF (29) is proposed, where the term $\int_{t-\tau}^{t} \boldsymbol{\delta}_1^T\mathbf{C}\mathbf{x}(s_1)d_{s_1}$ and $\int_{t-\tau}^{t} \boldsymbol{\delta}_2^T\mathbf{D}\mathbf{w}(s_2)d_{s_2}$ represent the energy of delay states and delay disturbances respectively.

Taking the L_1 performance index $\|\Delta\mathbf{z}\|_{L_1} < \gamma\|\mathbf{w}\|_{L_1}$ into account

$$\begin{aligned} J &= \int_0^\infty \|\Delta\mathbf{z}\|_{L_1} - \gamma\|\mathbf{w}\|_{L_1} d_t \\ &= \int_0^\infty \sum_{r_1=1}^{q} \Delta\mathbf{z}^{(r_1)} - \gamma\sum_{r_2=1}^{g} \mathbf{w}^{(r_2)} + \dot{V} - \dot{V} d_t \\ &= \int_0^\infty \sum_{r_1=1}^{q} \Delta\mathbf{z}^{(r_1)} - \gamma\sum_{r_2=1}^{g} \mathbf{w}^{(r_2)} + \dot{V} d_t + V(0) - V(\infty) \end{aligned} \tag{30}$$

where q is the dimension of $\mathbf{z}$, and g is the dimension of $\mathbf{w}$; $V(\infty)$ go back to the initial condition, so $V(0) = V(\infty)$. Then, the performance index is as follows:

$$
\begin{aligned}
J &= \int_0^\infty \sum_{r_1=1}^{q} \Delta \mathbf{z}^{(r_1)} - \gamma \sum_{r_2=1}^{g} \mathbf{w}^{(r_2)} + \dot{V} d_t \\
&= \int_0^\infty \mathbf{I}_q^T \Delta \mathbf{z} - \gamma \mathbf{I}_g^T \mathbf{w} + \dot{V} d_t \qquad (31)
\end{aligned}
$$

where $\mathbf{I}_q \in \Re^{q\times 1}$ and $\mathbf{I}_g \in \Re^{g\times 1}$ are vectors with all of the elements being 1.

According to the upper bounded of the augmented systems, the inequality of performance index J can be obtained as follows:

$$
\begin{aligned}
J &= \int_0^\infty \mathbf{I}_q^T \Delta \mathbf{z} - \gamma \mathbf{I}_g^T \mathbf{w} + \dot{V} d_t \\
&= \int_0^\infty \mathbf{I}_q^T \Delta \mathbf{z} - \gamma \mathbf{I}_g^T \mathbf{w} + \boldsymbol{\lambda}^T \dot{\boldsymbol{\eta}} + \boldsymbol{\delta}_1^T \mathbf{C}\mathbf{x} - \boldsymbol{\delta}_1^T \mathbf{C}\mathbf{x}_\tau \\
&\quad + \boldsymbol{\delta}_2^T \mathbf{D}\mathbf{w} - \boldsymbol{\delta}_2^T \mathbf{D}\mathbf{w}_\tau d_t \\
&\le \int_0^\infty \mathbf{I}_q^T \hat{\Delta \mathbf{z}} - \gamma \mathbf{I}_g^T \mathbf{w} + \boldsymbol{\lambda}^T \dot{\hat{\boldsymbol{\eta}}} + \boldsymbol{\delta}_1^T \mathbf{C}\mathbf{x} - \boldsymbol{\delta}_1^T \mathbf{C}\mathbf{x}_\tau \\
&\quad + \boldsymbol{\delta}_2^T \mathbf{D}\mathbf{w} - \boldsymbol{\delta}_2^T \mathbf{D}\mathbf{w}_\tau d_t \qquad (32)
\end{aligned}
$$

Taking (13) into (32), and introducing the equation $\sigma\boldsymbol{\delta}_2^T\mathbf{D}\mathbf{w} - \sigma\boldsymbol{\delta}_2^T\mathbf{D}\mathbf{w} = 0$, we have

$$
\begin{aligned}
J \le & \int_0^\infty \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y})(m_j(\mathbf{y}_{t_k})\times \\
& \{\mathbf{I}_q^T \tilde{\mathbf{C}}_{ij}(\mathbf{x},\mathbf{x}_f) + \boldsymbol{\lambda}^T \tilde{\mathbf{A}}_{ij}(\mathbf{x},\mathbf{x}_f) + \boldsymbol{\delta}_1^T \mathbf{C}[\mathbf{I}_{n\times n}\ \mathbf{0}_n]\}\boldsymbol{\eta} \\
& + \{\mathbf{I}_q^T \hat{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f) + \boldsymbol{\lambda}^T \hat{\tilde{\mathbf{A}}}_{\tau j}(\mathbf{x}_f) - \boldsymbol{\delta}_1^T \mathbf{C}\}\mathbf{x}_\tau \\
& + \{\mathbf{I}_q^T \hat{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f) + \boldsymbol{\lambda}^T \hat{\tilde{\mathbf{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) \\
& - [\gamma \mathbf{I}_g^T - (1-\sigma)\boldsymbol{\delta}_2^T\mathbf{D} \quad \sigma\boldsymbol{\delta}_2^T\mathbf{D}]\}\boldsymbol{\varphi} d_t \qquad (36)
\end{aligned}
$$

Then $J < 0$ can be guaranteed by the follow conditions:

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \boldsymbol{\Xi}_{1ij}(\mathbf{x},\mathbf{x}_f) \prec 0 \qquad (37)$$

$$\sum_{j=1}^{c} m_j(\mathbf{y}_{t_k}) \boldsymbol{\Xi}_{2j}(\mathbf{x}_f) \prec 0 \qquad (38)$$

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \boldsymbol{\Xi}_{3ij}(\mathbf{x},\mathbf{x}_f) \prec 0 \qquad (39)$$

where

$$
\begin{aligned}
\boldsymbol{\Xi}_{1ij}(\mathbf{x},\mathbf{x}_f) &= \mathbf{I}_q^T \tilde{\mathbf{C}}_{ij}(\mathbf{x},\mathbf{x}_f) + \boldsymbol{\lambda}^T \tilde{\mathbf{A}}_{ij}(\mathbf{x},\mathbf{x}_f) \\
&\quad + \boldsymbol{\delta}_1^T \mathbf{C}[\mathbf{I}_{n\times n}\ \mathbf{0}_n] \prec 0 \qquad (40) \\
\boldsymbol{\Xi}_{2j}(\mathbf{x}_f) &= \mathbf{I}_q^T \hat{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f) + \boldsymbol{\lambda}^T \hat{\tilde{\mathbf{A}}}_{\tau j}(\mathbf{x}_f) - \boldsymbol{\delta}_1^T \mathbf{C} \prec 0 \qquad (41) \\
\boldsymbol{\Xi}_{3ij}(\mathbf{x},\mathbf{x}_f) &= \mathbf{I}_q^T \hat{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f) + \boldsymbol{\lambda}^T \hat{\tilde{\mathbf{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) \\
&\quad - [\gamma \mathbf{I}_g^T - (1-\sigma)\boldsymbol{\delta}_2^T\mathbf{D} \quad \sigma\boldsymbol{\delta}_2^T\mathbf{D}] \prec 0 \qquad (42)
\end{aligned}
$$

Taking (14)-(21) into above conditions, and define:

$$
\mathbf{A}_{\lambda fj}(\mathbf{x}_f) = \begin{bmatrix} \boldsymbol{\lambda}_2^{(1)} \mathbf{A}_{fj}^{(1,:)}(\mathbf{x}_f); \\ \boldsymbol{\lambda}_2^{(2)} \mathbf{A}_{fj}^{(2,:)}(\mathbf{x}_f); \\ \vdots \\ \boldsymbol{\lambda}_2^{(n)} \mathbf{A}_{fj}^{(n,:)}(\mathbf{x}_f) \end{bmatrix}, \qquad (43)
$$

$$
\mathbf{B}_{\lambda fj}(\mathbf{x}_f) = \begin{bmatrix} \boldsymbol{\lambda}_2^{(1)} \mathbf{B}_{fj}^{(1,:)}(\mathbf{x}_f); \\ \boldsymbol{\lambda}_2^{(2)} \mathbf{B}_{fj}^{(2,:)}(\mathbf{x}_f); \\ \vdots \\ \boldsymbol{\lambda}_2^{(n)} \mathbf{B}_{fj}^{(n,:)}(\mathbf{x}_f) \end{bmatrix}. \qquad (44)
$$

Then the equivalent stability conditions are obtained:

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \tilde{\boldsymbol{\Xi}}_{1ij}(\mathbf{x},\mathbf{x}_f) \prec 0 \qquad (45)$$

$$\sum_{j=1}^{c} m_j(\mathbf{y}_{t_k}) \tilde{\boldsymbol{\Xi}}_{2j}(\mathbf{x}_f) \prec 0 \qquad (46)$$

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y}) m_j(\mathbf{y}_{t_k}) \tilde{\boldsymbol{\Xi}}_{3ij}(\mathbf{x},\mathbf{x}_f) \prec 0 \qquad (47)$$

where $\tilde{\boldsymbol{\Xi}}_{1ij}(\mathbf{x},\mathbf{x}_f)$, $\tilde{\boldsymbol{\Xi}}_{2j}(\mathbf{x}_f)$ and $\tilde{\boldsymbol{\Xi}}_{3ij}(\mathbf{x},\mathbf{x}_f)$ are defined at the bottom of this page.

Considering that the MFs are nonlinear functions, which are the obstacles preventing the above conditions from being calculated by the linear optimization toolbox, $J < 0$ is guaranteed by the following MFs independent conditions:

$$\tilde{\boldsymbol{\Xi}}_{1ij}(\mathbf{x},\mathbf{x}_f) \prec 0,\ \forall i \in \underline{p}, j \in \underline{c} \qquad (48)$$

$$\tilde{\boldsymbol{\Xi}}_{2j}(\mathbf{x}_f) \prec 0,\ \forall j \in \underline{c} \qquad (49)$$

$$\tilde{\boldsymbol{\Xi}}_{3ij}(\mathbf{x},\mathbf{x}_f) \prec 0,\ \forall i \in \underline{p}, j \in \underline{c} \qquad (50)$$

$$
\tilde{\boldsymbol{\Xi}}_{1ij}(\mathbf{x},\mathbf{x}_f) = \left[\begin{pmatrix} \mathbf{I}_q^T(\mathbf{C}_{fj}(\mathbf{x}_f) - \mathbf{E}_i(\mathbf{x})) \\ +\boldsymbol{\lambda}_1^T \mathbf{A}_i(\mathbf{x}) + \sum_{o=1}^{n} \mathbf{A}_{\lambda fj}^{(o,:)}(\mathbf{x}_f) - \boldsymbol{\lambda}_2^T \mathbf{A}_i(\mathbf{x}) \\ +\boldsymbol{\delta}_1^T \mathbf{C} \end{pmatrix} \quad \begin{pmatrix} \mathbf{I}_q^T \mathbf{C}_{fj}(\mathbf{x}_f) + \sum_{o=1}^{n} \mathbf{A}_{\lambda fj}^{(o,:)}(\mathbf{x}_f) \end{pmatrix} \right] \qquad (33)
$$

$$
\tilde{\boldsymbol{\Xi}}_{2j}(\mathbf{x},\mathbf{x}_f) = (1+\frac{\theta_2}{1-\theta_2})\mathbf{I}_q^T \mathbf{D}_{fj}(\mathbf{x}_f)\mathbf{C} + (1+\frac{\theta_2}{1-\theta_2})\sum_{o=1}^{n} \mathbf{B}_{\lambda fj}^{(o,:)}(\mathbf{x}_f)\mathbf{C} - \boldsymbol{\delta}_1^T \mathbf{C} \qquad (34)
$$

$$
\tilde{\boldsymbol{\Xi}}_{3ij}(\mathbf{x},\mathbf{x}_f) = \left[\begin{pmatrix} \sigma(1+\frac{\theta_2}{1-\theta_2})\mathbf{I}_q^T \mathbf{D}_{fj}(\mathbf{x}_f)\mathbf{D} - \mathbf{I}_q^T \mathbf{F}_i(\mathbf{x}) \\ +\boldsymbol{\lambda}_1^T \mathbf{B}_i(\mathbf{x}) + (1+\frac{\theta_2}{1-\theta_2})\sum_{o=1}^{n} \mathbf{B}_{\lambda fj}^{(o,:)}(\mathbf{x}_f)\mathbf{D} \\ -\boldsymbol{\lambda}_2^T \mathbf{B}_i(\mathbf{x}) - \gamma \mathbf{I}_g^T + (1-\sigma)\boldsymbol{\delta}_2^T \mathbf{D} \end{pmatrix} \quad \begin{pmatrix} \sigma(1+\frac{\theta_2}{1-\theta_2})\mathbf{I}_q^T \mathbf{D}_{fj}(\mathbf{x}_f)\mathbf{D} \\ +(1+\frac{\theta_2}{1-\theta_2})\sum_{o=1}^{n} \mathbf{B}_{\lambda fj}^{(o,:)}(\mathbf{x}_f)\mathbf{D} \\ -\sigma\boldsymbol{\delta}_2^T \mathbf{D} \end{pmatrix} \right] \qquad (35)
$$

C. Positivity Analysis

If the lower bounded augmented filter error systems (22) can be guaranteed to be positive, then the positivity of the augmented filter error system (10) can be guaranteed, so the positivity conditions are as follows:

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})\tilde{\mathbf{A}}_{ij}(\mathbf{x},\mathbf{x}_f)\text{is Metzler matrix;} \quad (51)$$

$$\sum_{j=1}^{c} m_j(\mathbf{y}_{t_k})\check{\tilde{\mathbf{A}}}_{\tau j}(\mathbf{x}_f) \succ 0; \quad (52)$$

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})\check{\tilde{\mathbf{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) \succ 0; \quad (53)$$

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})\tilde{\mathbf{C}}_{ij}(\mathbf{x},\mathbf{x}_f) \succ 0; \quad (54)$$

$$\sum_{j=1}^{c} m_j(\mathbf{y}_{t_k})\check{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f) \succ 0; \quad (55)$$

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})\check{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f) \succ 0. \quad (56)$$

To facilitate the simultaneous calculation of stability conditions and positivity conditions, pre-multiplying $\begin{bmatrix} \boldsymbol{\lambda}_2 & 0 \\ 0 & \boldsymbol{\lambda}_2 \end{bmatrix}$ to the positivity conditions (51), (52) and (53), then the positivity conditions are conditions (54)-(56) and the following conditions:

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})\check{\tilde{\mathfrak{A}}}_{ij}(\mathbf{x},\mathbf{x}_f)\text{is Metzler matrix;} \quad (57)$$

$$\sum_{j=1}^{c} m_j(\mathbf{y}_{t_k})\check{\tilde{\mathfrak{A}}}_{\tau j}(\mathbf{x}_f) \succ 0; \quad (58)$$

$$\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})\check{\tilde{\mathfrak{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) \succ 0; \quad (59)$$

where

$$\check{\tilde{\mathfrak{A}}}_{ij}(\mathbf{x},\mathbf{x}_f) = \begin{bmatrix} \boldsymbol{\lambda}_2\mathbf{A}_i(\mathbf{x}) & \mathbf{0} \\ \mathbf{A}_{\lambda fj}(\mathbf{x}_f) - \boldsymbol{\lambda}_2\mathbf{A}_i(\mathbf{x}) & \mathbf{A}_{\lambda fj}(\mathbf{x}_f) \end{bmatrix} \quad (60)$$

$$\check{\tilde{\mathfrak{A}}}_{\tau j}(\mathbf{x}_f) = \begin{bmatrix} \mathbf{0} \\ (1-\frac{\theta_1}{1+\theta_1})\mathbf{B}_{\lambda fj}(\mathbf{x}_f)\mathbf{C} \end{bmatrix} \quad (61)$$

$$\check{\tilde{\mathfrak{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) = \begin{bmatrix} \boldsymbol{\lambda}_2\mathbf{B}_i(\mathbf{x}) & \mathbf{0} \\ \boldsymbol{\Upsilon}_j(\mathbf{x}_f) - \boldsymbol{\lambda}_2\mathbf{B}_i(\mathbf{x}) & \boldsymbol{\Upsilon}_j(\mathbf{x}_f) \end{bmatrix} \quad (62)$$

$$\boldsymbol{\Upsilon}_j(\mathbf{x}_f) = \sigma(1-\frac{\theta_1}{1+\theta_1})\mathbf{B}_{\lambda fj}(\mathbf{x}_f)\mathbf{D} \quad (63)$$

The above positivity conditions cannot be calculated with the SOSTOOLS, because the MFs are nonlinear functions. To break through this barrier, the positivity of the above positivity conditions involving MFs are guaranteed by the positivity of each subcondition that does not contain MFs, the convex positivity conditions are obtained as follows:

$$\check{\tilde{\mathfrak{A}}}_{ij}(\mathbf{x},\mathbf{x}_f)\text{is Metzler matrix,} \quad \forall i \in \underline{p}, j \in \underline{c}; \quad (64)$$

$$\check{\tilde{\mathfrak{A}}}_{\tau j}(\mathbf{x}_f) \succ 0, \quad \forall j \in \underline{c}; \quad (65)$$

$$\check{\tilde{\mathfrak{B}}}_{ij}(\mathbf{x},\mathbf{x}_f) \succ 0, \quad \forall i \in \underline{p}, j \in \underline{c}; \quad (66)$$

$$\check{\tilde{\mathbf{C}}}_{ij}(\mathbf{x},\mathbf{x}_f) \succ 0, \quad \forall i \in \underline{p}, j \in \underline{c}; \quad (67)$$

$$\check{\tilde{\mathbf{C}}}_{\tau j}(\mathbf{x}_f) \succ 0; \quad \forall j \in \underline{c}; \quad (68)$$

$$\check{\tilde{\mathbf{D}}}_{ij}(\mathbf{x},\mathbf{x}_f) \succ 0, \quad \forall i \in \underline{p}, j \in \underline{c}; \quad (69)$$

According to the abovementioned stability analysis and positivity analysis, the polynomial fuzzy filter matrices can be obtained by calculating the positivity conditions and stability conditions, the SOS-form positivity conditions and stability conditions are summarized in the following theorem.

Theorem 1: For a PPFMB system (1), an event-triggered positive polynomial filter can be designed to make the polynomial fuzzy filter error system (10) asymptotically stable and positive while achieving optimal performance index γ, if given the predefined scalars θ_1, θ_2, σ, and there exist Lyapunov vectors $\boldsymbol{\lambda} \succ 0$, $\boldsymbol{\delta}_1 \succ 0$, $\boldsymbol{\delta}_2 \succ 0$, and polynomial filter decision matrices $\mathbf{A}_{\lambda fj}(\mathbf{x}_f)$, $\mathbf{B}_{\lambda fj}(\mathbf{x}_f)$, $\mathbf{C}_{fj}(\mathbf{x}_f)$, $\mathbf{D}_{fj}(\mathbf{x}_f)$, such that the following SOS-based conditions of optimization problem are satisfied for any $i \in \underline{p}, j \in \underline{c}$:

$$\begin{aligned}
&\min \gamma \\
&s.t. \\
&1)\ -\nu^T(\tilde{\boldsymbol{\Xi}}_{1ij}^{(1,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_1)\nu \text{ is SOS, } \forall \beta \in \underline{2n}; \\
&2)\ -\nu^T(\tilde{\boldsymbol{\Xi}}_{2j}^{(1,\beta)}(\mathbf{x}_f) - \varepsilon_2)\nu \text{ is SOS, } \forall \beta \in \underline{n}; \\
&3)\ -\nu^T(\tilde{\boldsymbol{\Xi}}_{3ij}^{(1,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_3)\nu \text{ is SOS, } \forall \beta \in \underline{2g}; \\
&4)\ \nu^T(\check{\tilde{\mathfrak{A}}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_4)\nu \text{ is SOS, } \forall \alpha \neq \beta \in \underline{2n}; \\
&5)\ \nu^T(\check{\tilde{\mathfrak{A}}}_{\tau j}^{(\alpha,\beta)}(\mathbf{x}_f) - \varepsilon_5)\nu \text{ is SOS, } \forall \alpha \in \underline{2n}, \beta \in \underline{n}; \\
&6)\ \nu^T(\check{\tilde{\mathfrak{B}}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_6)\nu \text{ is SOS, } \forall \alpha \in \underline{n+1}, \beta \in \underline{2g}; \\
&7)\ \nu^T(\tilde{\mathbf{C}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_7)\nu \text{ is SOS, } \forall \alpha \in \underline{m}, \beta \in \underline{2n}; \\
&8)\ \nu^T(\check{\tilde{\mathbf{C}}}_{\tau j}^{(\alpha,\beta)}(\mathbf{x}_f) - \varepsilon_8)\nu \text{ is SOS, } \forall \alpha \in \underline{q}, \beta \in \underline{n}; \\
&9)\ \nu^T(\check{\tilde{\mathbf{D}}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_9)\nu \text{ is SOS, } \forall \alpha \in \underline{m}, \beta \in \underline{2g}. \quad (70)
\end{aligned}$$

The filter matrices are obtained as $\mathbf{A}_{fj}(\mathbf{x}_f) = [\frac{\mathbf{A}_{\lambda fj}^{(1,:)}(\mathbf{x}_f)}{\lambda_2^{(1)}}; \frac{\mathbf{A}_{\lambda fj}^{(2,:)}(\mathbf{x}_f)}{\lambda_2^{(2)}}; \frac{\mathbf{A}_{\lambda fj}^{(n,:)}(\mathbf{x}_f)}{\lambda_2^{(n)}}]$, $\mathbf{B}_{fj}(\mathbf{x}_f) = [\frac{\mathbf{B}_{\lambda fj}^{(1,:)}(\mathbf{x}_f)}{\lambda_2^{(1)}}; \frac{\mathbf{B}_{\lambda fj}^{(2,:)}(\mathbf{x}_f)}{\lambda_2^{(2)}}; \frac{\mathbf{B}_{\lambda fj}^{(n,:)}(\mathbf{x}_f)}{\lambda_2^{(n)}}]$, $\mathbf{C}_{fj}(\mathbf{x}_f)$ and $\mathbf{D}_{fj}(\mathbf{x}_f)$.

IV. MFD Positivity and Stability Analysis

According to above stability analysis and positivity analysis in section III, the stability conditions (45)-(47) and positivity conditions (54)-(59) are obtained. Considering that the nonlinear MFs contained within these conditions cannot be handled by the linear optimization toolbox, their MFs independent subconditions (48)-(50) & (64)-(69) as their sufficient and unnecessary conditions are used to constrain them in Theorem 1. Although this is the easiest way to break the barriers posed by nonlinear MFs, these MFs independent conditions are too conservative due to the lack of MFs information. In this section, an improved IT2MFD analysis method is proposed to process nonlinear MFs and introduce their information into the analysis. The first step is to integrate the mismatched premise variables, then the multi-dimensional type-1 MFs are transformed into single-dimensional interval type-2 MFs; the second step is to perform IT2MFD stability analysis and positivity analysis by introducing the IT2 MFs information into the resultant conditions.

A. Integration of Mismatched Premise Variables

There are two types of MFs, $m_j(\mathbf{y}_{t_k})$ and $w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})$ in the stability conditions (45)-(47) and positivity conditions (54)-(59). To handle these nonlinear MFs, the piecewise linear membership functions dependent method [23] can be employed. However, this method treats difference premise variables as independent system symbols, which will ignore the relationship between the premise variables $\mathbf{y}$ and $\mathbf{y}_{t_k}$, resulting in increased conservatism and computational burden of analysis results. Thus, this MFD method is improved in this paper by establishing the relationship between these two premise variables. With the help of the time delay variable $\mathbf{y}_\tau$ and the event-triggered condition (7), the relationship between $\mathbf{y}$ and $\mathbf{y}_{t_k}$ are deeply excavated by the following algorithm:

Algorithm 1:

- ▶ The relationship between $\mathbf{y}$ and $\mathbf{y}_\tau$:
  According to the system dynamics, it can be found that $\mathbf{y} - \mathbf{y}_\tau = \int_{t-\tau(t)}^{t} \dot{\mathbf{y}}(s)ds$. Denote $\dot{\mathbf{y}}(\xi) \in \Re^m$ and $\dot{\mathbf{y}}_{max} \in \Re^m$ as one of value and the maximal of $\dot{\mathbf{y}}$ during the control process, respectively, then it can be included based on (4) that $\int_{t-\tau(t)}^{t} \dot{\mathbf{y}}(s) = (\mathbf{y} - \mathbf{y}_\tau) = \tau(t)\dot{\mathbf{y}}(\xi) \leq \tau_M \dot{\mathbf{y}}_{max}$. Thus, $\mathbf{y}_\tau$ is estimated to be in an interval about $\mathbf{y}$, that is, $\mathbf{y}_\tau \in [\mathbf{y} - \tau_M\dot{\mathbf{y}}_{max} \quad \mathbf{y} + \tau_M\dot{\mathbf{y}}_{max}]$.
- ▶ The relationship between $\mathbf{y}$ and $\mathbf{y}_{t_k}$:
  Based on the definition (5) and the event-triggered condition (7), it can be obtained that $\frac{1}{1+\theta_1}\min\{\mathbf{y}_\tau\} \preceq \mathbf{y}_{t_k} \preceq \frac{1}{1-\theta_2}\max\{\mathbf{y}_\tau\}$. Combining the relationship between $\mathbf{y}$ and $\mathbf{y}_\tau$, the value of $\mathbf{y}_{t_k}$ is estimated to be in an interval about $\mathbf{y}$, that is, $\mathbf{y}_{t_k} \in [\frac{1}{1+\theta_1}(\mathbf{y} - \tau_M\dot{\mathbf{y}}_{max}) \quad \frac{1}{1-\theta_2}(\mathbf{y} + \tau_M\dot{\mathbf{y}}_{max})]$.

Denote interval $[\frac{1}{1+\theta_1}(\mathbf{y} - \tau_M\dot{\mathbf{y}}_{max}) \quad \frac{1}{1-\theta_2}(\mathbf{y} + \tau_M\dot{\mathbf{y}}_{max})]$ as $\mathbf{\Phi}_{\mathbf{y}_{t_k}}(\mathbf{y})$. According to the relationship between the premise variables established above, the type-1 MFs $m_j(\mathbf{y}_{t_k})$ and $w_i(\mathbf{y})m_j(\mathbf{y}_{t_k})$ can be represented by the following single-dimensional interval type-2 MFs, respectively:

$$m_j(\mathbf{y}_{t_k}) = \bar{m}_j(\mathbf{y}) \equiv m_j(\mathbf{\Phi}_{\mathbf{y}_{t_k}}(\mathbf{y})), \tag{71}$$

$$w_i(\mathbf{y})m_j(\mathbf{y}_{t_k}) = h_{ij}(\mathbf{y}) \equiv w_i(\mathbf{y})m_j(\mathbf{\Phi}_{\mathbf{y}_{t_k}}(\mathbf{y})). \tag{72}$$

After the above transformation, the relationship between two premise variables $\mathbf{y}$ and $\mathbf{y}_{t_k}$ are established, more exact MFs information can be obtained in the following subsection to help reduce the conservatism and computational burden.

B. IT2MFD Positivity and Stability Analysis

As shown in Fig 1, after the transformation of MF in subsection IV-A, the multi-dimensional type-1 MF shown in the subfigure (a) is transformed into single-dimensional interval type-2 MF shown in the subfigure (b), where the dashed black line is the upper MF and the solid black line is the lower MF. In the following, the IT2MFD analysis method is employed to handle these interval type-2 MFs and derive the IT2MFD analysis results. The analysis process for the conditions containing MF $h_{ij}(\mathbf{y})$ is presented as follows.

Firstly, an embedded type-1 MF $\tilde{h}_{ij}(\mathbf{y})$ is randomly selected in the footprint of uncertainty of $h_{ij}(\mathbf{y})$ as the reference embedded type-1 MF, just like the dotted line in Fig 1, and the piecewise linear membership function approximation method is applied on this reference embedded type-1 MF, then its approximate form is represented as follows, as shown by the blue solid line in Fig 1,

$$\begin{aligned}\hat{\tilde{h}}_{ij}(\mathbf{y}) &= \sum_{\varsigma=1}^{K}\psi_\varsigma(\mathbf{y})\hat{\tilde{h}}_{ij\varsigma}(\mathbf{y}) \\ &= \sum_{\varsigma=1}^{K}\psi_\varsigma(\mathbf{y})\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_{\hat{n}}=1}^{2}\prod_{r=1}^{\hat{n}}\upsilon_{ri_r\varsigma}(y_{i_r})\chi_{iji_1i_2\dots i_{\hat{n}}},\end{aligned} \tag{73}$$

where K represents the number of the subspaces into which the whole operation domain is divided when $\tilde{h}_{ij}(\mathbf{y})$ is approximated; $\psi_\varsigma(\mathbf{y})$ is a boolean function that indicates whether the premise variables are in the ς-th operation subspaces; the predefined interpolation functions $\upsilon_{ri_r\varsigma}(\mathbf{y})$ satisfies properties $0 < \upsilon_{ri_r\varsigma}(\mathbf{y}) < 1$ and $\upsilon_{r1\varsigma}(\mathbf{y}) + \upsilon_{r2\varsigma}(\mathbf{y}) = 1$; $\chi_{iji_1i_2\dots i_{\hat{n}}}$ denotes the values of $\tilde{h}_{ij}(\mathbf{y})$ at the interpolation point $\mathbf{y} = [y_{i_1}, y_{i_2}, \dots, y_{i_{\hat{n}}}]$.

Then, the approximation error of the IT2 MF $h_{ij}(\mathbf{y})$ in the ς-th operation subspace is defined as $\Delta h_{ij\varsigma} = h_{ij\varsigma}(\mathbf{y}) - \hat{\tilde{h}}_{ij\varsigma}(\mathbf{y})$, and its upper and lower bounds are denoted as $\overline{\vartheta}_{ij\varsigma}$ and $\underline{\vartheta}_{ij\varsigma}$, respectively, just like the 'Uerror' and 'Lerror' in Fig 1.

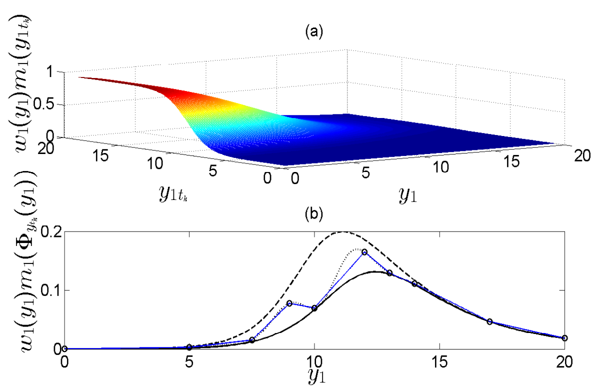

Fig. 1. The subfigure (a) is the membership function before the integration of premise variables, and the subfigure (b) is the membership function after the integration of premise variables.

Finally, with the help of the S-Procedure technology, the above approximated MFs information and the premise variables subspace boundaries are included in the analysis results, so that the non-convex stability condition (47) can be guaranteed by the following convex IT2MFD stability condition:

$$\sum_{\zeta=1}^{K}\psi_\varsigma(\mathbf{y})\sum_{i=1}^{p}\sum_{j=1}^{c}[(\hat{\tilde{h}}_{ij\varsigma}(\mathbf{y}) + \underline{\vartheta}_{ij\varsigma})\mathbf{\Xi}_{3ij}(\mathbf{x}, \mathbf{x}_f) + (\overline{\vartheta}_{ij\varsigma} - \underline{\vartheta}_{ij\varsigma})\mathbf{\Omega}_{3ij}(\mathbf{x}, \mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_{\hat{n}}=1}^{2}\times$$

$$\prod_{r=1}^{\hat{n}} \upsilon_{rir\varsigma}(y_{i_r})(y_{\hat{r}} - y_{\hat{r}\varsigma min})(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{R}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \prec 0. \tag{74}$$

where $\mathbf{\Omega}_{3ij}(\mathbf{x}, \mathbf{x}_f)$ and $\mathbf{R}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f)$ are the slack matrices which satisfy $\mathbf{\Omega}_{3ij}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{\Omega}_{3ij}(\mathbf{x}, \mathbf{x}_f) \succ \mathbf{\Xi}_{3ij}(\mathbf{x}, \mathbf{x}_f)$ and $\mathbf{R}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$. $\hat{n}$ is the number of the system state variables on which the membership function $\hat{\hat{h}}_{ij}$; $y_{\hat{r}}$ is the $\hat{r}$-th output; $y_{\hat{r}\varsigma max}$ and $y_{\hat{r}\varsigma min}$ are the maximum value and minimum value of $y_{\hat{r}}$ in the ς-th operation subspace, respectively.

According to the definition (73), the stability condition (74) can be equivalent to SOS sub-condition 6) in Theorem 2, which means that the overall stability of the system can be guaranteed only by system stability at all sample points. The derivation of other IT2MFD stability conditions follow the same line as the above stability condition, so the derivation process is omitted, and the obtained stability conditions all are summarized in Theorem 2.

In addition to IT2MFD stability conditions, convex IT2MFD positivity conditions also are obtained by following the similar line. Take positivity condition (59) as an example, the approximated MFs information, approximation error and subspace boundaries information of premise variables are adopted to derive the IT2MFD positivity conditions, then the convex form of the condition (59) is obtained as follows:

$$\sum_{\zeta=1}^{K} \psi_\varsigma(\mathbf{y}) \sum_{i=1}^{p}\sum_{j=1}^{c}[(\hat{\hat{h}}_{ij\varsigma}(\mathbf{y}) + \overline{\vartheta}_{ij\varsigma})\check{\check{\mathfrak{B}}}_{ij}(\mathbf{x}, \mathbf{x}_f) +$$
$$(\underline{\vartheta}_{ij\varsigma} - \overline{\vartheta}_{ij\varsigma})\mathbf{\Gamma}_{3ij}(\mathbf{x}, \mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\dots\sum_{i_{\hat{n}}=1}^{2} \times$$
$$\prod_{r=1}^{\hat{n}} \upsilon_{rir\varsigma}(y_{i_r})(y_{\hat{r}} - y_{\hat{r}\varsigma min})(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{G}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0. \tag{75}$$

where $\mathbf{\Gamma}_{3ij}(\mathbf{x}, \mathbf{x}_f)$ and $\mathbf{G}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f)$ are the slack matrices which satisfy $\mathbf{\Gamma}_{3ij}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{\Gamma}_{3ij}(\mathbf{x}, \mathbf{x}_f) \succ \check{\check{\mathfrak{B}}}_{ij}(\mathbf{x}, \mathbf{x}_f)$ and $\mathbf{G}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$. Considering the positivity of the $\psi_\varsigma(\mathbf{y})$ and $\upsilon_{rir\varsigma}(y_{i_r})$, the equivalent SOS condition of the positivity condition (75) is expressed as sub-condition 12) in Theorem 2. In addition, the other SOS positivity conditions also are summarized in the Theorem 2. IT2 MF $\bar{m}_j(\mathbf{y})$ in other stability conditions is handled by the same method as IT2 MF $h_{ij}(\mathbf{y})$, $\underline{\iota}_{j\varsigma}$ and $\overline{\iota}_{j\varsigma}$ are the lower and upper bounds of approximation error $\Delta\bar{m}_j = \bar{m}_j(\mathbf{y}) - \hat{\hat{m}}_j(\mathbf{y})$, and embedded type-1 MF $\hat{\hat{m}}_j(\mathbf{y})$ is approximated as follows:

$$\hat{\hat{m}}_j(\mathbf{y}) = \sum_{\varsigma=1}^{K}\psi_\varsigma(\mathbf{y})\hat{\hat{m}}_{j\varsigma}(\mathbf{y})$$
$$= \sum_{\varsigma=1}^{K}\psi_\varsigma(\mathbf{y})\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\dots\sum_{i_{\hat{n}}=1}^{2}\prod_{r=1}^{\hat{n}}\upsilon_{rir\varsigma}(y_{i_r})\pi_{ji_1i_2\dots i_{\hat{n}}}. \tag{76}$$

Theorem 2: For a positive PPFMB system (1), an event-triggered positive polynomial filter can be designed to make the polynomial fuzzy filter error system (10) asymptotically stable and positive while achieving optimal performance γ, if given the predefined scalars $\theta_1, \theta_2, \sigma$, and there exist Lyapunov vectors $\boldsymbol{\lambda} \succ 0$, $\boldsymbol{\delta}_1 \succ 0$, $\boldsymbol{\delta}_2 \succ 0$, and polynomial slack matrices $\mathbf{\Omega}_{1ij}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{\Omega}_{2j}(\mathbf{x}_f) \succ 0$, $\mathbf{\Omega}_{3ij}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{R}_{1\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{R}_{2\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{R}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{\Gamma}_{1ij}(\mathbf{x}, \mathbf{x}_f)$ is Metzler matrix, $\mathbf{\Gamma}_{2j}(\mathbf{x}_f) \succ 0$, $\mathbf{\Gamma}_{3ij}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{\Gamma}_{4ij}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{\Gamma}_{5j}(\mathbf{x}_f) \succ 0$, $\mathbf{\Gamma}_{6ij}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{G}_{1\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f)$ is Metzler matrix, $\mathbf{G}_{2\hat{r}\varsigma}(\mathbf{x}_f) \succ 0$, $\mathbf{G}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{G}_{4\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$, $\mathbf{G}_{5\hat{r}\varsigma}(\mathbf{x}_f) \succ 0$, $\mathbf{G}_{6\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f) \succ 0$, and polynomial filter decision matrices $\mathbf{A}_{\lambda fj}(\mathbf{x}_f)$ being Metzler matrix, $\mathbf{B}_{\lambda fj}(\mathbf{x}_f) \succ 0$, $\mathbf{C}_{fj}(\mathbf{x}_f) \succ 0$, $\mathbf{D}_{fj}(\mathbf{x}_f) \succ 0$, such that the following SOS-based conditions of optimization problem are satisfied:

$$\min \gamma$$
$$s.t.$$

1) $\nu^T(\mathbf{\Omega}_{1ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f) - \mathbf{\Xi}_{1ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f) - \varepsilon_1)\nu$ is SOS, $\forall\beta \in \underline{2n}$;

2) $$-\nu^T\{\sum_{i=1}^{p}\sum_{j=1}^{c}[(\chi_{ij\varsigma} + \underline{\vartheta}_{ij\varsigma})\mathbf{\Xi}_{1ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f)$$
$$+ (\overline{\vartheta}_{ij\varsigma} - \underline{\vartheta}_{ij\varsigma})\mathbf{\Omega}_{1ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})\times$$
$$(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{R}_{1\hat{r}\varsigma}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f) - \varepsilon_2\}\nu \text{ is SOS, } \forall\beta \in \underline{2n};$$

3) $\nu^T(\mathbf{\Omega}_{2j}^{(1,\beta)}(\mathbf{x}_f) - \mathbf{\Xi}_{2j}^{(1,\beta)}(\mathbf{x}_f) - \varepsilon_3)\nu$ is SOS, ; $\forall\beta \in \underline{2n}$;

4) $$-\nu^T\{\sum_{j=1}^{c}[(\pi_{j\varsigma} + \underline{\iota}_{j\varsigma})\mathbf{\Xi}_{2j}^{(1,\beta)}(\mathbf{x}_f) + (\overline{\iota}_{j\varsigma} - \underline{\iota}_{j\varsigma})\times$$
$$\mathbf{\Omega}_{2j}^{(1,\beta)}(\mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})(y_{\hat{r}\varsigma max} - y_{\hat{r}})\times$$
$$\mathbf{R}_{2\hat{r}\varsigma}^{(1,\beta)}(\mathbf{x}_f) - \varepsilon_4\}\nu \text{ is SOS, } \forall\beta \in \underline{2n};$$

5) $\nu^T(\mathbf{\Omega}_{3ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f) - \mathbf{\Xi}_{3ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f) - \varepsilon_5)\nu$ is SOS, $\alpha \in \underline{n}$;

6) $$-\nu^T\{\sum_{i=1}^{p}\sum_{j=1}^{c}[(\chi_{ij\varsigma} + \underline{\vartheta}_{ij\varsigma})\mathbf{\Xi}_{3ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f)$$
$$+ (\overline{\vartheta}_{ij\varsigma} - \underline{\vartheta}_{ij\varsigma})\mathbf{\Omega}_{3ij}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})\times$$
$$(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{R}_{3\hat{r}\varsigma}^{(1,\beta)}(\mathbf{x}, \mathbf{x}_f) - \varepsilon_6\}\nu \text{ is SOS, } \forall\beta \in \underline{2n};$$

7) $\nu^T(\mathbf{\Gamma}_{1ij}^{(\alpha,\beta)}(\mathbf{x}, \mathbf{x}_f) - \check{\check{\mathfrak{A}}}_{ij}^{(\alpha,\beta)}(\mathbf{x}, \mathbf{x}_f) - \varepsilon_7)\nu$ is SOS, $\forall\alpha \neq \beta \in \underline{2n}$;

8) $$\nu^T\{\sum_{i=1}^{p}\sum_{j=1}^{c}[(\chi_{ij\varsigma} + \overline{\vartheta}_{ij\varsigma})\check{\check{\mathfrak{A}}}_{ij}^{(\alpha,\beta)}(\mathbf{x}, \mathbf{x}_f)$$
$$+ (\underline{\vartheta}_{ij\varsigma} - \overline{\vartheta}_{ij\varsigma})\mathbf{\Gamma}_{1ij}^{(\alpha,\beta)}(\mathbf{x}, \mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})\times$$
$$(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{G}_{1\hat{r}\varsigma}^{(\alpha,\beta)}(\mathbf{x}, \mathbf{x}_f) - \varepsilon_8\}\nu \text{ is SOS, } \forall\alpha \neq \beta \in \underline{2n};$$

9) $\nu^T(\mathbf{\Gamma}_{2j}^{(\alpha,\beta)}(\mathbf{x}_f) - \check{\check{\mathfrak{A}}}_{\tau j}^{(\alpha,\beta)}(\mathbf{x}_f) - \varepsilon_9)\nu$ is SOS, $\forall\alpha \in \underline{n}, \beta \in \underline{n}$;

10) $$\nu^T\{\sum_{j=1}^{c}[(\pi_{j\varsigma} + \overline{\iota}_{j\varsigma})\check{\check{\mathfrak{A}}}_{\tau j}^{(\alpha,\beta)}(\mathbf{x}_f) + (\underline{\iota}_{j\varsigma} - \overline{\iota}_{j\varsigma})\times$$

$$\Gamma_{2j}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})(y_{\hat{r}\varsigma max} - y_{\hat{r}})\times$$
$$\mathbf{G}_{2\hat{r}\varsigma}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{10}\}\nu \text{ is SOS, } \forall\alpha\in\underline{2n},\beta\in\underline{n};$$
$$11)\ \nu^T(\Gamma_{3ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \check{\tilde{\mathfrak{B}}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{11})\nu \text{ is SOS,}$$
$$\forall\alpha\in\underline{n+1},\beta\in\underline{2g};$$
$$12)\ \nu^T\{\sum_{i=1}^{p}\sum_{j=1}^{c}[(\chi_{ij\varsigma} + \overline{\vartheta}_{ij\varsigma})\check{\tilde{\mathfrak{B}}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)$$
$$+ (\underline{\vartheta}_{ij\varsigma} - \overline{\vartheta}_{ij\varsigma})\Gamma_{3ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})\times$$
$$(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{G}_{3\hat{r}\varsigma}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{12}\}\nu \text{ is SOS,}$$
$$\forall\alpha\in\underline{n+1},\beta\in\underline{2g};$$
$$13)\ \nu^T(\Gamma_{4ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \tilde{\mathbf{C}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{13})\nu \text{ is SOS,}$$
$$\forall\alpha\in\underline{m},\beta\in\underline{2n};$$
$$14)\ \nu^T\{\sum_{i=1}^{p}\sum_{j=1}^{c}[(\chi_{ij\varsigma} + \overline{\vartheta}_{ij\varsigma})\tilde{\mathbf{C}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)$$
$$+ (\underline{\vartheta}_{ij\varsigma} - \overline{\vartheta}_{ij\varsigma})\Gamma_{4ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})\times$$
$$(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{G}_{4\hat{r}\varsigma}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{14}\}\nu \text{ is SOS,}$$
$$\forall\alpha\in\underline{m},\beta\in\underline{2n};$$
$$15)\ \nu^T(\Gamma_{5j}^{(\alpha,\beta)}(\mathbf{x}_f) - \check{\tilde{\mathbf{C}}}_{\tau j}^{(\alpha,\beta)}(\mathbf{x}_f) - \varepsilon_{15})\nu \text{ is SOS,}$$
$$\forall\alpha\in\underline{q},\beta\in\underline{n};$$
$$16)\ \nu^T\{\sum_{j=1}^{c}[(\pi_{j\varsigma} + \overline{\iota}_{j\varsigma})\check{\tilde{\mathbf{C}}}_{\tau j}^{(\alpha,\beta)}(\mathbf{x}_f) + (\underline{\iota}_{j\varsigma} - \overline{\iota}_{j\varsigma})\times$$
$$\Gamma_{5j}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})(y_{\hat{r}\varsigma max} - y_{\hat{r}})\times$$
$$\mathbf{G}_{5\hat{r}\varsigma}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{16}\}\nu \text{ is SOS, } \forall\alpha\in\underline{q},\beta\in\underline{n};$$
$$17)\ \nu^T(\Gamma_{6ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \check{\tilde{\mathbf{D}}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{17})\nu \text{ is SOS,}$$
$$\forall\alpha\in\underline{m},\beta\in\underline{2g};$$
$$18)\ \nu^T\{\sum_{i=1}^{p}\sum_{j=1}^{c}[(\chi_{ij\varsigma} + \overline{\vartheta}_{ij\varsigma})\check{\tilde{\mathbf{D}}}_{ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)\times$$
$$+ (\underline{\vartheta}_{ij\varsigma} - \overline{\vartheta}_{ij\varsigma})\Gamma_{6ij}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f)] + \sum_{\hat{r}=1}^{\hat{n}}(y_{\hat{r}} - y_{\hat{r}\varsigma min})\times$$
$$(y_{\hat{r}\varsigma max} - y_{\hat{r}})\mathbf{G}_{6\hat{r}\varsigma}^{(\alpha,\beta)}(\mathbf{x},\mathbf{x}_f) - \varepsilon_{18}\}\nu \text{ is SOS,}$$
$$\forall\alpha\in\underline{m},\beta\in\underline{2g}; \tag{77}$$

The filter matrices are obtained as $\mathbf{A}_{fj}(\mathbf{x}_f) = [\frac{\mathbf{A}_{\lambda fj}^{(1,:)}(\mathbf{x}_f)}{\lambda_2^{(1)}}; \frac{\mathbf{A}_{\lambda fj}^{(2,:)}(\mathbf{x}_f)}{\lambda_2^{(2)}}; \frac{\mathbf{A}_{\lambda fj}^{(n,:)}(\mathbf{x}_f)}{\lambda_2^{(n)}}]$, $\mathbf{B}_{fj}(\mathbf{x}_f) = [\frac{\mathbf{B}_{\lambda fj}^{(1,:)}(\mathbf{x}_f)}{\lambda_2^{(1)}}; \frac{\mathbf{B}_{\lambda fj}^{(2,:)}(\mathbf{x}_f)}{\lambda_2^{(2)}}; \frac{\mathbf{B}_{\lambda fj}^{(n,:)}(\mathbf{x}_f)}{\lambda_2^{(n)}}]$, $\mathbf{C}_{fj}(\mathbf{x}_f)$ and $\mathbf{D}_{fj}(\mathbf{x}_f)$.

Remark 3: In Theorem 2, sub-conditions 1)-6) are stability conditions, and sub-conditions 7)-18) are positivity conditions. It should be note that the positivity of the filter should be guaranteed by Metzler matrix $\sum_{j=1}^{p} m_j(\mathbf{y})\mathbf{A}_{fj}(\mathbf{x}_f)$, positivity matrices $\sum_{j=1}^{p} m_j(\mathbf{y})\mathbf{B}_{fj}(\mathbf{x}_f)$, $\sum_{j=1}^{p} m_j(\mathbf{y})\mathbf{C}_{fj}(\mathbf{x}_f)$, and $\sum_{j=1}^{p} m_j(\mathbf{y})\mathbf{D}_{fj}(\mathbf{x}_f)$. However, considering the computational burden of the results, they are not handled by the IT2MFD method, but degenerate to MF independent conditions: $\mathbf{A}_{\lambda fj}(\mathbf{x}_f)$ is Metzler matrix, $\mathbf{B}_{\lambda fj}(\mathbf{x}_f) \succ 0$, $\mathbf{C}_{fj}(\mathbf{x}_f) \succ 0$, $\mathbf{D}_{fj}(\mathbf{x}_f) \succ 0$, $\forall j \in \underline{c}$.

V. Simulation

In this section, a simulation example is adopted to illustrate the effectiveness of the proposed design strategy of event-triggered polynomial fuzzy filter.

Considering a three-rules polynomial fuzzy model based system, the system and input matrices are as follows:

$$\mathbf{A}_1(x_1) = \begin{bmatrix} -6 - 0.7x_1^2 & 0.5 \\ 0.35 & -6 - 1.2x_1^2 - x_1 \end{bmatrix},$$
$$\mathbf{A}_2(x_1) = \begin{bmatrix} -5 - 1.1x_1^2 & 0.45 \\ 0.3 & -5 - 0.5x_1^2 - x_1 \end{bmatrix},$$
$$\mathbf{A}_3(x_1) = \begin{bmatrix} -5.3 - 0.9x_1^2 & 0.5 \\ 0.32 & -5.5 - 0.9x_1^2 - x_1 \end{bmatrix},$$
$$\mathbf{B}_1(x_1) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad \mathbf{B}_2(x_1) = \begin{bmatrix} 0.2 \\ 0.25 \end{bmatrix},$$
$$\mathbf{B}_3(x_1) = \begin{bmatrix} 0.16 \\ 0.2 \end{bmatrix}, \quad \mathbf{C}(x_1) = \begin{bmatrix} 0.3 + 0.01x_1^2 & 0.4 \end{bmatrix},$$
$$\mathbf{D}(x_1) = 0.2, \quad \mathbf{E}_1(x_1) = [0.2 \quad 0.1], \quad \mathbf{F}_1(x_1) = 0.3,$$
$$\mathbf{E}_2(x_1) = [0.2 \quad 0.2], \quad \mathbf{F}_2(x_1) = 0.4,$$
$$\mathbf{E}_3(x_1) = [0.35 \quad 0.25], \quad \mathbf{F}_3(x_1) = 0.37.$$

The disturbance is chosen as $\mathbf{w} = 0.5e^{-t}|cos(2t)|$. The MFs of the polynomial fuzzy model based system are $w_1(y_1) = 1 - \frac{1}{1+e^{-(y_1-8)/3}}$, $w_3(y_1) = \frac{1}{1+e^{-(y_1-12)/3}}$, and $w_2(y_1) = 1 - w_1(y_1) - w_3(y_1)$, where $y_1 = (0.3 + 0.01x_1^2)x_1 + 0.2\mathbf{w}$. According to Lemma 1, the above system is a PPFMB system.

A. The Effectiveness of the Built Augmented System and Proposed Novel LCLF

In order to verify the effectiveness of the built augmented system and the proposed LCLF in filter design, the Theorem 1 is employed to calculate the values of filter system matrices $\mathbf{A}_{fj}, \mathbf{B}_{fj}, \mathbf{C}_{fj}, \mathbf{D}_{fj}$ in event-triggered polynomial fuzzy filter (9). In this paper, the MFs of the polynomial fuzzy filter are allowed to be different from the MFs of the PPFMB systems, and they are chosen as $m_1(y_{1t_k}) = 1 - \frac{1}{1+e^{y_{1t_k}-10}}$ and $m_2(y_{1t_k}) = 1 - m_1(y_{1t_k})$. When the Theorem 1 is used to calculate the decision variables, the event-triggered coefficients θ_1 and θ_2 are chosen as 0.28 and 0.26, the disturbance weight coefficient is chosen as $\sigma = 0.72$, $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_9$ all are set as 10^{-3}, the variable $\boldsymbol{\nu}$ is defined as a system symbol. Then, the filter system matrices and the Lyapunov vectors are obtained, and they are shown in supplementary file due to page limit. Meanwhile, the minimum allowable performance index is optimized to $\gamma = 0.88$.

The time response of system states, signal to be estimated, filter states and estimated signal are shown in Fig. 2, 3 and 4. In each figure, the black curves represent the system states and the signal to be estimated, the red curves represent the filter states in Fig 2 and 3, and the estimated signal in Fig

4. It can be founded that the system and the designed filter are asymptotic stability and positivity, and the signal $\mathbf{z}$ can be roughly estimated by the signal $\mathbf{z}_f$, which means that the filter design method based on the built augmented system and the proposed novel LCLF are effective.

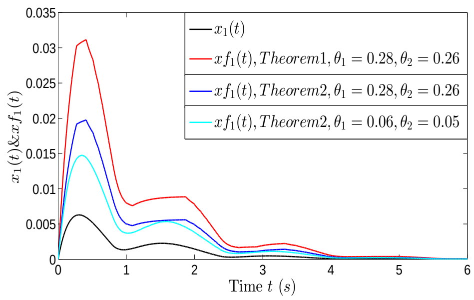

Fig. 2. Time response of system state $x_1(t)$ (black curve) and filter state $x_{f1}(t)$ ('red curve', 'blue curve' and 'cyan curve' represent the states of filters designed by Theorem 1 with $\theta_1, \theta_2 = 0.28, 0.26$, Theorem 2 with $\theta_1, \theta_2 = 0.28, 0.26$ and Theorem 2 with $\theta_1, \theta_2 = 0.06, 0.05$, respectively).

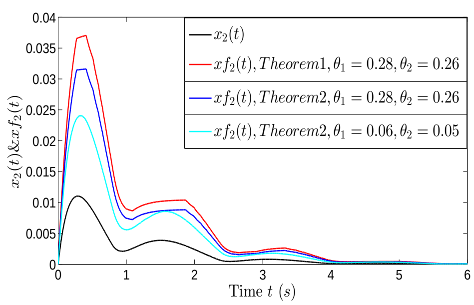

Fig. 3. Time response of system state $x_2(t)$ (black curve) and filter state $x_{f2}(t)$ ('red curve', 'blue curve' and 'cyan curve' represent the states of filters designed by Theorem 1 with $\theta_1, \theta_2 = 0.28, 0.26$, Theorem 2 with $\theta_1, \theta_2 = 0.28, 0.26$ and Theorem 2 with $\theta_1, \theta_2 = 0.06, 0.05$, respectively).

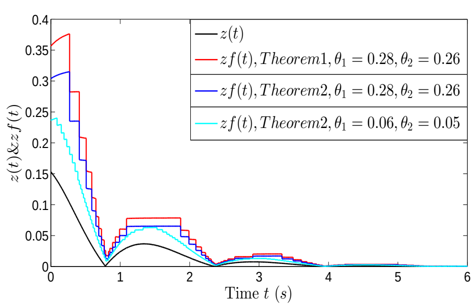

Fig. 4. Time response of signal $z(t)$ (black curve) and the estimated signal $z_f(t)$ ('red curve', 'blue curve' and 'cyan curve' represent the estimated output signals obtained by Theorem 1 with $\theta_1, \theta_2 = 0.28, 0.26$, Theorem 2 with $\theta_1, \theta_2 = 0.28, 0.26$ and Theorem 2 with $\theta_1, \theta_2 = 0.06, 0.05$, respectively).

B. The Effectiveness and Superiority of the Improved IT2MFD Method with Premise Integration

The above results are obtained by calculating Theorem 1, the mismatched premise variables information and MF information are ignored. In order to verify the validity of the proposed IT2MFD analysis method in optimizing performance index, the mismatched premise variables are integrated by the Algorithm 1, the premise variable y_{1t_k} is represented by the interval of the premise variable y_1 as follows:

$$\mathbf{\Phi}_{\mathbf{y}_{t_k}}(y_1) \in [\frac{1}{1+\theta_1}(y_1 - \tau_M \dot{y}_{1max}) \quad \frac{1}{1-\theta_2}(y_1 + \tau_M \dot{y}_{1max})]$$

where event-triggered coefficients θ_1 and θ_2 are chosen as 0.28 and 0.26, respectively, and $\dot{y}_{1max}$ are estimated as 5; the sampling period is $h = 0.02s$, the maximum communication delay is $\bar{\tau} = 0.01s$, and $\tau_M = \bar{\tau} + h$. Based on the above transformation, the premise variables of plant model and filter are integrated into one premise variable, and the initial type-1 filter MFs $m_1(y_{1t_k})$ and $m_2(y_{1t_k})$ in subsection V-A are transformed into the following IT2 MFs:

$$m_1(\mathbf{\Phi}_{\mathbf{y}_{t_k}}(y_1)) = 1 - \frac{1}{1 + e^{\frac{1}{\mathbf{\Phi}_{\mathbf{y}_{t_k}}(y_1) - 10}}},$$
$$m_2(\mathbf{\Phi}_{\mathbf{y}_{t_k}}(y_1)) = 1 - m_1(\mathbf{\Phi}_{\mathbf{y}_{t_k}}(y_1)),$$

In order to verify the effectiveness of the proposed improved IT2MFD method with premise integration in optimizing performance index, the Theorem 2 is employed to design the filter gains and other decision variables, the degree of the slack matrices $\mathbf{\Gamma}_{3ij}(\mathbf{x}, \mathbf{x}_f)$, $\mathbf{\Gamma}_{6ij}(\mathbf{x}, \mathbf{x}_f)$, $\mathbf{G}_{3\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f)$ and $\mathbf{G}_{6\hat{r}\varsigma}(\mathbf{x}, \mathbf{x}_f)$ are chosen as 0, the degree of the other slack matrices are chosen as 0 to 2. When the embedded MFs are approximated, the sample points are chosen at $y_1 \in \{0, 4, 8, \ldots, 20, \ldots\}$. The disturbance weight coefficient is chosen as $\sigma = 0.72$, $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{18}$ all are set as 10^{-3}. Then, the Lyapunov vectors and the filter system matrices are obtained, and they are shown in supplementary file due to page limit. Meanwhile, the minimum allowable performance index is optimized to $\gamma = 0.68$.

The above results have shown that the proposed improved IT2MFD method with premise integration is helpful to reduce the performance index γ. In order to verify the effectiveness of these analysis methods more intuitively, the time response of filter states $x_{f1}(t)$ and $x_{f2}(t)$ are shown with blue curves in Figs.2 and 3, the estimated output signal $z_f(t)$ is shown with blue curve in Fig. 4. The comparison of red curve and blue curve in Fig. 4 shows the error between signal $\mathbf{z}$ and the estimated signal $\mathbf{z}_f$ is significantly reduced by using the improved IT2MFD method with premise integration, which more directly reflects the advantage of this improved method in improving filter performance.

C. The Relationship between the Network Bandwidth Saving Efficiency and Performance Index

The event-triggered mechanism is used to design filter in this paper for saving the network bandwidth resource, the output signals only be released when the event-triggered condition is violated. For the simulation in subsection V-B, the sampled-data release instants and the amplitude of the output signals are shown in Fig. 5 (a), only 21% output signals are transmitted from the sensor side to the filter side, the networked bandwidth resources are saved.

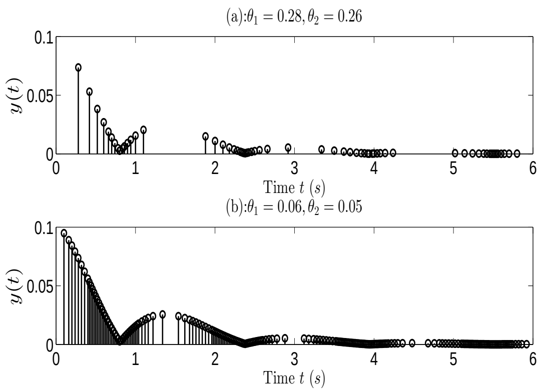

Fig. 5. Release instants and amplitude of the output signals.

To explore the relationship between the network bandwidth saving efficiency and performance index, the event-triggered coefficients θ_1 and θ_2 are set as 0.06 and 0.05, other parameter settings are the same with them in subsection V-B. Then the optimized performance index $\gamma = 0.28$ can be obtained by Theorem 2, the obtained filter matrices and Lyapunov vectors are shown in supplementary file due to page limiit. The time responses of system states, filter states and estimated signal are shown with cyan curves in Figs 2, 3, 4. The sampled-data release instants and the amplitude of the output signals are shown in Fig. 5 (b). In this case, 62% output signals are transmitted from the sensor side to the filter side. The comparison of blue curve and cyan curve in each Fig 2, 3, 4 and the comparison of (a) and (b) in Fig. 5 show that the estimated performance is improved by reduce the values of event-triggered coefficients, but the transmitted data will increase, which means less network bandwidth resources saving efficiency.

VI. Conclusion

In this paper, an event-triggered positive fuzzy filter was designed for PPFMB systems under the comprehensive consideration of disturbance of output signal, transmission delay and positive constrains. In view of the above system characteristics, an augmented system containing the filter error system was reconstructed under a novel linear event-triggered condition, also an novel LCLF was proposed to perform the stability analysis with the help of the upper bounds of the augmented system, and the necessary resultant conditions for filter design were obtained along with the implementation of positivity analysis. In addition, the mismatched premise variables caused by event-triggered mechanism were integrated into interval type-2 premise variables, and the IT2MFD method was utilized to further optimize the L_1 performance of the designed filter. The simulation results have verified that the bandwidth resources can be saved through the designed event-triggered condition, and the positive fuzzy filter designed by the filter design strategies proposed in this paper has good performance index. In the future, other network control related issues, such as quantization error, network attacks, will be considered, and more effective membership function dependent methods will be proposed to reduce the influence of these network-induced problems on control performance.

References

[1] W. Haddad, V. Chellaboina, and Q. Hui, *Nonnegative and Compartmental Dynamical Systems*, 2010.

[2] X. Wang, Y. Tao, and X. Song, "Mathematical model for the control of a pest population with impulsive perturbations on diseased pest," *Applied Mathematical Modelling*, vol. 33, no. 7, pp. 3099–3106, 2009.

[3] R. Shorten, D. Leith, J. Foy, and R. Kilduff, "Analysis and design of aimd congestion control algorithms in communication networks," *Automatica*, vol. 41, no. 4, pp. 725–730, 2005.

[4] A. Benzaouia, F. Mesquine, M. Benhayoun, H. Schulte, and S. Georg, "Stabilization of positive constrained T-S fuzzy systems: Application to a buck converter," *Journal of the Franklin Institute*, vol. 351, no. 8, pp. 4111–4123, 2014.

[5] L. Farina and S. Rinaldi, *Positive linear systems: theory and applications*. John Wiley & Sons, 2011.

[6] M. Han, H. Lam, F. Liu, and Y. Tang, "More relaxed stability analysis and positivity analysis for positive polynomial fuzzy systems via membership functions dependent method," *Fuzzy Sets and Systems*, vol. 432, pp. 111–131, 2022.

[7] N. Otsuka and D. Kakehi, "Interval full-order switched positive observers for uncertain switched positive linear systems," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 6465–6470, 2020.

[8] H. Trinh, D. C. Huong, L. V. Hien, and S. Nahavandi, "Design of reduced-order positive linear functional observers for positive time-delay systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 64, no. 5, pp. 555–559, 2017.

[9] I. Ghous and J. Lu, "Robust observer design for two-dimensional discrete positive switched systems with delays," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 12, pp. 3297–3301, 2020.

[10] W. Qi, J. H. Park, G. Zong, J. Cao, and J. Cheng, "Filter for positive stochastic nonlinear switching systems with phase-type semi-markov parameters and application," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 4, pp. 2225–2236, 2022.

[11] A. Spagolla, C. F. Morais, R. C. L. F. Oliveira, and P. L. D. Peres, "Reduced order positive filter design for positive uncertain discrete-time linear systems," *IEEE Control Systems Letters*, vol. 6, pp. 1148–1153, 2022.

[12] E. Ahmadi, J. Zarei, and R. Razavi-Far, "Robust ℓ_1-controller design for discrete-time positive T–S fuzzy systems using dual approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 2, pp. 706–715, 2022.

[13] X. Li, C. Liu, H. K. Lam, F. Liu, and X. Zhao, "Stability analysis of fuzzy polynomial positive systems with time delay," in *2014 International Conference on Fuzzy Theory and Its Applications (iFUZZY2014)*, 2014, pp. 24–28.

[14] Z. Bao, H. K. Lam, Y. Peng, F. Liu, and K. Mehran, "Stability and stabilization of polynomial fuzzy-model-based switched nonlinear systems under mdadt switching signal," *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 1, pp. 1–13, 2023.

[15] A. Meng, H. K. Lam, Z. Wang, and F. Liu, "Static output feedback fault tolerant control and relaxed stability analysis of positive polynomial fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 9, pp. 3928–3939, 2022.

[16] M. Han, H. Lam, Y. Li, F. Liu, and C. Zhang, "Observer-based control of positive polynomial fuzzy systems with unknown time delay," *Neurocomputing*, vol. 349, pp. 77–90, 2019.

[17] L. Fu, H. K. Lam, F. Liu, B. Xiao, and Z. Zhong, "Static output-feedback tracking control for positive polynomial fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 6, pp. 1722–1733, 2022.

[18] M. Han, H. K. Lam, F. Liu, Y. Tang, B. Han, and H. Zhou, "Stabilization analysis and impulsive controller design for positive interval type-2 polynomial fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 9, pp. 3952–3966, 2022.

[19] D. Zhang, J. Xie, and B. Ning., "Network-based filtering for positive systems with random communication delays and deception attacks," *Neurocomputing*, vol. 400, pp. 450–457, 2020.

[20] P. Li, J. Lam, and Z. Shu, "H_∞ positive filtering for positive linear discrete-time systems: An augmentation approach," *IEEE Transactions on Automatic Control*, vol. 55, no. 10, pp. 2337–2342, 2010.

[21] S. Zhu, Q.-L. Han, and C. Zhang., "l_1-gain performance analysis and positive filter design for positive discrete-time markov jump linear systems: A linear programming approach," *Automatica*, vol. 50, no. 8, pp. 2098–2107, 2014.

[22] X. Chen, J. Lam, and H.-K. Lam., "Positive filtering for positive Takagi-Sugeno fuzzy systems under ℓ_1 performance," *Information Sciences*, vol. 299, pp. 32–41, 2015.

[23] A. Meng, H. K. Lam, F. Liu, and Y. Yang, "Filter design for positive T–S fuzzy continuous-time systems with time delay using piecewise-linear membership functions," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 9, pp. 2521–2531, 2021.

[24] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.

[25] L. Zhang, B. Wang, Y. Zheng, A. Zemouche, X. Zhao, and C. Shen, "Robust packetized MPC for networked systems subject to packet dropouts and input saturation with quantized feedback," *IEEE Transactions on Cybernetics*, 2022 (In press).

[26] E. Tian, Z. Wang, L. Zou, and D. Yue, "Chance-constrained H_∞ control for a class of time-varying systems with stochastic nonlinearities: The finite-horizon case," *Automatica*, vol. 107, pp. 296–305, 2019.

[27] J. Falk, H. Geppert, F. Durr, S. Bhowmik, and K. Rothermel, "Dynamic QoS-aware traffic planning for time-triggered flows in the real-time data plane," *IEEE Transactions on Network and Service Management*, vol. 19, no. 2, pp. 1807–1825, 2022.

[28] F. Cheng, H. Wang, L. Zhang, A. Ahmad, and N. Xu, "Decentralized adaptive neural two-bit-triggered control for nonstrict-feedback nonlinear systems with actuator failures," *Neurocomputing*, vol. 500, pp. 856–867, 2022.

[29] L. Ding, Q.-L. Han, X. Ge, and X.-M. Zhang, "An overview of recent advances in event-triggered consensus of multiagent systems," *IEEE Transactions on Cybernetics*, vol. 48, no. 4, pp. 1110–1123, 2018.

[30] I. Ahmed, M. Rehan, N. Iqbal, and C. K. Ahn, "A novel event-triggered consensus approach for generic linear multi-agents under heterogeneous sector-restricted input nonlinearities," *IEEE Transactions on Network Science and Engineering*, 2023 (In press).

[31] F. Tang, H. Wang, X.-H. Chang, L. Zhang, and K. H. Alharbi, "Dynamic event-triggered control for discrete-time nonlinear markov jump systems using policy iteration-based adaptive dynamic programming," *Nonlinear Analysis: Hybrid Systems*, vol. 49, p. 101338, 2023.

[32] J. Zhang, G. Zheng, Y. Feng, and Y. Chen, "Event-triggered state-feedback and dynamic output-feedback control of positive markovian jump systems with intermittent faults," *IEEE Transactions on Automatic Control*, 2022 (In press).

[33] F. Lin, J. Zhang, X. Jia, and X. Zhou., "Adaptive event-triggering distributed filter of positive markovian jump systems based on disturbance observer," *Journal of the Franklin Institute*, vol. 360, pp. 2507–2537, 2023.

[34] K. Yin and D. Yang, "Output feedback l_1 control of positive markov jump systems: A dynamic event-triggered method," *Journal of the Franklin Institute*, vol. 359, no. 8, pp. 3631–3655, 2022.

[35] S. Xiao, Y. Zhang, Q. Xu, and B. Zhang, "Event-triggered network-based ℓ_1-gain filtering for positive linear systems," *International Journal of Systems Science*, vol. 48, no. 6, pp. 1281–1290, 2017.

[36] S. Xiao, Y. Zhang, and B. Zhang, "Event-triggered network-based state observer design of positive systems," *Information Sciences*, vol. 469, pp. 30–43, 2018.

[37] X. Deng, J. Zhang, and T. Raissi., "Event-triggered positive l_1-gain non-fragile filter design for positive markov jump systems," *Information Sciences*, vol. 573, pp. 562–584, 2021.

[38] C. Zhang, J. Hu, J. Qiu, and Q. Chen, "Event-triggered nonsynchronized $\mathcal{H}_\infty$ filtering for discrete-time T-S fuzzy systems based on piecewise lyapunov functions," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 2330–2341, 2017.

[39] X.-L. Wang and G.-H. Yang, "Event-triggered H_∞ filtering for discrete-time T-S fuzzy systems via network delay optimization technique," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 10, pp. 2026–2035, 2019.

[40] H. Li, Z. Chen, L. Wu, H. K. Lam, and H. Du, "Event-triggered fault detection of nonlinear networked systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 4, pp. 1041–1052, 2017.

[41] Y. Pan and G.-H. Yang, "Event-triggered fault detection filter design for nonlinear networked systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 11, pp. 1851–1862, 2018.

[42] X.-L. Wang and G.-H. Yang, "Event-triggered H_∞ control for T-S fuzzy systems via new asynchronous premise reconstruction approach," *IEEE Transactions on Cybernetics*, vol. 51, no. 6, pp. 3062–3070, 2021.

[43] M. Han, H. K. Lam, F. Liu, Y. Tang, and H. Zhou, "Estimation of domain of attraction for discrete-time positive interval type-2 polynomial fuzzy systems with input saturation," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 2, pp. 397–411, 2022.

[44] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.