DOI:
10.1016/j.econlet.2022.110494

Document Version
Peer reviewed version

The Impact of Operational Delay on Irreversible Investment under Knightian Uncertainty

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Abstract

Lags between investing in a venture and when it comes into operation have a significant effect on the operative capacity of a firm. However, with a few exceptions, such lags are ignored in the modelling of irreversible investment under uncertainty. In this paper I examine the impact such lags have on the optimal investment strategy of a firm in the context of Knightian uncertainty (KU) and an ambiguity-averse investor. I find the effect of such lags to be significant in this context and, for high enough levels of KU, its widely reported negative impact on the threshold is reversed entirely by the delay. Furthermore, when KU and lags are jointly incorporated into a problem of irreversible investment under uncertainty, volatility impacts the NPV from investing (in the absence of either, it has no impact).

Keywords: Knightian uncertainty, operational delay, Irreversible investment.

JEL Classification Numbers: C61; D81

1 Introduction

Lags between the initiation of an investment project and when it becomes productive have a significant effect on the attainable productive options and the operative capacity faced by any competitive firm (cf. [3] and [1]). Moreover, in empirical models, investment demand is usually estimated on the basis of lagged variables. As such, it was pointed out in [1] that it is surprising how little attention has been paid to such lags in the theory of irreversible investment under uncertainty. However, apart from their studies and the study by [18] whose model provides a link between these two papers, alongside a more

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recent contribution by [17], delivery lags, or operational delay, are not accounted for in the economic literature on irreversible investment under uncertainty.

This is a gap in the literature that ought to be filled and, in this paper, I demonstrate the important consequences that operational delay can have on the value of the investment project and on the decision bounds in the context of Knightian uncertainty (KU) ([8]) and ambiguity aversion.

A standard result in the literature (cf. for example, [14] or [20]) is that, when the payoff is received as a fixed lump-sum, an ambiguity-averse investor, who maximises her expected profit over the "worst" element in the set of probability measures characterising KU, will invest at a lower threshold relative to a risk-neutral decision-maker (cf. [7]). The reasoning is that ambiguity aversion erodes the opportunity cost of investing and, therefore, the value of waiting. As such, exercising the investment option resolves the investor's KU making early investment attractive. However, when there is a delay in information between the time the decision to invest is undertaken, say t and the time the investment becomes productive, say $t + \delta$, the KU is not resolved once the investment is initiated. As such, it is plausible to presume that this operational delay will have an impact on the optimal strategy for an ambiguity-averse investor. In this paper, I investigate how the interaction of both impact on the optimal investment strategy.

It is worth noting that the aforementioned effect of KU on the optimal investment strategy is true in the case where the payoff is received as a fixed lump sum. However, [15] and [11] investigate the case in which the payoff is received as a perpetual flow and show that higher levels of KU may delay investment because it is not resolved after the investment has been undertaken. [16] incorporate KU into a model of capacity choice and expansion. They too find that the optimal investment threshold under KU is substantially higher, but the payoff in the model is also a flow and, as such, the KU is not resolved upon stopping. The focus of this paper is on the lump sum case.

The literature on irreversible investment under uncertainty has grown substantially since the contribution by [4]. In particular, many facets of investment and management problems are incorporated into their basic framework (see [2] and [9] for relatively recent and comprehensive reviews). As I pointed out, the impact of operational delay is under-researched but the impact of KU has gained traction on both the theoretical and empirical fronts ([6]; [15]; [20]; [11]; [19]; [16] and [5] are just a limited sample).

A standard result in the literature is that KU accelerates investment when the payoff is received as a fixed lump sum (see, for example, [14] and [20]). A further standard result is that operational delay also accelerates investment if it is anticipated that the state variable representing the payoff will increase during the delay period; i.e., if the growth rate of the stochastic payoff is positive (see, for example, [17]).

While individually these features accelerate investment, when both features are incorporated into the investment problem, the investment threshold *increases* in KU if the extent of operational delay or KU is sufficiently high. By accounting for operational delay, KU impacts the expected NPV from investing, whereas in the absence of such a delay, it only impacts the optimal investment strategy via its impact on the convenience yield. On the other hand, KU lowers the anticipated increase in the payoff during the delay period relative to the case of no KU. Together, these effects combine to reverse their individual impacts on the optimal investment threshold. To the best of my knowledge, this is the first study to examine this interaction and find such a result.

Finally, I also examine the effect of volatility on the optimal investment threshold in the presence of KU and operational delay. The standard and widely reported result in the literature is that higher levels of volatility increase the value of the investment option and, consequently, delays investment (i.e., the relationship is negative).

However, in the case of KU, the effect may be positive when there is no operational delay and will always be positive when there is a delay. This is a result that is not reported in the literature, to the best of my knowledge, and is contrary to the standard result. Instead, it is shown that the relationship between risk σ^2 and investment, in the presence of KU, is positive (cf. [15]). However, given that studies typically analyse the effect of σ (rather than σ^2) on the optimal investment threshold, this result ought to be highlighted and given more acknowledgement in the literature. This is particularly important because, in the presence of operational delay, the effect of σ on the optimal threshold is via its effect on the NPV from investing rather than on its effect on the value of waiting. However, it is widely understood that this volatility parameter has no effect on the NPV from investing, irrespective of whether the payoff is received as a lump sum or a perpetual flow and that its impact on the optimal investment strategy can only be via its impact on the value of the investment option (cf. [4]).

Overall, this paper highlights some important considerations about the interplay between KU and delivery lags which ought to be acknowledged and addressed to a greater extent in future research in the area of irreversible investment under uncertainty.

2 Model and Solution

Consider an investor with the option to invest in a venture by incurring a sunk cost $I > 0$. Future output is uncertain and the investor is ambiguous over the true probability measure governing the future dynamics of this output. To model her ambiguity, I follow [15] and assume she is averse to this ambiguity in the sense that she maximises her expected profit over the "worst" element in the set of probability measures characterising

Knightian uncertainty (KU).

Uncertainty about future cash flows are modelled on a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, P^\kappa)$, with expectation operator E_v. The stochastic evolution of the state variable $(V_t)_{t\geq 0}$ is modelled as a geometric Brownian motion and its dynamics under P^κ are given by

$$\frac{dV_t}{V_{t-}} = (\mu - \sigma\kappa)dt + \sigma dB_t^\kappa, \tag{1}$$

where $(B_t)_{t\geq 0}$ is a standard Brownian motion with $B_0 = 0$, P^κ-a.s. and the parameters μ and $\sigma > 0$ are assumed to be constant. The parameter $\kappa > 0$ is the upper-rim density generator under κ-ignorance; in other words, it is the density generator from the strongly rectangular set Θ of density generators that yields the lowest growth rate to the investment. Importantly, it represents the extent of KU (or ambiguity) faced by the investor.

The firm discounts future cash flow at a constant rate $\rho > \max\{\mu, 0\}$ and the *characteristic operator* of V is given by

$$\mathcal{L}_v\phi(v) = \frac{1}{2}\sigma^2 v^2 \phi''(v) + (\mu - \sigma\kappa)v\phi(v). \tag{2}$$

Suppose that there is a time lag δ between the time the decision to invest is taken τ and the time the investment becomes productive; i.e., the time the payoff is realised. Given this, she must restrict herself to the set $\mathcal{T}_\delta$ of *delayed stopping times*; i.e., functions $\tau_\delta : \Omega \to [0, \infty]$ such that $\{\omega \in \Omega | \tau_\delta(\omega) \leq t\} \in \mathcal{F}_{t-\delta}$ (cf. [17]).

The firm's problem is to choose a stopping time τ_δ at which to invest given (1); i.e.,

$$\begin{aligned} F_\delta^*(v) &= \sup_{\tau_\delta \in T_\delta} E_v\left[e^{-\rho\tau_\delta}(V_{\tau_\delta} - I)\right] \\ &= \sup_{\tau_\delta \in T_\delta} E_v\left[e^{-\rho\tau_\delta}\left(ve^{-(\sigma\kappa-\mu)\tau_\delta} - I\right)\right] \end{aligned} \tag{3}$$

Applying Theorem 2.1 in [17], we can re-write $F_\delta^*(v)$ as follows

$$F_\delta^*(v) = \sup_{\tau \in \mathcal{T}_0} E_v\left[e^{-\rho\delta} F(V_\delta)\right] \tag{4}$$

where $\mathcal{T}_0$ denotes the set of non-delayed stopping times and

$$F(v) = v - I > 0.$$

By applying the standard approach to solve this problem as outlined in, for example,

[4], let $\phi \geq F$ be a function such that

$$\mathcal{L}_v \phi - \rho\phi = 0$$

and $\phi(0) = 0$. Thus,

$$\phi(v) = Av^{\beta_1}$$

where

$$\beta_1 = \frac{1}{2} - \frac{\mu - \sigma\kappa}{\sigma^2} + \frac{1}{\sigma^2}\sqrt{\left(\mu - \sigma\kappa - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2\rho} > 1.$$

The optimal stopping trigger is then found by applying the value matching and smooth pasting conditions

$$\phi(V^*) = F(V^*) \text{ and } \phi'(V^*) = F'(V^*)$$

to give

$$(V^*)^{(\delta,\kappa)} = (V^*)^{(0,\kappa)} e^{(\sigma\kappa - \mu)\delta} \tag{5}$$

where

$$(V^*)^{(0,\kappa)} = \frac{\beta_1^\kappa}{\beta_1^\kappa - 1} I$$

denotes the threshold in the absence of delay ($\delta = 0$).

3 Results

Let $(V^*)^{(0,0)}$ denote the standard investment threshold in the absence of delay and KU. A standard result in the literature (cf., for example, [14] and [20]) is that $(V^*)^{(0,0)} > (V^*)^{(0,\kappa)}$ (when the payoff is received as a lump sum); in other words, ambiguity-averse investors with higher degrees of KU are more likely to exercise the investment option and thereby resolve uncertainty.

It is easily established from Eq. (5) that $(V^*)^{(0,0)} > (V^*)^{(\delta,0)}$ for $\mu > 0$. This implies that, in the absence of KU, operational delay also speeds up investment. This is driven by the effect of $e^{-\mu\delta}$ on V^* (since β_1^0 does not depend on δ). Specifically, for $\mu > 0$, it is anticipated that V_t is will increase during the delay period δ and, as such, it will be optimal to invest sooner (cf. [17]).

However, the following proposition shows that when KU and operational delay interact to affect the optimal investment strategy, for a high enough level of KU and/or operational delay, the optimal strategy is to invest *later* than the standard real options rule implies. The result is plotted in Fig. 1 below.

Proposition 1. *For any level of operational delay δ, there will always exist a κ^* such* *that for all $\kappa \geq \kappa^*$,*

$$(V^*)^{(0,\kappa)} < (V^*)^{(0,0)} < (V^*)^{(\delta,\kappa)},$$

where $(V^*)^{(0,0)}$ is the standard optimal investment threshold in the absence of ambigity and delay.

Proof. See Appendix A.

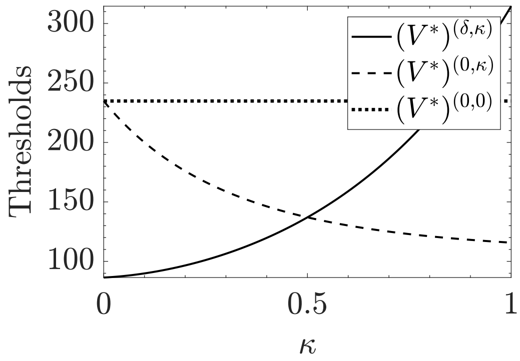

Figure 1: The effect of κ and δ on the optimal investment trigger V^*. The parameter values are given by $(\rho, \mu, \sigma, I) = (0.2, 0.1, 0.2, 100)$.

To understand the economic drivers underpinning the result, I first consider the case in which there is no KU, but there is operational delay. Assuming the drift of the payoff process is positive, the expected NPV of the investment will increase during the delay period (cf. Eq. (3) for $\kappa = 0$). Moreover, the extent of delay does not impact the value of waiting to invest and, as such, the critical threshold above which it is optimal to invest is lower because of the anticipation that V_t will increase during the delay period.

Next consider the case in which there is KU, but no operational delay. The required rate of return from holding the project is ρ, of which $\mu - \sigma\kappa$ comes in the form of an expected capital gain. Indeed, this is the lowest possible expected capital gain since we incorporate ambiguity aversion by determining the value of the project using the probability measure associated with the least favourable geometric Brownian motion process; in other words, the "worst" case scenario. Therefore, the difference between ρ and $\mu - \sigma\kappa$, otherwise known as the convenience yield, represents the opportunity cost of keeping the option alive. The greater the extent of KU (κ), the higher is this opportunity cost. On the other hand, in the absence of operational delay, the extent of KU does not impact on the expected NPV from exercising the option because, in this model, the payoff is received as a fixed lump sum. As such, the effect of KU is to speed up investment relative to the case in which there is no KU because of its positive effect on the convenience yield accrued from holding the investment opportunity alive.

However, when the effects of KU and operational delay interact, higher levels of KU imply later investment is optimal. The economic reasoning is that during the delay period, the present value of the payoff to be received is discounted by a rate of $\sigma\kappa - \mu$ (cf. Eq. (3)). Therefore, the greater the extent of KU, the more heavily is the expected NPV from investing discounted. Furthermore, higher levels of KU reduce the anticipated increase in V_t during the delay period. The combination of these effects dominate the effect of KU on the opportunity cost of waiting and, as such, the independent (negative) effects of δ and κ on V^* are reversed. It has already been emphasised by [1], [10], [12] and [13] that delivery lags have a significant effect on the optimal irreversible investment policies of rational investors facing uncertainty over future payoff. My paper supports this claim by highlighting the impact of delivery lags in the face of KU, which is a novel contribution to the literature.

3.1 The Impact of Volatility σ

The partial derivative of $(V^*)^{(\delta,\kappa)}$ with respect to σ is given by

$$\frac{\partial (V^*)^{(\delta,\kappa)}}{\partial \sigma} = \underbrace{\delta\kappa \frac{\beta_1^\kappa}{\beta_1^k - 1} I e^{(\sigma\kappa-\mu)\delta}}_{\text{PV effect}>0} + \underbrace{\frac{-1}{(\beta_1^\kappa - 1)^2} \frac{\partial \beta_1^\kappa}{\partial \sigma} e^{(\sigma\kappa-\mu)\delta}}_{\text{Option value effect: } > / < 0} . \tag{6}$$

The option value effect will be positive iff $\partial\beta_1^\kappa/\partial\sigma < 0 \Longleftrightarrow \sigma(\beta_1^\kappa - 1) > \kappa$. For $\kappa = 0$, this holds and gives the standard result. However, when κ is high, the option value effect is negative for low values of σ since β_1^κ increases in σ (see Fig. 2) below.

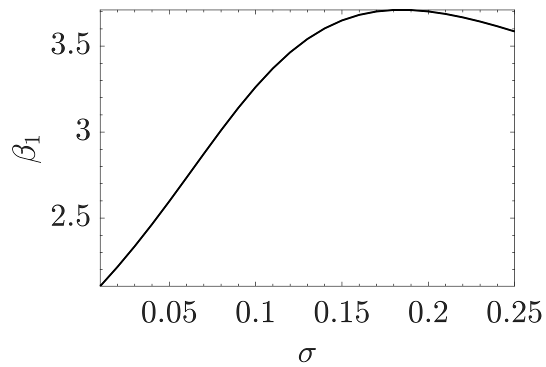

Figure 2: The effect of σ and β_1^κ. The parameter values are given by $(\rho, \mu, \kappa, I) = (0.2, 0.1, 0.5, 100)$.

First assume that there is no operational delay; i.e., $\delta = 0$ *and* $\partial\beta_1^\kappa/\partial\sigma > 0$ (i.e., σ is low and κ is high). This implies a higher σ leads to a lower optimal investment threshold $(V^*)^{(0,\kappa)}$ because the value of the investment opportunity decreases in the extent of volatility. This is contrary to the standard result and is driven by the effect of

KU, specifically, the term $\kappa\sigma$. A higher value of σ implies a lower perceived growth rate and, consequently, a lower valuation on the investment opportunity because the chance of the absorbing barrier being reached is higher. However, when we account for delay, the PV effect once again dominates and leads to a higher threshold $(V^*)^{(\delta,\kappa)}$, *ceteris paribas* (see Fig. 3 below).

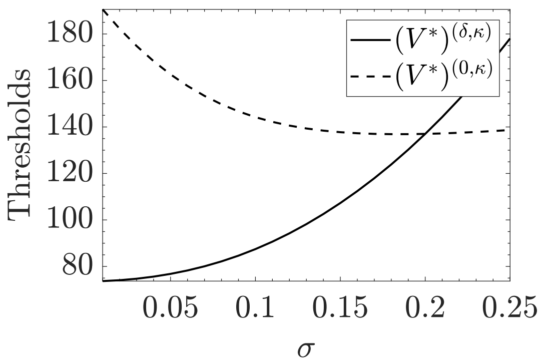

Figure 3: The effect of σ on $(V^*)^{(\delta,\kappa)}$ and $(V^*)^{(0,\kappa)}$.

4 Concluding Remarks

In this paper I show that the presence of lags between the time the investment is initiated and when it becomes productive has a profound impact on the optimal investment threshold for an ambiguity-averse investor who receives her payoff as a fixed lump sum.

Relative to the standard (no KU and no delay case), KU alone is widely reported to speed up investment. Moreover, operational delay alone is also reported to speed up investment. However, when the interplay between these features are considered, for high enough levels of either, investment will be delayed relative to the standard case.

Furthermore, the interplay induces an impact of volatility on the NPV of the investment and, finally, in the absence of delay but in the presence of KU, more volatility may speed up investment, which is a result that, to the best of my knowledge, not been addressed in the literature to date.

Appendix

A Proof of Proposition 1

It is easily established that $\partial\beta_1^\kappa/\partial\kappa > 0$. As such,

$$\frac{\partial(V^*)^{(0,\kappa)}}{\partial\kappa} < 0 \Longrightarrow (V^*)^{(0,\kappa)} < (V^*)^{(0,0)}.$$

When we account for operational delay,

$$\begin{aligned}\frac{\partial(V^*)^{(\delta,\kappa)}}{\partial\kappa} < 0 \Longleftrightarrow & \frac{\partial\beta_1^\kappa}{\partial\kappa} > \sigma\delta\beta_1^\kappa(\beta_1^\kappa - 1)\\ \Longleftrightarrow & \frac{1}{\sigma^2(\beta_1 - 1/2) + \mu - \sigma\kappa} > \delta(\beta_1^\kappa - 1)\\ \Longleftrightarrow & (\mu - \sigma\kappa - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\rho > \delta^2(\beta_1^\kappa - 1)^2\end{aligned} \tag{A.1}$$

However, since $\frac{\partial\beta_1^\kappa}{\partial\kappa} > 0$, this cannot hold if δ and/or κ are particularly high. Therefore, in that case,

$$\frac{\partial(V^*)^{(\delta,\kappa)}}{\partial\kappa} > 0$$

and, moreover, for κ and/or δ sufficiently high, it will hold that $(V^*)^{(0,0)} < (V^*)^{(\delta,\kappa)}$.

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