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A New Sampled-Data Output Feedback Controller Design of Nonlinear Systems via Fuzzy-Affine-Models

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***Abstract*—This paper focuses on the sampled-data output feedback control problem for nonlinear systems represented by Takagi-Sugeno fuzzy affine models. An input delay approach is adopted to describe the sample-and-hold behavior of the measurement output. Via augmenting the system states with the control input, the resulting closed-loop system is converted into a singular system firstly. Based on piecewise quadratic Lyapunov-Krasovskii functionals, some novel results on the sampled-data piecewise affine output feedback controller design are attained by employing some convexification techniques. Simulation studies are presented to illustrate the effectiveness of the proposed scheme.**

***Index Terms*—Fuzzy control; sampled-data control; output feedback; nonlinear systems; convex optimization.**

I. INTRODUCTION

Many real-world systems and industrial processes are instinctively nonlinear, and all these inherent nonlinearities in various forms always introduce many difficulties in controller synthesis and stability analysis. Notice that the methods relying on T-S fuzzy models have been widely adopted in steering of complex processes with nonlinearities in past years [1]–[12]. The overall dynamical features of T-S fuzzy model can be described by a set of local linear/affine models, which are in smooth connection by fuzzy membership functions [13]–[15]. It has been proved that T-S fuzzy model can serve as a technically efficient tool to approximate smooth nonlinear systems to arbitrary degrees of accuracy within any convex compact set [16]. As a consequence, plenty of efforts have been devoted to T-S fuzzy systems aiming at industrial applications [17], [18] and theoretical study [19], [20].

Generally, most early results on analysis and design of T-S fuzzy systems were obtained through a common quadratic Lyapunov function (CQLF), for instance, [15], [21], [22]. Nevertheless, these results have already been shown to be conservative for many fuzzy systems subject to high nonlinearity, as a common Lyapunov matrix may not exist. To reduce the conservatism, some significant results were proposed via fuzzy quadratic Lyapunov functions (FQLFs) [23]–[25], and piecewise quadratic Lyapunov functions (PQLFs) [16], [26], [27], respectively. Since CQLF can be viewed as a special case of the more general FQLFs/PQLFs, then the improved FQLFs/PQLFs-based analysis methods can handle a boarder class of fuzzy systems.

Sampled-data control has been well developed since digital computer technology is extensively utilized in engineering applications. More recently, various schemes to handle the sampled-data control problem have been proposed, for instance, the impulsive system approach [28], the discrete-time approach [29], and the input delay approach [30]. Specifically, in [28], a linear impulsive system was constructed to characterize the uncertain sampled-data system, and discontinuous Lyapunov functions at impulse times were adopted to derive the exponential stability analysis conditions. The authors in [29] studied the sampled-data control issue for linear systems subject to uncertain time-varying sampling intervals by a discrete-time approach, and the sampled-data system was described by a time-invariant discrete-time system. Specifically, the input delay approach possesses much enhanced capability to tackle the uncertain sampling periods or uncertain system matrices [31]–[33]. In general, the sampled-data information is firstly converted into a delayed control input, and then the stability analysis for the sampled-data control systems is conducted via using some relaxed inequality techniques. Thus, great efforts have been devoted to sampled-data control systems via the input delay approach in past years [31], [34], [35].

Recently, there have also been some remarkable results on T-S fuzzy sampled-data control systems [36]–[41]. [36] was the first attempt investigate sampled-data fuzzy control systems using input delay approach that the imperfect premise matching concept. [37] was proposed to deal with the mismatched membership functions between fuzzy model and fuzzy controller. The robust sampled-data control problem was studied for uncertain T-S fuzzy systems subject to time-delay in [38]. The authors in [41] designed fuzzy sampled-data controllers for chaotic systems without/with input constraints, respectively. Notice that the early fuzzy sampled-data control results [38]–[41] were basically obtained through the full state feedback method. Unfortunately, the full system state variables are not always accessible in plenty of engineering applications [42]–[44]. Therefore, some results on sampled-data output feedback control have been reported [45], [46]. [45] was

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a first attempt investigating sampled-data polynomial fuzzy control system. The authors in [45] designed a sampled-data output feedback controller for polynomial fuzzy systems. [46] proposed an estimator-based sampled-data output feedback control scheme for uncertain active suspension systems via the input delay method and Jensen's inequality. Nevertheless, the existing results in [38]–[41], [45], [46] were proposed merely for T-S fuzzy systems with linear local models through a CQLF, while the T-S fuzzy affine dynamic models are with substantially improved function approximation capability [16], [27]. To our best knowledge, it is challenging and important to further investigate the output feedback control (OFC) issue for T-S fuzzy affine sampled-data systems in piecewise Lyapunov-Krasovskii functionals (PLKFs) framework. This motivates our current work.

This work is concerned with the sampled-data OFC problem for nonlinear systems by T-S fuzzy affine models. Via augmenting the system states with the control input, the closed-loop system is firstly converted into a singular system. Through PLKFs and a new integral inequality, the robust $\mathscr{H}_\infty$ performance is analyzed, and the sampled-data piecewise affine (PWA) static output feedback controller synthesis results are proposed through convexification procedures. Simulation studies are given to present the effectiveness of the proposed approach.

The rest of the current work is given as follows. The preliminaries are presented in Section II. The PWA controller synthesis results are given in Section III. Simulation studies are given to illustrate the validity of the developed schemes within Section IV. Section V is devoted to conclusions.

Notations. $\mathbb{Z}_+$ denotes the set of non-negative integers. $\mathtt{Sym}\{S\}$ is short for $S+S^{\mathrm{T}}$.

II. Preliminaries

Consider a T-S fuzzy affine system characterized by fuzzy **IF-THEN** rules as,

Plant Rule $\mathscr{R}^l$: IF $\theta_1(x(t))$ **is** $\mathscr{F}_1^l$ **and** $\cdots$ **and** $\theta_\varphi(x(t))$ **is** $\mathscr{F}_\varphi^l$**, THEN**

$$\begin{cases} \dot{x}(t) = (A_l+\Delta A_l)\,x(t) + a_l + \Delta a_l + (B_l+\Delta B_l)\,u(t) \\ \qquad\quad +D_{l1}w(t) \\ y(t) = (C+\Delta C)x(t) + D_2 w(t) \\ z(t) = L_l x(t),\ l\in\mathscr{L}=\{1,\cdots,r\} \end{cases} \tag{1}$$

where $\mathscr{F}_\phi^l(\phi=1,\cdots,\varphi)$ represent fuzzy sets; $\mathscr{R}^l$ denotes the l-th fuzzy inference rule; $\theta(x(t)) := [\theta_1(x(t)),\cdots,\theta_\varphi(x(t))]$ are the measurable premise variables; r is the number of inference rules; $x(t)\in\Re^{n_x}$ denotes the system states; $y(t)\in\Re^{n_y}$ represents the system measurement output; $u(t)\in\Re^{n_u}$ is the control input; $z(t)\in\Re^{n_z}$ is the regulated output; $w(t)\in\Re^{n_w}$ represents the external disturbance and $w(t)\in L_2[0,\infty)$; ΔA_l, Δa_l, ΔB_l, and ΔC denote the uncertainty terms satisfying

$$\begin{aligned} \begin{bmatrix} \Delta A_l & \Delta a_l & \Delta B_l \end{bmatrix} &= U_{l1}\Delta_{l1}(t)\begin{bmatrix} W_{l1} & W_{l2} & W_{l3} \end{bmatrix}, \\ \Delta C &= U_2\Delta_2(t)W_4,\ l\in\mathscr{L} \end{aligned} \tag{2}$$

with U_{l1}, U_2, W_{l1}, W_{l2}, W_{l3}, and W_4 being known real-valued matrices. $\Delta_{l1}(t)\in\Re^{s_1\times s_2}$, $\Delta_2(t)\in\Re^{s_3\times s_2}$ represent unknown time-varying matrices satisfying

$$\begin{aligned} \Delta_{l1}^{\mathrm{T}}(t)\Delta_{l1}(t) &\le \mathbf{I},\ l\in\mathscr{L}, \\ \Delta_2^{\mathrm{T}}(t)\Delta_2(t) &\le \mathbf{I}. \end{aligned} \tag{3}$$

Denote $\mu_l[\theta(x(t))]$ as the normalized membership function (MF),

$$\mu_l[\theta(x(t))] := \frac{\prod_{\phi=1}^{\varphi}\mu_{l\phi}\left[\theta_\phi(x(t))\right]}{\sum_{\nu=1}^{r}\prod_{\phi=1}^{\varphi}\mu_{\nu\phi}\left[\theta_\phi(x(t))\right]} \ge 0, \quad \sum_{l=1}^{r}\mu_l[\theta(x(t))] = 1 \tag{4}$$

with $\mu_{l\phi}\left[\theta_\phi(x(t))\right]$ being the grade of membership of $\theta_\phi(x(t))$ in $\mathscr{F}_\phi^l$. For brevity, define $\mu_l := \mu_l[\theta(x(t))]$.

Via a center-average defuzzifier, product inference, and singleton fuzzifier, one has the subsequent T-S fuzzy affine model,

$$\begin{cases} \dot{x}(t) = (A(\mu)+\Delta A(\mu))\,x(t) + a(\mu) + \Delta a(\mu) \\ \qquad\quad + (B(\mu)+\Delta B(\mu))\,u(t) + D_1(\mu)w(t) \\ y(t) = (C+\Delta C)x(t) + D_2 w(t) \\ z(t) = L(\mu)x(t) \end{cases} \tag{5}$$

where

$$\begin{cases} A(\mu) = \sum\limits_{l=1}^{r}\mu_l A_l, \Delta A(\mu) = \sum\limits_{l=1}^{r}\mu_l \Delta A_l, \\ a(\mu) = \sum\limits_{l=1}^{r}\mu_l a_l, \Delta a(\mu) = \sum\limits_{l=1}^{r}\mu_l \Delta a_l, \\ B(\mu) = \sum\limits_{l=1}^{r}\mu_l B_l, \Delta B(\mu) = \sum\limits_{l=1}^{r}\mu_l \Delta B_l, \\ D_1(\mu) = \sum\limits_{l=1}^{r}\mu_l D_{l1}, L(\mu) = \sum\limits_{l=1}^{r}\mu_l L_l. \end{cases} \tag{6}$$

Since a polyhedral decomposition of the system state-space is induced by MFs and fuzzy rules, the global model in (5) can be deemed as a convex combination of several local models in individual regions. According to [16], the premise variable space is divided into crisp subspaces and fuzzy subspaces. The crisp subspace is the region with merely one rule. The fuzzy subspaces are the regions subject to $0<\mu_l<1$.

Specify the indices of subspaces $\mathcal{I} = \mathcal{I}_0\bigcup\mathcal{I}_1$, and $\mathcal{I}_0$ includes the index set of subspaces covering the origin, while $\mathcal{I}_1$ denotes the index set of subspaces without the origin. The following set

$$\mathscr{N}(i) := \{m|\mu_m[x(t)]>0,\ m\in\mathscr{L},\ x(t)\in S_i,\ i\in\mathcal{I}\} \tag{7}$$

is introduced to characterize the indices in each subspace S_i.

With (7), the system (5) is formulated as

$$\begin{cases} \dot{x}(t) = (\mathcal{A}_i+\Delta\mathcal{A}_i)\,x(t) + \mathrm{a}_i + \Delta\mathrm{a}_i + (\mathcal{B}_i+\Delta\mathcal{B}_i)\,u(t) \\ \qquad\quad +\mathcal{D}_{i1}w(t) \\ y(t) = (C+\Delta C)\,x(t) + D_2 w(t) \\ z(t) = \mathcal{L}_i x(t),\ i\in\mathcal{I} \end{cases} \tag{8}$$

where

$$\begin{cases} \mathcal{A}_i := \sum\limits_{m\in\mathscr{N}(i)}\mu_m A_m, \Delta\mathcal{A}_i := \sum\limits_{m\in\mathscr{N}(i)}\mu_m \Delta A_m, \\ \mathrm{a}_i := \sum\limits_{m\in\mathscr{N}(i)}\mu_m a_m, \Delta\mathrm{a}_i := \sum\limits_{m\in\mathscr{N}(i)}\mu_m \Delta a_m, \\ \mathcal{B}_i := \sum\limits_{m\in\mathscr{N}(i)}\mu_m B_m, \Delta\mathcal{B}_i := \sum\limits_{m\in\mathscr{N}(i)}\mu_m \Delta B_m, \end{cases}$$

$$\left\{ \mathcal{D}_{i1} := \sum_{m\in\mathscr{N}(i)} \mu_m D_{m1}, \mathcal{L}_i := \sum_{m\in\mathscr{N}(i)} \mu_m L_m \right. \tag{9}$$

with $\sum_{m\in\mathscr{N}(i)} \mu_m = 1$, $0 < \mu_m \leq 1$.

In this work, merely the sampled measurement output information of $y(t)$ is assumed to be available for controller design purpose. Particularly, the sampled measurement outputs $y(t_j)$, $j \in \mathbb{Z}_+$ at the sampling instant t_j are generally kept constant by a zero-order hold (ZOH), and

$$0 = t_0 < t_1 < t_2 < \cdots < t_j < \cdots \tag{10}$$

with $\lim_{j\to\infty} t_j = \infty$.

Then for system (1), design a sampled-data piecewise affine (PWA) static output feedback controller as

$$u(t) = K_i y(t_j) + k_i,\ t \in [t_j, t_{j+1}),\ i \in \mathcal{I} \tag{11}$$

with $K_i \in \Re^{n_u\times n_y}$ and $k_i \in \Re^{n_u\times 1}$ being controller gains to be determined, and $k_i = 0$ when $i \in \mathcal{I}_0$. Note that when with $k_i \equiv 0$, the PWA controller (11) reduces to a piecewise linear (PWL) one, and the PWA controller (11) is more powerful for the sampled-data control of system (1) than the PWL one.

Consequently, with the controller in (11), the resulting closed-loop system is

$$\left\{ \begin{array}{l} \dot{1} = 0 \\ \dot{x}(t) = (\mathcal{A}_i + \Delta\mathcal{A}_i)\, x(t) + \mathrm{a}_i + \Delta\mathrm{a}_i \\ \qquad + (\mathcal{B}_i + \Delta\mathcal{B}_i)\, u(t) + \mathcal{D}_{i1} w(t), \\ 0 \times \dot{u}(t) = K_i\,(C + \Delta C)\, x(t_j) + k_i - u(t) + K_i D_2 w(t_j) \\ z(t) = \mathcal{L}_i x(t),\ t \in [t_j, t_{j+1}),\ i \in \mathcal{I}. \end{array} \right. \tag{12}$$

Reformulate the system (12) as

$$\left\{ \begin{array}{l} \tilde{E}\dot{\tilde{x}}(t) = \tilde{\mathbf{A}}_i \tilde{x}(t) + \tilde{\mathbf{C}}_i \tilde{H}_1 \tilde{x}(t_j) + \tilde{\mathbf{D}}_i \tilde{w}(t) \\ z(t) = \tilde{\mathcal{L}}_i \tilde{x}(t),\ t \in [t_j, t_{j+1}),\ i \in \mathcal{I} \end{array} \right. \tag{13}$$

where

$$\left\{ \begin{array}{l} \tilde{w}(t) = \begin{bmatrix} w^{\mathrm{T}}(t) & w^{\mathrm{T}}(t_j) \end{bmatrix}^{\mathrm{T}}, \\ \left. \begin{array}{l} \tilde{E} = \mathrm{diag}\{\mathbf{I}_{n_x}, \mathbf{0}_{n_u\times n_u}\}, \\ \tilde{x}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & u^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \\ \tilde{\mathbf{A}}_i = \begin{bmatrix} \mathcal{A}_i + \Delta\mathcal{A}_i & \mathcal{B}_i + \Delta\mathcal{B}_i \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}, \\ \tilde{\mathbf{C}}_i = \begin{bmatrix} \mathbf{0} \\ K_i(C + \Delta C) \end{bmatrix}, \\ \tilde{\mathbf{D}}_i = \begin{bmatrix} \mathcal{D}_{i1} & \mathbf{0} \\ \mathbf{0} & K_i D_2 \end{bmatrix}, \\ \tilde{\mathcal{L}}_i = \begin{bmatrix} \mathcal{L}_i & \mathbf{0} \end{bmatrix}, \tilde{H}_1 = \begin{bmatrix} \mathbf{I}_{n_x} & \mathbf{0} \end{bmatrix}, \end{array} \right\} \text{if } i \in \mathcal{I}_0, \\ \left. \begin{array}{l} \tilde{E} = \mathrm{diag}\{\mathbf{I}_{(1+n_x)}, \mathbf{0}_{n_u\times n_u}\}, \\ \tilde{x}(t) = \begin{bmatrix} 1 & x^{\mathrm{T}}(t) & u^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \\ \tilde{\mathbf{A}}_i = \\ \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} \\ \mathrm{a}_i + \Delta\mathrm{a}_i & \mathcal{A}_i + \Delta\mathcal{A}_i & \mathcal{B}_i + \Delta\mathcal{B}_i \\ k_i & \mathbf{0} & -\mathbf{I} \end{bmatrix}, \\ \tilde{\mathbf{C}}_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ K_i(C + \Delta C) \end{bmatrix}, \\ \tilde{\mathbf{D}}_i = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathcal{D}_{i1} & \mathbf{0} \\ \mathbf{0} & K_i D_2 \end{bmatrix}, \\ \tilde{\mathcal{L}}_i = \begin{bmatrix} \mathbf{0} & \mathcal{L}_i & \mathbf{0} \end{bmatrix}, \tilde{H}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{n_x} & \mathbf{0} \end{bmatrix}, \end{array} \right\} \text{if } i \in \mathcal{I}_1. \end{array} \right. \tag{14}$$

Remark 2.1. Note that the controller gains K_i and k_i have been decoupled from the control input matrices via a singular-system-based analysis process presented in (13)-(14). Compared with the method given in [16], this feature also permits the output matrices and the control input channels to contain parameter uncertainties, and extra structural constraints on the Lyapunov matrices are avoided for PWA controller design purpose, which will be demonstrated in details in the subsequent section.

Denote

$$d(t) = t - t_j \leq h, t \in [t_j, t_{j+1}), \tag{15}$$

and then rewrite system (13) as

$$\left\{ \begin{array}{l} \tilde{E}\dot{\tilde{x}}(t) = \tilde{\mathbf{A}}_i \tilde{x}(t) + \tilde{\mathbf{C}}_i \tilde{H}_1 \tilde{x}(t - d(t)) + \tilde{\mathbf{D}}_i \tilde{w}(t) \\ z(t) = \tilde{\mathbf{L}}_i \tilde{x}(t),\ i \in \mathcal{I}. \end{array} \right. \tag{16}$$

This work will synthesize a PWA static output feedback controller relying on the sampled-data measurement output for fuzzy system (1). For a prescribed disturbance attenuation level $\gamma > 0$, the resulting PWA closed-loop system is guaranteed to be asymptotically stable with $\tilde{w}(t) \equiv 0$. It is also satisfied that

$$\mathscr{H}_\infty^{zw} = \sup \frac{\|z\|_2}{\|\tilde{w}\|_2} < \gamma \tag{17}$$

with zero initial condition for nonzero $\tilde{w}(t) \in L_2[0,\ \infty)$.

III. Main Results

This section will propose some new stability analysis results for the closed-loop system (16) through novel piecewise Lyapunov-Krasovskii functionals (PLKFs), and an integral inequality is adopted to handle the coupling issues in the quadratic crossing terms, such that the conservatism can be further reduced. Then the sampled-data PWA controller synthesis results will be proposed in a convex optimization framework.

A. Stability Analysis Through PLKFs

To guarantee the PLKFs continuous among boundaries of the subspace, according to [27], establish matrices $\tilde{F}_i = \begin{bmatrix} f_i & F_i \end{bmatrix}$, $i \in \mathcal{I}$, with $f_i = 0$ for $i \in \mathcal{I}_0$ to depict the boundary across the subspaces,

$$\tilde{F}_i \begin{bmatrix} 1 \\ x(t) \end{bmatrix} = \tilde{F}_s \begin{bmatrix} 1 \\ x(t) \end{bmatrix}, x(t) \in S_i \bigcap S_s,\ i, s \in \mathcal{I}. \tag{18}$$

For further conservatism reduction, we will also adopt the S-procedure by establishing matrices $\tilde{G}_i = \begin{bmatrix} g_i & G_i \end{bmatrix}, i \in \mathcal{I}$, with $g_i = 0$ for $i \in \mathcal{I}_0$ and

$$\tilde{G}_i \begin{bmatrix} 1 \\ x(t) \end{bmatrix} \succeq 0, x(t) \in S_i, i \in \mathcal{I} \tag{19}$$

with $\succeq$ indicating each entry being nonnegative.

Theorem 3.1. For a given disturbance attenuation level γ, the system (16) is asymptotically stable, if matrices $0 < \{Q, M\} \in \Re^{n_x\times n_x}$, $X = X^{\mathrm{T}}$, $Y_i = Y_i^{\mathrm{T}} \succeq 0$, $T_i = T_i^{\mathrm{T}} \succeq 0$, $P_{i2} \in \Re^{n_u\times n_u}$, $i \in \mathcal{I}$, $\tilde{P}_{i3} \in \Re^{n_u\times n_x}$, $\tilde{N} \in \Re^{(5n_x+n_u+2n_w)\times 4n_x}$, for $i \in \mathcal{I}_0$, $\tilde{P}_{i3} \in \Re^{n_u\times(1+n_x)}$,

$\tilde{N} \in \Re^{(1+5n_x+n_u+2n_w)\times 4n_x}$, for $i \in \mathcal{I}_1$, exist such that the conditions in (20)-(21) hold,

$$\tilde{F}_i^{\mathrm{T}} X \tilde{F}_i - \tilde{G}_i^{\mathrm{T}} Y_i \tilde{G}_i > 0, i \in \mathcal{I}, \tag{20}$$

$$\begin{aligned} &\mathtt{Sym}\{e_1^{\mathrm{T}} \tilde{P}_i^{\mathrm{T}} \bar{\mathcal{A}}_i + \tilde{N}\Sigma_2\} + (1-\sigma(n))h\bar{\mathcal{A}}_i^{\mathrm{T}} \tilde{H}_1^{\mathrm{T}} M \tilde{H}_1 \bar{\mathcal{A}}_i \\ &+\Sigma_1^{\mathrm{T}} \bar{Q}\Sigma_1 + \sigma(n)h\tilde{N}\mathrm{diag}\{M, 3M, 5M, 7M\}^{-1}\tilde{N}^{\mathrm{T}} \\ &+e_1^{\mathrm{T}} \tilde{H}_0^{\mathrm{T}} \tilde{G}_i^{\mathrm{T}} T_i \tilde{G}_i \tilde{H}_0 e_1 + e_1^{\mathrm{T}} \tilde{\mathcal{L}}_i^{\mathrm{T}} \tilde{\mathcal{L}}_i e_1 - \gamma^2 e_6^{\mathrm{T}} e_6 < 0, \\ &n = 1, 2,\ i \in \mathcal{I} \end{aligned} \tag{21}$$

where

$$\left\{\begin{array}{l} \sigma(1) = 0, \sigma(2) = 1, \\ \tilde{P}_i = \begin{bmatrix} \tilde{F}_i^{\mathrm{T}} X \tilde{F}_i & \mathbf{0} \\ \tilde{P}_{i3} & P_{i2} \end{bmatrix}, \\ \bar{Q} = \mathrm{diag}\{\tilde{H}_1^{\mathrm{T}} Q \tilde{H}_1, -Q\}, \\ \Sigma_1 = \begin{bmatrix} e_1^{\mathrm{T}} & e_2^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ \Sigma_2 = \left[\begin{array}{l} e_1^{\mathrm{T}} \tilde{H}_1^{\mathrm{T}} - e_2^{\mathrm{T}} \quad e_1^{\mathrm{T}} \tilde{H}_1^{\mathrm{T}} + e_2^{\mathrm{T}} - 2e_3^{\mathrm{T}} \\ e_1^{\mathrm{T}} \tilde{H}_1^{\mathrm{T}} - e_2^{\mathrm{T}} - 6e_3^{\mathrm{T}} + 6e_4^{\mathrm{T}} \\ e_1^{\mathrm{T}} \tilde{H}_1^{\mathrm{T}} + e_2^{\mathrm{T}} - 12e_3^{\mathrm{T}} + 30e_4^{\mathrm{T}} - 20e_5^{\mathrm{T}} \end{array}\right]^{\mathrm{T}}, \\ \left.\begin{array}{l} \bar{\mathcal{A}}_i = \begin{bmatrix} \tilde{\mathbf{A}}_i & \tilde{\mathbf{C}}_i & \mathbf{0}_{(n_x+n_u)\times 3n_x} & \tilde{\mathbf{D}}_i \end{bmatrix}, \\ e_1 = \begin{bmatrix} \mathbf{I}_{(n_x+n_u)} & \mathbf{0}_{(n_x+n_u)\times(4n_x+2n_w)} \end{bmatrix}, \\ e_s = \begin{bmatrix} \underbrace{\mathbf{0}\cdots\mathbf{0}}_{s-1} & \mathbf{I}_{\varrho(s)} & \underbrace{\mathbf{0}\cdots\mathbf{0}}_{6-s} \end{bmatrix} \\ \quad \in \Re^{\varrho(s)\times(5n_x+n_u+2n_w)}, s = 2, \cdots, 6, \\ \tilde{H}_0 = \begin{bmatrix} \mathbf{I}_{n_x} & \mathbf{0}_{n_x\times n_u} \end{bmatrix}, \\ \tilde{H}_1 = \begin{bmatrix} \mathbf{I}_{n_x} & \mathbf{0} \end{bmatrix}, \end{array}\right\} \text{if } i \in \mathcal{I}_0, \\ \left.\begin{array}{l} \bar{\mathcal{A}}_i = \begin{bmatrix} \tilde{\mathbf{A}}_i & \tilde{\mathbf{C}}_i & \mathbf{0}_{(1+n_x+n_u)\times 3n_x} & \tilde{\mathbf{D}}_i \end{bmatrix}, \\ e_1 = \begin{bmatrix} \mathbf{I}_{(1+n_x+n_u)} & \mathbf{0}_{(1+n_x+n_u)\times(4n_x+2n_w)} \end{bmatrix}, \\ e_s = \begin{bmatrix} \underbrace{\mathbf{0}\cdots\mathbf{0}}_{s-1} & \mathbf{I}_{\varrho(s)} & \underbrace{\mathbf{0}\cdots\mathbf{0}}_{6-s} \end{bmatrix} \\ \quad \in \Re^{\varrho(s)\times(1+5n_x+n_u+2n_w)}, s = 2, \cdots, 6, \\ \tilde{H}_0 = \begin{bmatrix} \mathbf{I}_{1+n_x} & \mathbf{0}_{(1+n_x)\times n_u} \end{bmatrix}, \\ \tilde{H}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{n_x} & \mathbf{0} \end{bmatrix}, \end{array}\right\} \text{if } i \in \mathcal{I}_1 \end{array}\right. \tag{22}$$

with h given in (15), and

$$\varrho(s) = \begin{cases} n_x, s = 2, \cdots, 5, \\ 2n_w, s = 6. \end{cases} \tag{23}$$

Proof: Consider the following PLKFs being continuous among the boundary of the subspaces

$$V(t) = \sum_{q=1}^{3} V_q(t) \tag{24}$$

and

$$\begin{cases} V_1(t) = \tilde{x}^{\mathrm{T}}(t)\tilde{E}^{\mathrm{T}} \tilde{P}_i \tilde{x}(t), \\ V_2(t) = \int_{t-d(t)}^{t} \tilde{x}^{\mathrm{T}}(s)\tilde{H}_1^{\mathrm{T}} Q \tilde{H}_1 \tilde{x}(s)ds \\ \quad +(h-d(t))x^{\mathrm{T}}(t-d(t))Qx(t-d(t)), \\ V_3(t) = (h-d(t))\int_{t-d(t)}^{t} \dot{\tilde{x}}^{\mathrm{T}}(s)\tilde{E}^{\mathrm{T}} \tilde{H}_1^{\mathrm{T}} M \tilde{H}_1 \tilde{E}\dot{\tilde{x}}(s)ds, \end{cases} \tag{25}$$

where

$$\tilde{P}_i = \begin{bmatrix} \tilde{F}_i^{\mathrm{T}} X \tilde{F}_i & \mathbf{0} \\ \tilde{P}_{i3} & P_{i2} \end{bmatrix} \tag{26}$$

and $0 < \{Q, M\} \in \Re^{n_x\times n_x}$, $X = X^{\mathrm{T}}$, and $P_{i2} \in \Re^{n_u\times n_u}$, $i \in \mathcal{I}$, are arbitrary invertible matrices, $\tilde{P}_{i3} \in \Re^{n_u\times n_x}$, for $i \in \mathcal{I}_0$, $\tilde{P}_{i3} \in \Re^{n_u\times(1+n_x)}$, for $i \in \mathcal{I}_1$.

Define

$$\left\{\begin{array}{l} \zeta(t) = \left[\begin{array}{l} \tilde{x}^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-d(t)) \quad \frac{1}{d(t)}\int_{t-d(t)}^{t} x^{\mathrm{T}}(s)ds \\ \quad \frac{2}{d^2(t)}\int_{t-d(t)}^{t}\int_{t-d(t)}^{\alpha} x^{\mathrm{T}}(s)dsd\alpha \\ \frac{6}{d^3(t)}\int_{t-d(t)}^{t}\int_{t-d(t)}^{\beta}\int_{t-d(t)}^{\alpha} x^{\mathrm{T}}(s)dsd\alpha d\beta \quad \tilde{w}^{\mathrm{T}}(t) \end{array}\right]^{\mathrm{T}}, \\ \left.\begin{array}{l} e_1 = \begin{bmatrix} \mathbf{I}_{(n_x+n_u)} & \mathbf{0}_{(n_x+n_u)\times(4n_x+2n_w)} \end{bmatrix}, \\ e_s = \begin{bmatrix} \underbrace{\mathbf{0}\cdots\mathbf{0}}_{s-1} & \mathbf{I}_{\varrho(s)} & \underbrace{\mathbf{0}\cdots\mathbf{0}}_{6-s} \end{bmatrix}, \\ \quad \in \Re^{\varrho(s)\times(5n_x+n_u+2n_w)}, s = 2, \cdots, 6, \end{array}\right\} \text{if } i \in \mathcal{I}_0, \\ \left.\begin{array}{l} e_1 = \begin{bmatrix} \mathbf{I}_{(1+n_x+n_u)} & \mathbf{0}_{(1+n_x+n_u)\times(4n_x+2n_w)} \end{bmatrix}, \\ e_s = \begin{bmatrix} \underbrace{\mathbf{0}\cdots\mathbf{0}}_{s-1} & \mathbf{I}_{\varrho(s)} & \underbrace{\mathbf{0}\cdots\mathbf{0}}_{6-s} \end{bmatrix}, \\ \quad \in \Re^{\varrho(s)\times(1+5n_x+n_u+2n_w)}, s = 2, \cdots, 6, \end{array}\right\} \text{if } i \in \mathcal{I}_1, \\ \varrho(s) = \begin{cases} n_x, s = 2, \cdots, 5, \\ 2n_w, s = 6. \end{cases} \end{array}\right. \tag{27}$$

Obviously, $\tilde{x}(t) = e_1\zeta(t)$, $x(t) = \tilde{H}_1 e_1\zeta(t)$, $x(t-d(t)) = e_2\zeta(t)$, and $\tilde{w}(t) = e_6\zeta(t)$.

The inequality (20) guarantees the positive definiteness of PLKFs within each local subspace. With the PLKFs given in (24)-(25), if

$$\dot{V}(t) + z^{\mathrm{T}}(t)z(t) - \gamma^2\tilde{w}^{\mathrm{T}}(t)\tilde{w}(t) < 0, \tag{28}$$

the closed-loop system (16) is asymptotically stable in an $\mathscr{H}_\infty = \gamma$ setup.

Along the system (16), the time-derivative of $V_1(t)$ is

$$\begin{aligned} \dot{V}_1(t) &= 2\tilde{x}^{\mathrm{T}}(t)\tilde{P}_i^{\mathrm{T}}\tilde{E}\dot{\tilde{x}}(t) \\ &= 2\zeta^{\mathrm{T}}(t)e_1^{\mathrm{T}}\tilde{P}_i^{\mathrm{T}}\bar{\mathcal{A}}_i\zeta(t) \end{aligned} \tag{29}$$

where

$$\begin{cases} \bar{\mathcal{A}}_i = \begin{bmatrix} \tilde{\mathbf{A}}_i & \tilde{\mathbf{C}}_i & \mathbf{0}_{(n_x+n_u)\times 3n_x} & \tilde{\mathbf{D}}_i \end{bmatrix},\ i \in \mathcal{I}_0, \\ \bar{\mathcal{A}}_i = \begin{bmatrix} \tilde{\mathbf{A}}_i & \tilde{\mathbf{C}}_i & \mathbf{0}_{(1+n_x+n_u)\times 3n_x} & \tilde{\mathbf{D}}_i \end{bmatrix},\ i \in \mathcal{I}_1 \end{cases} \tag{30}$$

with $\tilde{\mathbf{A}}_i$, $\tilde{\mathbf{C}}_i$, and $\tilde{\mathbf{D}}_i$ given in (14).

The time-derivative of $V_2(t)$ is

$$\begin{aligned} \dot{V}_2(t) &= \tilde{x}^{\mathrm{T}}(t)\tilde{H}_1^{\mathrm{T}}Q\tilde{H}_1\tilde{x}(t) - x^{\mathrm{T}}(t-d(t))Qx(t-d(t)) \\ &= \zeta^{\mathrm{T}}(t)\Sigma_1^{\mathrm{T}}\bar{Q}\Sigma_1\zeta(t) \end{aligned} \tag{31}$$

where

$$\begin{cases} \Sigma_1 = \begin{bmatrix} e_1^{\mathrm{T}} & e_2^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ \bar{Q} = \mathrm{diag}\{\tilde{H}_1^{\mathrm{T}}Q\tilde{H}_1, -Q\}. \end{cases} \tag{32}$$

The time-derivative of $V_3(t)$ is

$$\begin{aligned} \dot{V}_3(t) = &(h-d(t))\dot{\tilde{x}}^{\mathrm{T}}(t)\tilde{E}^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}}M\tilde{H}_1\tilde{E}\dot{\tilde{x}}(t) \\ &-\int_{t-d(t)}^{t}\dot{\tilde{x}}^{\mathrm{T}}(s)\tilde{E}^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}}M\tilde{H}_1\tilde{E}\dot{\tilde{x}}(s)ds. \end{aligned} \tag{33}$$

Employing Lemma 1 given in the appendix to tackle the second term within the right hand side of (33), one has

$$\begin{aligned} &-\int_{t-d(t)}^{t}\dot{\tilde{x}}^{\mathrm{T}}(s)\tilde{E}^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}}M\tilde{H}_1\tilde{E}\dot{\tilde{x}}(s)ds \\ &\le \zeta^{\mathrm{T}}(t)\Big(\mathtt{Sym}\{\tilde{N}\Sigma_2\} \\ &\qquad +d(t)\tilde{N}\mathrm{diag}\{M, 3M, 5M, 7M\}^{-1}\tilde{N}^{\mathrm{T}}\Big)\zeta(t) \end{aligned} \tag{34}$$

where $\tilde{N} \in \Re^{(5n_x+n_u+2n_w)\times 4n_x}$, for $i \in \mathcal{I}_0$, $\tilde{N} \in \Re^{(1+5n_x+n_u+2n_w)\times 4n_x}$, for $i \in \mathcal{I}_1$, and

$$\Sigma_2 = \begin{bmatrix} e_1^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}} - e_2^{\mathrm{T}} & e_1^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}} + e_2^{\mathrm{T}} - 2e_3^{\mathrm{T}} \\ e_1^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}} - e_2^{\mathrm{T}} - 6e_3^{\mathrm{T}} + 6e_4^{\mathrm{T}} \\ e_1^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}} + e_2^{\mathrm{T}} - 12e_3^{\mathrm{T}} + 30e_4^{\mathrm{T}} - 20e_5^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \tag{35}$$

Based on (33)-(35), one has

$$\dot{V}_3(t) \le \zeta^{\mathrm{T}}(t)\Big((h-d(t))\bar{\mathcal{A}}_i^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}}M\tilde{H}_1\bar{\mathcal{A}}_i + \mathtt{Sym}\{\tilde{N}\Sigma_2\} + d(t)\tilde{N}\mathrm{diag}\{M,3M,5M,7M\}^{-1}\tilde{N}^{\mathrm{T}}\Big)\zeta(t). \tag{36}$$

Using the S-procedure based on (19), and considering (29), (31), and (36), the following inequality implies (28),

$$\zeta^{\mathrm{T}}(t)\Lambda_i(d(t))\zeta(t) < 0, i \in \mathcal{I} \tag{37}$$

where

$$\begin{cases} \Lambda_i(d(t)) = \mathtt{Sym}\{e_1^{\mathrm{T}}\tilde{P}_i^{\mathrm{T}}\bar{\mathcal{A}}_i + \tilde{N}\Sigma_2\} + \Sigma_1^{\mathrm{T}}\bar{Q}\Sigma_1 \\ \qquad +(h-d(t))\bar{\mathcal{A}}_i^{\mathrm{T}}\tilde{H}_1^{\mathrm{T}}M\tilde{H}_1\bar{\mathcal{A}}_i \\ \qquad +d(t)\tilde{N}\mathrm{diag}\{M,3M,5M,7M\}^{-1}\tilde{N}^{\mathrm{T}} \\ \qquad +e_1^{\mathrm{T}}\tilde{H}_0^{\mathrm{T}}\tilde{G}_i^{\mathrm{T}}T_i\tilde{G}_i\tilde{H}_0e_1 + e_1^{\mathrm{T}}\tilde{\mathcal{L}}_i^{\mathrm{T}}\tilde{\mathcal{L}}_ie_1 - \gamma^2e_6^{\mathrm{T}}e_6, \\ \tilde{H}_0 = \begin{bmatrix} \mathbf{I}_{n_x} & \mathbf{0}_{n_x\times n_u} \end{bmatrix}, \ i \in \mathcal{I}_0, \\ \tilde{H}_0 = \begin{bmatrix} \mathbf{I}_{1+n_x} & \mathbf{0}_{(1+n_x)\times n_u} \end{bmatrix}, \ i \in \mathcal{I}_1 \end{cases} \tag{38}$$

with $T_i \succeq 0$, $i \in \mathcal{I}$.

Then the subsequent inequality implies (37)

$$\Lambda_i(d(t)) < 0, i \in \mathcal{I} \tag{39}$$

which indicates that the system (16) is asymptotically stable with disturbance attenuation level γ.

Note that a time-varying delay $d(t)$ exists in the condition (39) and satisfies

$$0 \le d(t) \le h \tag{40}$$

which indicates that $d(t)$ can be constructed as,

$$d(t) = \lambda \cdot 0 + (1-\lambda)h \tag{41}$$

with $0 \le \lambda \le 1$.

Since $d(t)$ in (41) is a linear function of the variable λ, then it can be seen that (39) holds for $\lambda = 1$ and $\lambda = 0$, respectively, which implies (21). The proof is completed. ■

Remark 3.1. Most existing fuzzy sampled-data OFC results, such as [45], [46], require that the parameter uncertainties are not allowed to exist in the control input channels. However, the PWA output feedback controller design approach proposed in this paper has released this restriction. In addition, the measurement output matrices are also allowed to contain parameter uncertainties. Therefore, the proposed sampled-data PWA controller design method tends to be more applicable.

Remark 3.2. By utilizing a PLKFs-based method and an elegant integral inequality given in Lemma 1, new sufficient criteria for the stability analysis of the system (16) are proposed in Theorem 3.1. Differing from the conventional stability analysis approaches, a singular system is formulated to deal with the coupling issues in the control input matrices with the controller gains, and through some convexification procedures, the stability analysis of the closed-loop system has been conducted. Note that the PLKFs-based scheme developed in Theorem 3.1 also results in the conservatism reduction. The aforementioned features also make our results different from those in [45], [46].

B. Sampled-Data PWA Controller Synthesis

Theorem 3.2. For the fuzzy affine system (1), the system (16) is asymptotically stable in an $\mathscr{H}_\infty = \gamma$ setup, if matrices $0 < \{Q, M\} \in \Re^{n_x\times n_x}$, $X = X^{\mathrm{T}}$, $Y_i = Y_i^{\mathrm{T}} \succeq 0$, $T_i = T_i^{\mathrm{T}} \succeq 0$, $P_{i2} \in \Re^{n_u\times n_u}$, $\bar{K}_i \in \Re^{n_u\times n_y}$, $i \in \mathcal{I}$, $\tilde{N} \in \Re^{(5n_x+n_u+2n_w)\times 4n_x}$, for $i \in \mathcal{I}_0$, $\bar{k}_i \in \Re^{n_u}$, $\tilde{N} \in \Re^{(1+5n_x+n_u+2n_w)\times 4n_x}$, for $i \in \mathcal{I}_1$, and scalars $\varepsilon_i > 0$, $i \in \mathcal{I}$, exist such that (20) and the following inequality holds,

$$\begin{bmatrix} \Upsilon_1 & \sqrt{\sigma(n)h}\tilde{N} & \sqrt{(1-\sigma(n))h}\bar{A}_m^{\mathrm{T}}\tilde{H}_4^{\mathrm{T}}M & \Upsilon_2 & e_1^{\mathrm{T}}\bar{J}^{\mathrm{T}}\bar{K}_iU_{m2} \\ \star & -\Upsilon_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & -M & \sqrt{(1-\sigma(n))h}MU_{m1} & \mathbf{0} \\ \star & \star & \star & -\varepsilon_i\mathbf{I} & \mathbf{0} \\ \star & \star & \star & \star & -\varepsilon_i\mathbf{I} \end{bmatrix} < 0, \quad n = 1,2, \ m \in \mathscr{N}(i), \ i \in \mathcal{I} \tag{42}$$

where $\tilde{H}_1$ is given in (14), and Σ_1 and Σ_2 are given in (22), and

$$\begin{cases} \Upsilon_1 = \mathtt{Sym}\left\{e_1^{\mathrm{T}}\left(\begin{bmatrix} \tilde{F}_i^{\mathrm{T}}X\tilde{F}_i \\ \mathbf{0} \end{bmatrix}\bar{A}_m + \bar{J}^{\mathrm{T}}\bar{C}_i\right) + \tilde{N}\Sigma_2\right\} \\ \qquad +\Sigma_1^{\mathrm{T}}\bar{Q}\Sigma_1 + e_1^{\mathrm{T}}\tilde{H}_0^{\mathrm{T}}\tilde{G}_i^{\mathrm{T}}T_i\tilde{G}_i\tilde{H}_0e_1 \\ \qquad +e_1^{\mathrm{T}}\tilde{L}_m^{\mathrm{T}}\tilde{L}_me_1 - \gamma^2e_6^{\mathrm{T}}e_6 + \varepsilon_i\bar{W}_m^{\mathrm{T}}\bar{W}_m, \\ \Upsilon_2 = e_1^{\mathrm{T}}\begin{bmatrix} \tilde{F}_i^{\mathrm{T}}X\tilde{F}_i \\ \mathbf{0} \end{bmatrix}\tilde{H}_3U_{m1}, \\ \Upsilon_3 = \mathrm{diag}\{M,3M,5M,7M\}, \\ \bar{C}_i = \\ \begin{bmatrix} \bar{k}_i\tilde{H}_2 & -P_{i2}^{\mathrm{T}} & \bar{K}_iC & \mathbf{0}_{n_u\times(3n_x+n_w)} & \bar{J}^{\mathrm{T}}\bar{K}_iD_2 \end{bmatrix}, \\ \left.\begin{array}{l} \bar{A}_m = \begin{bmatrix} A_m & B_m & \mathbf{0}_{n_x\times 4n_x} & D_{m1} & \mathbf{0}_{n_x\times n_w} \end{bmatrix}, \\ \tilde{H}_2 = \mathbf{0}_{1\times n_x}, \ \tilde{H}_3 = \mathbf{I}_{n_x}, \ \tilde{H}_4 = \mathbf{I}_{n_x}, \\ \tilde{L}_m = \begin{bmatrix} L_m & \mathbf{0}_{n_z\times n_u} \end{bmatrix}, \\ \bar{W}_m = \begin{bmatrix} W_{m1} & W_{m3} & \mathbf{0} & \mathbf{0}_{n_{s_2}\times(3n_x+2n_w)} \\ \mathbf{0} & \mathbf{0} & W_4 & \mathbf{0}_{n_{s_2}\times(3n_x+2n_w)} \end{bmatrix}, \\ \qquad \text{if } i \in \mathcal{I}_0, \end{array}\right\} \\ \left.\begin{array}{l} \bar{A}_m = \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} & \mathbf{0}_{1\times 4n_x} & \mathbf{0} & \mathbf{0} \\ a_m & A_m & B_m & \mathbf{0}_{n_x\times 4n_x} & D_{m1} & \mathbf{0} \end{bmatrix}, \\ \tilde{H}_2 = \begin{bmatrix} 1 & \mathbf{0}_{1\times n_x} \end{bmatrix}, \\ \tilde{H}_3 = \begin{bmatrix} \mathbf{0}_{1\times n_x} \\ \mathbf{I}_{n_x} \end{bmatrix}, \\ \tilde{H}_4 = \begin{bmatrix} \mathbf{0}_{n_x\times 1} & \mathbf{I}_{n_x} \end{bmatrix}, \\ \tilde{L}_m = \begin{bmatrix} \mathbf{0}_{n_z\times 1} & L_m & \mathbf{0}_{n_z\times n_u} \end{bmatrix}, \\ \bar{W}_m = \\ \begin{bmatrix} W_{m2} & W_{m1} & W_{m3} & \mathbf{0} & \mathbf{0}_{n_{s_2}\times(3n_x+2n_w)} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & W_4 & \mathbf{0}_{n_{s_2}\times(3n_x+2n_w)} \end{bmatrix}, \\ \qquad \text{if } i \in \mathcal{I}_1 \end{array}\right\} \end{cases} \tag{43}$$

with $\tilde{J} \in \Re^{n_u\times n_x}$, for $i \in \mathcal{I}_0$, and $\tilde{J} \in \Re^{n_u\times(1+n_x)}$, for $i \in \mathcal{I}_1$, being arbitrary matrices, and $\bar{J} = \begin{bmatrix} \tilde{J} & \mathbf{I}_{n_u} \end{bmatrix}$.

In addition, the controller gains can be attained by

$$\begin{cases} K_i = P_{i2}^{-\mathrm{T}}\bar{K}_i,\ i \in \mathcal{I}, \\ k_i = P_{i2}^{-\mathrm{T}}\bar{k}_i,\ i \in \mathcal{I}_1. \end{cases} \tag{44}$$

Proof: Extracting the fuzzy MFs, the following inequality implies (21),

$$\begin{aligned} &\mathtt{Sym}\Big\{ e_1^{\mathrm{T}}\Big(\tilde{P}_i^{\mathrm{T}}(\bar{A}_{im0}+\Delta\bar{A}_{im0}) \\ &\qquad + \begin{bmatrix} \tilde{P}_{i3}^{\mathrm{T}} \\ P_{i2}^{\mathrm{T}} \end{bmatrix}(\bar{C}_{i0}+\Delta\bar{C}_{i0})\Big) + \tilde{N}\Sigma_2 \Big\} \\ &+\Sigma_1^{\mathrm{T}}\bar{Q}\Sigma_1 + \sigma(n)h\tilde{N}\mathrm{diag}\{M,3M,5M,7M\}^{-1}\tilde{N}^{\mathrm{T}} \\ &+(1-\sigma(n))h(\bar{A}_{im0}^{\mathrm{T}}+\Delta\bar{A}_{im0}^{\mathrm{T}})\tilde{H}_1^{\mathrm{T}}M\tilde{H}_1(\bar{A}_{im0}+\Delta\bar{A}_{im0}) \\ &+e_1^{\mathrm{T}}\tilde{H}_0^{\mathrm{T}}\tilde{G}_i^{\mathrm{T}}T_i\tilde{G}_i\tilde{H}_0e_1 + e_1^{\mathrm{T}}\tilde{L}_m^{\mathrm{T}}\tilde{L}_me_1 - \gamma^2e_6^{\mathrm{T}}e_6 < 0, \\ &\qquad\qquad n=1,2,\ m\in\mathscr{N}(i),\ i\in\mathcal{I} \end{aligned} \tag{45}$$

where

$$\begin{cases} \tilde{P}_i = \begin{bmatrix} \tilde{F}_i^{\mathrm{T}}X\tilde{F}_i & \mathbf{0} \\ \tilde{P}_{i3} & P_{i2} \end{bmatrix}, \\ \left.\begin{aligned} &\bar{A}_{im0}+\Delta\bar{A}_{im0} = \\ &\begin{bmatrix} A_m+\Delta A_m & B_m+\Delta B_m & \mathbf{0}_{n_x\times 4n_x} & D_{m1} & \mathbf{0}_{3n_x\times n_w} \\ \mathbf{0} & -\mathbf{I} & \mathbf{0}_{n_u\times 4n_x} & \mathbf{0}_{n_u\times n_w} & \mathbf{0}_{3n_x\times n_w} \end{bmatrix}, \\ &\bar{C}_{i0}+\Delta\bar{C}_{i0} = \begin{bmatrix} \mathbf{0}_{n_u\times(n_x+n_u)} & K_i(C+\Delta C) & \mathbf{0}_{n_u\times(3n_x+n_w)} & K_iD_2 \end{bmatrix}, \\ &\tilde{L}_m = \begin{bmatrix} L_m & \mathbf{0}_{n_z\times n_u} \end{bmatrix}, \end{aligned}\right\} \text{if } i\in\mathcal{I}_0, \\ \left.\begin{aligned} &\bar{A}_{im0}+\Delta\bar{A}_{im0} = \\ &\begin{bmatrix} 0 & \mathbf{0}_{1\times n_x} & \mathbf{0}_{1\times n_u} & \mathbf{0}_{1\times 4n_x} & \mathbf{0}_{1\times n_w} & \mathbf{0}_{1\times n_w} \\ a_m+\Delta a_m & A_m+\Delta A_m & B_m+\Delta B_m & \mathbf{0}_{n_x\times 4n_x} & D_{m1} & \mathbf{0}_{n_x\times n_w} \\ k_i & \mathbf{0} & -\mathbf{I} & \mathbf{0}_{n_u\times 4n_x} & \mathbf{0}_{n_u\times n_w} & \mathbf{0}_{n_u\times n_w} \end{bmatrix}, \\ &\bar{C}_{i0}+\Delta\bar{C}_{i0} = \begin{bmatrix} \mathbf{0}_{n_u\times(1+n_x+n_u)} & K_i(C+\Delta C) & \mathbf{0}_{n_u\times(3n_x+n_w)} & K_iD_2 \end{bmatrix}, \\ &\tilde{L}_m = \begin{bmatrix} \mathbf{0}_{n_z\times 1} & L_m & \mathbf{0}_{n_z\times n_u} \end{bmatrix}, \end{aligned}\right\} \text{if } i\in\mathcal{I}_1. \end{cases} \tag{46}$$

Define

$$\begin{cases} \bar{K}_i = P_{i2}^{\mathrm{T}}K_i,\ i\in\mathcal{I}, \\ \bar{k}_i = P_{i2}^{\mathrm{T}}k_i,\ i\in\mathcal{I}_0. \end{cases} \tag{47}$$

Inspecting the explicit structural features of the system matrices in (46), one can find that the controller gains do not appear in the first and second rows of $\bar{A}_{im0}+\Delta\bar{A}_{im0}$. For numerical tractability, specify $\tilde{P}_{i3}$ as

$$\tilde{P}_{i3} = P_{i2}\tilde{J} \tag{48}$$

where $\tilde{J}\in\Re^{n_u\times n_x}$, for $i\in\mathcal{I}_0$, and $\tilde{J}\in\Re^{n_u\times(1+n_x)}$, for $i\in\mathcal{I}_1$, are arbitrary matrices.

Based on (47)-(48) and adopting Lemma 2 given in the appendix to tackle the parameter uncertainties presented in (2), the subsequent inequality indicates (45) for scalars $\varepsilon_i>0$, $i\in\mathcal{I}$ relying on Schur complement,

$$\begin{aligned} &\begin{bmatrix} \Psi_1 & \sqrt{\sigma(n)h}\tilde{N} & \sqrt{(1-\sigma(n))h}\bar{A}_m^{\mathrm{T}}\tilde{H}_4^{\mathrm{T}}M \\ \star & -\Psi_2 & \mathbf{0} \\ \star & \star & -M \end{bmatrix} \\ &+\varepsilon_i^{-1}\begin{bmatrix} \Psi_3 & \Psi_4 \\ \mathbf{0} & \mathbf{0} \\ \Psi_5 & \mathbf{0} \end{bmatrix}(\star) + \varepsilon_i\begin{bmatrix} \bar{W}_{m1}^{\mathrm{T}} & \bar{W}_{m2}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}(\star) < 0, \\ &\qquad\qquad n=1,2, m\in\mathscr{N}(i), i\in\mathcal{I} \end{aligned} \tag{49}$$

where

$$\begin{cases} \Psi_1 = \mathtt{Sym}\Big\{e_1^{\mathrm{T}}\Big(\begin{bmatrix} \tilde{F}_i^{\mathrm{T}}X\tilde{F}_i \\ \mathbf{0} \end{bmatrix}\bar{A}_m + \bar{J}^{\mathrm{T}}\bar{C}_i\Big) + \tilde{N}\Sigma_2\Big\} \\ \quad +\Sigma_1^{\mathrm{T}}\bar{Q}\Sigma_1 + e_1^{\mathrm{T}}\tilde{H}_0^{\mathrm{T}}\tilde{G}_i^{\mathrm{T}}T_i\tilde{G}_i\tilde{H}_0e_1 \\ \quad +e_1^{\mathrm{T}}\tilde{L}_m^{\mathrm{T}}\tilde{L}_me_1 - \gamma^2e_6^{\mathrm{T}}e_6, \\ \Psi_2 = \mathrm{diag}\{M,3M,5M,7M\}, \\ \Psi_3 = e_1^{\mathrm{T}}\begin{bmatrix} \tilde{F}_i^{\mathrm{T}}X\tilde{F}_i \\ \mathbf{0} \end{bmatrix}\tilde{H}_3U_{m1}, \\ \Psi_4 = e_1^{\mathrm{T}}\bar{J}^{\mathrm{T}}\bar{K}_iU_{m2}, \\ \Psi_5 = \sqrt{(1-\sigma(n))h}MU_{m1}, \\ \bar{J} = \begin{bmatrix} \tilde{J} & \mathbf{I}_{n_u} \end{bmatrix}, \\ \bar{C}_i = \begin{bmatrix} \bar{k}_i\tilde{H}_2 & -P_{i2}^{\mathrm{T}} & \bar{K}_iC & \mathbf{0}_{n_u\times(3n_x+n_w)} & \bar{K}_iD_2 \end{bmatrix}, \\ \left.\begin{aligned} &\bar{A}_m = \begin{bmatrix} A_m & B_m & \mathbf{0}_{n_x\times 4n_x} & D_{m1} & \mathbf{0}_{n_x\times n_w} \end{bmatrix}, \\ &\tilde{H}_2 = \mathbf{0}_{1\times n_x},\ \tilde{H}_3 = \mathbf{I}_{n_x},\ \tilde{H}_4 = \mathbf{I}_{n_x}, \\ &\tilde{L}_m = \begin{bmatrix} L_m & \mathbf{0}_{n_z\times n_u} \end{bmatrix}, \\ &\bar{W}_{m1} = \begin{bmatrix} W_{m1} & W_{m3} & \mathbf{0}_{n_{s_2}\times(4n_x+2n_w)} \end{bmatrix}, \\ &\bar{W}_{m2} = \begin{bmatrix} \mathbf{0}_{n_{s_2}\times(n_x+n_u)} & W_4 & \mathbf{0}_{n_{s_2}\times(3n_x+2n_w)} \end{bmatrix}, \end{aligned}\right\} \text{if } i\in\mathcal{I}_0, \\ \left.\begin{aligned} &\bar{A}_m = \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} & \mathbf{0}_{1\times 4n_x} & \mathbf{0} & \mathbf{0} \\ a_m & A_m & B_m & \mathbf{0}_{n_x\times 4n_x} & D_{m1} & \mathbf{0} \end{bmatrix}, \\ &\tilde{H}_2 = \begin{bmatrix} 1 & \mathbf{0}_{1\times n_x} \end{bmatrix},\ \tilde{H}_3 = \begin{bmatrix} \mathbf{0}_{1\times n_x} \\ \mathbf{I}_{n_x} \end{bmatrix}, \\ &\tilde{H}_4 = \begin{bmatrix} \mathbf{0}_{n_x\times 1} & \mathbf{I}_{n_x} \end{bmatrix}, \\ &\tilde{L}_m = \begin{bmatrix} \mathbf{0}_{n_z\times 1} & L_m & \mathbf{0}_{n_z\times n_u} \end{bmatrix}, \\ &\bar{W}_{m1} = \begin{bmatrix} W_{m2} & W_{m1} & W_{m3} & \mathbf{0}_{n_{s_2}\times(4n_x+2n_w)} \end{bmatrix}, \\ &\bar{W}_{m2} = \begin{bmatrix} \mathbf{0}_{n_{s_2}\times(1+n_x+n_u)} & W_4 & \mathbf{0}_{n_{s_2}\times(3n_x+2n_w)} \end{bmatrix}, \end{aligned}\right\} \text{if } i\in\mathcal{I}_1. \end{cases} \tag{50}$$

Thus inequality (42) implies (49) based on Schur complement.

Note that the conditions in (42) imply $-P_{i2}-P_{i2}^{\mathrm{T}}<0$, which indicates the invertibility of P_{i2}. Consequently, the controller gains can be calculated via (44). The proof is accomplished. ■

The stability of the closed-loop system is assured by Theorem 3.1 and the controller gains can be designed based on Theorem 3.2.

The detailed procedures on computing the sampled-data controller gains are given as follows.

Step 1. Based on the normalized membership functions and space partitions of the system (1), calculate the matrices $\tilde{G}_i$ and $\tilde{F}_i$ for Theorem 3.2 via (18)-(19).

Step 2. For given matrices $\tilde{J}$, together with those parameters computed in Step 1, solve the LMI problems given in Theorem 3.2 over the matrix variables Q, M, X, Y_i, T_i, P_{i2}, $\bar{K}_i$, $\bar{k}_i$, and $\tilde{N}$, and scalars $\{\varepsilon_i>0, i\in\mathcal{I}\}$.

Step 3. Based on the obtained matrices P_{i2}, $\bar{k}_i$ and $\bar{K}_i$ in Step 2, the controller gains can be then computed via (44).

IV. Simulation Studies

Example 4.1. As shown in Fig. 1, consider a nonlinear circuit containing nonlinear resistor and parasitic capacitor [48].

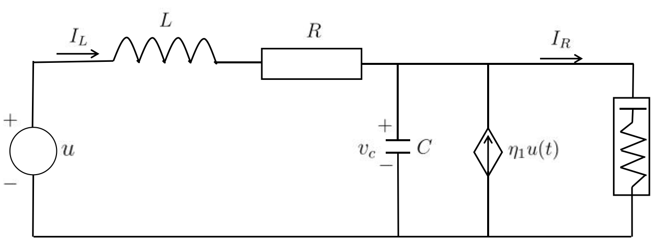


Fig. 1. Nonlinear circuit with nonlinear resistor and parasitic capacitor

The v_c-I_R characteristic of the nonlinear resistor is $I_R = 0.2(v_c^3 - v_c)$. Based on the Kirchoff voltage and current law, one can acquire the state equation of the circuit system as

$$\begin{cases} C\dot{v}_c(t) = 0.2(v_c - v_c^3) + I_L + \eta_1 u(t) + \eta_2 w(t) \\ L\dot{I}_L(t) = -v_c - I_L R + u(t) \end{cases} \tag{51}$$

where $\eta_1 = 0.1$, $\eta_2 = 0.4$, $C = 0.2$F, $R = 10\Omega$, $L = 1$H, and $w(t)$ represents the external nonlinear disturbance. Denote $x_1(t) = v_c(t)$, $x_2(t) = I_L(t)$. Linearize the nonlinear system (51) at three operating points $(0,0)^{\mathrm{T}}$ and $(\pm 1, 0)^{\mathrm{T}}$, and then one can describe the system (51) via the subsequent T-S fuzzy affine model with three rules,

Plant Rule $\mathscr{R}^l$: IF $x_1(t)$ is $\mathscr{F}_1^l$, THEN

$$\begin{cases} \dot{x}(t) = (A_l + \Delta A_l)\, x(t) + a_l + \Delta a_l + (B_l + \Delta B_l)\, u(t) \\ \qquad\quad + D_{l1} w(t) \\ y(t) = (C + \Delta C) x(t) + D_2 w(t) \\ z(t) = L_l x(t), \ \ l \in \{1, 2, 3\} \end{cases} \tag{52}$$

where

$$\begin{cases} A_1 = A_3 = \begin{bmatrix} -2 & 5 \\ -1 & -10 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 5 \\ -1 & -10 \end{bmatrix}, \\ a_1 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, a_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \\ B_l = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ D_{l1} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, D_2 = 2, \\ L_l = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, l = \{1, 2, 3\} \end{cases} \tag{53}$$

and the parameter uncertainties ΔA_l, Δa_l, ΔB_l and ΔC are given in the form of (2) with

$$\begin{cases} U_{l1} = \begin{bmatrix} 0.15 \\ 0 \end{bmatrix}, U_2 = 0.1, W_{l1} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \\ W_{l2} = 0.05, W_{l3} = 0.1, W_4 = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, l = \{1, 2, 3\}. \end{cases} \tag{54}$$

The normalized MFs are presented in Fig. 2, where $d_1 = -5$, $d_2 = -0.8$, $d_3 = 0.8$ and $d_4 = 5$. The premise variable space is divided as,

$$\begin{aligned} S_1 &= \{x \in \Re^2 | d_1 \le x_1 \le d_2\}, \\ S_2 &= \{x \in \Re^2 | d_2 \le x_1 \le d_3\}, \\ S_3 &= \{x \in \Re^2 | d_3 \le x_1 \le d_4\}. \end{aligned} \tag{55}$$

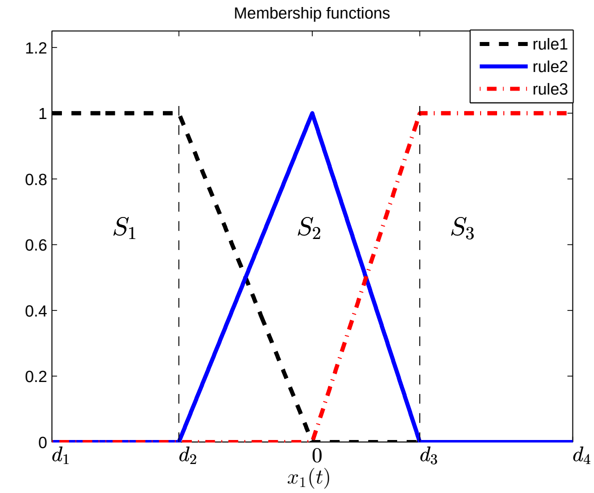


Fig. 2. Membership Functions

We attempt to synthesize a sampled-data PWA output feedback controller (11) to ensure the closed-loop system to be asymptotically stable with robust performance γ. *Assume that the system full states in this example are not available and the measurement output is in a sampled-data form with given maximum sampling interval $h = 0.2s$. Furthermore, the existence of the affine terms $a_l + \Delta a_l$ has also introduced much difficulty for sampled-data controller synthesis. Thus, the design methods given in [16], [26], [45], [46] are not applicable*. Fortunately, applying Theorem 3.2 with $\tilde{J} = \begin{bmatrix} 3 & 3 \end{bmatrix}$, for $i \in \mathcal{I}_0$, and $\tilde{J} = \begin{bmatrix} 3 & 3 & 3 \end{bmatrix}$, for $i \in \mathcal{I}_1$, one can obtain the feasible solutions with robust $\mathscr{H}_\infty$ performance $\gamma_{\min} = 1.5459$, and the controller gains are

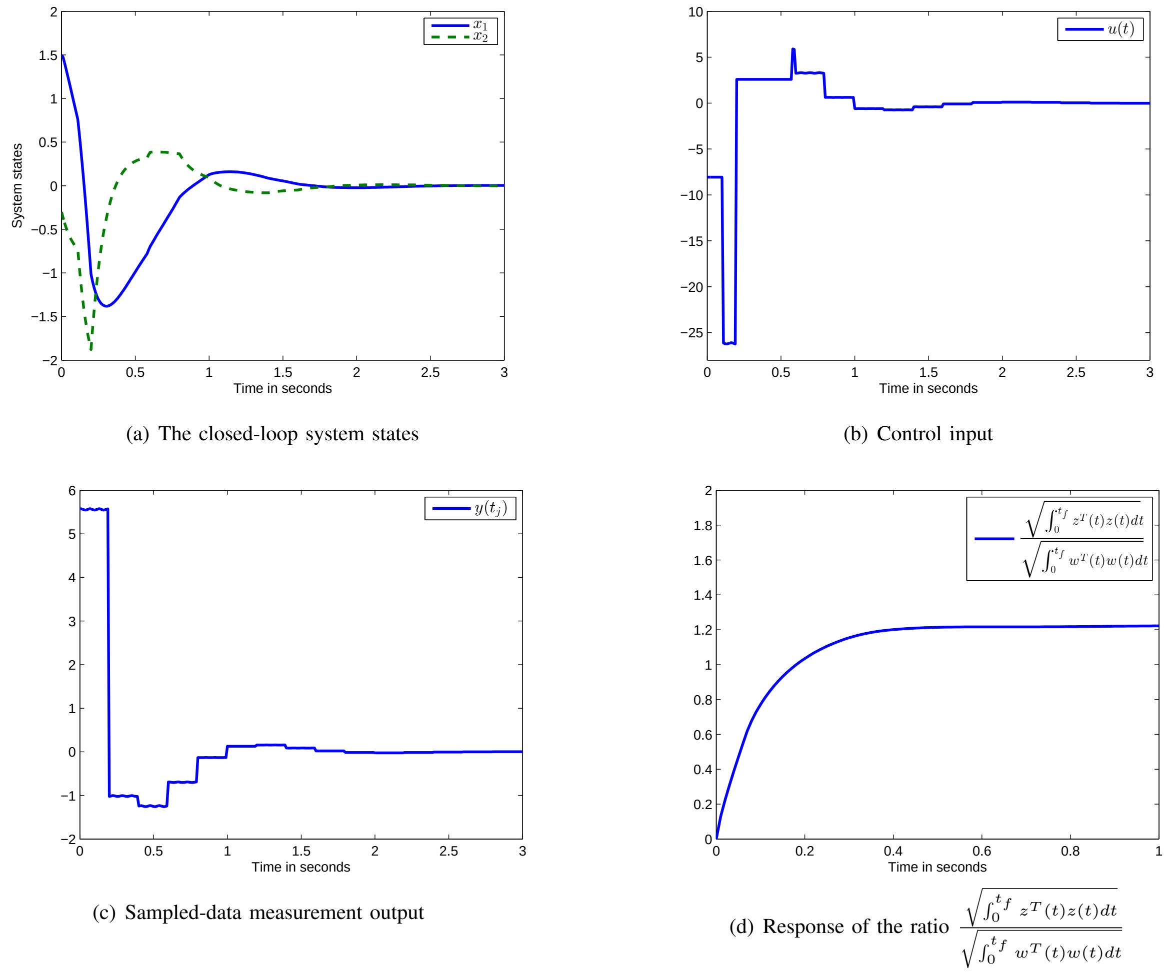
$$\begin{cases} K_1 = 0.0057, \ k_1 = 2.5907, \\ K_2 = -4.7080, \\ K_3 = 0.0351, \ k_3 = -8.2608. \end{cases} \tag{56}$$

Notice that when using Theorem 3.2 with PLKFs in (24), one can calculate $\tilde{F}_i$ and $\tilde{G}_i$ via

$$\begin{aligned} &\left[\begin{array}{c|c|c|c|c|c} F_1 & F_2 & F_3 & f_1 & f_2 & f_3 \end{array}\right] = \\ &\left[\begin{array}{c|c|c|c|c|c} -\rho & \mathbf{0}_{1\times n_x} & \mathbf{0}_{1\times n_x} & d_2 & 0 & 0 \\ \mathbf{0}_{1\times n_x} & \mathbf{0}_{1\times n_x} & \rho & 0 & 0 & -d_3 \\ \mathbf{I}_{n_x} & \mathbf{I}_{n_x} & \mathbf{I}_{n_x} & \mathbf{0}_{n_x\times 1} & \mathbf{0}_{n_x\times 1} & \mathbf{0}_{n_x\times 1} \end{array}\right], \\ &\left[\begin{array}{c|c|c|c|c|c} G_1 & G_2 & G_3 & g_1 & g_2 & g_3 \end{array}\right] = \\ &\left[\begin{array}{c|c|c|c|c|c} \rho & \mathbf{0}_{1\times n_x} & \rho & -d_1 & 0 & -d_3 \\ -\rho & \mathbf{0}_{1\times n_x} & -\rho & d_2 & 0 & d_4 \end{array}\right] \end{aligned} \tag{57}$$

with $n_x = 2$ and $\rho = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

To verify the effectiveness of the developed scheme, simulations are conducted. Under initial condition $x_0 = \begin{bmatrix} 1.5 & -0.3 \end{bmatrix}^{\mathrm{T}}$ and exogenous disturbance function $w(t) = 5\mathrm{e}^{-2t}\cos(2\pi t)$, by using the sampled-data PWA output feedback controller in (11), Fig. 3(a) and Fig. 3(b) show the system states and control input, respectively. The sampled-data measurement output is demonstrated in Fig. 3(c). Under zero initial condition, Fig. 3(d) shows the $\mathscr{H}_\infty$ performance. The ratio $\frac{\sqrt{\int_0^{t_f} z^T(t) z(t) dt}}{\sqrt{\int_0^{t_f} w^T(t) w(t) dt}}$ is about 1.21, which is lower than



(a) The closed-loop system states

(b) Control input

(c) Sampled-data measurement output

(d) Response of the ratio $\frac{\sqrt{\int_0^{t_f} z^T(t)z(t)dt}}{\sqrt{\int_0^{t_f} w^T(t)w(t)dt}}$

Fig. 3. Time responses of the closed-loop system in Example 4.1.

$\gamma_{\min} = 1.5459$.

To further show the advantages of PWA controller over PWL controller, in the sequel, consider another numerical example.

Example 4.2. Consider a fuzzy affine system (1) involving three rules and

$$\left\{\begin{array}{l} A_1 = \begin{bmatrix} 0.1 & 0.6 \\ -0.5 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} -0.2 & 0.7 \\ 0.1 & 0.1 \end{bmatrix}, \\ A_3 = \begin{bmatrix} -0.3 & 0.5 \\ 0.2 & 0.1 \end{bmatrix}, \\ a_1 = \begin{bmatrix} 0 \\ -0.3 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, a_3 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 1 \\ 1.2 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_3 = \begin{bmatrix} 1.4 \\ 1 \end{bmatrix}, \\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{l1} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, D_2 = 1, \\ L_l = \begin{bmatrix} 0.005 & 0 \end{bmatrix}, l \in \{1,2,3\} \end{array}\right. \tag{58}$$

and the parameter uncertainties ΔA_l, Δa_l, ΔB_l and ΔC are given in the form of (2) with

$$\left\{\begin{array}{l} U_{l1} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}, U_2 = 0.1, W_{l1} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}, \\ W_{l2} = 0.2, W_{l3} = 0.08, W_4 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}, l = \{1,2,3\}. \end{array}\right. \tag{59}$$

The MFs are given in Fig. 2 and $-d_2 = d_3 = 20$, $-d_1 = d_4 = 200$. The premise variable space is divided into three regions as in (55). The characteristic matrices $\tilde{F}_i$ and $\tilde{G}_i$ can also be calculated via (57) with $n_x = 2$ and $\rho = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

We attempt to synthesize a sampled-data PWA/PWL controller (11) for the T-S fuzzy affine system (1) in an $\mathscr{H}_\infty = \gamma$ framework. Obviously, a PWL controller can be attained by setting $k_i \equiv 0$ in (11). Adopting Theorem 3.2 for the cases that $k_i \neq 0$ and $k_i \equiv 0$, a detailed comparison of the robust $\mathscr{H}_\infty$ performance $\gamma_{\min}$ for the sampled-data PWA and PWL controllers is given in Table I. With the observation of Table I, one could easily conclude that the $\mathscr{H}_\infty$ performance $\gamma_{\min}$ relying on PWA controllers are generally better than that relying on PWL ones.

TABLE I
COMPARISON OF ROBUST $\mathscr{H}_\infty$ PERFORMANCE $\gamma_{\min}$ FOR PWL AND PWA CONTROLLERS IN EXAMPLE 4.2

Controllers	$h = 0.2$	$h = 0.4$	$h = 0.7$
PWA controller	1.4954	2.4761	2.8446
PWL controller	1.9595	3.6817	4.0191

V. Conclusions

This paper has investigated the issue of sampled-data output feedback control for nonlinear systems through T-S fuzzy affine models. Via augmenting the system states with the control input and based on an input delay technique, the closed-loop system is formulated into a singular system with time-varying delay. By utilizing a PLKFs-based method with an integral inequality, some new sampled-data PWA output feedback controller synthesis results are proposed under a convex optimization framework. Simulation studies are provided to verify the effectiveness of the proposed approach. It is noted that in practical applications, control systems are always subject to input constraints, which would have a great impact on the performance of the closed-loop control systems. Thus, one interesting future research work is the study of fuzzy sampled-data output feedback control for nonlinear systems with actuator saturation [42], [44]. Another future work is the investigation of fuzzy sampled-data output feedback control for stochastic nonaffine nonlinear systems [8], [9].

VI. Appendix

Lemma 1 [47]. ρ denotes a differentiable function: $[t_1,\ t_2] \rightarrow \Re^p$. For a positive define matrix $0 < M \in \Re^{p\times p}$ and matrix $N \in \Re^{5p\times 4p}$, the subsequent inequality holds,

$$-\int_{t_1}^{t_2} \dot{\rho}^{\mathrm{T}}(s) Q \dot{\rho}(s) ds \leq \xi^{\mathrm{T}} \Omega \xi$$

where $d = t_2 - t_1$, and

$$\begin{cases} \Omega = d\cdot N\mathrm{diag}\{M, 3M, 5M, 7M\}^{-1}N^{\mathrm{T}} + \mathtt{Sym}\{N\cdot\Pi\}, \\ \Pi = \left[\begin{array}{cccc} \Pi_1^{\mathrm{T}} & \Pi_2^{\mathrm{T}} & \Pi_3^{\mathrm{T}} & \Pi_4^{\mathrm{T}} \end{array}\right]^{\mathrm{T}}, \\ \xi = \Big[\begin{array}{ccc} \rho^{\mathrm{T}}(t_2) & \rho^{\mathrm{T}}(t_1) & \frac{1}{d}\int_{t_1}^{t_2}\rho^{\mathrm{T}}(s)ds \end{array} \\ \quad \begin{array}{cc} \frac{2}{d^2}\int_{t_1}^{t_2}\int_{t_1}^{s}\rho^{\mathrm{T}}(\alpha)d\alpha ds & \frac{6}{d^3}\int_{t_1}^{t_2}\int_{t_1}^{s}\int_{t_1}^{\alpha}\rho^{\mathrm{T}}(\beta)d\beta d\alpha ds \end{array}\Big]^{\mathrm{T}}, \\ \Pi_1 = e_1 - e_2, \Pi_2 = e_1 + e_2 - 2e_3, \\ \Pi_3 = e_1 - e_2 + 6e_3 - 6e_4, \\ \Pi_4 = e_1 + e_2 - 12e_3 + 30e_4 - 20e_5, \\ e_n = \left[\begin{array}{ccc} \mathbf{0}_{p\times(n-1)p} & \mathbf{I}_p & \mathbf{0}_{p\times(5-n)p} \end{array}\right],\ n = 1,\cdots,5. \end{cases}$$

Lemma 2 [27]. Given real matrices $M = M^{\mathrm{T}}$, U, W, and $\Delta(t)$, the subsequent inequality holds for all $\|\Delta(t)\| \leq \mathbf{I}$,

$$M + \mathtt{Sym}\{U\Delta(t)W\} < 0$$

if and only if, for some parameter $\varepsilon > 0$,

$$M + \varepsilon U^{\mathrm{T}}U + \varepsilon^{-1}W^{\mathrm{T}}W < 0$$

holds.

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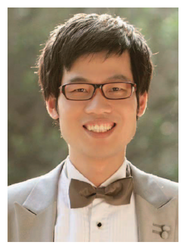

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