Robust Stability Analysis and Feedback Control for Uncertain Systems with Time-Delay and External Disturbance

Wei Zheng, *Member, IEEE*, Hak-Keung Lam, *Fellow, IEEE*, Fuchun Sun, *Fellow, IEEE*, and Shuhuan, Wen, *Member, IEEE*

***Abstract*—This paper addresses the delay-dependent Takagi-Sugeno (T-S) fuzzy state feedback control and exponential admissibility analysis for a class of T-S fuzzy singular uncertain systems. Firstly, the T-S fuzzy model is employed to approximate the singular uncertain system with time-varying delay, saturation input and unmatched disturbance. Secondly, the delay-dependent T-S fuzzy state feedback controller is designed by employing the T-S fuzzy model. Thirdly, the free-weighting matrices and delay-dependent Lyapunov-Krasovskii functional with multiple integral terms are employed to derive the delay-dependent exponential admissibility conditions and prescribed H-infinity performance is guaranteed. Compared with previous works, the delay-dependent T-S fuzzy state feedback controller is designed for the T-S fuzzy singular uncertain system to relax system design conditions. The convex hull lemma is employed to convert the closed-loop system with saturation input into the closed-loop system without saturation input to enhance controller design flexibility. The Schur complement lemma and Gronwall Bellman lemma are employed to derive the less conservative delay-dependent stability conditions for determining controller gain matrices. The exact invariant set with less conservativeness is employed to convert the controller design problem into linear matrix inequalities (LMIs) optimization constraints to reduce computation complexity of solving LMIs. Finally, simulation examples are presented to show the effectiveness of the proposed methods.**

***Index Terms*—Free-weighting matrices, Lyapunov-Krasovskii functional, linear matrix inequalities, disturbance.**

I. INTRODUCTION

ROBUST stability analysis and feedback control for the dynamic systems are fundamental and complex problems in the control theory. The stability analysis and controller design methods are challenging and significant because the system plants are subjected by different kinds of uncertainties in the engineering applications, such as the exogenous unknown inputs, measurement noises, unknown parameters, unmodeled dynamics and so on [1, 2]. Moreover, many practical systems are the hybrid systems and contain the time-delay and/or external disturbance, such as the economic systems, power systems, urban traffic systems and robotic control systems [3, 4]. It is well known that the time-delay and/or external disturbance are often encountered in the various systems [5, 6]. The robust control of the systems has become the important research topic and many fresh investigations have been discussed [7, 8]. The nonlinearities will arise because of the time-varying delays and external disturbances. Firstly, although there are many literatures investigate the time-delay systems, few literatures have solved the problem of unmatched disturbance in the control system. Secondly, compared with previous works, most of the literatures focused on the matched disturbances. The "unmatched disturbances" means that the external disturbances and control inputs are in the different channel and this constraint may be limited in the practical systems. Thus, this paper investigates the controller design and stability analysis problem for the singular uncertain systems with time-varying delay, saturation input and unmatched disturbance.

Fuzzy control is an extraordinary technology in the soft computing, and it has the better ability to offer the significant industrial applications [9]. Moreover, fuzzy control is a specific artificial intelligence technology, and it widely used in automatic control field [10]. Compared with other artificial intelligence technologies, such as neural networks control, fuzzy control is more flexible because it can incorporate the knowledge and experience of designer [11]. Model-based fuzzy control and optimal tuning represent two viable directions to the systematic design of fuzzy controllers. Model-based fuzzy control and optimal optimization represent two feasible fields of fuzzy system design. However, both fields require to guarantee the stability of fuzzy control system [11, 12]. For example, it has been shown that the T-S fuzzy system was asymptotically stable if there exist the common positive definite solutions to a set of LMIs [13]. Besides, the asymptotic stability was guaranteed, even if the system plant subjected to the parameter uncertainties captured by membership functions [14]. In this paper, the singular uncertain system will contain more uncertainties, because the solutions are not only determined by past results and current inputs, but also related to future inputs of the system, such as the noncausal characteristics. The conservative admissibility conditions will limit the flexibility of controller design, thus some conservative stability conditions may arise in the exponential admissibility analysis of the singular uncertain system.

It is important to obtain the state variables information in practice, thus the state feedback control has been proposed [15]. For example, the adaptive robust state feedback controller was designed for a class of nonlinear single-input and single-output systems, and the fuzzy model was employed to approximate the unmeasured states and unstructured uncertainties [16]. The adaptive robust state feedback controller was designed for a class of nonlinear multi-input and multi-output systems, and the fuzzy state observer was employed to estimate the state variables for guaranteeing the asymptotic stability [17]. Compared with output feedback, the state feedback has better ability to reflect the internal characteristics of state variables. Moreover, the state feedback does not change the controllability of system, but may change the observability of system [18].

It is well known that the delay-independent state output

This paper was financially supported by National Natural Science Foundation of China (91848206); Natural Science Foundation of Hebei Province (F2021203083, F2021203104); Natural Science Foundation for High Education College Science and Technology Plan of Hebei Province (QN2021138).

Wei Zheng and Shuhuan Wen are with the School of Electrical Engineering, Yanshan University, Qinhuangdao, 066004, China (e-mail: weizheng@ysu.edu.cn; wenshuhuau@sohu.com)

Hak-Keung Lam is with the Department of Engineering, King's College London, Strand Campus, Strand, London, WC2R 2LS, United Kingdom (Corresponding author e-mail: hak-keung.lam@kcl.ac.uk).

Fuchun Sun is with the School of Information Science and Technology, Tsinghua University, Beijing, 100084, China (e-mail: fcsun@tsinghua.edu.cn).

feedback control is easy to implement, but some strict design conditions should be considered [18, 19]. For example, the fuzzy delay-dependent state feedback controller was designed for a class of T-S fuzzy systems to achieve strict (Q, S, R)-α-dissipativity [20]. With the help of above literature, the research motivations in this paper are presented below. (i) The T-S fuzzy model has better ability to approximate the singular uncertain systems [21]. (ii) The delay-dependent state feedback control has better dependent characteristic than the delay-independent state feedback control to reflect the system internal features [21]. (iii) Most of the existing literatures are investigated based on the conventional Lyapunov-Krasovskii functional and few literatures employ the free-weighting matrices and delay-dependent Lyapunov-Krasovskii functional for the exponential admissibility analysis [21]. (iv) γ is the H-infinity performance index and used to investigate the prescribed H-infinity performance in the controller design [22]. In addition, this paper addresses the delay-dependent T-S fuzzy state feedback control by formulating the closed-loop system with saturation input into a closed-loop system without saturation input for enhancing the design flexibility.

TIn this paper, the contributions can be summarized below. (i) It is challenging to achieve the delay-dependent T-S fuzzy state feedback control and investigate the exponential admissibility analysis of singular uncertain system, because the time-varying delay, saturation input and unmatched disturbance are all considered system plant. The T-S fuzzy modeling technique is used to denote the system plant as an average weighted sum of semilinear subsystems, and the system plant can be approximated effectively. (ii) In the delay-dependent design concept, the conservativeness of stability conditions can be relaxed to some extent by considering the upper bounds information and lower bounds information of time-delays. In order to simplify the closed-loop system, this paper employs convex hull lemma to convert the closed-loop system with saturation input into closed-loop system without saturation input. (iii) The free-weighting matrices and delay-dependent Lyapunov-Krasovskii functional are designed, and it is able to reduce the system design conservatism. Moreover, the computational complexity of controller design is reduced via Schur complement lemma. Especially, Gronwall-Bellman lemma allows obtaining the simple design conditions to exponentially stabilize the uncertain systems.

Notations: $\mathbb{R}^n$ denotes the *n*-dimensional Euclidean space, $\mathbb{R}^{n\times n}$ denotes the set of the $n\times n$-dimensional matrix. $M>0(\geq 0)$ and $M<0(\leq 0)$ denote the positive definite (semi-positive definite) matrix and negative definite (semi-negative definite) matrix with appropriate dimensions, respectively. " I " and " 0 " denote the identity matrix and zero matrix with appropriate dimensions, respectively. $rank(E)$ denotes the rank of E. $\sum_{i=1}^{r} h_i(\bullet)$ denotes the sum of $h_i(\bullet)$ with $i=1,\ 2,\ \dots,\ r$, $\prod_{j=1}^{g}\mathcal{M}_j(\bullet)$ denotes the product of $\mathcal{M}_j(\bullet)$ with $j=1,\ 2,\ \dots,\ g$. $sat(\bullet)$ denotes the saturation function of " $\bullet$ ", $sup(\bullet)$ denotes the supremum of " $\bullet$ ". $max(\bullet)$ and $min(\bullet)$ denote the maximum value and minimum value of " $\bullet$ ", respectively. $\|\bullet\|$ denotes the Euclid norm of " $\bullet$ ", $diag\{\cdots\}$ denotes the block diagonal matrix, and " * " denotes the vector term that is induced by symmetry.

II. System Description

Applying the T-S fuzzy model, the T-S fuzzy singular uncertain system is given below.

Plant rule i: if $x_1(t)$ *is* $\mathcal{M}_{i1}$, $x_2(t)$ *is* $\mathcal{M}_{i2}$, ... , *and* $x_g(t)$ *is* $\mathcal{M}_{ig}$, *then*

$$\begin{cases} E\dot{x}(t)=\left(A_i+\Delta A_i(t)\right)x(t)+\left(A_{di}+\Delta A_{di}(t)\right)x\left(t-d(t)\right) \\ \qquad +B_i sat\left(u(t)\right)+B_{wi}w(t) \\ z(t)=\left(C_i+\Delta C_i(t)\right)x(t)+\left(C_{di}+\Delta C_{di}(t)\right)x\left(t-d(t)\right) \\ x(t)=\varphi(t),\quad t\in[-d_2,\ 0] \end{cases} \tag{1}$$

where $x_1(t),\ x_2(t),\ \dots,\ x_g(t)$ are the premise variables of the system, $\mathcal{M}_{ij}$ ($i=1,\ 2,\ \dots,\ r$, $j=1,\ 2,\ \dots,\ g$) is the fuzzy set of the system, *r* is the number of the fuzzy rules and *g* is the number of the premise variables. A_i, A_{di}, B_i, B_{wi}, C_i and C_{di} are the system gain matrices with appropriate dimensions, $x(t)\in\mathbb{R}^n$, $z(t)\in\mathbb{R}^p$, $u(t)\in\mathbb{R}^n$ are the state variable, control output, and control input, respectively. $\varphi(t)$ is the initial condition for $t\in[-d_2\ \ 0]$, and d_2 is defined in (5).

$E\in\mathbb{R}^{n\times n}$ is a singular matrix satisfying

$$rank(E)<n \tag{2}$$

$\Delta A_i(t)$, $\Delta A_{di}(t)$, $\Delta C_i(t)$ and $\Delta C_{di}(t)$ are the Lesbegue uncertainties

$$\begin{cases} \Delta A_i(t)=U_{i1}\Delta_{i1}(t)V_{i1}, & \Delta A_{di}(t)=U_{i2}\Delta_{i2}(t)V_{i2} \\ \Delta C_i(t)=U_{i3}\Delta_{i3}(t)V_{i3}, & \Delta C_{di}(t)=U_{i4}\Delta_{i4}(t)V_{i4} \end{cases} \tag{3}$$

where U_{i1}, U_{i2}, U_{i3}, U_{i4}, V_{i1}, V_{i2}, V_{i3} and V_{i4} are the known constant matrices with appropriate dimensions, $\Delta_{i1}(t)$, $\Delta_{i2}(t)$, $\Delta_{i3}(t)$ and $\Delta_{i4}(t)$ satisfy

$$\begin{cases} \Delta_{i1}^T(t)\Delta_{i1}(t)\leq I, & \Delta_{i2}^T(t)\Delta_{i2}(t)\leq I \\ \Delta_{i3}^T(t)\Delta_{i3}(t)\leq I, & \Delta_{i4}^T(t)\Delta_{i4}(t)\leq I \end{cases} \tag{4}$$

$d(t)$ is the time-varying delay satisfying

$$0<d_1\leq d(t)\leq d_2,\quad \dot{d}(t)\leq\bar{d} \tag{5}$$

where d_1, d_2 and $\bar{d}$ are the lower bound of $d(t)$, upper bound of $d(t)$ and upper bound of $\dot{d}(t)$, respectively.

$sat\left(u(t)\right):\mathbb{R}^n\to\mathbb{R}^n$ is the saturation input and described as

$$\begin{cases} u(t)=\left[u_1(t)\quad u_2(t)\quad \dots\quad u_n(t)\right]^T \\ sat\left(u(t)\right)=\left[sat\left(u_1(t)\right)\quad sat\left(u_2(t)\right)\quad \dots\quad sat\left(u_n(t)\right)\right]^T \end{cases} \tag{6}$$

with

$$\begin{cases} sat\left(u_1(t)\right)=sign\left(u_1(t)\right)min\left\{1,\ \left\|u_1(t)\right\|\right\} \\ sat\left(u_2(t)\right)=sign\left(u_2(t)\right)min\left\{1,\ \left\|u_2(t)\right\|\right\} \\ \qquad\vdots \\ sat\left(u_n(t)\right)=sign\left(u_n(t)\right)min\left\{1,\ \left\|u_n(t)\right\|\right\} \end{cases} \tag{7}$$

where $u_1(t)$, $u_2(t)$, … and $u_n(t)$ are the vector elements of $u(t)$. $sat\left(u_1(t)\right)$, $sat\left(u_2(t)\right)$, … and $sat\left(u_n(t)\right)$ are the vector elements of $sat\left(u(t)\right)$. $sign\left(u_1(t)\right)$, $sign\left(u_2(t)\right)$, … and $sign\left(u_n(t)\right)$ are the sign functions of $u_1(t)$, $u_2(t)$ and $u_n(t)$, respectively.

$min\{1,\ \|u_1(t)\|\}$, $min\{1,\ \|u_2(t)\|\}$, … and $min\{1,\ \|u_n(t)\|\}$ are the minimum values of $\{1,\ \|u_1(t)\|\}$, $\{1,\ \|u_2(t)\|\}$, … and $\{1,\ \|u_n(t)\|\}$, respectively. $\|u_1(t)\|$, $\|u_2(t)\|$, … and $\|u_n(t)\|$ are the Euclidean norms of $u_1(t)$, $u_2(t)$, … and $u_n(t)$, respectively.

$w(t)$ is the unmatched disturbance satisfying

$$w_1 \le w(t) \le w_2 \tag{8}$$

where w_1, w_2 and $\bar{w}$ are the lower bound of $w(t)$, upper bound of $w(t)$ and upper bound of $\dot{w}(t)$, respectively.

Applying T-S fuzzy inference to (1), one can obtain

$$\begin{cases} E\dot{x}(t) = \left(A_i(h)+\Delta A_i(h)\right)x(t)+\left(A_{di}(h)+\Delta A_{di}(h)\right)x\left(t-d(t)\right) \\ \qquad +B_i(h)sat\left(u(t)\right)+B_{wi}(h)w(t) \\ z(t) = \left(C_i(h)+\Delta C_i(h)\right)x(t)+\left(C_{di}(h)+\Delta C_{di}(h)\right)x\left(t-d(t)\right) \\ x(t)=\varphi(t), \quad t\in[-d_2,\ 0] \end{cases} \tag{9}$$

with

$$\begin{cases} A_i(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)A_i, & A_{di}(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)A_{di} \\ B_i(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)B_i, & B_{wi}(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)B_{wi} \\ C_i(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)C_i, & C_{di}(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)C_{di} \\ \Delta A_i(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)\Delta A_i(t), & \Delta A_{di}(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)\Delta A_{di}(t) \\ \Delta C_i(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)\Delta C_i(t), & \Delta C_{di}(h)=\sum_{i=1}^{r}h_i\left(x(t)\right)\Delta C_{di}(t) \end{cases} \tag{10}$$

where $h_i\left(x(t)\right)$ is the membership function and described as

$$h_i\left(x(t)\right)=\prod_{j=1}^{g}\mathcal{M}_{ij}\left(x_j(t)\right)\Big/\sum_{i=1}^{r}\prod_{j=1}^{g}\mathcal{M}_{ij}\left(x_j(t)\right) \tag{11}$$

in which

$$h_i\left(x(t)\right)\ge 0, \quad \sum_{i=1}^{r}h_i\left(x(t)\right)=1 \tag{12}$$

where $\mathcal{M}_{ij}\left(x_j(t)\right)$ is the grade of $h_i\left(x(t)\right)$.

From (3) and (10), one has

$$\begin{cases} \Delta A_i(h)=\bar{U}_{i1}\Delta_{i1}(t)V_{i1}, & \Delta A_{di}(h)=\bar{U}_{i2}\Delta_{i2}(t)V_{i2} \\ \Delta C_i(h)=\bar{U}_{i3}\Delta_{i3}(t)V_{i3}, & \Delta C_{di}(h)=\bar{U}_{i4}\Delta_{i4}(t)V_{i4} \end{cases} \tag{13}$$

with

$$\begin{cases} \bar{U}_{i1}=\sum_{i=1}^{r}h_i\left(x(t)\right)U_{i1}, & \bar{U}_{i2}=\sum_{i=1}^{r}h_i\left(x(t)\right)U_{i2} \\ \bar{U}_{i3}=\sum_{i=1}^{r}h_i\left(x(t)\right)U_{i3}, & \bar{U}_{i4}=\sum_{i=1}^{r}h_i\left(x(t)\right)U_{i4} \end{cases} \tag{14}$$

Remark 1. The finite-time dissipative control and stochastic mean-square stability analysis were proposed for a class of singular Markovian jumping systems with saturation input [23]. The sliding mode control and H-infinity stability analysis were proposed for a class of singular uncertain systems with time-varying delay [24]. Compared with [23, 24], the time-varying delay, saturation input and unmatched disturbance are all considered for the singular uncertain system in this paper. Besides, the T-S fuzzy model has nice ability to approximate system plant, because it offers a distinctive framework to describe system plant as an average weighted sum of semilinear subsystems [25]. Thus, the T-S fuzzy model is employed to approximate the singular uncertain system in this section.

III. Controller Design

Applying T-S fuzzy model, the delay-dependent T-S fuzzy state feedback controller is designed.

Controller rule i: if $x_1(t)$ *is* $\mathcal{N}_{i1}$, $x_2(t)$ *is* $\mathcal{N}_{i2}$, … , *and* $x_g(t)$ *is* $\mathcal{N}_{ig}$, *then*

$$u(t)=K_i x(t)+K_{hi}x\left(t-h(t)\right) \tag{15}$$

where $x_1(t)$, $x_2(t)$, … and $x_g(t)$ are the premise variables of the controller, $\mathcal{N}_{ij}$ ($i=1,\ 2,\ \dots,\ r$, $j=1,\ 2,\ \dots,\ g$) is the fuzzy set of the controller, *r* is the number of fuzzy rules and *g* is the number of premise variables. K_i and K_{hi} are the controller gain matrices, $h(t)$ is the time-varying delay satisfying

$$0<h_1\le h(t)\le h_2, \quad \dot{h}(t)\le \bar{h} \tag{16}$$

where h_1, h_2 and $\bar{h}$ are the lower bound of $h(t)$, upper bound of $h(t)$ and upper bound of $\dot{h}(t)$, respectively.

Applying T-S fuzzy inference to (15), one can obtain

$$u(t)=K_i(\omega)x(t)+K_{hi}(\omega)x\left(t-h(t)\right) \tag{17}$$

with

$$K_i(\omega)=\sum_{i=1}^{r}\omega_i\left(x(t)\right)K_i, \quad K_{hi}(\omega)=\sum_{i=1}^{r}\omega_i\left(x(t)\right)K_{hi} \tag{18}$$

where $\omega_i\left(x(t)\right)$ is the membership function and described as

$$\omega_i\left(x(t)\right)=\prod_{j=1}^{g}\mathcal{N}_{ij}\left(x_j(t)\right)\Big/\sum_{i=1}^{r}\prod_{j=1}^{g}\mathcal{N}_{ij}\left(x_j(t)\right) \tag{19}$$

in which

$$\omega_i\left(x(t)\right)\ge 0, \quad \sum_{i=1}^{r}\omega_i\left(x(t)\right)=1 \tag{20}$$

where $\mathcal{N}_{ij}\left(x_j(t)\right)$ is the grade of $\omega_i\left(x(t)\right)$.

Applying (17) to (9), the closed-loop system is obtained

$$\begin{cases} E\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(x(t)\right)\omega_j\left(x(t)\right)\Big(\left(A_i+\Delta A_i(t)\right)x(t) \\ \qquad +\left(A_{di}+\Delta A_{di}(t)\right)x\left(t-d(t)\right) \\ \qquad +B_i sat\left(K_j x(t)+K_{hj}x\left(t-h(t)\right)\right)+B_{wi}w(t)\Big) \\ z(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(x(t)\right)\omega_j\left(x(t)\right)\Big(\left(C_i+\Delta C_i(t)\right)x(t) \\ \qquad +\left(C_{di}+\Delta C_{di}(t)\right)x\left(t-d(t)\right)\Big) \\ x(t)=\varphi(t), \quad t\in\left[-max(d_2,\ h_2),\ 0\right] \end{cases} \tag{21}$$

Let us define diagonal matrices $D_1\in\mathbb{R}^{n\times n}$, $D_2\in\mathbb{R}^{n\times n}$, … and $D_m\in\mathbb{R}^{n\times n}$ with $m=1,2,\dots,2^n$, and the diagonal elements are 1 or 0. $\mathcal{D}$ is the set of D_1, D_2, … and D_m, where $\bar{D}_m=I-D_m$ and $\bar{D}_m\in\mathcal{D}$.

Substituting (15) in to (6), one has

$$sat\left(u(t)\right)=sat\left(K_i x(t)+K_{hi}x\left(t-h(t)\right)\right) \tag{22}$$

Applying *Lemma 1* (conves hull lemma) to (22), one has

$$\begin{aligned} sat\left(K_i x(t)+K_{hi}x\left(t-h(t)\right)\right)=\sum_{m=1}^{2^n}\alpha_m\Big(&\left(D_m K_i+\bar{D}_m K_H\right)x(t) \\ &+\left(D_m K_{hi}+\bar{D}_m K_{Hh}\right)x\left(t-h(t)\right)\Big) \end{aligned} \tag{23}$$

where K_H and K_{Hh} are the matrices with appropriate dimensions.

Applying (23) to (21), the closed-loop system (21) is rewritten

$$\begin{cases} E\dot{x}(t)=\sum_{m=1}^{2^n}\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x(t))\omega_j(x(t))\alpha_m\bar{A}_m x(t) \\ \quad +\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x(t))\omega_j(x(t))\bar{A}_{di}x(t-d(t)) \\ \quad +\sum_{m=1}^{2^n}\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x(t))\omega_j(x(t))\alpha_m\bar{B}_m x(t-h(t)) \\ \quad +\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x(t))\omega_j(x(t))B_{wi}w(t) \\ z(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x(t))\omega_j(x(t))\left(\bar{C}_i x(t)+\bar{C}_{di}x(t-d(t))\right) \\ x(t)=\varphi(t),\quad t\in\left[-max(d_2,\ h_2),\ 0\right] \end{cases} \tag{24}$$

with

$$\begin{cases} \bar{A}_m=A_m+\Delta A_i(t),\quad \bar{A}_{di}=A_{di}+\Delta A_{di}(t) \\ \bar{B}_m=B_iD_mK_{hj}+B_i\bar{D}_mK_{Hh} \\ \bar{C}_i=C_i+\Delta C_i(t),\quad \bar{C}_{di}=C_{di}+\Delta C_{di}(t) \end{cases} \tag{25}$$

where

$$A_m=A_i+B_iD_mK_j+B_i\bar{D}_mK_{hj} \tag{26}$$

Definitions 1-6 and *Lemmas 1-9* are given to derive the main results in *Section IV*.

Consider a class of singular systems as follows [27]

$$\begin{cases} E\dot{x}(t)=Ax(t)+A_dx(t-d(t)) \\ z(t)=Cx(t) \\ x(t)=\varphi(t),\quad t\in[-d_2,\ 0] \end{cases} \tag{27}$$

where A, A_d and C are the system gain matrices with appropriate dimensions, $E\in\mathbb{R}^{n\times n}$ is a singular matrix satisfying $rank(E)<n$. $x(t)\in\mathbb{R}^n$ is the state variable, $\varphi(t)$ is the initial condition with $t\in[-d_2,\ 0]$, $d(t)$ is the time-varying delay satisfying $0<d_1\le d(t)\le d_2$ and $0\le\dot{d}(t)\le\bar{d}$. d_1, d_2 and $\bar{d}$ are the lower bound of $d(t)$, upper bound of $d(t)$ and upper bound of $\dot{d}(t)$, respectively.

Definition 1 [26]. For a given scalar $\rho\ge0$, singular matrix $E\in\mathbb{R}^{n\times n}$ and matrix $P=P^T>0$ with appropriate dimension, the attraction domain $\mathcal{E}(E^TPE,\ \rho)$ can be defined

$$\mathcal{E}(E^TPE,\ \rho)=\left\{x(t)\in\mathbb{R}^n \,\middle|\, x^T(t)E^TPEx(t)\le\rho,\quad \rho\ge0\right\} \tag{28}$$

where $\mathcal{E}(E^TPE,\ \rho)$ is an ellipsoid including state variables trajectory, and the saturation does not occur in $\mathcal{E}(E^TPE,\ \rho)$.

Definition 2 [26]. For the given matrices K_i and K_{hi} with appropriate dimensions, k_{mi} is the *m*-th row of K_i, k_{mhi} is the *m*-th row of K_{hi}, and $\mathcal{L}(K_i,\ K_{hi})$ can be defined as follows

$$\mathcal{L}(K_i,\ K_{hi})=\left\{x(t)\in\mathbb{R}^n \,\middle|\, \left\|k_{mi}x(t)+k_{mhi}x(t-h(t))\right\|\le1\right\} \tag{29}$$

where $\mathcal{L}(K_i,\ K_{hi})$ is a polyhedral including state variables trajectory, and the saturation does not occur in $\mathcal{L}(K_i,\ K_{hi})$.

Definition 3 (regular impulse free [27]). The system (27) is said to be regular impulse free if $(E,\ A)$ is regular impulse free, i.e., the state variables trajectory is included in $\mathcal{E}(E^TPE,\ \rho)$.

Definition 4 (exponentially stable [27]). Under any initial condition, the system (27) is said to be exponentially stable if there exist $\hat{\zeta}>0$ and $\hat{\lambda}>0$ satisfying

$$\|x(t)\|\le\hat{\zeta}e^{-\hat{\lambda}t}\|\varphi(t)\|_{d_2} \tag{30}$$

Definition 5 (exponentially admissible [27]). Under any initial condition, the system (27) is said to be exponentially admissible if the system is regular impulse free and exponentially stable.

Definition 6 (H-infinite performance [28]). Under zero initial condition and $w(t)\ne0$, the prescribed H-infinity performance is guaranteed if

$$\int_0^\infty z^T(t)z(t)dt\le\int_0^\infty\gamma^2w^T(t)w(t)dt,\quad w(t)\ne0 \tag{31}$$

where $\gamma>0$ is the H-infinity performance index.

Lemma 1 (convex hull lemma [29]). For the given matrices $\hat{F}$ and $\hat{K}$ and $x(t)\in\mathbb{R}^n$, if there exists α_s satisfying

$$\sum_{m=1}^{2^n}\alpha_m=1,\quad 0\le\alpha_m\le1 \tag{32}$$

then the following conditions hold

$$sat(\hat{F}x(t))\in co\left\{D_m\hat{F}x(t)+\bar{D}_m\hat{K}x(t)\right\},\quad m=1,\ 2,\ \ldots,\ 2^n \tag{33}$$

or

$$sat(\hat{F}x(t))=\sum_{m=1}^{2^n}\alpha_m\left(D_m\hat{F}+\bar{D}_m\hat{K}\right)x(t),\quad D_m\in\mathcal{D} \tag{34}$$

Lemma 2 (Schur complement lemma [30]). For the given matrices $\hat{M}_{11}$, $\hat{M}_{12}$, $\hat{M}_{21}$ and $\hat{M}_{22}$ satisfying $\hat{M}_{11}=\hat{M}_{11}^T$ and $\hat{M}_{22}=\hat{M}_{22}^T$, the inequality (35)

$$\begin{bmatrix}\hat{M}_{11} & \hat{M}_{12}\\ \hat{M}_{21} & \hat{M}_{22}\end{bmatrix}<0 \tag{35}$$

is equivalent to

$$\hat{M}_{22}<0,\quad \hat{M}_{11}-\hat{M}_{12}\hat{M}_{22}^{-1}\hat{M}_{21}^T<0 \tag{36}$$

Lemma 3 (Jensen integral inequality [31]). For the given scalars $\hat{r}_1<\hat{r}_2$ and matrix $\hat{M}$ satisfying $\hat{r}_1<\hat{r}_2$ and $\hat{M}=\hat{M}^T>0$, the following inequality holds

$$\left(\int_{\hat{r}_1}^{\hat{r}_2}\hat{w}(t)dt\right)^T\hat{M}\left(\int_{\hat{r}_1}^{\hat{r}_2}\hat{w}(t)dt\right)\le(\hat{r}_2-\hat{r}_1)\int_{\hat{r}_1}^{\hat{r}_2}\hat{w}^T(t)\hat{M}\hat{w}(t)dt \tag{37}$$

where $\hat{w}(t)$ is a vector function.

Lemma 4 [32]. For the given scalars $0\le\hat{d}_1\le\hat{d}_2$, $\hat{h}$ and matrices $\hat{X}$, $\hat{E}$, $\hat{M}_1$, $\hat{M}_2$ satisfying $\hat{X}=\hat{X}^T>0$, the following inequality holds

$$-\int_{t-\hat{d}_2}^{t-\hat{d}_1}\dot{x}^T(t)\hat{E}^T\hat{X}\hat{E}\dot{x}(t)dt\le\hat{\xi}^T(t)\hat{\gamma}\hat{\xi}(t)+\hat{h}\hat{\xi}^T(t)\hat{Y}^T\hat{X}^{-1}\hat{Y}\hat{\xi}(t) \tag{38}$$

where

$$\begin{cases}\hat{\xi}(t)=\left[x^T(t-\hat{d}_1)\quad x^T(t-\hat{d}_2)\right]^T,\quad \hat{Y}=\left[\hat{M}_1\quad \hat{M}_2\right]\\ \hat{\gamma}=\begin{bmatrix}\hat{M}_1^T\hat{E}+\hat{E}^T\hat{M}_1 & -\hat{M}_1^T\hat{E}+\hat{E}^T\hat{M}_2\\ * & -\hat{M}_2^T\hat{E}-\hat{E}^T\hat{M}_2\end{bmatrix}\end{cases} \tag{39}$$

Lemma 5 [32]. For the given scalars $0\le\hat{d}_1\le\hat{d}_2$ and matrices $\hat{M}=\hat{M}^T>0$, $\hat{E}$ with appropriate dimensions, the following inequality holds

$$\begin{aligned}&-\frac{\hat{d}_2^2-\hat{d}_1^2}{2}\int_{-\hat{d}_2}^{-\hat{d}_1}\int_{t+\theta}^{t}\left(\hat{E}\dot{x}(t)\right)^T\hat{M}\hat{E}\dot{x}(t)dtd\theta\\ &\le\begin{bmatrix}x(t)\\ \int_{t-\hat{d}_2}^{t-\hat{d}_1}x(t)dt\end{bmatrix}^T\begin{bmatrix}-\hat{d}_1^2\hat{E}^T\hat{M}\hat{E} & \hat{d}_2\hat{E}^T\hat{M}\hat{E}\\ \hat{d}_2\hat{E}^T\hat{M}\hat{E} & -\hat{E}^T\hat{M}\hat{E}\end{bmatrix}\begin{bmatrix}x(\theta)\\ \int_{t-\hat{d}_2}^{t-\hat{d}_1}x(\theta)d\theta\end{bmatrix}\end{aligned} \tag{40}$$

where $x(t)\in\mathbb{R}^n$ and $\dot{x}(t)$: $[-d_2,\ 0]\to\mathbb{R}^n$.

Proof. It can be verified that

$$\begin{bmatrix} \left(\hat{E}\dot{x}(t)\right)^T \hat{M}\hat{E}\dot{x}(t) & \left(\hat{E}\dot{x}(t)\right)^T \\ * & \hat{M}^{-1}\end{bmatrix} = \begin{bmatrix} \left(\hat{E}\dot{x}(t)\right)^T\hat{M}^{\frac{1}{2}} & 0 \\ \hat{M}^{-\frac{1}{2}} & 0\end{bmatrix}\begin{bmatrix}\hat{M}^{\frac{1}{2}}\hat{E}\dot{x}(t) & \hat{M}^{-\frac{1}{2}}\\ 0 & 0\end{bmatrix} \geq 0 \tag{41}$$

From (41), one has

$$\begin{bmatrix} \int_{-\hat{d}_2}^{-\hat{d}_1}\int_{t+\theta}^{t}\left(\hat{E}\dot{x}(t)\right)^T \hat{M}\hat{E}\dot{x}(t)\,dtd\theta & \int_{-\hat{d}_2}^{-\hat{d}_1}\int_{t+\theta}^{t}\left(\hat{E}\dot{x}(t)\right)^T dtd\theta \\ * & \int_{-\hat{d}_2}^{-\hat{d}_1}\int_{t+\theta}^{t}\hat{M}^{-1}dtd\theta \end{bmatrix} \geq 0 \tag{42}$$

Applying *Lemma 2* (Schur complement lemma), the inequality (40) holds. The proof of *Lemma 5* is completed.

Lemma 6 (Gronwall Bellman lemma [33]). For the given scalars $0 \leq \hat{\alpha} < \hat{\beta}$, $0 \leq \hat{a} < \hat{b}$, $\hat{k} > 0$ and integer scalar $\hat{c} > 1$ satisfying

$$\begin{cases} \int_{\hat{\alpha}}^{\hat{\beta}} f(t)\,dt > 0, & \hat{\alpha},\ \hat{\beta} \in \left[\hat{a},\ \hat{b}\right] \\ x(t) \leq \hat{k} + \int_{\hat{a}}^{t} f(t)\left(x(t)\right)^{\hat{c}}\,dt, & t \in \left[\hat{a},\ \hat{b}\right] \\ 1-(\hat{c}-1)\hat{k}^{(\hat{c}-1)}\int_{\hat{a}}^{\hat{b}} f(t)\,dt > 0, & t \in \left[\hat{a},\ \hat{b}\right] \end{cases} \tag{43}$$

the following inequality holds

$$x(t) \leq \frac{\hat{k}}{\left(1-(\hat{c}-1)\hat{k}^{\hat{c}-1}\int_{\hat{a}}^{t} f(t)\,dt\right)^{\frac{1}{\hat{c}-1}}}, \quad t \in \left[\hat{a},\ \hat{b}\right] \tag{44}$$

where $f(t)$ is a vector function.

Lemma 7 [27]. For the given scalars $0 < \hat{\zeta}_1 < 1$, $\hat{\zeta}_2 > 0$ and $\hat{\lambda} > 0$ satisfying

$$\begin{cases} \hat{\zeta}_1 e^{\hat{\lambda}\tau} < 1 \\ f(s) \leq \hat{\zeta}_1 \sup_{t-\tau\leq s\leq t}\left(f(s)\right) + \hat{\zeta}_2 e^{-\hat{\lambda}s}, \quad \tau = max(d_2,\ h_2) \end{cases} \tag{45}$$

the following inequality holds

$$f(s) \leq \sup_{-\tau\leq s\leq 0}\left(f(s)\right)e^{-\hat{\lambda}s} + \frac{\hat{\zeta}_2}{1-\hat{\zeta}_1 e^{\hat{\lambda}\tau}}e^{-\hat{\lambda}s} \tag{46}$$

Lemma 8 [34]. For the given scalar $\hat{\varepsilon} > 0$ and matrices $\hat{U}$, $\hat{V}$, $\hat{Q}^0 = \left(\hat{Q}^0\right)^T > 0$ with appropriate dimensions satisfying

$$\hat{\varepsilon}\hat{U}\hat{U}^T + \hat{\varepsilon}^{-1}\hat{V}^T\hat{V} + \hat{Q}^0 < 0 \tag{47}$$

the following inequality holds

$$\hat{U}\hat{\Delta}\hat{V} + \hat{V}^T\hat{\Delta}^T\hat{V}^T + \hat{Q}^0 < 0 \tag{48}$$

$$\hat{\Delta}^T\hat{\Delta} \leq I \tag{49}$$

Lemma 9 [35]. For the given matrices $\hat{X}$, $\hat{Y} = \hat{Y}^T > 0$, $\hat{Z}$ with appropriate dimensions, the following inequality holds

$$-\hat{Z}^T\hat{Y}\hat{Z} \leq \hat{X}^T\hat{Z} + \hat{Z}^T\hat{X} + \hat{X}^T\hat{Y}^{-1}\hat{X} \tag{50}$$

Remark 2. The objective in this paper is to design controller (17) for system (9) such that (i) the closed-loop system is regular impulse free according to *Definition 3*; (ii) the closed-loop system is exponentially admissible according to *Definition 5*; (iii) the prescribed H-infinity performance is guaranteed according to *Definition 6*; (iv) the controller gain matrices K_i and K_{hi} can be determined by employing the proposed methods.

Remark 3. The sliding model filter control and asynchronous dissipative analysis were proposed for a class of singular Markov jump systems [36], without considering state feedback control. The state observer and stochastic stability analysis were proposed for a class of singular systems [37], without considering admissibility analysis. Compared with [36, 37], the exponential admissibility analysis are investigated in this paper. Besides, the state feedback control is easy to implement [38]. Combining the T-S fuzzy modelling technique and state feedback control, the delay-dependent T-S fuzzy state feedback controller is designed. Moreover, convex hull lemma can be used to achieve the controller design for enhancing design flexibility [39]. Thus, the convex hull lemma is employed to convert the closed-loop system with saturation input into the closed-loop system without saturation input. Additionally, the schematic diagram of control scheme is presented as follows.

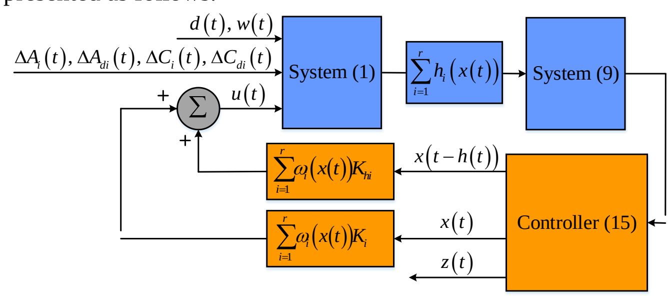

Fig. 1 The schematic diagram of control scheme.

IV. Main Results

A. Stability conditions

In this section, the delay-dependent stability conditions are derived by employing the free-weighting matrices and delay-dependent Lyapunov-Krasovskii functional.

Theorem 1. For the T-S fuzzy singular uncertain system (9) under controller (17), the closed-loop system (24) is regular impulse free, exponentially admissible and prescribed H-infinity performance is guaranteed if for the given scalars $d_1 > 0$, $0 < h_1 \leq h_2$, $0 \leq \bar{d} < 1$, $0 \leq \bar{h} < 1$, $\rho \geq 0$, $\gamma > 0$ and matrices $P_i = P_i^T > 0$, $Q_{i1} = Q_{i1}^T > 0$, $Q_{i2} = Q_{i2}^T > 0$, $Q_{i3} = Q_{i3}^T > 0$, $R_i = R_i^T > 0$, $R_{i1} = R_{i1}^T > 0$, $R_{i2} = R_{i2}^T > 0$, $R_{i3} = R_{i3}^T > 0$, $Z_{i1} = Z_{i1}^T > 0$, $Z_{i2} = Z_{i2}^T > 0$, $Z_{i3} = Z_{i3}^T > 0$, $Z_{i4} = Z_{i4}^T > 0$, $Y_{i1} = Y_{i1}^T > 0$, $Y_{i2} = Y_{i2}^T > 0$, $Y_{i3} = Y_{i3}^T > 0$, $Y_{i4} = Y_{i4}^T > 0$, $M_{i1} = M_{i1}^T > 0$, $M_{i2} = M_{i2}^T > 0$, $N_{i1} = N_{i1}^T > 0$, $N_{i2} = N_{i2}^T > 0$, there exist matrices S, S_{i11}, S_{i12}, S_{i21}, S_{i22}, S_{i31}, S_{i32}, S_{i41} and S_{i42} with appropriate dimensions satisfying

$$\mathcal{T}_i = \begin{bmatrix} T_{i1} & T_{i2} & T_{i4} & T_{i6} & T_{i9} \\ * & T_{i3} & 0 & T_{i7} & T_{i10} \\ * & * & T_{i5} & 0 & 0 \\ * & * & * & T_{i8} & 0 \\ * & * & * & * & T_{i11} \end{bmatrix} < 0, \quad i = 1,\ 2,\ \ldots,\ r \tag{51}$$

with

$$T_{i1} = \begin{bmatrix} T_{i111} & \left(E^TP_i + SR_i^T\right)\bar{A}_{di} & -d_1S_{i11}^TE + d_1E^TS_{i12} & 0 \\ * & \left(\bar{d}-1\right)Q_{i3} & 0 & 0 \\ * & * & T_{i12} & -d_2S_{i21}^TE + d_2E^TS_{i22} \\ * & * & * & T_{i13} \end{bmatrix}$$

$$\begin{cases} T_{i2} = \begin{bmatrix} \left(E^T P_i + SR_i^T\right)\bar{B}_m & -h_1 S_{i31}^T E + h_1 E^T S_{i32} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ T_{i3} = \begin{bmatrix} \left(\bar{h}-1\right)R_{i3} & 0 & 0 \\ * & T_{i31} & -h_2 S_{i41}^T E + h_2 E^T S_{i42} \\ * & * & T_{i32} \end{bmatrix} \end{cases} \quad (52)$$

$$T_{i4} = \begin{bmatrix} d_1 E^T M_{i1} E & d_2 E^T M_{i2} E & h_1 E^T N_{i1} E & h_2 E^T N_{i2} E \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} T_{i5} = diag\{-Z_{i3} - E^T M_{i1} E, \quad -Z_{i4} - E^T M_{i2} E, \\ \qquad -Y_{i3} - E^T N_{i1} E, \quad -Y_{i4} - E^T N_{i2} E\} \\ T_{i6} = \begin{bmatrix} \left(E^T P_i + SR_i^T\right)B_{wi} & \bar{A}_m^T L_i^T \\ 0 & \bar{A}_{di}^T L_i^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad T_{i7} = \begin{bmatrix} 0 & \bar{B}_m^T L_i^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ T_{i8} = \begin{bmatrix} -\gamma^2 I & B_{wi}^T L_i^T \\ * & -L_i \end{bmatrix} \end{cases} \quad (53)$$

$$\begin{cases} T_{i9} = \begin{bmatrix} d_1 S_{i11}^T & 0 & h_1 S_{i31}^T & 0 & \bar{C}_i^T \\ 0 & 0 & 0 & 0 & \bar{C}_{di}^T \\ d_1 S_{i12}^T & d_2 S_{i21}^T & 0 & 0 & 0 \\ 0 & d_2 S_{i22}^T & 0 & 0 & 0 \end{bmatrix} \\ T_{i10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_1 S_{i32}^T & h_2 S_{i41}^T & 0 \\ 0 & 0 & 0 & h_2 S_{i42}^T & 0 \end{bmatrix} \\ T_{i11} = diag\{-Z_{i1}, \quad -Z_{i2}, \quad -Y_{i1}, \quad -Y_{i2}, \quad I\} \end{cases} \quad (54)$$

where

$$\begin{cases} T_{i111} = \left(E^T P_i + SR_i^T\right)\bar{A}_m + \bar{A}_m^T \left(E^T P_i + SR_i^T\right)^T \\ \quad + Q_{i1} + Q_{i3} + R_{i1} + R_{i3} + d_1^2 Z_{i3} + h_1^2 Y_{i3} + d_2^2 Z_{i4} + h_2^2 Y_{i4} \\ \quad + d_1 S_{i11}^T E + d_1 E^T S_{i11} + h_1 S_{i31}^T E + h_1 E^T S_{i31} - d_1^2 E^T M_{i1} E \\ \quad - h_1^2 E^T N_{i1} E - d_2^2 E^T M_{i2} E - h_2^2 E^T N_{i2} E \\ T_{i12} = -Q_{i1} + Q_{i2} - d_1 S_{i12}^T E - d_1 E^T S_{i12} + d_2 S_{i21}^T E + d_2 E^T S_{i21} \\ T_{i13} = -Q_{i2} - d_2 S_{i22}^T E - d_2 E^T S_{i22} \\ T_{i31} = -R_{i1} + R_{i2} - h_1 S_{i32}^T E - h_1 E^T S_{i32} + h_2 S_{i41}^T E + h_2 E^T S_{i41} \\ T_{i32} = -R_{i2} - h_2 S_{i42}^T E - h_2 E^T S_{i42} \end{cases} \quad (55)$$

$$\begin{cases} \mathcal{E}\left(E^T P_i E, \ \gamma^2 \rho\right) \subset \mathcal{L}\left(K_H, \ K_{Hh}\right) \\ E^T R_i = 0, \quad rank\left(R_i\right) = n - r, \quad R_i \in \mathbb{R}^{n\times(n-r)} \\ L_i = d_1^2 Z_{i1} + d_2^2 Z_{i2} + h_1^2 Y_{i1} + h_2^2 Y_{i2} + \frac{d_1^4}{4} M_{i1} + d_c^2 M_{i2} + \frac{h_1^4}{4} N_{i1} + h_c^2 N_{i2} \end{cases} \quad (56)$$

in which

$$d_c = \frac{\left(d_2^2 - d_1^2\right)}{2}, \quad h_c = \frac{\left(h_2^2 - h_1^2\right)}{2} \quad (57)$$

Proof. The proof in *Theorem 1* is divided into *Steps 1-3*.

Step 1. The proof of objective (i) in *Remark 2*: the closed-loop system (24) is regular impulse free.

From (32) and (51), one has

$$\sum_{m=1}^{2^n} \alpha_m \mathcal{T}_i < 0 \quad (58)$$

From (58), it can be verified that

$$\sum_{m=1}^{2^n} \alpha_m T_{i111} < 0 \quad (59)$$

The free-weighting matrices M and N are defined as follows

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad N^T S = \begin{bmatrix} S_{i11} \\ S_{i22} \end{bmatrix}, \quad M\sum_{m=1}^{n} \alpha_m \bar{A}_m N = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \quad (60)$$

where $I_r \in \mathbb{R}^{r\times r}$ is an identity matrix. S_{i11}, S_{i22}, A_{i11}, A_{i12}, A_{i21} and A_{i22} are the matrices with appropriate dimensions.

From (56) and (60), one has

$$M^T R_i = \begin{bmatrix} 0 \\ K_i \end{bmatrix} \quad (61)$$

Multiplying N^T and N into Pre- and Post- of (59), one has

$$S_{i22} K_i^T A_{i22} + A_{i22}^T K_i S_{i22}^T < 0 \quad (62)$$

From (62), it can be seen that A_{i22} is a nonsingular matrix, this means $\left(E, \ \sum_{m=1}^{2^n} \alpha_m \bar{A}_m\right)$ is regular impulse free, thus the closed-loop system is regular impulse free according to *Definition 3*. The objective (i) in *Remark 2* is achieved, and the proof of *Step 1* is completed.

Step 2. The proof of objective (ii) in *Remark 2*: the closed-loop system is exponentially admissible according to *Definition 5*.

For (24), consider the delay-dependent Lyapunov-Krasovskii functional with multiple integral terms as follows

$$V\left(x_t\right) = V_1\left(x_t\right) + V_2\left(x_t\right) + V_3\left(x_t\right) + V_4\left(x_t\right) + V_5\left(x_t\right) + V_6\left(x_t\right) + V_7\left(x_t\right) \quad (63)$$

with

$$\begin{cases} V_1\left(x_t\right) = x^T(t) E^T P_i E x(t) \\ V_2\left(x_t\right) = \int_{t-d_1}^{t} x^T(t) Q_{i1} x(t) dt + \int_{t-d_2}^{t-d_1} x^T(t) Q_{i2} x(t) dt \\ \qquad + \int_{t-d(t)}^{t} x^T(t) Q_{i3} x(t) dt \\ V_3\left(x_t\right) = \int_{t-h_1}^{t} x^T(t) R_{i1} x(t) dt + \int_{t-h_2}^{t-h_1} x^T(t) R_{i2} x(t) dt \\ \qquad + \int_{t-h(t)}^{t} x^T(t) R_{i3} x(t) dt \\ V_4\left(x_t\right) = \int_{-d_1}^{0}\int_{t+\theta}^{t} d_1 \left(E\dot{x}(t)\right)^T Z_{i1} E\dot{x}(t) dt d\theta \\ \qquad + \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} d_2 \left(E\dot{x}(t)\right)^T Z_{i2} E\dot{x}(t) dt d\theta \\ \qquad + \int_{-d_1}^{0}\int_{t+\theta}^{t} d_1 x^T(t) Z_{i3} x(t) dt d\theta \\ \qquad + \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} d_2 x^T(t) Z_{i4} x(t) dt d\theta \\ V_5\left(x_t\right) = \int_{-h_1}^{0}\int_{t+\theta}^{t} h_1 \left(E\dot{x}(t)\right)^T Y_{i1} E\dot{x}(t) dt d\theta \\ \qquad + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} h_2 \left(E\dot{x}(t)\right)^T Y_{i2} E\dot{x}(t) dt d\theta \\ \qquad + \int_{-h_1}^{0}\int_{t+\theta}^{t} h_1 x^T(t) Y_{i3} x(t) dt d\theta \\ \qquad + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} h_2 x^T(t) Y_{i4} x(t) dt d\theta \\ V_6\left(x_t\right) = \int_{-d_1}^{0}\int_{\theta}^{0}\int_{t+\vartheta}^{t} \frac{d_1^2}{2} \left(E\dot{x}(t)\right)^T M_{i1} E\dot{x}(t) dt d\vartheta d\theta \\ \qquad + \int_{-d_2}^{-d_1}\int_{\theta}^{0}\int_{t+\vartheta}^{t} d_c \left(E\dot{x}(t)\right)^T M_{i2} E\dot{x}(t) dt d\vartheta d\theta \\ V_7\left(x_t\right) = \int_{-h_1}^{0}\int_{\theta}^{0}\int_{t+\vartheta}^{t} \frac{h_1^2}{2} \left(E\dot{x}(t)\right)^T N_{i1} E\dot{x}(t) dt d\vartheta d\theta \\ \qquad + \int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{t+\vartheta}^{t} d_c \left(E\dot{x}(t)\right)^T N_{i2} E\dot{x}(t) dt d\vartheta d\theta \end{cases} \quad (64)$$

Taking the time derivative of (64) along (24), one has

$$\begin{cases}\dot V_1(x_t) = 2x^T(t)E^T P_i E\dot x(t)\\ \dot V_2(x_t) \le x^T(t)Q_{i1}x(t) - x^T(t-d_1)Q_{i1}x(t-d_1) + x^T(t-d_1)\\ \qquad \times Q_{i2}x(t-d_1) - x^T(t-d_2)Q_{i2}x(t-d_2)\\ \qquad + x^T(t)Q_{i3}x(t) - (1-\bar d)x^T(t-d(t))Q_{i3}x(t-d(t))\\ \dot V_3(x_t) \le x^T(t)R_{i1}x(t) - x^T(t-h_1)R_{i1}x(t-h_1) + x^T(t-h_1)\\ \qquad \times R_{i2}x(t-h_1) - x^T(t-h_2)R_{i2}x(t-d_2)\\ \qquad + x^T(t)R_{i3}x(t) - (1-\bar h)x^T(t-h(t))R_{i3}x(t-h(t))\\ \dot V_4(x_t) \le d_1^2(E\dot x(t))^T Z_{i1}E\dot x(t) - d_1\int_{t-d_1}^{t}(E\dot x(t))^T Z_{i1}E\dot x(t)dt\\ \qquad + d_2^2(E\dot x(t))^T Z_{i2}E\dot x(t) - d_2\int_{t-d_2}^{t-d_1}(E\dot x(t))^T Z_{i2}E\dot x(t)dt\\ \qquad + d_1^2 x^T(t)Z_{i3}x(t) - d_1\int_{t-d_1}^{t}x^T(t)Z_{i3}x(t)dt\\ \qquad + d_2^2 x^T(t)Z_{i4}x(t) - d_2\int_{t-d_2}^{t-d_1}x^T(t)Z_{i4}x(t)dt\end{cases} \tag{65}$$

$$\begin{cases}\dot V_5(x_t) \le h_1^2(E\dot x(t))^T Y_{i1}E\dot x(t) - h_1\int_{t-h_1}^{t}(E\dot x(t))^T Y_{i1}E\dot x(v)dt\\ \qquad + h_2^2(E\dot x(t))^T Y_{i2}E\dot x(t) - h_2\int_{t-h_2}^{t-h_1}(E\dot x(t))^T Y_{i2}E\dot x(t)dt\\ \qquad + h_1^2 x^T(t)Y_{i3}x(t) - h_1\int_{t-h_1}^{t}x^T(t)Y_{i3}x(t)dt\\ \qquad + h_2^2 x^T(t)Y_{i4}x(t) - h_2\int_{t-h_2}^{t-h_1}x^T(t)Y_{i4}x(t)dt\\ \dot V_6(x_t) \le \frac{d_1^4}{4}(E\dot x(t))^T M_{i1}E\dot x(t) - \frac{d_1^2}{2}\int_{-d_1}^{0}\int_{t+\theta}^{t}(E\dot x(t))^T M_{i1}E\dot x(t)dtd\theta\\ \qquad + d_c^2(E\dot x(t))^T M_{i2}E\dot x(t) - d_c\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}(E\dot x(t))^T M_{i2}E\dot x(t)dtd\theta\\ \dot V_7(x_t) \le \frac{h_1^4}{4}(E\dot x(t))^T N_{i1}E\dot x(t) - \frac{h_1^2}{2}\int_{-h_1}^{0}\int_{t+\theta}^{t}(E\dot x(t))^T N_{i1}E\dot x(t)dtd\theta\\ \qquad + h_c^2(E\dot x(t))^T N_{i2}E\dot x(t) - h_c\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}(E\dot x(t))^T N_{i2}E\dot x(t)dtd\theta\end{cases} \tag{66}$$

Applying *Lemmas 3-5* to (65) and (66), one has

$$\dot V(x_t) \le \varsigma^T(t)(\Gamma_{i1}+\Gamma_{i2})\varsigma(t) \tag{67}$$

with

$$\begin{cases}\varsigma^T(t) = \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-d_1) & x^T(t-d_2)\\ x^T(t-h_1) & x^T(t-h_2) & \int_{t-d_1}^{t}x^T(t)dt & \int_{t-d_2}^{t-d_1}x^T(t)dt\\ x^T(t-h(t)) & 0 & 0 \;\dots\; 0 & \\ \int_{t-h_1}^{t}x^T(t)dt & \int_{t-h_2}^{t-h_1}x^T(t)dt & \underbrace{0 \;\dots\; 0}_{5} & \end{bmatrix}\\ \Gamma_{i1} = \begin{bmatrix} T_{i1} & T_{i2} & T_{i3}\\ * & T_{i4} & 0\\ * & * & T_{i5}\end{bmatrix}, \quad \Gamma_{i2} = \Lambda_i^T L_i \Lambda_i\end{cases} \tag{68}$$

where

$$\Lambda_i = \begin{bmatrix}\bar A_m & \bar A_{di} & 0 & 0 & \bar B_m & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{69}$$

It can be verified that

$$\dot V(x_t) \le -\beta\|x(t)\|^2, \quad t \ge \tau \tag{70}$$

where $\beta > 0$ is a scalar.

Then, from (63) and (64), one has

$$V(x_t) \le \beta_1\|x(t)\|^2 + \beta_2\int_{t-\tau}^{t}\|x(t)\|^2 dt + \beta_3\int_{t-\tau}^{t}\|x(t-\tau)\|^2 dt, \quad t \ge \tau \tag{71}$$

where $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_3 > 0$ are the scalars.

From (24), one has

$$\|x(t)\| \le \|x(0)\| + \int_0^t \mu_1\|x(t)\|dt + \int_0^t \mu_2\|x(t-d(t))\|dt + \int_0^t \mu_3\|x(t-h(t))\|dt \tag{72}$$

where $\mu_1 > 0$, $\mu_2 > 0$ and $\mu_3 > 0$ are the scalars.

From (72), it can be verified that

$$\|x(t)\| \le \|x(0)\| + \mu_4\int_{-\tau}^{t}\|x(t)\|dt, \quad t \ge \tau \tag{73}$$

where

$$\mu_4 = \mu_1 + \mu_2 + \mu_3$$

one can obtain

$$\sup_{t\le\tau}(\|x(t)\|) \le (1+\mu_4\tau)\|\varphi(t)\|_\tau + \mu_4\int_0^t \sup_{t\le\tau}(\|x(t)\|)dt, \quad t \le \tau \tag{74}$$

Applying *Lemma 6* (Gronwall Bellman lemma) to (74), one has

$$\sup_{t\le\tau}(\|x(t)\|) \le (1+\mu_4\tau)\|\varphi(t)\|_\tau e^{\mu_4\tau} = \mu_5\|\varphi(t)\|_\tau, \quad t \le \tau \tag{75}$$

where

$$\mu_5 = (1+\mu_4\tau)e^{\mu_4\tau}$$

With above analysis, one has

$$V(x_t) \le \beta_4\|\varphi(t)\|_\tau^2, \quad t \le \tau \tag{76}$$

where $\beta_4 > 0$ is a scalar.

From *Step 1*, one knows that the closed-loop system (24) is regular impulse free, and the free-weighting matrices $\bar M$ and $\bar N$ can be defined as follows

$$\bar M E \bar N = \begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix}, \quad \bar M \bar A_m \bar N = \begin{bmatrix} A_{m1} & 0\\ 0 & I_{n-r}\end{bmatrix} \tag{77}$$

where $I_r \in \mathbb{R}^{r\times r}$ and $I_{n-r} \in \mathbb{R}^{(n-r)\times(n-r)}$ are the identity matrices, A_{m1} is a matrix with appropriate dimension.

From (76), let us define

$$\begin{cases}\bar M \bar A_{di}\bar N = \begin{bmatrix} A_{di1} & A_{di2}\\ A_{di3} & A_{di4}\end{bmatrix}, \quad \bar M \bar B_m \bar N = \begin{bmatrix} B_{m1} & B_{m2}\\ B_{m3} & B_{m4}\end{bmatrix}\\ \bar M^{-T}P_i\bar N = \begin{bmatrix} P_{i11} & P_{i12}\\ * & P_{i22}\end{bmatrix}, \quad \bar N Q_i \bar N = \begin{bmatrix} Q_{i11} & Q_{i12}\\ * & Q_{i22}\end{bmatrix}\end{cases} \tag{78}$$

$$\xi(t) = \bar N^{-1}x(t) = \begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix} \tag{79}$$

where A_{di1}, A_{di2}, A_{di3}, A_{di4}, B_{m1}, B_{m2}, B_{m3}, B_{m4}, P_i, P_{i11}, P_{i12}, P_{i22}, Q_i, Q_{i11}, Q_{i12} and Q_{i22} are the matrices with appropriate dimensions, $\xi_1(t)$ and $\xi_2(t)$ are the vector elements of $\xi(t)$.

Applying (79) to (24), the closed-loop system (24) can be rewritten

$$\begin{cases}\dot\xi_1(t) = \sum_{m=1}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_m A_{m1}\xi_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}A_{di1}\xi_1(t-d(t))\\ \qquad + \sum_{i=1}^{r}\sum_{j=1}^{r}A_{di2}\xi_2(t-d(t)) + \sum_{m=1}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_m B_{m1}\xi_1(t-h(t))\\ \qquad + \sum_{m=1}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_m B_{m2}\xi_2(t-h(t))\\ \xi_2(t) = -\sum_{i=1}^{r}\sum_{j=1}^{r}A_{di3}\xi_1(t-d(t)) - \sum_{i=1}^{r}\sum_{j=1}^{r}A_{di4}\xi_2(t-d(t))\\ \qquad - \sum_{m=1}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_m B_{m3}\xi_1(t-h(t)) - \sum_{m=1}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_m B_{m4}\xi_2(t-h(t))\\ \xi(t) = \phi(t) = \bar N^{-1}\varphi(t), \quad t \in [-\tau, 0]\end{cases} \tag{80}$$

Remark 4. Firstly, from (24) and (80), it can be seen that the proof of exponential stability for (24) can be converted into the proof of exponential stability for (80). Secondly, from (80), it can be seen that the closed-loop system (80) includes subsystems $\dot{\xi}_1(t)$ and $\xi_2(t)$. Thus, the proof of exponential stability for (80) can be converted into the proof of exponential stability for $\dot{\xi}_1(t)$ and $\xi_2(t)$. The proof of exponential stability for (80) is divided into *Steps 2.1-2.2*.

Step 2.1. The proof of exponential stability for $\dot{\xi}_1(t)$.

For $\dot{\xi}_1(t)$, consider the delay-dependent Lyapunov-Krasovskii functional with multiple integral terms

$$W(x_t) = e^{\varepsilon t}V(x_t) > 0, \quad t \ge \tau \tag{81}$$

where $\varepsilon > 0$ is a scalar and $\tau = max\{d_2, \ h_2\}$.

From (81), one knows that $t \ge \tau$, one has

$$\begin{aligned} W(x_t) - W(x_\tau) &\le \int_\tau^t e^{\varepsilon t}\left(\varepsilon V(x_t) - \beta\|x(t)\|^2\right)dt \\ &\le \int_\tau^t e^{\varepsilon t}\left(\varepsilon\beta_1\|x(t)\|^2 + \varepsilon\beta_2\int_{t-\tau}^t\|x(t)\|^2 dt\right. \\ &\quad \left. + \varepsilon\beta_3\int_{t-\tau}^t\|x(t-\tau)\|^2 dt - \beta\|x(t)\|^2\right)dt, \quad t \ge \tau \end{aligned} \tag{82}$$

and it can be verified that

$$\begin{cases} \int_\tau^t e^{\varepsilon t}dt\int_{t-\tau}^t\|x(t)\|^2 dt \le \tau e^{\varepsilon\tau}\int_0^t e^{\varepsilon t}\|x(t)\|^2 dt \\ \le \tau e^{\varepsilon\tau}\int_\tau^t e^{\varepsilon t}\|x(t)\|^2 dt + \tau^2 e^{2\varepsilon\tau}\mu_5^2\|\varphi(t)\|_\tau^2 \\ \int_\tau^t e^{\varepsilon t}dt\int_{t-\tau}^t\|x(t\text{-}\tau)\|^2 dt \le \tau e^{\varepsilon\tau}\int_0^t e^{\varepsilon t}\|x(t\text{-}\tau)\|^2 dt \\ \le \tau e^{2\varepsilon\tau}\int_\tau^t e^{\varepsilon t}\|x(t)\|^2 dt + 2\tau^2 e^{3\varepsilon\tau}\mu_5^2\|\varphi(t)\|_\tau^2, \quad t \ge \tau \end{cases} \tag{83}$$

From (82) and (83), one has

$$\begin{aligned} W(x_t) - W(x_\tau) &\le \left(\int_\tau^t e^{\varepsilon t}\|x(t)\|^2 dt\right)\left(\varepsilon\beta_1 + \varepsilon\tau e^{\varepsilon\tau}\beta_2 + \varepsilon\tau e^{2\varepsilon\tau}\beta_3 - \beta\right) \\ &\quad + \left(\tau^2 e^{2\varepsilon\tau} + 2\tau^2 e^{3\varepsilon\tau}\right)\mu_5^2\|\varphi(t)\|_\tau^2, \quad t \ge \tau \end{aligned} \tag{84}$$

From (70), (71) and (81), one knows that $\beta > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $\beta_3 > 0$, $\varepsilon > 0$ and $\tau = max\{d_2, \ h_2\}$, one can obtain

$$\varepsilon\beta_1 + \varepsilon\tau e^{\varepsilon\tau}\beta_2 + \varepsilon\tau e^{2\varepsilon\tau}\beta_3 - \beta \le 0 \tag{85}$$

Next, from (84) and (85), one has

$$x^T(t)E^T P_i Ex(t) \le W(x_t) \le \kappa e^{-\varepsilon t}\|\varphi(t)\|_\tau^2, \quad t \ge \tau \tag{86}$$

where $\kappa > 0$ is a scalar.

With above analysis, it can be verified that

$$\|\xi_1(t)\| \le \sqrt{\frac{\kappa e^{-\varepsilon t}}{\lambda_1 P_{i11}}}\|\varphi(t)\|_\tau = \sqrt{\frac{\kappa}{\lambda_1 P_{i11}}}e^{-\frac{\varepsilon t}{2}}\|\varphi(t)\|_\tau, \quad t \ge \tau \tag{87}$$

where $\lambda_1 > 0$ is a scalar.

Thus, from (87), it can be seen that $\xi_1(t)$ is exponentially stable with $t \ge \tau$. This means the subsystem $\dot{\xi}_1(t)$ is exponentially stable with $t \ge \tau$. Via a similar method, the subsystem $\dot{\xi}_1(t)$ is also exponentially stable with $t < \tau$. The proof of *Step 2.1* is completed.

Step 2.2. The proof of exponential stability for $\xi_2(t)$.

For $\xi_2(t)$, consider $J_{\xi_2}(t)$ as follows

$$\begin{aligned} J_{\xi_2}(t) &= \xi_2^T(t)J_{11}\xi_2(t) - \left(1-\bar{d}\right)\xi_2^T(t-d(t))Q_{i3}\xi_2(t-d(t)) \\ &\quad - \left(1-\bar{h}\right)\xi_2^T(t-h(t))R_{i3}\xi_2(t-h(t)) \end{aligned} \tag{88}$$

where

$$J_{11} = Q_{i1} + Q_{i3} + R_{i1} + R_{i3} + h_1^2 Y_{i3} + h_2^2 Y_{i4} + d_1^2 Z_{i3} + d_2^2 Z_{i4} \tag{89}$$

Multiplying $\xi_2^T(t)S_{i21}K_i^T$ into Post- of $\xi_2(t)$, one has

$$\begin{aligned} &\xi_2^T(t)S_{i21}K_i^T\xi_2(t) \\ &= -\xi_2^T(t)S_{i21}K_i^T\sum_{i=1}^r\sum_{j=1}^r h_i(x(t))\omega_j(x(t))A_{di4}\xi_2(t-d(t)) \\ &\quad - \xi_2^T(t)S_{i21}K_i^T\sum_{m=1}^n\sum_{i=1}^r\sum_{j=1}^r h_i(x(t))\omega_j(x(t))\alpha_m B_{m4} \\ &\quad \times\xi_2(t-h(t)) - \xi_2^T(t)S_{i21}K_i^T\sum_{i=1}^r\sum_{j=1}^r h_i(x(t))\omega_j(x(t))q(t) \end{aligned} \tag{90}$$

where

$$q(t) = A_{di3}\xi_1(t-d(t)) + \sum_{m=1}^n\alpha_m B_{m3}\xi_1(t-h(t)) \le \sqrt{x^T(t)E^T P_i Ex(t)} \tag{91}$$

From (87) and (91), one has

$$\|q(t)\|^2 \le \kappa e^{-\varepsilon t}\|\varphi(t)\|_\tau^2, \quad t > 0 \tag{92}$$

where $\kappa > 0$ is a scalar.

From (88), (90) and (92), one can obtain

$$\begin{aligned} J_{\xi_2}(t) &= \xi_2^T(t)\left(S_{i21}K_i^T + K_i^T S_{i21}^T + J_{11}\right)\xi_2(t) + 2\xi_2^T(t)S_{i21}K_i^T \\ &\quad \times A_{di4}\xi_2(t-d(t)) + 2\xi_2^T(t)S_{i21}K_i^T\sum_{m=1}^n\alpha_m B_{m4}\xi_2(t-h(t)) \\ &\quad + 2\xi_2^T(t)S_{i21}K_i^T q(t) - \left(1-\bar{d}\right)\xi_2^T(t-d(t))Q_{i3}\xi_2(t-d(t)) \\ &\quad - \left(1-\bar{h}\right)\times\xi_2^T(t-h(t))R_{i3}\xi_2(t-h(t)) \\ &\le \begin{bmatrix} \xi_2(t) \\ \xi_2(t-d(t)) \\ \xi_2(t-h(t)) \end{bmatrix}^T \Theta \begin{bmatrix} \xi_2(t) \\ \xi_2(t-d(t)) \\ \xi_2(t-h(t)) \end{bmatrix} \\ &\quad + \eta_1\xi_2^T(t)\xi_2(t) + \eta_1^{-1}q^T(t)K_i^T S_{i21}^T S_{i21}K_i^T q(t) \end{aligned} \tag{93}$$

where $\eta_1 > 0$ is a scalar and

$$\Theta = \begin{bmatrix} S_{i22}K_i^T + K_i^T S_{i22}^T + J_{11} & S_{i22}K_i^T A_{di4} & S_{i22}K_i^T\sum_{m=1}^n\alpha_m B_{m4} \\ * & -\left(1-\bar{d}\right)Q_{i3} & 0 \\ * & * & -\left(1-\bar{h}\right)Q_{i3} \end{bmatrix} \tag{94}$$

From (51), it can be verified that

$$\begin{bmatrix} \sum_{m=1}^n\alpha_m T_{i111} & \left(E^T P_i + SR_i^T\right)\bar{A}_{di} & * & * & \left(E^T P_i + SR_i^T\right)\sum_{m=1}^n\alpha_m\bar{B}_m \\ \bar{A}_{di}^T\left(P_i^T E + R_i S^T\right) & -\left(1-\bar{d}\right)Q_{i3} & * & * & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ \sum_{m=1}^n\alpha_m\bar{B}_m^T\left(P_i^T E + R_i S^T\right) & 0 & * & * & -\left(1-\bar{h}\right)R_{i3} \end{bmatrix} < 0 \tag{95}$$

For (95), let us define

$$\mathcal{N} = \begin{bmatrix} N & 0 & 0 \\ 0 & N & 0 \\ 0 & 0 & N \end{bmatrix} \tag{96}$$

Multiplying $\mathcal{N}^T$ and $\mathcal{N}$ into Pre- and Post- of (95), one has

$$\mathcal{N}^T \begin{bmatrix} \sum_{m=1}^{n}\alpha_m T_{i111} & \left(E^T P_i + S R_i^T\right)\bar{A}_{di} & * & * & \left(E^T P_i + S R_i^T\right)\sum_{m=1}^{n}\alpha_m \bar{B}_m \\ \bar{A}_{di}^T\left(P_i^T E + R_i S^T\right) & -\left(1-\bar{d}\right)Q_{i3} & * & * & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ \sum_{m=1}^{n}\alpha_m \bar{B}_m^T\left(P_i^T E + R_i S^T\right) & 0 & * & * & -\left(1-\bar{h}\right)R_{i3} \end{bmatrix}\mathcal{N} < 0 \tag{97}$$

Applying *Lemma 2* (Schur complement lemma) to (97), one has

$$\Theta \le diag\left\{-\eta_2 I, \quad 0, \quad 0\right\} \tag{98}$$

where $\eta_2 > 0$ is a scalar satisfying

$$\eta_2 - \eta_1 > 0 \tag{99}$$

From (88), (93) and (99), one has

$$\begin{aligned} J_{\xi_2}(t) &= \xi_2^T(t) J_{11}\xi_2(t) - \left(1-\bar{d}\right)\xi_2^T\left(t-d(t)\right)Q_{i3}\xi_2\left(t-d(t)\right) \\ &\quad -\left(1-\bar{h}\right)\xi_2^T\left(t-h(t)\right)R_{i3}\xi_2\left(t-h(t)\right) \\ &\le \left(\eta_1-\eta_2\right)\xi_2^T(t)\xi_2(t) + \eta_1^{-1}q^T(t)K_i S_{i22}^T S_{i22} K_i^T q(t) \end{aligned} \tag{100}$$

From (89) and (99), one has

$$J_{11} + \left(\eta_2 - \eta_1\right)I \ge \eta_3\left(Q_{i3} + R_{i3}\right) \tag{101}$$

where $\eta_3 > 1$ is a scalar.

Next, from (100) and (101), one has

$$\begin{aligned} &\xi_2^T(t)\left(Q_{i3}+R_{i3}\right)\xi_2(t) \\ &\le \left(1-\bar{d}\right)\eta_3^{-1}\xi_2^T\left(t-d(t)\right)Q_{i3}\xi_2\left(t-d(t)\right) + \left(1-\bar{h}\right)\eta_3^{-1} \\ &\times \xi_2^T\left(t-h(t)\right)\eta_2 Q_{i3}\xi_2\left(t-h(t)\right) + \eta_3^{-1}\eta_1^{-1}q^T(t)K_i S_{i22}^T S_{i22} K_i^T q(t) \end{aligned} \tag{102}$$

which infers

$$f(t) \le \chi_1 \sup_{t-\tau \le s \le t} f(s) + \chi_2 e^{-\delta t} \tag{103}$$

where

$$\begin{cases} f(t) = \xi_2^T(t)\left(Q_{i3}+R_{i3}\right)\xi_2(t) \\ \chi_1 = max\left\{\left(1-\bar{d}\right)\eta_3^{-1}, \quad \left(1-\bar{h}\right)\eta_3^{-1}\right\} \in (0, \ 1) \\ \chi_2 = \left(\eta_1\eta_3\right)^{-1}\bar{h}\left\|S_{i22}\hat{K}^T\right\|^2\left\|\varphi(t)\right\|_\tau^2 > 0 \\ 0 < \delta < min\left\{2\lambda_1, \quad \tau^{-1}ln\left(\chi_1^{-1}\right)\right\} \end{cases} \tag{104}$$

Applying *Lemma 7* to (103), one has

$$\begin{aligned} \left\|\xi_2(t)\right\|^2 &\le \lambda_1^{-1}\left(Q_{i3}+R_{i3}\right)\lambda_2\left(Q_{i3}+R_{i3}\right)\left\|\xi_2(t)\right\|_\tau^2 e^{-\delta t} \\ &\quad + \frac{\lambda_1^{-1}\left(Q_{i3}+R_{i3}\right)\chi_2}{1-\chi_1 e^{\delta\tau}}e^{-\delta t} \end{aligned} \tag{105}$$

where $\lambda_2 > \lambda_1 > 0$ are the scalars.

Thus, from (105), it can be seen that the subsystem $\xi_2(t)$ is exponentially stable. The proof of *Step 2.2* is completed.

From *Step 2.1* and *Step 2.2*, it can be seen that the closed-loop system is exponentially stable. Since the closed-loop system is regular impulse free and exponentially stable, the closed-loop system is exponentially admissible. The objective (ii) in *Remark 2* is achieved, and the proof of *Step 2* is completed.

Step 3. The proof of objective (iii) in *Remark 2*: the prescribed H-infinity performance is guaranteed according to *Definition 6*. For (24), consider the H-infinity performance functional $J_w(t)$

$$J_w(t) = \int_0^t\left(z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right)dt, \quad w(t) \ne 0, \quad t > 0 \tag{106}$$

From (32) and (51), one has

$$\int_0^t \hat{\varsigma}^T(t)\sum_{m=1}^{n}\alpha_m \mathcal{T}_i\hat{\varsigma}(t)dt < 0, \qquad t > 0 \tag{107}$$

where

$$\hat{\varsigma}^T(t) = \begin{bmatrix} x^T(t) & x^T\left(t-d(t)\right) & x^T\left(t-d_1\right) & x^T\left(t-d_2\right) & x^T\left(t-h_2\right) & \int_{t-d_1}^{t}x^T(t)dt & \int_{t-d_2}^{t-d_1}x^T(t)dt & \int_{t-h_1}^{t}x^T(t)dt & 0 \ \ldots \ 0 \\ 0 & x^T\left(t-h(t)\right) & x^T\left(t-h_1\right) & 0 & 0 & \int_{t-h_2}^{t-h_1}x^T(t)dt & w^T(t) & 0 & \underbrace{0 \ \ldots \ 0}_{10} \end{bmatrix} \tag{108}$$

From (67) and (107), one has

$$\begin{aligned} &\int_0^t\left(\dot{V}(x_t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right)dt \\ &\le \int_0^t \hat{\varsigma}^T(t)\sum_{m=1}^{n}\alpha_m \mathcal{T}_i\hat{\varsigma}(t)dt, \quad w(t) \ne 0, \quad t > 0 \end{aligned} \tag{109}$$

From (107) and (109), one has

$$\begin{aligned} &\int_0^t\left(\dot{V}(x_t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right)dt \\ &= \int_0^t \dot{V}(x_t)dt + \int_0^t\left(z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right)dt < 0, \ \ w(t) \ne 0, \ \ t > 0 \end{aligned} \tag{110}$$

From (63), one has

$$\int_0^t \dot{V}(x_t)dt = V(x_t) > 0 \tag{111}$$

From (110) and (111), one has

$$J_w(t) = \int_0^t\left(z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right)dt < 0, \quad w(t) \ne 0, \quad t > 0 \tag{112}$$

Considering (112) as $t \to \infty$, one has

$$\int_0^\infty z^T(t)z(t)dt \le \int_0^\infty \gamma^2 w^T(t)w(t)dt, \quad w(t) \ne 0 \tag{113}$$

From (113), it can be seen that the prescribed H-infinity performance is guaranteed. The objective (iii) in *Remark 2* is achieved, and the proof of *Step 3* is completed.

From *Step 1-3*, it can be seen that the closed-loop system is regular impulse free, exponentially admissible and prescribed H-infinity performance is guaranteed. The proof of *Theorem 1* is completed.

Remark 5. The free-weighting matrices can handle the quadratic double integral items by designing some matrix variables without model transformation [40]. Thus, M and N are employed to relax the design conditions, and the closed-loop system is regular impulse free. Besides, the delay-dependent Lyapunov-Krasovskii functional can derive the less conservative stability conditions by adjusting delay-dependent parameters [23]. Thus, $V(x_t)$ is constructed to derive the stability conditions. Moreover, the free-weighting matrices $\bar{M}$ and $\bar{N}$ are employed to convert the closed-loop system (24) into (80). Then, the proof of exponential stability for (80) is converted into the proof of exponential stability for $\dot{\xi}_1(t)$ and $\xi_2(t)$, such that the proof of exponential stability is simplified. $J_w(t)$ is designed in (106), and the prescribed H-infinity performance is guaranteed via $V(x_t)$. It should be noticed that the exponential admissibility analysis is presented via LMIs (51)-(57).

Remark 6. The classical Lyapunov-Krasovskii method often requires the time-varying delay $d(t)$ to satisfy some conservative conditions, such as $0 \le d(t) < \infty$ and $0 \le \dot{d}(t) < 1$ [41]. However, the aforementioned restrictions from the Lyapunov-Krasovskii method can be avoided by using the Lyapunov-Razumikhin method [41]. Thus, the Lyapunov-Razumikhin method will be considered for the controller design of the hybrid T-S fuzzy systems with multiple time-varying delays and unmatched disturbances in the future.

B. *Less conservative stability conditions*

In this section, the less conservative delay-dependent stability conditions are derived to determine K_i and K_{hi}.

Theorem 2. For the T-S fuzzy singular uncertain system (9) under controller (17), the closed-loop system (24) is solvable in $\mathcal{E}\left(E^T P_i E,\ \gamma^2\rho\right)$ if for the given scalars $a>0$, $b>0$, $d_1>0$, $0<h_1\le h_2$, $0\le \bar{d}<1$, $0\le \bar{h}<1$, $\rho\ge 0$, $\gamma>0$ and matrices $P_i=P_i^T$, $\tilde{Q}_{i1}=\tilde{Q}_{i1}^T>0$, $\tilde{Q}_{i2}=\tilde{Q}_{i2}^T>0$, $\tilde{Q}_{i3}=\tilde{Q}_{i3}^T>0$, $R_i=R_i^T>0$, $\tilde{R}_{i1}=\tilde{R}_{i1}^T>0$, $\tilde{R}_{i2}=\tilde{R}_{i2}^T>0$, $\tilde{R}_{i3}=\tilde{R}_{i3}^T>0$, $\tilde{Z}_{i1}=\tilde{Z}_{i1}^T>0$, $\tilde{Z}_{i2}=\tilde{Z}_{i2}^T>0$, $\tilde{Z}_{i3}=\tilde{Z}_{i3}^T>0$, $\tilde{Z}_{i4}=\tilde{Z}_{i4}^T>0$, $\tilde{Y}_{i1}=\tilde{Y}_{i1}^T>0$, $\tilde{Y}_{i2}=\tilde{Y}_{i2}^T>0$, $\tilde{Y}_{i3}=\tilde{Y}_{i3}^T>0$, $\tilde{Y}_{i4}=\tilde{Y}_{i4}^T>0$, $\tilde{M}_{i1}=\tilde{M}_{i1}^T>0$, $\tilde{M}_{i2}=\tilde{M}_{i2}^T>0$, $\tilde{N}_{i1}=\tilde{N}_{i1}^T>0$, $\tilde{N}_{i2}=\tilde{N}_{i2}^T>0$, there exist matrices X_i, Y_i, Y_{hi}, Y_H, Y_{Hh}, S, $\tilde{S}_{i11}$, $\tilde{S}_{i12}$, $\tilde{S}_{i21}$, $\tilde{S}_{i22}$, $\tilde{S}_{i31}$, $\tilde{S}_{i32}$, $\tilde{S}_{i41}$ and $\tilde{S}_{i42}$ with appropriate dimensions satisfying

$$\begin{bmatrix} -\dfrac{1}{2\gamma^2\rho} & Y_H & Y_{Hh} \\ * & -EX_i & 0 \\ * & * & -EX_i \end{bmatrix} \le 0,\quad i=1,2,\ldots,r \tag{114}$$

$$\Xi_i = \begin{bmatrix} \tilde{T}_{i1} & \tilde{T}_{i2} & \tilde{T}_{i4} & \tilde{T}_{i6} & \tilde{T}_{i9} & \tilde{T}_{i12} & a\tilde{T}_{i15} \\ * & \tilde{T}_{i3} & 0 & \tilde{T}_{i7} & \tilde{T}_{i10} & 0 & 0 \\ * & * & \tilde{T}_{i5} & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{T}_{i8} & 0 & \tilde{T}_{i13} & 0 \\ * & * & * & * & \tilde{T}_{i11} & \tilde{T}_{i14} & 0 \\ * & * & * & * & * & -aI & 0 \\ * & * & * & * & * & * & -aI \end{bmatrix} < 0,\quad i=1,2,\ldots,r \tag{115}$$

with

$$\begin{cases} \tilde{T}_{i1} = \begin{bmatrix} \tilde{T}_{i111} & \bar{A}_{di}X_i & \tilde{T}_{i112} & 0 \\ * & \left(\bar{d}-1\right)\tilde{Q}_{i3} & 0 & 0 \\ * & * & \tilde{T}_{i113} & \tilde{T}_{i114} \\ * & * & * & \tilde{T}_{i115} \end{bmatrix} \\ \tilde{T}_{i2} = \begin{bmatrix} \tilde{T}_{i21} & \tilde{T}_{i22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \tilde{T}_{i3} = \begin{bmatrix} \left(\bar{h}-1\right)\tilde{R}_{i3} & 0 & 0 \\ * & \tilde{T}_{i31} & \tilde{T}_{i32} \\ * & * & \tilde{T}_{i33} \end{bmatrix} \\ \tilde{T}_{i4} = \begin{bmatrix} d_1E\tilde{M}_{i1}E^T & d_2E\tilde{M}_{i2}E^T & h_1E\tilde{N}_{i1}E^T & h_2E\tilde{N}_{i2}E^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \tilde{T}_{i5} = diag\left\{-\tilde{Z}_{i3}-E\tilde{M}_{i1}E^T,\ -\tilde{Z}_{i4}-E\tilde{M}_{i2}E^T,\right. \\ \qquad\qquad \left. -\tilde{Y}_{i3}-E\tilde{N}_{i1}E^T,\ -\tilde{Y}_{i4}-E\tilde{N}_{i2}E^T\right\} \end{cases} \tag{116}$$

$$\begin{cases} \tilde{T}_{i6} = \begin{bmatrix} B_{wi} & \tilde{T}_{i61} & \cdots & \tilde{T}_{i61} \\ 0 & X_i\bar{A}_{di}^T & \cdots & X_i\bar{A}_{di}^T \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix},\quad \tilde{T}_{i8} = \begin{bmatrix} -\gamma^2 I & \tilde{T}_{i81} \\ * & \tilde{T}_{i82} \end{bmatrix} \\ \tilde{T}_{i7} = \begin{bmatrix} 0 & \tilde{T}_{i71} & \cdots & \tilde{T}_{i71} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix},\ \tilde{T}_{i10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_1\tilde{S}_{i32}^T & h_2\tilde{S}_{i41}^T & 0 \\ 0 & 0 & 0 & h_2\tilde{S}_{i42}^T & 0 \end{bmatrix} \\ \tilde{T}_{i9} = \begin{bmatrix} d_1\tilde{S}_{i11}^T & 0 & h_1\tilde{S}_{i31}^T & 0 & X_i\bar{C}_i^T \\ 0 & 0 & 0 & 0 & X_i\bar{C}_{di}^T \\ d_1\tilde{S}_{i12}^T & d_2\tilde{S}_{i21}^T & 0 & 0 & 0 \\ 0 & d_2\tilde{S}_{i22}^T & 0 & 0 & 0 \end{bmatrix} \end{cases} \tag{117}$$

$$\begin{cases} \tilde{T}_{i11} = diag\left(-\tilde{Z}_{i1},\ -\tilde{Z}_{i2},\ -\tilde{Y}_{i1},\ \tilde{Y}_{i2}\quad -I\right) \\ \tilde{T}_{i12} = \begin{bmatrix} \bar{U}_{i1} & \bar{U}_{i2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \tilde{T}_{i13} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \bar{U}_{i1} & \bar{U}_{i2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \bar{U}_{i1} & \bar{U}_{i2} & 0 & 0 \end{bmatrix} \\ \tilde{T}_{i14} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{U}_{i3} & \bar{U}_{i4} \end{bmatrix}, \tilde{T}_{i15} = \begin{bmatrix} X_iV_{i1}^T & 0 & X_iV_{i3}^T & 0 \\ 0 & X_iV_{i2}^T & 0 & X_iV_{i4}^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{cases} \tag{118}$$

where

$$\begin{cases} \tilde{T}_{i111} = A_iX_i + B_iD_mY_i + B_i\bar{D}_mY_H + X_iA_i^T + Y_i^TD_m^TB_i^T + Y_H^T\bar{D}_m^TB_i^T \\ \quad + \tilde{Q}_{i1} + \tilde{R}_{i1} + \tilde{Q}_{i3} + \tilde{R}_{i3} + d_1^2\tilde{Z}_{i3} + h_1^2\tilde{Y}_{i3} + d_2^2\tilde{Z}_{i4} + h_2^2\tilde{Y}_{i4}^T \\ \quad + d_1\tilde{S}_{i11}^TE^T + d_1E\tilde{S}_{i11} + h_1\tilde{S}_{i31}^TE^T + h_1E\tilde{S}_{i31} - d_1^2E\tilde{M}_{i1}E^T \\ \quad - h_1^2E\tilde{N}_{i1}E^T - d_2^2E\tilde{M}_{i2}E^T - h_2^2E\tilde{N}_{i2}E^T \\ \tilde{T}_{i112} = -d_1\tilde{S}_{i11}^TE^T + d_1E\tilde{S}_{i12},\quad \tilde{T}_{i115} = -\tilde{Q}_{i2} - d_2\tilde{S}_{i22}^TE^T - d_2E\tilde{S}_{i22} \\ \tilde{T}_{i113} = -\tilde{Q}_{i1} + \tilde{Q}_{i2} - d_1\tilde{S}_{i12}^TE^T - d_1E\tilde{S}_{i12} + d_2\tilde{S}_{i21}^TE^T + d_2E\tilde{S}_{i21} \\ \tilde{T}_{i114} = -d_2\tilde{S}_{i21}^TE^T + d_2E\tilde{S}_{i22} \end{cases} \tag{119}$$

$$\tilde{T}_{i21} = B_iD_mY_{hi} + B_i\bar{D}_mY_{Hh},\quad \tilde{T}_{i22} = -h_1\tilde{S}_{i31}^TE^T + h_1E\tilde{S}_{i32} \tag{120}$$

$$\begin{cases} \tilde{T}_{i31} = -\tilde{R}_{i1} + \tilde{R}_{i2} - h_1\tilde{S}_{i32}^TE^T - h_1E\tilde{S}_{i32} + h_2\tilde{S}_{i41}^TE^T + h_2E\tilde{S}_{i41} \\ \tilde{T}_{i32} = -h_2\tilde{S}_{i41}^TE^T + h_2E\tilde{S}_{i42},\quad \tilde{T}_{i33} = -\tilde{R}_{i2} - h_2\tilde{S}_{i42}^TE^T - h_2E\tilde{S}_{i42} \end{cases} \tag{121}$$

$$\begin{cases} \tilde{T}_{i61} = X_iA_i^T + Y_i^TD_m^TB_i^T + Y_H^T\bar{D}_m^TB_i^T,\ \tilde{T}_{i71} = Y_{hi}^TD_m^TB_i^T + Y_{Hh}^T\bar{D}_m^TB_i^T \\ \tilde{T}_{i81} = \Big[\underbrace{B_{wi}^T,\ \ldots,\ B_{wi}^T}_{8}\Big] \\ \tilde{T}_{i82} = diag\left\{-\dfrac{2b}{d_1}X_i + b^2\tilde{Z}_{i1},\ -\dfrac{2b}{d_2}X_i + b^2\tilde{Z}_{i2},\right. \\ \qquad -\dfrac{2b}{h_1}X_i + b^2\tilde{Y}_{i1},\ -\dfrac{2b}{h_2}X_i + b^2\tilde{Y}_{i2}, \\ \qquad -\dfrac{4b}{d_1^2}X_i + b^2\tilde{M}_{i1}\ \ -\dfrac{2b}{d_c}X_i + b^2\tilde{M}_{i2} \\ \qquad \left. -\dfrac{4b}{h_1^2}X_i + b^2\tilde{N}_{i1},\ -\dfrac{2b}{h_c}X_i + b^2\tilde{N}_{i2}\right\} \end{cases} \tag{122}$$

and K_i, K_{hi} can be determined as follows

$$K_i = Y_iX_i^{-1},\quad K_{hi} = Y_{hi}X_i^{-1} \tag{123}$$

Proof. Firstly, let us define

$$x(t) = N\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},\ K_HN = \begin{bmatrix} K_{H_1} & K_{H_2} \end{bmatrix},\ K_{Hh}N = \begin{bmatrix} K_{Hh_1} & K_{Hh_2} \end{bmatrix} \tag{124}$$

where K_{H_1}, K_{H_2}, K_{Hh_1} and K_{Hh_2} are the matrices with appropriate dimensions.

From (56), one knows that $\mathcal{E}\left(E^T P_i E,\ \gamma^2\rho\right) \subset \mathcal{L}\left(K_H,\ K_{Hh}\right)$. Then, consider (124) and $\mathcal{E}\left(E^T P_i E,\ \gamma^2\rho\right) \subset \mathcal{L}\left(K_H,\ K_{Hh}\right)$, one can obtain $K_{H_2}=0$ and $K_{Hh_2}=0$.

Secondly, let us define

$$x_1(t)=0, \quad \left\|k_{H_{2m}}x_2(t)+k_{Hh_{2m}}x_2\left(t-h(t)\right)\right\| > \gamma\rho^{\frac{1}{2}} \tag{125}$$

where $k_{H_{2m}}$ is the m-th row of K_{H_2}, and $k_{Hh_{2m}}$ is the m-th row of K_{Hh_2}.

Then, one has

$$x^T(t)E^TP_iEx(t)=0, \quad \left\|k_{H_m}x(t)+k_{Hh_m}x\left(t-h(t)\right)\right\| > \gamma\rho^{\frac{1}{2}} \tag{126}$$

where k_{H_m} is the m-th row of K_H, and k_{Hh_m} is the m-th row of K_{Hh}. From (126), it can be seen that (126) and $\mathcal{E}\left(E^T P_i E,\ \gamma^2\rho\right) \subset \mathcal{L}\left(K_H,\ K_{Hh}\right)$ are contradictory, one has

$$\begin{cases} K_H x(t)=K_{H_1}x_1(t), & K_{Hh}x(t)=K_{Hh_1}x_1(t) \\ x^T(t)E^TP_iEx(t)\le\gamma^2\rho, & x^T\left(t-h(t)\right)E^TP_iEx\left(t-h(t)\right)\le\gamma^2\rho \end{cases} \tag{127}$$

One knows that $\mathcal{E}\left(E^T P_i E,\ \gamma^2\rho\right) \subset \mathcal{L}\left(K_H,\ K_{Hh}\right)$ is equivalent to

$$\begin{bmatrix} k_{H_{1m}} & k_{Hh_{1m}} \end{bmatrix} P_i^{-1} \begin{bmatrix} k_{H_{1m}} & k_{Hh_{1m}} \end{bmatrix}^T \le \frac{1}{\gamma^2\rho}, \quad m=1,\ 2,\ \ldots,\ n \tag{128}$$

where $k_{H_{1m}}$ is the m-th row of K_{H_1}, $k_{Hh_{1m}}$ is the m-th row of K_{Hh_1}, and

$$P_i=\begin{bmatrix} P_{i11} & 0 \\ 0 & P_{i11} \end{bmatrix} \tag{129}$$

with

$$P_{i11}=P_{i11}^T>0 \tag{130}$$

Applying *Lemma 2* (Schur complement lemma) to (128)

$$\begin{bmatrix} -\frac{1}{\gamma^2\rho} & \begin{bmatrix} k_{H_{1m}} & k_{Hh_{1m}} \end{bmatrix} \\ \begin{bmatrix} k_{H_{1m}} & k_{Hh_{1m}} \end{bmatrix}^T & -P_i \end{bmatrix} \le 0, \quad m=1,\ 2,\ \ldots,\ n \tag{131}$$

which is equivalent to

$$\begin{bmatrix} -\frac{1}{2\gamma^2\rho} & \begin{bmatrix} k_{H_{1m}} & 0 \end{bmatrix} & \begin{bmatrix} k_{Hh_{1m}} & 0 \end{bmatrix} \\ * & -\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} P_{i11} & 0 \\ 0 & P_{i11} \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} & 0 \\ * & * & -\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} P_{i11} & 0 \\ 0 & P_{i11} \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \le 0, \quad m=1,\ 2,\ \ldots,\ n \tag{132}$$

Choose appropriate matrices P_i and R_i to obtain (133)

$$E^TP_i+SR_i^T=\left(E^TP_i+SR_i^T\right)^T \tag{133}$$

Let us define

$$\begin{cases} X_i=\left(E^TP_i+SR_i^T\right)^{-1}, & Y_i=K_iX_i, \quad Y_{hi}=K_{hi}X_i \\ Y_H=K_HX_i, \quad Y_{Hh}=K_{Hh}X_i \end{cases} \tag{134}$$

$$Y_HN=\begin{bmatrix} Y_{H_1} & 0 \end{bmatrix}, \quad Y_{Hh}N=\begin{bmatrix} Y_{Hh_1} & 0 \end{bmatrix} \tag{135}$$

Multiplying $diag\left(1,\ \ X_iN^{-T},\ \ X_iN^{-T}\right)$ and $diag\left(1,\ \ N^{-1}X_i,\ \ N^{-1}X_i\right)$ into Pre- and Post- of (132), one has

$$\begin{bmatrix} -\frac{1}{2\gamma^2\rho} & y_{H_{1m}} & y_{Hh_{1m}} \\ * & -EX_i & 0 \\ * & * & -EX_i \end{bmatrix} \le 0, \quad m=1,\ 2,\ \ldots,\ n \tag{136}$$

where $y_{H_{1m}}$ is the m-th row of Y_{H_1}, and $y_{Hh_{1m}}$ is the m-th row of Y_{Hh_1}.

Applying *Lemma 8* to (51), one has

$$\mathcal{T}_i=\mathcal{T}_{i0}+O_{i1}O_{i2}O_{i3}+\left(O_{i1}O_{i2}O_{i3}\right)^T\le\mathcal{T}_{i0}+a^{-1}O_{i1}O_{i1}^T+aO_{i3}^TO_{i3}<0 \tag{137}$$

with

$$\begin{cases} \mathcal{T}_{i0}=\begin{bmatrix} \bar{T}_{i1} & T_{i2} & T_{i4} & \bar{T}_{i6} & \bar{T}_{i9} \\ * & T_{i3} & 0 & T_{i7} & T_{i10} \\ * & * & T_{i5} & 0 & 0 \\ * & * & * & T_{i8} & 0 \\ * & * & * & * & T_{i11} \end{bmatrix}, \quad O_{i3}=\begin{bmatrix} V_{i1} & 0 & 0 & \cdots & 0 \\ 0 & V_{i2} & 0 & \cdots & 0 \\ V_{i3} & 0 & 0 & \cdots & 0 \\ 0 & V_{i4} & 0 & \cdots & 0 \end{bmatrix} \\ O_{i1}=\begin{bmatrix} \left(E^TP_i+SR_i^T\right)\bar{U}_{i1} & \left(E^TP_i+SR_i^T\right)\bar{U}_{i2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \\ \bar{U}_{i1} & \bar{U}_{i2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \bar{U}_{i1} & \bar{U}_{i2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ O_{i2}=diag\left\{\Delta_{i1}(t),\ \ \Delta_{i2}(t),\ \ \Delta_{i3}(t),\ \ \Delta_{i4}(t)\right\} \end{cases} \tag{138}$$

where

$$\begin{cases} \bar{T}_{i1}=\begin{bmatrix} \bar{T}_{i111} & \left(E^TP_i+SR_i^T\right)\bar{A}_{di} & -d_1S_{i11}^TE+d_1E^TS_{i12} & 0 \\ * & \left(\bar{d}-1\right)Q_{i3} & 0 & 0 \\ * & * & T_{i12} & -d_2S_{i21}^TE+d_2E^TS_{i22} \\ * & * & * & T_{i13} \end{bmatrix}, \quad \bar{T}_{i6}=\begin{bmatrix} \left(E^TP_i+SR_i^T\right)B_{wi} & \bar{A}_m^TL_i \\ 0 & \bar{A}_{di}^TL_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \bar{T}_{i9}=\begin{bmatrix} d_1S_{i11}^T & 0 & h_1S_{i31}^T & 0 & \bar{C}_i^T \\ 0 & 0 & 0 & 0 & \bar{C}_{di}^T \\ d_1S_{i12}^T & d_2S_{i21}^T & 0 & 0 & 0 \\ 0 & d_2S_{i22}^T & 0 & 0 & 0 \end{bmatrix} \end{cases} \tag{139}$$

in which

$$\begin{aligned} \bar{T}_{i111}&=\left(E^TP_i+SR_i^T\right)\bar{A}_m+\bar{A}_m^T\left(E^TP_i+SR_i^T\right)^T+Q_{i1}+Q_{i3}+R_{i1}+R_{i3} \\ &+d_1^2Z_{i3}+h_1^2Y_{i3}+d_2^2Z_{i4}+h_2^2Y_{i4}+d_1S_{i11}^TE+d_1E^TS_{i11}+h_1S_{i31}^TE \\ &+h_1E^TS_{i31}-d_1^2E^TM_{i1}E-h_1^2E^TN_{i1}E-d_2^2E^TM_{i2}E-h_2^2E^TN_{i2}E \end{aligned} \tag{140}$$

Next, let us define

$$\begin{cases} \tilde{Q}_{i1}=X_iQ_{i1}X_i, \ \tilde{Q}_{i2}=X_iQ_{i2}X_i, \ \tilde{Q}_{i3}=X_iQ_{i3}X_i, \ \tilde{R}_{i1}=X_iR_{i1}X_i \\ \tilde{R}_{i2}=X_iR_{i2}X_i, \ \tilde{R}_{i3}=X_iR_{i3}X_i, \ \tilde{Z}_{i1}=X_iZ_{i1}X_i, \ \tilde{Z}_{i2}=X_iZ_{i2}X_i \\ \tilde{Z}_{i3}=X_iZ_{i3}X_i, \ \tilde{Z}_{i4}=X_iZ_{i4}X_i, \ \tilde{Y}_{i1}=X_iY_{i1}X_i, \ \tilde{Y}_{i2}=X_iY_{i2}X_i \\ \tilde{Y}_{i3}=X_iY_{i3}X_i, \ \tilde{Y}_{i4}=X_iY_{i4}X_i, \ \tilde{M}_{i1}=X_iM_{i1}X_i, \ \tilde{M}_{i2}=X_iM_{i2}X_i \\ \tilde{N}_{i1}=X_iN_{i1}X_i, \ \tilde{N}_{i2}=X_iN_{i2}X_i, \ \tilde{S}_{i11}=X_iS_{i11}X_i, \ \tilde{S}_{i12}=X_iS_{i12}X_i \\ \tilde{S}_{i21}=X_iS_{i21}X_i, \ \tilde{S}_{i22}=X_iS_{i22}X_i, \ \tilde{S}_{i31}=X_iS_{i31}X_i, \ \tilde{S}_{i32}=X_iS_{i32}X_i \\ \tilde{S}_{i41}=X_iS_{i41}X_i, \ \tilde{S}_{i42}=X_iS_{i42}X_i \end{cases} \tag{141}$$

$$\mathcal{X} = diag\{\underbrace{X_i, \ \dots, \ X_i}_{11} \ \underbrace{I, \ \dots, \ I}_{9} \ \underbrace{X_i, \ \dots, \ X_i}_{4} \ \underbrace{I, \ \dots, \ I}_{9}\} \quad (142)$$

Multiplying $\mathcal{X}^T$ and $\mathcal{X}$ into Pre- and Post- of (137), one has

$$\mathcal{X}^T\left(\mathcal{T}_{i0} + O_{i1}O_{i2}O_{i3} + \left(O_{i1}O_{i2}O_{i3}\right)^T\right)\mathcal{X} < 0 \quad (143)$$

Applying *Lemma 5* to (143), one has

$$-\left(d_1^2 Z_{i1}\right)^{-1} = -d_1^{-1}Z_{i1}^{-1}d_1^{-1} \le -d_1^{-1}bX_i - d_1^{-1}bX_i + b^2 X_i Z_{i1} X_i$$

Via a similar method, the inequality (115) holds. Thus, the controller (17) is solvable in $\mathcal{E}\left(E^T P_i E, \ \gamma^2\rho\right)$, K_i and K_{hi} can be determined. The proof of *Theorem 2* is completed.

Remark 7. In *Theorem 2*, Schur complement lemma and Gronwall Bellman lemma are employed to determine the controller gain matrices K_i and K_{hi}. Thus, the controller design problem is converted into the LMIs optimization constraints to reduce the computation complexity of solving LMIs in *Corollary 1*.

Remark 8. From *Theorem 2*, it can be seen that there exist Y_H and Y_{Hh} satisfying the inequality (114) and the solutions of (114) and (115) can be solved based on the appropriate selections of a, b, d_1, h_1, h_2, $\bar{d}$, $\bar{h}$, ρ and γ. With the purpose to obtain the upper bounds d_2 for the control objective in this paper, one needs to perform a line search in the scalars a, b, d_1, h_1, h_2, $\bar{d}$, $\bar{h}$, ρ and γ. Besides, the scalars a, b, d_1, h_1, h_2, $\bar{d}$, $\bar{h}$, ρ and γ should lie inside the finite interval to assure the feasibility solutions for the inequalities (114) and (115).

C. LMIs optimization constraints

Corollary 1. For (24), the exact invariant set with less conservativeness can be described as

$$\begin{cases} max \ v_1 \\ s.t. \begin{cases} \text{(a)} \quad v_1 X_R \in \mathcal{E}\left(E^T P_i E, \ \gamma^2\rho\right) \\ \text{(b)} \quad inequality \ (114) \\ \text{(c)} \quad inequality \ (115) \end{cases} \end{cases} \quad (144)$$

with

$$X_R = co\{x_0(t)\}, \quad x_0(t) = \begin{bmatrix} x_{01}(t) & x_{02}(t) & \dots & x_{0n}(t) \end{bmatrix}^T \quad (145)$$

where $v_1 > 0$ is a scalar, $x_{01}(t)$, $x_{02}(t)$, … and $x_{0n}(t)$ are the vector elements of $x_0(t)$.

Note that the constraint (a) is equivalent to

$$x_r^T(t)E^T P_i E x_r(t) \le \frac{\gamma^2\rho}{v_1^2}, \quad r = 1, \ 2, \ \dots, \ n \quad (146)$$

According to *Lemma 2* (Schur complement lemma) and *Lemma 9*, (146) is equivalent to (147)

$$\begin{bmatrix} -v_2 & x_r^T(t)E^T & x_r^T(t) \\ * & -2I & 0 \\ * & * & -X_i^T - X_i + 0.5I \end{bmatrix} \le 0, \quad r = 1, \ 2, \ \dots, \ n \quad (147)$$

where

$$v_2 = \frac{\gamma^2 n}{v_1^2}$$

With above analysis, (144) can be converted into LMIs optimization constraint (148)

$$\begin{cases} min \ v_2 \\ s.t. \begin{cases} \text{(a)} \quad inequality \ (114) \\ \text{(b)} \quad inequality \ (115) \\ \text{(c)} \quad inequality \ (147) \end{cases} \end{cases} \quad (148)$$

Remark 9. From (144) and (148), it can be seen that the parameter values can be solved to optimize the objective values. For this purpose, one should choose the appropriate scalars $c > 0$, $0 < d_1 \le d_2$, $0 < h_1 \le h_2$, $0 \le \bar{d} < 1$, $0 \le \bar{h} < 1$, $\rho \ge 0$ and $\gamma > 0$ to guarantee the controller is solvable in $\mathcal{E}\left(E^T P_i E, \ \gamma^2\rho\right)$. Moreover, the LMIs optimization constraint can be nonstrict LMIs or strict LMIs, such as the nonstrict LMIs constraint (a), (c) and strict LMIs constraint (b).

Remark 10. It should be noticed that reducing the computation complexity of solving LMIs is another performance index in the control system design. Thus, the controller design problem is converted into LMIs optimization constraint problem of solving LMIs in this section, such that the obtained LMIs can be solved effectively and the computation complexity can be reduced via the optimization algorithm. Moreover, if some specific conditions are considered in the time delay system, such as it is difficult to obtain the state variables information, one can employ the dynamic output feedback technique for the delay-dependent controller design in the future.

V. Simulation Examples

A. Example 1

Consider a class of 2-dimensions T-S fuzzy singular uncertain systems

Plant rule i: if $x_1(t)$ *is* $\mathcal{M}_{i1}$, $x_2(t)$ *is* $\mathcal{M}_{i2}$, … , *and* $x_g(t)$ *is* $\mathcal{M}_{ig}$, *then*

$$\begin{cases} E\dot{x}(t) = \left(A_i + \Delta A_i(t)\right)x(t) + \left(A_{di} + \Delta A_{di}(t)\right)x\left(t - d(t)\right) \\ \qquad + B_i sat\left(u(t)\right) + B_{wi}w(t) \\ z(t) = \left(C_i + \Delta C_i(t)\right)x(t) + \left(C_{di} + \Delta C_{di}(t)\right)x\left(t - d(t)\right) \\ x(t) = \varphi(t), \quad t \in [-d_2, \ 0] \end{cases} \quad (149)$$

Applying T-S fuzzy inference, one can obtain

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{2} h_i\left(x(t)\right)\left(\left(A_i + \Delta A_i(t)\right)x(t) + \left(A_{di} + \Delta A_{di}(t)\right)\right. \\ \qquad \left. \times x\left(t - d(t)\right) + B_i sat\left(u(t)\right) + B_{wi}w(t)\right) \\ z(t) = \sum_{i=1}^{2} h_i\left(x(t)\right)\left(\left(C_i + \Delta C_i(t)\right)x(t) + \left(C_{di} + \Delta C_{di}(t)\right)\right. \\ \qquad \left. \times x\left(t - d(t)\right)\right) \\ x(t) = \varphi(t), \quad t \in [-d_2, \ 0] \end{cases} \quad (150)$$

with

$$\begin{cases} \Delta A_1(t) = U_{11}\Delta_{11}(t)V_{11}, & \Delta A_{d1}(t) = U_{12}\Delta_{12}(t)V_{12} \\ \Delta C_1(t) = U_{13}\Delta_{13}(t)V_{13}, & \Delta C_{d1}(t) = U_{14}\Delta_{14}(t)V_{14} \\ \Delta A_2(t) = U_{21}\Delta_{21}(t)V_{21}, & \Delta A_{d2}(t) = U_{22}\Delta_{22}(t)V_{22} \\ \Delta C_2(t) = U_{23}\Delta_{23}(t)V_{23}, & \Delta C_{d2}(t) = U_{24}\Delta_{24}(t)V_{24} \end{cases} \quad (151)$$

For (150), the controller is designed as follows

Controller rule i: if $x_1(t)$ *is* $\mathcal{N}_{i1}$, $x_2(t)$ *is* $\mathcal{N}_{i2}$, … , *and* $x_g(t)$ *is* $\mathcal{N}_{ig}$, *then*

$$u(t) = K_i x(t) + K_{hi} x\left(t - h(t)\right) \quad (152)$$

Applying T-S fuzzy inference, a 2-rule T-S fuzzy model is employed and one has

$$u(t) = \sum_{i=1}^{2} \omega_i\left(x(t)\right)\left(K_i x(t) + K_{hi} x\left(t - h(t)\right)\right) \quad (153)$$

$$\begin{cases} K_1 = Y_1 X_1^{-1}, & K_{h1} = Y_{h1} X_1^{-1} \\ K_2 = Y_2 X_2^{-1}, & K_{h2} = Y_{h2} X_2^{-1} \end{cases} \quad (154)$$

$\omega_1(x(t))$ and $\omega_2(x(t))$ are the membership functions and

$$\begin{cases} \omega_1(x(t)) = 1 - \dfrac{sin(x(t))}{3}, \quad 0 \le x(t) \le \pi \\ \omega_2(x(t)) = 1 - \omega_1(x(t)) \end{cases} \tag{155}$$

Solving the LMIs in *Theorem 2* with $a = 0.60$, $b = 1.50$, $d_1 = 0.30$, $d_2 = 0.80$, $h_1 = 0.30$, $h_2 = 0.60$, $\bar{d} = 0.26$, $\bar{h} = 0.26$, $\rho = 1.60$ and $\gamma = 0.45$, where a, b, d_1, d_2, h_1, h_2, $\bar{d}$, $\bar{h}$, ρ and γ satisfy (114) and (115), then X_1, X_2, Y_1, Y_2, Y_{h1} and Y_{h2} are solved as follows

$$\begin{cases} X_1 = \begin{bmatrix} 0.2580 & -0.0003 \\ -0.0003 & 1.7600 \end{bmatrix}, & X_2 = \begin{bmatrix} 0.3605 & -0.0015 \\ -0.0018 & 1.0168 \end{bmatrix} \\ Y_1 = \begin{bmatrix} 0.8974 & 1.5292 \\ 0.1816 & -2.1015 \end{bmatrix}, & Y_2 = \begin{bmatrix} 0.4319 & 1.7913 \\ 0.5282 & -2.1655 \end{bmatrix} \\ Y_{h1} = \begin{bmatrix} -0.1697 & 0.0301 \\ 0.0332 & -0.1558 \end{bmatrix}, & Y_{h2} = \begin{bmatrix} -0.1590 & 0.0207 \\ 0.0719 & -0.1009 \end{bmatrix} \end{cases}$$

Substituting X_1, X_2, Y_1, Y_2, Y_{h1} and Y_{h2} into (154), one has

$$\begin{cases} K_1 = \begin{bmatrix} 3.4793 & 0.8695 \\ 0.7025 & -1.1939 \end{bmatrix}, & K_2 = \begin{bmatrix} 1.2069 & 1.7635 \\ 1.4546 & -2.1276 \end{bmatrix} \\ K_{h1} = \begin{bmatrix} -0.6577 & 0.0170 \\ 0.1286 & -0.0885 \end{bmatrix}, & K_{h2} = \begin{bmatrix} -0.4410 & 0.0197 \\ 0.1990 & -0.0989 \end{bmatrix} \end{cases}$$

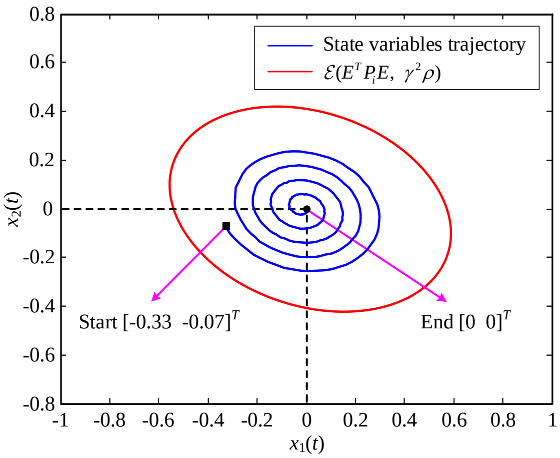

Fig. 2 The state variables trajectory in $\mathcal{E}(E^T P_i E,\ \gamma^2\rho)$ for the closed-loop system.

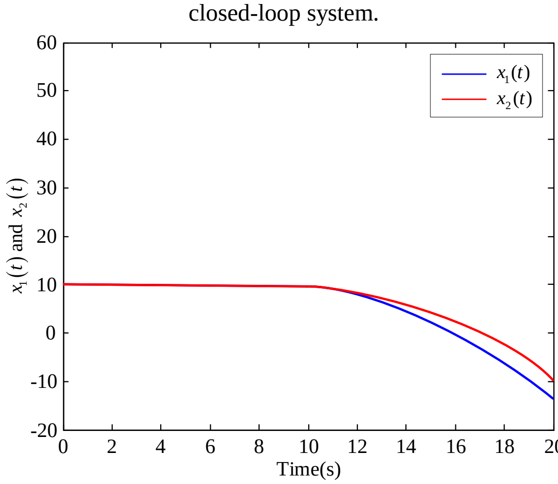

Fig. 3 The responses of $x_1(t)$ and $x_2(t)$ for the open-loop system.

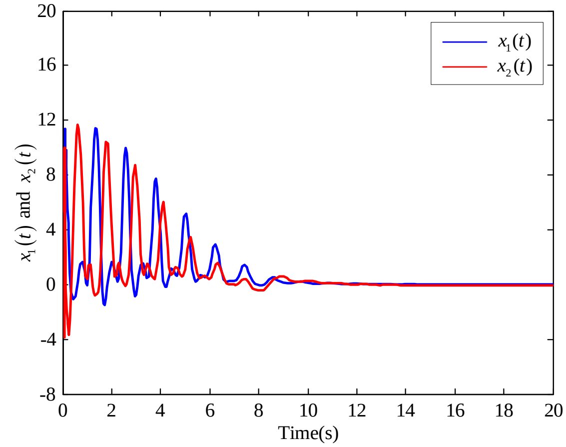

Fig. 4 The responses of $x_1(t)$ and $x_2(t)$ for the closed-loop system.

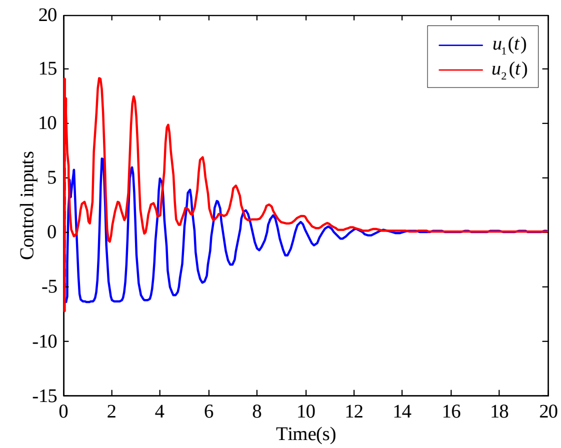

Fig. 5 The responses of control inputs $u_1(t)$ and $u_2(t)$.

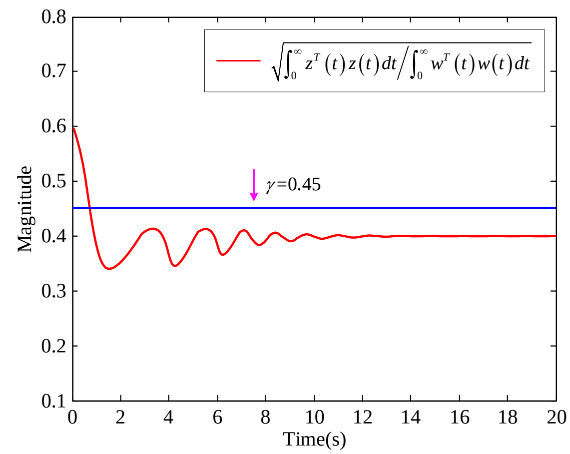

Fig. 6 The response of $\sqrt{\int_0^\infty z^T(t)z(t)dt \Big/ \int_0^\infty w^T(t)w(t)dt}$ for the closed-loop system.

In the simulations, the time-varying delay is given as $d(t) = 0.08|cos(t)|$, the external disturbance is given as $w(t) = 0.36|sin(t)|$, and the given H-infinity performance index is given as $\gamma = 0.45$. The state variables trajectory in $\mathcal{E}(E^T P_i E,\ \gamma^2\rho)$ is shown in Fig. 2. The responses of $x_1(t)$ and $x_2(t)$ for the open-loop system are shown in Fig. 3. The responses of $x_1(t)$ and $x_2(t)$ for the closed-loop system are shown in Fig. 4. The responses of control inputs are shown in Fig. 5. The response of

$\sqrt{\int_0^\infty z^T(t)z(t)dt \Big/ \int_0^\infty w^T(t)w(t)dt}$ for the closed-loop system is shown in Fig. 6. From Fig. 2, it can be seen that the state variables trajectory for the closed-loop system is included in $\mathcal{E}\left(E^T P_i E,\ \gamma^2\rho\right)$. From Figs. 3-4, it can be seen that the closed-loop system is exponentially stable. Thus, the closed-loop system is exponentially admissible because the closed-loop system is regular impulse free and exponentially stable. From Figs. 4-5, it can be seen that the state variables and control inputs are convergent. From Fig. 6, the prescribed H-infinity performance is guaranteed.

Remark 11. The H-infinity performance γ is often used to investigate the prescribed H-infinity performance for the minimum sensitivity problem of the systems with disturbance. The prescribed H-infinity performance reflects the robustness of the system against external disturbances. From Fig. 6, it can be seen that response of $\sqrt{\int_0^\infty z^T(t)z(t)dt \Big/ \int_0^\infty w^T(t)w(t)dt}$ for the closed-loop system are smaller than given H-infinity performance index $\gamma = 0.45$. This means the closed-loop system has strong robustness.

Remark 12. For the controller design and stability analysis of the time delay systems, the delay-dependent methods are generally less conservative than delay-independent methods, especially when the size of time delay is small in the practical application system. Moreover, from (149), it can be seen that the time-varying delay, saturation input and unmatched disturbance are all considered, which makes the considered model more general and the results can be applied to a wilder practical system. In this section, the example 1 is presented to show the effectiveness of proposed methods, i.e., the closed-loop system is regular impulse free, exponentially admissible and prescribed H-infinity performance is guaranteed. The example 2 is presented to show the applicability of proposed methods in the practical application system, thus the practical inverted pendulum system model is considered in example 2.

B. Example 2

Note Detailed analysis process and simulation results of "Example 2" are presented in the "Supplementary Materials", available online.

VI. CONCLUSION

The T-S fuzzy model is employed to approximate the singular uncertain system, and the difficulties caused by the time-varying delay, saturation input and unmatched disturbance are approximated effectively. The time-varying delay and T-S fuzzy model are employed to design the delay-dependent T-S fuzzy state feedback controller, and the design conditions are relaxed. The free-weighting matrices and delay-dependent Lyapunov-Krasovskii functional are employed, and the closed-loop system is regular impulse free, exponentially admissible and prescribed H-infinity performance is guaranteed. The convex hull lemma is employed to convert the closed-loop system with saturation input into closed-loop system without saturation input, and the design flexibility is enhanced. The Schur complement lemma and Gronwall Bellman lemma are employed in the stability analysis, the less conservative exponential admissibility conditions are derived and controller gain matrices are determined. The controller design problem is converted into the LMIs optimization constraints, and the computation complexity of solving LMIs is reduced.

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