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Sub-predictor-based fuzzy control design for a reaction-diffusion equation

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Abstract—This paper addresses the stabilization of the nonlinear reaction-diffusion equation with an unknown but bounded delay. Based on Takagi-Sugeno (T-S) fuzzy rules, a T-S fuzzy PDE model is constructed to describe the nonlinear distributed parameter system. More importantly, we contribute to the large delay compensation and the future state prediction by introducing a chain of sub-predictors for the T-S fuzzy PDE model, and the convergence of observation errors is guaranteed as well. Different from the conventional approaches, the proposed observer can compensate a larger delay. To stabilize the system, for the first time, a new sub-predictor-based T-S fuzzy control law is suggested by using the proposed observer, which is composed of elementary observers connected in series. The implementation of the sub-predictor feedback is simpler than the classical methods that only one of the elementary observers is employed. Via Lyapunov-Krasovskii approach, sufficient exponential stability conditions are established in the framework of linear matrix inequalities (LMIs). A numerical example is provided to illustrate the effectiveness of theoretical results.

Index Terms—Time delay, sub-predictors, observer design, fuzzy control.

I. INTRODUCTION

Time-delay is universal and inevitable in real life, and it often occurs in many aspects, such as networks, electricity, engineering, medicine. Sometimes it may negatively affect the systems as a source of instability. Thus, effective control strategies need to be proposed to stabilize delayed systems. Researchers have investigated many fruitful results on delayed systems (see e.g. [1]-[3]). Especially, to compensate time delays, the effectiveness of the predictor-free/predictor-based feedback method has been demonstrated in recent years. Predictor-free feedback control law has been proposed to compensate small enough delays (see e.g. [4]). Predictor-based feedback control law has been applied for ODE/PDE systems and cascaded ODE-PDE/PDE-PDE systems to compensate large delays (see e.g. [5]-[7]).

As for the predictor, there are many major breakthroughs in previous studies, such as Smith predictor, finite spectrum assignment and Artstein-Kwon-Pierson “reduction” approach

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in [8]-[11]. Predictor-based feedback controllers have been discussed in a single plant (see e.g. [12]) and interconnected systems (see e.g. [7], [13]) using the prediction of the future state values. However, in the process of compensating large delays, it may be difficult to integrate input signals in control design over historical time (see e.g. [14]). As for the sub-predictor, related results on the compensation of time delays can be found in [15]-[18]. This method has the advantages of dealing with large delays sequentially which can be divided into small pieces and guaranteeing the convergence of observation errors.

For ODE systems, some detailed works on sub-predictors have been presented to deal with output delay in [15] and input delay in [16]. In [17], both output and input delays have been considered in networked control systems. For PDE systems, a similar scheme has been further extended to semi-linear diffusion PDE with arbitrary output delay (see e.g. [18]). The sub-predictors mentioned in [15]-[18] were constructed by a chain of cascade observers. In [37], another type of sub-predictors has been studied to deal with the large delay by using a recursive observer structure for a wide class of nonlinear systems. Exponential convergence was guaranteed by using a globally drift-observability condition and Gronwall’s lemma.

More and more practical problems are modeled as complex nonlinear systems. The T-S fuzzy model was presented in [19] for the sake of simplifying the nonlinear part by establishing “IF-THEN” fuzzy rules, which have been successfully applied widely in the fields of engineering, robotics, finance, medicine and so on. Based on a great number of studies for finite dimensional systems, the T-S fuzzy model has been researched in theoretical analysis and practical applications (see e.g. [19]-[22]). It also has been developed for infinite dimensional cases (see e.g. [23]-[27]). For the stabilization of T-S fuzzy PDEs, corresponding control strategies based on the state feedback in [24] and state observer in [25] have been proposed to stabilize the one-dimensional reaction-diffusion systems. However, the parameter uncertainties were not considered in the above literature [19]-[27]. For the case that the grades of membership contain uncertain information, the interval type-2 (IT2) fuzzy model was proposed. In the framework of IT2 fuzzy model, lots of research topics on fault estimation (see e.g. [28]) and observer-based control design (see e.g. [29]) have been studied.

In comparison to the previous study [25], we find that [25] focuses on compensating a small input delay. However, the proposed method is inapplicable for the case of large delay. Motivated by [30], when a simple observer was not enough

to compensate a large delay, sub-predictors were shown to be more efficient and effective by simulation examples, which inspires our work. For the first time, we introduce a chain of sub-predictors into T-S fuzzy PDE model to relax the limitation of delay. Different from the classical predictor-based controller that usually relies on the backstepping transformation (see e.g. [7]), our new control design based on sub-predictors is much simpler that only one of the elementary observers is employed. The designed predictor in [12] was effective for the delay with an upper bound, while the sub-predictors proposed in the present work can be applicable for a larger delay. This work is different from the existing results presented in [18]. Here we consider a more complex situation involving T-S fuzzy sets and Dirac delta function, which bring challenges to our analysis. Note that [17] studies the sub-predictor-based controller design for ODE systems. However, to the best of our knowledge, it is still an open research question in the context of nonlinear PDE systems. Our study focuses on the stabilization of the nonlinear reaction-diffusion equation with a large delay. Sufficient conditions based on the Lyapunov stability theory in the shape of LMIs are derived for the assurance on exponential stability. We face here two main difficulties:

- The nonlinearity and delay from the PDE model bring challenges to the sub-predictor-based T-S fuzzy control law design.
- The stability analysis is non-trivial due to the complex interconnection of the sub-predictors.

To summarize, here are the contributions of the present work:

- A chain of sub-predictors is novelly introduced for T-S fuzzy PDE system to achieve the future state prediction. In particular, the proposed method can be applicable for compensation of a large delay, compared with the conventional approach (see e.g. [25]).
- The control design based on the sub-predictors has been discussed for ODE systems (see e.g. [17]). For the first time, a new sub-predictor-based T-S fuzzy controller is suggested to tackle the stabilization problem of the nonlinear parabolic distributed parameter system. It is much simpler for implementation than the traditional one (see e.g. [7]).
- The problem of sub-predictor-based T-S fuzzy control design for nonlinear reaction-diffusion equation is formulated as an LMI problem which can be solved by choosing appropriate number of sub-predictors.

The following is the organization of this paper. The description and preparations for considered system are provided in Section II. To handle the nonlinear term, a T-S fuzzy PDE model is presented firstly and a chain of sub-predictors are introduced afterward by using point measurements in Section III. In Section IV, the corresponding sub-predictor-based fuzzy control law is proposed for exponential stabilization. To demonstrate the theoretical results, an example is provided in Section V. Conclusions and suggestions are presented in Section VI.

Notation. P is a positive definite for any matrix $P > 0$. In partitioned matrices, use the symbol $*$ to represent the sym-

metric blocks. I is identity matrix with appropriate dimension. $L_n^2(0, c) \triangleq L^2((0, c); \mathbb{R}^n)$ stands for the space of measurable squared-integrable functions over $(0, c)$ with the corresponding norm:

$$\|y\|_{L_n^2(0,c)}^2 = \int_0^c |y(x)|^2 dx.$$

Denote $H_n^1(0, c) \triangleq \{y \in L_n^2(0, c), y' \in L_n^2(0, c)\}$ as the Sobolev space with the norm

$$\|y\|_{H_n^1(0,c)}^2 = \|y\|_{L_n^2(0,c)}^2 + \|y'\|_{L_n^2(0,c)}^2.$$

For a function $y(x, t)$, the subscripts x and t represent the partial derivative, i.e. $y_x(x, t) = \frac{\partial y}{\partial x}$, $y_{xx}(x, t) = \frac{\partial^2 y}{\partial x^2}$ and $y_t(x, t) = \frac{\partial y}{\partial t}$.

II. PRELIMINARIES AND PROBLEM FORMULATION

Some useful inequalities are presented as follows:

Lemma 1. (*Wirtinger's inequality [35]*): For $l_a < l_b$, let $\xi \in H_n^1(l_a, l_b)$ with $\xi(l_a) = 0$ or $\xi(l_b) = 0$. Then

$$\|\xi(x)\|_{L_n^2(l_a, l_b)}^2 \leq \frac{4(l_b - l_a)^2}{\pi^2} \left\| \frac{d\xi}{dx}(x) \right\|_{L_n^2(l_a, l_b)}^2.$$

Moreover, if $\xi(l_a) = \xi(l_b) = 0$, then

$$\|\xi(x)\|_{L_n^2(l_a, l_b)}^2 \leq \frac{(l_b - l_a)^2}{\pi^2} \left\| \frac{d\xi}{dx}(x) \right\|_{L_n^2(l_a, l_b)}^2.$$

Lemma 2. (*Poincaré's inequality [36]*): For a scalar function $\xi \in H_n^1(l_a, l_b)$ satisfying $\int_{l_a}^{l_b} \xi(x) dx = 0$, the following inequality holds:

$$\|\xi(x)\|_{L_n^2(l_a, l_b)}^2 \leq \frac{(l_b - l_a)^2}{\pi^2} \left\| \frac{d\xi}{dx}(x) \right\|_{L_n^2(l_a, l_b)}^2.$$

Consider the one-dimensional reaction-diffusion equation with delayed input as follows:

$$\begin{cases} y_t(x, t) = \gamma y_{xx}(x, t) + f(y(x, t)) + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r), \\ d_L y(0, t) + (1 - d_L) y_x(0, t) = 0, \\ d_R y(1, t) + (1 - d_R) y_x(1, t) = 0, \\ y(x, 0) = y_0(x), \end{cases} \quad (1)$$

where $x \in (0, 1)$, $y(x, t) \in \mathbb{R}^n$ is the state, $r > 0$ is input delay, $\gamma > 0$ is viscosity and control inputs are $u_j(t) \in \mathbb{R}^n$ ($j = 1, \dots, N$) for the entire system. $\delta(x)$ represents Dirac delta function with purpose of achieving specific point actuations. $f(\cdot)$ denotes a sufficiently smooth nonlinear function satisfying $f(0) = 0$. Different values of $d_L, d_R \in \{0, 1\}$ denote different boundary conditions - Dirichlet, Neumann or mixed conditions.

Assume that $\{\Omega_j\}_{j=1}^N$ is a partition of $[0, 1]$. Each subinterval is denoted as $\Omega_j \triangleq [x_{j-1}, x_j]$ whose length satisfies $0 < |\Omega_j| \leq \Delta_u$, where Δ_u is the corresponding upper bound. The inputs $u_j(t)$ enter the system (1) through $\delta(x)$ at the points

$$\bar{x}_j = \frac{x_{j-1} + x_j}{2} \in \Omega_j.$$

We denote sampling time instants by $0 = t_0 < t_1 < \dots < t_k \dots$, $\lim_{k \rightarrow \infty} t_k = \infty$ and sampling subdomains are bounded:

$$t_{k+1} - t_k \leq h.$$

We suppose that delayed point measurements provided by sensors are given as follows:

$$\nu_j(t) = y(\bar{x}_j, t_k - r)$$

for $j = 1, 2, \dots, N$ and $t \in [t_k, t_{k+1})$, $k = 0, 1, 2, \dots$.

III. SUB-PREDICTOR-BASED OBSERVER DESIGN

A. T-S fuzzy model

According to [26] and [27], we propose the following IF-THEN fuzzy rules:

Plant Rule α :

IF $\kappa_1(x, t)$ is T_1^α , and $\kappa_2(x, t)$ is T_2^α, \dots , and $\kappa_l(x, t)$ is T_l^α , **THEN**

$$\begin{cases} y_t(x, t) = \gamma y_{xx}(x, t) + A_\alpha y(x, t) + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r), \\ d_L y(0, t) + (1 - d_L) y_x(0, t) = 0, \\ d_R y(1, t) + (1 - d_R) y_x(1, t) = 0, \\ y(x, 0) = y_0(x), \end{cases}$$

where $A_\alpha \in \mathbb{R}^{n \times n}$, $\alpha \in \mathcal{S} \triangleq \{1, 2, \dots, s\}$, $\beta = \{1, 2, \dots, l\}$. T_β^α denote fuzzy sets, the total number of fuzzy rules is s and $\kappa_\beta(x, t)$ are known functions of $y(x, t)$.

Then one gets the system

$$\begin{cases} y_t(x, t) = \gamma y_{xx}(x, t) + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t)) A_\alpha y(x, t) \\ \quad + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r), \\ d_L y(0, t) + (1 - d_L) y_x(0, t) = 0, \\ d_R y(1, t) + (1 - d_R) y_x(1, t) = 0, \\ y(x, 0) = y_0(x), \end{cases} \quad (2)$$

where $\kappa(x, t) = [\kappa_1(x, t), \kappa_2(x, t), \dots, \kappa_l(x, t)]^T$ and

$$h_\alpha(\kappa(x, t)) = \frac{\prod_{\beta=1}^l T_\beta^\alpha(\kappa_\beta(x, t))}{\sum_{\alpha=1}^s \prod_{\beta=1}^l T_\beta^\alpha(\kappa_\beta(x, t))}, \alpha \in \mathcal{S}.$$

The term T_β^α are grades of the membership of $\kappa_\beta(x, t)$ in T_β^α . For $x \in (0, 1)$ and $t \geq 0$, it is assumed that

$$\prod_{\beta=1}^l T_\beta^\alpha(\kappa_\beta(x, t)) \geq 0 \text{ and } \sum_{\alpha=1}^s \prod_{\beta=1}^l T_\beta^\alpha(\kappa_\beta(x, t)) > 0.$$

Hence, the following properties hold

$$h_\alpha(\kappa(x, t)) \geq 0, \alpha \in \mathcal{S} \text{ and } \sum_{\alpha=1}^s h_\alpha(\kappa(x, t)) = 1, \quad (3)$$

which are significant in the simplification and derivation of the results later.

B. Observer design

We introduce the following sub-predictors to predict future state: $\hat{y}^1(x, t - r) \rightarrow \hat{y}^2(x, t - \frac{M-1}{M}r) \rightarrow \dots \rightarrow \hat{y}^M(x, t - \frac{1}{M}r) \rightarrow y(x, t)$ and define the following estimation errors:

$$\begin{cases} e^1(x, t) = \hat{y}^1(x, t - r) - \hat{y}^2(x, t - \frac{M-1}{M}r), \\ e^2(x, t) = \hat{y}^2(x, t - \frac{M-1}{M}r) - \hat{y}^3(x, t - \frac{M-2}{M}r), \\ \quad \vdots \\ e^{M-1}(x, t) = \hat{y}^{M-1}(x, t - \frac{2}{M}r) - \hat{y}^M(x, t - \frac{1}{M}r), \\ e^M(x, t) = \hat{y}^M(x, t - \frac{1}{M}r) - y(x, t), \end{cases} \quad (4)$$

subject to

$$\begin{cases} d_L e^i(0, t) + (1 - d_L) e_x^i(0, t) = 0, \\ d_R e^i(1, t) + (1 - d_R) e_x^i(1, t) = 0, \end{cases} \quad i = 1, 2, \dots, M.$$

As in [32], we introduce the following shape functions

$$b_j(x) = \begin{cases} 1, & x \in [x_{j-1}, x_j], \\ 0, & \text{otherwise,} \end{cases} \quad j = 1, 2, \dots, N. \quad (5)$$

We design a chain of sub-predictor-based observers for $t \in [t_k - r, t_{k+1} - r)$ as follows:

$$\begin{cases} \hat{y}_t^1(x, t) = \gamma \hat{y}_{xx}^1(x, t) + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t)) A_\alpha \hat{y}^1(x, t) \\ \quad + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t) - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t)) \\ \quad \times L_\alpha [\hat{y}^1(\bar{x}_j, t - \frac{1}{M}r) - \hat{y}^2(\bar{x}_j, t)], \\ \hat{y}_t^2(x, t) = \gamma \hat{y}_{xx}^2(x, t) + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t)) A_\alpha \hat{y}^2(x, t) \\ \quad + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - \frac{1}{M}r) - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t)) \\ \quad \times L_\alpha [\hat{y}^2(\bar{x}_j, t - \frac{1}{M}r) - \hat{y}^3(\bar{x}_j, t)], \\ \quad \vdots \\ \hat{y}_t^{M-1}(x, t) = \gamma \hat{y}_{xx}^{M-1}(x, t) + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t)) A_\alpha \hat{y}^{M-1}(x, t) \\ \quad + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - \frac{M-2}{M}r) - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t)) \\ \quad \times L_\alpha [\hat{y}^{M-1}(\bar{x}_j, t - \frac{1}{M}r) - \hat{y}^M(\bar{x}_j, t)], \\ \hat{y}_t^M(x, t) = \gamma \hat{y}_{xx}^M(x, t) + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t)) A_\alpha \hat{y}^M(x, t) \\ \quad + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - \frac{M-1}{M}r) - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t)) \\ \quad \times L_\alpha [\hat{y}^M(\bar{x}_j, t - \frac{1}{M}r) - y(\bar{x}_j, t)], \end{cases} \quad (6)$$

subject to

$$\begin{cases} d_L \hat{y}^i(0, t) + (1 - d_L) \hat{y}_x^i(0, t) = 0, \\ d_R \hat{y}^i(1, t) + (1 - d_R) \hat{y}_x^i(1, t) = 0, \end{cases} \quad i = 1, 2, \dots, M,$$

where $L_\alpha \in \mathbb{R}^{n \times n}$, $\alpha \in \mathcal{S}$ are observer gains to be determined. Furthermore, from (6), one gets

$$\left\{ \begin{aligned} & \hat{y}_t^1(x, t - r) = \gamma \hat{y}_{xx}^1(x, t - r) \\ & + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t - r)) A_\alpha \hat{y}^1(x, t - r) \\ & + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r) \\ & - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - r)) \\ & \times L_\alpha [\hat{y}^1(\bar{x}_j, t - \frac{M+1}{M}r) - \hat{y}^2(\bar{x}_j, t - r)], \\ & \hat{y}_t^2(x, t - \frac{M-1}{M}r) = \gamma \hat{y}_{xx}^2(x, t - \frac{M-1}{M}r) \\ & + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t - \frac{M-1}{M}r)) A_\alpha \hat{y}^2(x, t - \frac{M-1}{M}r) \\ & + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r) \\ & - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{M-1}{M}r)) \\ & \times L_\alpha [\hat{y}^2(\bar{x}_j, t - r) - \hat{y}^3(\bar{x}_j, t - \frac{M-1}{M}r)], \\ & \vdots \\ & \hat{y}_t^{M-1}(x, t - \frac{2}{M}r) = \gamma \hat{y}_{xx}^{M-1}(x, t - \frac{2}{M}r) \\ & + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t - \frac{2}{M}r)) A_\alpha \hat{y}^{M-1}(x, t - \frac{2}{M}r) \\ & + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r) \\ & - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{2}{M}r)) \\ & \times L_\alpha [\hat{y}^{M-1}(\bar{x}_j, t - \frac{3}{M}r) - \hat{y}^M(\bar{x}_j, t - \frac{2}{M}r)], \\ & \hat{y}_t^M(x, t - \frac{1}{M}r) = \gamma \hat{y}_{xx}^M(x, t - \frac{1}{M}r) \\ & + \sum_{\alpha=1}^s h_\alpha(\kappa(x, t - \frac{1}{M}r)) A_\alpha \hat{y}^M(x, t - \frac{1}{M}r) \\ & + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r) \\ & - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{1}{M}r)) \\ & \times L_\alpha [\hat{y}^M(\bar{x}_j, t - \frac{2}{M}r) - y(\bar{x}_j, t - \frac{1}{M}r)], \end{aligned} \right. \quad (7)$$

where $t \in [t_k - \frac{M-1}{M}r, t_{k+1} - \frac{M-1}{M}r)$, $k = 0, 1, 2, \dots$ and $\hat{y}^i(x, t) = 0$ for $t \leq 0$, $i = 1, 2, \dots, M$.

Define $F(e^i, x, t)$ as follows:

$$F(e^i, x, t) = \begin{cases} f(\hat{y}^i, x, t) - f(\hat{y}^{i+1}, x, t), \\ \quad \quad \quad i = 1, 2, \dots, M-1, \\ f(\hat{y}^i, x, t) - f(y, x, t), \quad i = M, \end{cases}$$

where $f(y, x, t) = \sum_{\alpha=1}^s h_\alpha(\kappa(x, t)) A_\alpha y(x, t)$ and

$$\begin{aligned} & f(\hat{y}^i, x, t) \\ & = \sum_{\alpha=1}^s h_\alpha(\kappa(x, t - \frac{M-i+1}{M}r)) A_\alpha \hat{y}^i(x, t - \frac{M-i+1}{M}r) \end{aligned}$$

for $i = 1, 2, \dots, M$.

Define $w(e^i, x, t)$ as follows:

$$w(e^i, x, t) = \begin{cases} \int_0^1 f_{\hat{y}^i}(\hat{y}^i, x, t) d\theta \\ = \int_0^1 f_{\hat{y}^{i+1+\theta e^i}}(\hat{y}^{i+1+\theta e^i}, x, t) d\theta \\ = \int_0^1 \frac{f_{\hat{y}^{i+1+\theta e^i}}(\hat{y}^{i+1+\theta e^i}, x, t)}{e^i} dy, \\ \quad \quad \quad i = 1, 2, \dots, M-1, \\ \int_0^1 f_{\hat{y}^i}(\hat{y}^i, x, t) d\theta \\ = \int_0^1 f_{y+\theta e^i}(y + \theta e^i, x, t) d\theta \\ = \int_0^1 \frac{f_{y+\theta e^i}(y + \theta e^i, x, t)}{e^i} dy, \\ \quad \quad \quad i = M. \end{cases}$$

where $f_{\hat{y}^i}(\hat{y}^i, x, t)$ represents the partial derivative of $f(\hat{y}^i, x, t)$ with respect to the variable \hat{y}^i . Then we obtain

$$w(e^i, x, t) e^i = \begin{cases} f(\hat{y}^{i+1} + e^i, x, t) - f(\hat{y}^{i+1}, x, t) \\ = f(\hat{y}^i, x, t) - f(\hat{y}^{i+1}, x, t), \\ \quad \quad \quad i = 1, 2, \dots, M-1, \\ f(\hat{y}^i, x, t) - f(y, x, t), \quad i = M. \end{cases} \quad (10)$$

As in [31], we make the following assumption:

Assumption 1. Suppose that the membership functions $h_\alpha(\kappa(x, t))$, $\alpha \in \mathcal{S}$ are known and the variation ranges of $w(e^i, x, t)$ satisfy

$$|w(e^i, x, t)| \leq \varepsilon_i, \quad i = 1, 2, \dots, M, \quad (11)$$

where ε_i are positive scalars.

Remark 1. Assumption 1 is reasonable because the membership functions $h_\alpha(\kappa(x, t))$, $\alpha \in \mathcal{S}$ are known generally for any $x \in [0, 1]$, $t > 0$. Fuzzy rules are always established in advance by human or control engineers so that $w(e^i, x, t)$ satisfies the boundedness property reasonably. The values of ε_i satisfying (11) can be obtained through theoretical analysis or numerical simulation.

From (7) and (10), one has the error system

$$\left\{ \begin{array}{l} e_t^1(x, t) = \gamma e_{xx}^1(x, t) + w(e^1, x, t)e^1(x, t) \\ \quad - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t-r)) L_\alpha e^1(\bar{x}_j, t - \frac{1}{M}r) \\ \quad + \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{M-1}{M}r)) L_\alpha e^2(\bar{x}_j, t - \frac{1}{M}r), \\ e_t^2(x, t) = \gamma e_{xx}^2(x, t) + w(e^2, x, t)e^2(x, t) \\ \quad - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{M-1}{M}r)) L_\alpha e^2(\bar{x}_j, t - \frac{1}{M}r) \\ \quad + \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{M-2}{M}r)) L_\alpha e^3(\bar{x}_j, t - \frac{1}{M}r), \\ \quad \vdots \\ e_t^{M-1}(x, t) = \gamma e_{xx}^{M-1}(x, t) + w(e^{M-1}, x, t)e^{M-1}(x, t) \\ \quad - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{2}{M}r)) L_\alpha e^{M-1}(\bar{x}_j, t - \frac{1}{M}r) \\ \quad + \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{1}{M}r)) L_\alpha e^M(\bar{x}_j, t - \frac{1}{M}r), \\ e_t^M(x, t) = \gamma e_{xx}^M(x, t) + w(e^M, x, t)e^M(x, t) \\ \quad - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{1}{M}r)) L_\alpha e^M(\bar{x}_j, t - \frac{1}{M}r), \end{array} \right. \quad (12)$$

where $t \in [t_k - \frac{M-1}{M}r, t_{k+1} - \frac{M-1}{M}r)$, $k = 0, 1, 2, \dots$.

Theorem 1. For error system governed by (12), given scalars $\Delta_u > 0$, $r > 0$, $0 < \delta < 1$, if there exist $0 < P \in \mathbb{R}^{n \times n}$, $0 < P_i \in \mathbb{R}^{n \times n}$ ($i = 1, 2, 3, 4, 5$), $W_\alpha \in \mathbb{R}^{n \times n}$ ($\alpha \in \mathcal{S}$), and $\lambda \geq 0$ satisfying LMIs as follows:

$$\Psi \triangleq -2\gamma(1-\delta)P_2 + P_5 + \max\{d_L, d_R\}P \leq 0, \quad (13)$$

$$-e^{-2\delta \frac{r}{M}} P_5 + \lambda I_n \leq 0, \quad (14)$$

$$\Phi = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} & \chi_{14} \\ * & \chi_{22} & \chi_{23} & \chi_{24} \\ * & * & \chi_{33} & 0 \\ * & * & * & -\frac{\pi^2}{\Delta_u} \lambda I_n \end{bmatrix} \leq 0, \quad (15)$$

where

$$\begin{aligned} \chi_{11} &= -\max\{d_L, d_R\} \frac{\pi^2}{4-3d_L d_R} P - \frac{M}{r} e^{-2\delta \frac{r}{M}} P_4 \\ &\quad + 2\delta P_1 + P_3 + 2P_2 E, \\ \chi_{12} &= P_1 + P_2 E - P_2, \\ \chi_{13} &= \frac{M}{r} e^{-2\delta \frac{r}{M}} P_4 - W_\alpha, \\ \chi_{14} &= W_\alpha, \\ \chi_{22} &= -2P_2 + \frac{r}{M} P_4, \\ \chi_{23} &= -\chi_{24} = -W_\alpha, \\ \chi_{33} &= -e^{-2\delta \frac{r}{M}} (P_3 + \frac{M}{r} P_4), \end{aligned}$$

then error system governed by (12) is exponentially stable with

decay rate δ in the sense that

$$\|e^i(\cdot, t)\|_{H_n^1(0,1)}^2 \leq c e^{-2\delta t} \|e^i(\cdot, 0)\|_{H_n^1(0,1)}^2, \quad (16)$$

holds for $i = 1, \dots, M$ with a positive constant

$$c \triangleq \frac{\max\{\lambda_{\max}(P_1) + \frac{r}{M} \lambda_{\max}(P_3), \gamma \lambda_{\max}(P_2) + \frac{r}{M} \lambda_{\max}(P_5)\}}{\min\{\lambda_{\min}(P_1), \gamma \lambda_{\min}(P_2)\}}. \quad (17)$$

Moreover, the observer gain matrices are given by

$$L_\alpha = P_2^{-1} W_\alpha, \quad \alpha \in \mathcal{S}. \quad (18)$$

Proof. Step 1: For the case of $i = M$, consider the Lyapunov-Krasovskii functional:

$$\begin{aligned} V_M(t) &= \int_0^1 [e^M(x, t)]^T P_1 e^M(x, t) dx \\ &\quad + \gamma \int_0^1 [e_x^M(x, t)]^T P_2 e_x^M(x, t) dx \\ &\quad + \int_0^1 \int_{t-\frac{r}{M}}^t e^{2\delta(s-t)} [e^M(x, s)]^T P_3 e^M(x, s) ds dx \\ &\quad + \int_0^1 \int_{-\frac{r}{M}}^0 \int_{t+\theta}^t e^{2\delta(s-t)} [e_s^M(x, s)]^T P_4 e_s^M(x, s) ds d\theta dx \\ &\quad + \int_0^1 \int_{t-\frac{r}{M}}^t e^{2\delta(s-t)} [e_x^M(x, s)]^T P_5 e_x^M(x, s) ds dx \end{aligned} \quad (19)$$

where $P_i \in \mathbb{R}^{n \times n}$ ($i = 1, 2, 3, 4, 5$) are positive definite.

Differentiating $V_M(t)$ along (12), one gets

$$\begin{aligned} \dot{V}_M(t) + 2\delta V_M(t) &= \int_0^1 [e_t^M(x, t)]^T P_1 e^M(x, t) dx \\ &\quad + \int_0^1 [e^M(x, t)]^T P_1 e_t^M(x, t) dx \\ &\quad + \gamma \int_0^1 [e_{xt}^M(x, t)]^T P_2 e_x^M(x, t) dx \\ &\quad + \gamma \int_0^1 [e_x^M(x, t)]^T P_2 e_{xt}^M(x, t) dx \\ &\quad + 2\delta \int_0^1 [e^M(x, t)]^T P_1 e^M(x, t) dx \\ &\quad + 2\delta \gamma \int_0^1 [e_x^M(x, t)]^T P_2 e_x^M(x, t) dx \\ &\quad + \int_0^1 [e^M(x, t)]^T P_3 e^M(x, t) dx \\ &\quad + \frac{r}{M} \int_0^1 [e_t^M(x, t)]^T P_4 e_t^M(x, t) dx \\ &\quad - e^{-2\delta \frac{r}{M}} \int_0^1 [e^M(x, t - \frac{r}{M})]^T P_3 e^M(x, t - \frac{r}{M}) dx \\ &\quad - \int_0^1 \int_{t-\frac{r}{M}}^t e^{2\delta(s-t)} [e_s^M(x, s)]^T P_4 e_s^M(x, s) ds dx \\ &\quad + \int_0^1 [e_x^M(x, t)]^T P_5 e_x^M(x, t) dx \\ &\quad - e^{-2\delta \frac{r}{M}} \int_0^1 [e_x^M(x, t - \frac{r}{M})]^T P_5 e_x^M(x, t - \frac{r}{M}) dx. \end{aligned} \quad (20)$$

Relying on Jensen's inequality, it can be obtained that

$$\begin{aligned}
& - \int_0^1 \int_{t-\frac{r}{M}}^t e^{2\delta(s-t)} [e_s^M(x, s)]^T P_4 e_s^M(x, s) ds dx \\
& \leq -\frac{M}{r} e^{-2\delta \frac{r}{M}} \int_0^1 \left[\int_{t-\frac{r}{M}}^t e_s^M(x, s) ds \right]^T \\
& \quad \times P_4 \left[\int_{t-\frac{r}{M}}^t e_s^M(x, s) ds \right] dx \\
& = -\frac{M}{r} e^{-2\delta \frac{r}{M}} \int_0^1 \left[e^M(x, t) - e^M(x, t - \frac{r}{M}) \right]^T \\
& \quad \times P_4 \left[e^M(x, t) - e^M(x, t - \frac{r}{M}) \right] dx \\
& \leq -\frac{M}{r} e^{-2\delta \frac{r}{M}} \int_0^1 \left[[e^M(x, t)]^T P_4 e^M(x, t) \right. \\
& \quad \left. + [e^M(x, t - \frac{r}{M})]^T P_4 e^M(x, t - \frac{r}{M}) \right. \\
& \quad \left. - 2[e^M(x, t)]^T P_4 e^M(x, t - \frac{r}{M}) \right] dx.
\end{aligned} \tag{21}$$

Using the descriptor method, from (12), we obtain

$$\begin{aligned}
0 & = 2 \int_0^1 [[e^M(x, t)]^T + [e_t^M(x, t)]^T] P_2 \\
& \quad \times \left[-e_t^M(x, t) + \gamma e_{xx}^M(x, t) + w(e^M(x, t) e^M(x, t) \right. \\
& \quad \left. - \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{1}{M}r)) L_\alpha e^M(\bar{x}_j, t - \frac{1}{M}r) \right] dx.
\end{aligned} \tag{22}$$

Integrating by parts, one has

$$\begin{aligned}
2\gamma \int_0^1 [e^M(x, t)]^T P_2 e_{xx}^M(x, t) dx \\
= -2\gamma \int_0^1 [e_x^M(x, t)]^T P_2 e_x^M(x, t) dx,
\end{aligned} \tag{23}$$

$$\begin{aligned}
2\gamma \int_0^1 [e_t^M(x, t)]^T P_2 e_{xx}^M(x, t) dx \\
= -2\gamma \int_0^1 [e_{xt}^M(x, t)]^T P_2 e_x^M(x, t) dx.
\end{aligned} \tag{24}$$

By Wirtinger's inequality from lemma 1, one has

$$\begin{aligned}
\max\{d_L, d_R\} \times \left[\int_0^1 [e_x^M(x, t)]^T P e_x^M(x, t) dx \right. \\
\left. - \frac{\pi^2}{4 - 3d_L d_R} \int_0^1 [e^M(x, t)]^T P e^M(x, t) dx \right] \geq 0
\end{aligned}$$

for any $P > 0$.

To analyze the stability, we denote

$$f_j(x, t) \triangleq e^M(x, t - \frac{1}{M}r) - e^M(\bar{x}_j, t - \frac{1}{M}r).$$

Hence,

$$\sum_{j=1}^N e^M(\bar{x}_j, t - \frac{1}{M}r) = \sum_{j=1}^N \left[e^M(x, t - \frac{1}{M}r) - f_j(x, t) \right]. \tag{26}$$

Therefore, some terms of (22) can be rewritten as

$$\begin{aligned}
0 & = 2 \int_0^1 [[e^M(x, t)]^T + [e_t^M(x, t)]^T] P_2 \\
& \quad \times \left[-\sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{1}{M}r)) L_\alpha e^M(\bar{x}_j, t - \frac{1}{M}r) \right] dx \\
& = -2 \int_0^1 [[e^M(x, t)]^T + [e_t^M(x, t)]^T] P_2 \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) \\
& \quad \times h_\alpha(\kappa(x, t - \frac{1}{M}r)) L_\alpha \left[e^M(x, t - \frac{1}{M}r) - f_j(x, t) \right] dx.
\end{aligned} \tag{27}$$

Due to $\int_{\Omega_j} f_j(x, t) dx = 0$, by Poincaré's inequality from Lemma 2, we have

$$\begin{aligned}
& \int_{\Omega_j} f_j^T(x, t) f_j(x, t) dx \\
& \leq \frac{\Delta_u^2}{\pi^2} \int_{\Omega_j} \left[e_x^M(x, t - \frac{1}{M}r) \right]^T e_x^M(x, t - \frac{1}{M}r) dx,
\end{aligned} \tag{28}$$

which brings

$$\begin{aligned}
& \lambda \sum_{j=1}^N \left[\int_{\Omega_j} \left[e_x^M(x, t - \frac{1}{M}r) \right]^T e_x^M(x, t - \frac{1}{M}r) dx \right. \\
& \quad \left. - \frac{\pi^2}{\Delta_u^2} \int_{\Omega_j} f_j^T(x, t) f_j(x, t) dx \right] \geq 0.
\end{aligned} \tag{29}$$

for any $\lambda \geq 0$.

From (11), using (21), (23), (24), (26) and adding (22), (25), (29) into (20) lead to

$$\begin{aligned}
\dot{V}_M(t) + 2\delta V_M(t) & \leq \int_0^1 [e_x^M(x, t)]^T \Psi e_x^M(x, t) dx \\
& \quad + \int_0^1 \left[e_x^M(x, t - \frac{1}{M}r) \right]^T (-e^{2\delta \frac{r}{M}} P_5 + \lambda I_n) e_x^M(x, t - \frac{1}{M}r) dx \\
& \quad + \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t - \frac{1}{M}r)) \eta^T(x, t) \Phi \eta(x, t) dx,
\end{aligned} \tag{30}$$

where

$$\eta(x, t) = \text{col}\{e^M(x, t), e_t^M(x, t), e^M(x, t - \frac{r}{M}), f_j(x, t)\}$$

and $W_\alpha = P_2 L_\alpha$, $\alpha \in \mathcal{S}$. Therefore, if (13)-(15) hold, then $\dot{V}_M(t) + 2\delta V_M(t) \leq 0$.

Step 2: For the cases of $i = 1, \dots, M - 1$, consider the Lyapunov-Krasovskii functional:

$$\begin{aligned}
V_i(t) & = \int_0^1 [e^i(x, t)]^T P_1 e^i(x, t) dx \\
& \quad + \gamma \int_0^1 [e_x^i(x, t)]^T P_2 e_x^i(x, t) dx \\
& \quad + \int_0^1 \int_{t-\frac{r}{M}}^t e^{2\delta(s-t)} [e^i(x, s)]^T P_3 e^i(x, s) ds dx \\
& \quad + \int_0^1 \int_{-\frac{r}{M}}^0 \int_{t+\theta}^t e^{2\delta(s-t)} [e_s^i(x, s)]^T P_4 e_s^i(x, s) ds d\theta dx \\
& \quad + \int_0^1 \int_{t-\frac{r}{M}}^t e^{2\delta(s-t)} [e_x^i(x, s)]^T P_5 e_x^i(x, s) ds dx.
\end{aligned} \tag{31}$$

If (13)-(15) hold, then

$$\dot{V}_i(t) + (2\delta - \epsilon\chi)V_i(t) - \chi V_{i+1}(t) \leq 0$$

where $i = 1, \dots, M-1$, ϵ is small enough, χ is large enough and $2\delta - \epsilon\chi > 0$.

At last, construct the Lyapunov-Krasovskii functional

$$V(t) = \sum_{i=1}^M \epsilon^{M-i} V_i(t). \quad (32)$$

Then we have $\dot{V}(t) + (2\delta - \epsilon\chi)V(t) \leq 0$ meaning that

$$V(t) \leq e^{-2\delta t} V(0). \quad (33)$$

Furthermore, (19), (31) and (33) yield

$$\begin{aligned} \min \{ \lambda_{\min}(P_1), \gamma \lambda_{\min}(P_2) \} \cdot \|e^i(\cdot, t)\|_{H_n^1(0,1)}^2 \\ \leq V_i(t) \leq e^{-2\delta t} V_i(0) \leq \bar{c} e^{-2\delta t} \|e^i(\cdot, 0)\|_{H_n^1(0,1)}^2, \end{aligned} \quad (34)$$

for $i = 1, \dots, M$ where

$$\bar{c} = \max \left\{ \lambda_{\max}(P_1) + \frac{r}{M} \lambda_{\max}(P_3), \gamma \lambda_{\max}(P_2) + \frac{r}{M} \lambda_{\max}(P_5) \right\}.$$

Thus, the inequality (16) holds with a positive constant

$$c \triangleq \frac{\bar{c}}{\min \{ \lambda_{\min}(P_1), \gamma \lambda_{\min}(P_2) \}}.$$

□

Remark 2. We notice that LMI conditions in (13)-(15) are convex in parameter r . That is, if the decision variable remains the same, for any constant delay $r_0 \in [0, r]$, (13)-(15) are also satisfied. Hence, the exponential convergence can be guaranteed when the time delay r_0 is uncertain but bounded in $[0, r]$.

Remark 3. LMI conditions in (13)-(15) depend on the value $r_{max} = r/M$. It is obvious that if (13)-(15) are feasible for r_{max} , they are also feasible for any positive integer $M \geq r/r_{max}$. For each time delay r , a sufficiently large M can always be found by Yalimp. Meanwhile, the exponential convergence can be guaranteed.

Remark 4. The computational complexity of Theorem 1 can be estimated as being proportional to $N_d^3 N_l$, where $N_d = 3n(n+1) + sn^2 + 1$ is the total number of scalar decision variables, and $N_l = 6$ is the total row size of LMIs.

IV. OBSERVER-BASED CONTROLLER DESIGN

Next, for stabilization of the system (2), we propose the following observer-based fuzzy feedback control law:

$$u_j(t) = - \sum_{\alpha=1}^s K_\alpha \int_{\Omega_j} h_\alpha(\kappa(x, t)) \hat{y}^1(x, t) dx, \quad (35)$$

where K_α ($\alpha = 1, 2, \dots, s$) will be determined later.

From (7) and (12), under the control law (35), the actuated

system becomes

$$\left\{ \begin{aligned} \hat{y}_t^1(x, t-r) &= \gamma \hat{y}_{xx}^1(x, t-r) \\ &+ \sum_{\alpha=1}^s h_\alpha(\kappa(x, t-r)) A_\alpha \hat{y}^1(x, t-r) \\ &- \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t-r)) L_\alpha e^1(\bar{x}_j, t - \frac{1}{M}r) \\ &+ \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t-r), \\ e_t^1(x, t) &= \gamma e_{xx}^1(x, t) + w(e^1, x, t) e^1(x, t) \\ &- \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t-r)) L_\alpha e^1(\bar{x}_j, t - \frac{1}{M}r) \\ &+ \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{M-1}{M}r)) L_\alpha e^2(\bar{x}_j, t - \frac{1}{M}r), \\ e_t^2(x, t) &= \gamma e_{xx}^2(x, t) + w(e^2, x, t) e^2(x, t) \\ &- \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{M-1}{M}r)) L_\alpha e^2(\bar{x}_j, t - \frac{1}{M}r) \\ &+ \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{M-2}{M}r)) L_\alpha e^3(\bar{x}_j, t - \frac{1}{M}r), \\ &\vdots \\ e_t^{M-1}(x, t) &= \gamma e_{xx}^{M-1}(x, t) + w(e^{M-1}, x, t) e^{M-1}(x, t) \\ &- \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{2}{M}r)) L_\alpha e^{M-1}(\bar{x}_j, t - \frac{1}{M}r) \\ &+ \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{1}{M}r)) L_\alpha e^M(\bar{x}_j, t - \frac{1}{M}r), \\ e_t^M(x, t) &= \gamma e_{xx}^M(x, t) + w(e^M, x, t) e^M(x, t) \\ &- \sum_{\alpha=1}^s \sum_{j=1}^N b_j(x) h_\alpha(\kappa(x, t - \frac{1}{M}r)) L_\alpha e^M(\bar{x}_j, t - \frac{1}{M}r). \end{aligned} \right. \quad (36)$$

Theorem 2. For the actuated system (36) under the designed sub-predictor-based fuzzy control law (35), given scalars $\Delta_u > 0$, $r > 0$, $\mu > 0$, $0 < \delta_0 < \delta < 1$, if there exist $0 < P \in \mathbb{R}^{n \times n}$, $0 < P_i \in \mathbb{R}^{n \times n}$, ($i = 1, 2, 3, 4, 5$), $\lambda \geq 0$, $\lambda_1 \geq 0$, $R_\alpha \in \mathbb{R}^{n \times n}$, $0 < W_\alpha \in \mathbb{R}^{n \times n}$ ($\alpha \in \mathcal{S}$) satisfying (13), (14), (15) and the following LMIs:

$$-2\gamma P_2 + \lambda_1 I_n \leq 0, \quad (37)$$

$$\Theta = \begin{bmatrix} \theta_{11} & -R_\alpha \\ * & -\frac{\pi^2}{\Delta_u^2} I_n \end{bmatrix} \leq 0, \quad (38)$$

where

$$\theta_{11} = A_\alpha^T P_2 + P_2 A_\alpha - (R_\alpha^T + R_\alpha) + 2\delta_0 P_2 + \mu W_\alpha,$$

then the designed controller (35) exponentially stabilize the

system (36) with decay rate δ_0 in the sense that

$$\|\hat{y}^1(\cdot, t-r)\|_{L_n^2(0,1)}^2 \leq C_0 e^{-2\delta_0 t} \|y(\cdot, 0)\|_{H_n^1(0,1)}^2 \quad (39)$$

holds with a positive constant

$$C_0 = \frac{\max\{\frac{1}{\mu}, \frac{\Delta_u^2}{\mu\pi^2}\} \cdot \max_{1 \leq \alpha \leq s} \{\lambda_{\max}(W_\alpha)\} c}{(\delta - \delta_0) \cdot \lambda_{\min}(P_2)}$$

and c satisfies (17).

Moreover, the controller gain matrices are given by

$$K_\alpha = P_2^{-1} R_\alpha, \quad \alpha \in \mathcal{S}, \quad (40)$$

and the observer gain matrices are given by (18).

Proof. Choose the following Lyapunov-Krasovskii functional:

$$V_0(t) = \int_0^1 [\hat{y}^1(x, t-r)]^T P_2 \hat{y}^1(x, t-r) dx. \quad (41)$$

Set $R_\alpha = P_2 K_\alpha$, $\alpha \in \mathcal{S}$. Differentiating (41) along (36) and (35), integrating by parts, we arrive at

$$\begin{aligned} \dot{V}_0(t) + 2\delta_0 V_0(t) &= -2\gamma \int_0^1 [\hat{y}_x^1(x, t-r)]^T P_2 \hat{y}_x^1(x, t-r) dx \\ &+ \int_0^1 \sum_{\alpha=1}^s h_\alpha(\kappa(x, t-r)) [\hat{y}^1(x, t-r)]^T [A_\alpha^T P_2 + P_2 A_\alpha] \\ &\times \hat{y}^1(x, t-r) dx - \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) [\hat{y}^1(x, t-r)]^T \\ &\times [L_\alpha^T P_2 + P_2 L_\alpha] e^1(\bar{x}_j, t - \frac{1}{M}r) dx - \sum_{\alpha=1}^s \sum_{j=1}^N [\hat{y}^1(\bar{x}_j, t-r)]^T \\ &\times [R_\alpha^T + R_\alpha] \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) \hat{y}^1(x, t-r) dx \\ &+ 2\delta_0 \int_0^1 [\hat{y}^1(x, t-r)]^T P_2 \hat{y}^1(x, t-r) dx. \end{aligned} \quad (42)$$

Set $W_\alpha = P_2 L_\alpha$, $\alpha \in \mathcal{S}$ and $W_\alpha > 0$. From Young's inequality, one gets

$$\begin{aligned} &- \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) [\hat{y}^1(x, t-r)]^T \\ &\times [L_\alpha^T P_2 + P_2 L_\alpha] e^1(\bar{x}_j, t - \frac{1}{M}r) dx \\ &\leq \mu \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) [\hat{y}^1(x, t-r)]^T \\ &\times W_\alpha \hat{y}^1(x, t-r) dx \\ &+ \frac{1}{\mu} \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) \left[e^1(\bar{x}_j, t - \frac{1}{M}r) \right]^T \\ &\times W_\alpha e^1(\bar{x}_j, t - \frac{1}{M}r) dx \end{aligned} \quad (43)$$

for any constant $\mu > 0$.

From (26) and (28), we have

$$\begin{aligned} &\frac{1}{\mu} \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) \left[e^1(\bar{x}_j, t - \frac{1}{M}r) \right]^T \\ &\times W_\alpha e^1(\bar{x}_j, t - \frac{1}{M}r) dx \end{aligned}$$

$$\begin{aligned} &\leq \frac{2}{\mu} \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) \left[e^1(x, t - \frac{1}{M}r) \right]^T \\ &\times W_\alpha e^1(x, t - \frac{1}{M}r) dx \\ &+ \frac{2\Delta_u^2}{\mu\pi^2} \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) \left[e_x^1(x, t - \frac{1}{M}r) \right]^T \\ &\times W_\alpha e_x^1(x, t - \frac{1}{M}r) dx \\ &\leq \max\{\frac{2}{\mu}, \frac{2\Delta_u^2}{\mu\pi^2}\} \cdot \max_{1 \leq \alpha \leq s} \{\lambda_{\max}(W_\alpha)\} \\ &\times \|e^1(\cdot, t - \frac{1}{M}r)\|_{H_n^1(0,1)}^2, \end{aligned} \quad (44)$$

Substituting (16) of Theorem 1 into (44) yields

$$\begin{aligned} &\frac{1}{\mu} \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) \left[e^1(\bar{x}_j, t - \frac{1}{M}r) \right]^T \\ &\times W_\alpha e^1(\bar{x}_j, t - \frac{1}{M}r) dx \\ &\leq C e^{-2\delta t} \|e^1(\cdot, 0)\|_{H_n^1(0,1)}^2, \end{aligned} \quad (45)$$

where $C \triangleq \max\{\frac{2}{\mu}, \frac{2\Delta_u^2}{\mu\pi^2}\} \cdot \max_{1 \leq \alpha \leq s} \{\lambda_{\max}(W_\alpha)\} c$ and the constant c satisfies (17).

Denote

$$g_j(x, t) \triangleq \hat{y}^1(\bar{x}_j, t-r) - \hat{y}^1(x, t-r). \quad (46)$$

Applying Poincaré's inequality from Lemma 2, we have

$$\begin{aligned} &\lambda_1 \sum_{j=1}^N \left[\int_{\Omega_j} [\hat{y}_x^1(x, t-r)]^T \hat{y}_x^1(x, t-r) dx \right. \\ &\left. - \frac{\pi^2}{\Delta_u^2} \int_{\Omega_j} g_j^T(x, t) g_j(x, t) dx \right] \geq 0 \end{aligned} \quad (47)$$

for any constant $\lambda_1 \geq 0$.

Adding (47) into (42), from (43)-(46), one has

$$\begin{aligned} &\dot{V}_0(t) + 2\delta_0 V_0(t) \\ &\leq \int_0^1 [\hat{y}_x^1(x, t-r)]^T (-2\gamma P_2 + \lambda_1 I_n) \hat{y}_x^1(x, t-r) dx \\ &+ \sum_{\alpha=1}^s \sum_{j=1}^N \int_{\Omega_j} h_\alpha(\kappa(x, t-r)) \varphi^T(x, t) \Theta \varphi(x, t) dx \\ &+ C e^{-2\delta t} \|e^1(\cdot, 0)\|_{H_n^1(0,1)}^2 \end{aligned} \quad (48)$$

where $\varphi(x, t) = \text{col}\{\hat{y}^1(x, t-r), g_j(x, t)\}$.

Therefore, if (13)-(15), (37), (38) are satisfied, then

$$\dot{V}_0(t) + 2\delta_0 V_0(t) \leq C e^{-2\delta t} \|e^1(\cdot, 0)\|_{H_n^1(0,1)}^2,$$

which implies

$$V_0(t) \leq e^{-2\delta_0 t} V_0(0) + \frac{C e^{-2\delta_0 t}}{2(\delta - \delta_0)} \|e^1(\cdot, 0)\|_{H_n^1(0,1)}^2. \quad (49)$$

Since $\hat{y}^1(x, t) = 0$ for $t \leq 0$, from (41) and (49), we derive

$$\begin{aligned} &\lambda_{\min}(P_2) \cdot \|\hat{y}^1(\cdot, t-r)\|_{L_n^2(0,1)}^2 \\ &\leq V_0(t) \leq \frac{C e^{-2\delta_0 t}}{2(\delta - \delta_0)} \|e^1(\cdot, 0)\|_{H_n^1(0,1)}^2. \end{aligned} \quad (50)$$

Thus, the inequality (39) holds with a positive constant

$$C_0 = \frac{C}{2(\delta - \delta_0) \cdot \lambda_{\min}(P_2)}.$$

□

Remark 5. In [6], depending on the backstepping method, the scheme for treating ODE/PDE systems with input delays as PDE-ODE/PDE-PDE cascade systems with boundary control is introduced. In comparison with [6], this paper has the merit that effective sub-predictors design and LMI approach are applied for T-S fuzzy system to compensate both input and output delay.

Remark 6. The computational complexity of Theorem 2 can be estimated as being proportional to $N_d^3 N_l$, where $N_d = 3n(n+1) + 2sn^2 + 2$ is the total number of scalar decision variables, and $N_l = 9$ is the total row size of LMIs.

Remark 7. It is noted that the designed observer (6) has the same premise membership functions as the fuzzy system (2). As for the case of the membership functions depending on the observer state, the corresponding result can be also obtained. In this case, the observer (6) becomes

Observer Rule q :

IF $\hat{\kappa}_1(x, t)$ is T_1^q , and $\hat{\kappa}_2(x, t)$ is T_2^q, \dots , and $\hat{\kappa}_l(x, t)$ is T_l^q , **THEN**

$$\begin{aligned} \hat{y}_t^i(x, t) &= \gamma \hat{y}_{xx}^i(x, t) + \sum_{q=1}^s h_q(\hat{\kappa}(x, t)) A_q \hat{y}^i(x, t) \\ &+ \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - \frac{i-1}{M}r) - \sum_{q=1}^s \sum_{j=1}^N b_j(x) h_q(\hat{\kappa}(x, t)) \\ &\times L_q [\hat{y}^i(\bar{x}_j, t - \frac{1}{M}r) - \hat{y}^{i+1}(\bar{x}_j, t)], \quad i = 1, \dots, M-1, \\ \hat{y}_t^M(x, t) &= \gamma \hat{y}_{xx}^M(x, t) + \sum_{q=1}^s h_q(\hat{\kappa}(x, t)) A_q \hat{y}^M(x, t) \\ &+ \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - \frac{M-1}{M}r) - \sum_{q=1}^s \sum_{j=1}^N b_j(x) h_q(\hat{\kappa}(x, t)) \\ &\times L_q [\hat{y}^M(\bar{x}_j, t - \frac{1}{M}r) - y(\bar{x}_j, t)], \quad i = M, \end{aligned}$$

where $L_q \in \mathbb{R}^{n \times n}$, $q \in \mathcal{S}$ are observer gains to be determined. Then the error system (12) becomes

$$\begin{aligned} e_t^i(x, t) &= \gamma e_{xx}^i(x, t) + w(e^i, x, t) e^i(x, t) \\ &- \sum_{q=1}^s \sum_{j=1}^N b_j(x) h_q(\hat{\kappa}(x, t-r)) L_q e^i(\bar{x}_j, t - \frac{1}{M}r) \\ &+ \sum_{q=1}^s \sum_{j=1}^N b_j(x) h_q(\hat{\kappa}(x, t - \frac{M-1}{M}r)) L_q e^{i+1}(\bar{x}_j, t - \frac{1}{M}r), \\ &i = 1, \dots, M-1, \\ e_t^M(x, t) &= \gamma e_{xx}^M(x, t) + w(e^M, x, t) e^M(x, t) \\ &- \sum_{q=1}^s \sum_{j=1}^N b_j(x) h_q(\hat{\kappa}(x, t - \frac{1}{M}r)) L_q e^M(\bar{x}_j, t - \frac{1}{M}r) \\ &+ \sum_{q=1}^s \sum_{\alpha=1}^s h_q(\hat{\kappa}(x, t)) h_\alpha(\kappa(x, t)) \\ &\times (A_q - A_\alpha) \left[\hat{y}^M(x, t - \frac{r}{M}) - e^M(x, t) \right], \quad i = M. \end{aligned}$$

The above will lead some changes of (15) as follows:

$$\begin{bmatrix} \chi_{11} - 2P_{q\alpha} & \chi_{12} - P_{q\alpha} & \chi_{13} & \chi_{14} & P_{q\alpha} & 0 \\ * & \chi_{22} & \chi_{23} & \chi_{24} & P_{q\alpha} & 0 \\ * & * & \chi_{33} & 0 & 0 & 0 \\ * & * & * & -\frac{\pi^2}{\Delta_u^2} \lambda I_n & 0 & 0 \\ * & * & * & * & \theta_{11} & -R_q \\ * & * & * & * & * & -\frac{\pi^2}{\Delta_u^2} I_n \end{bmatrix} \leq 0$$

where

$$\begin{aligned} P_{q\alpha} &= P_2(A_q - A_\alpha), \quad q \in \mathcal{S}, \quad \alpha \in \mathcal{S}, \\ \chi_{11} &= -\max\{d_L, d_R\} \frac{\pi^2}{4 - 3d_L d_R} P - \frac{M}{r} e^{-2\delta \frac{r}{M}} P_4 + 2\delta P_1 \\ &\quad + P_3 + 2P_2 E, \\ \chi_{12} &= P_1 + P_2 E - P_2, \quad \chi_{13} = \frac{M}{r} e^{-2\delta \frac{r}{M}} P_4 - W_q, \\ \chi_{14} &= -\chi_{23} = \chi_{24} = W_q, \\ \chi_{22} &= -2P_2 + \frac{r}{M} P_4, \quad \chi_{33} = -e^{-2\delta \frac{r}{M}} (P_3 + \frac{M}{r} P_4), \\ \theta_{11} &= A_q^T P_2 + P_2 A_q - (R_q^T + R_q) + 2\delta_0 P_2 + \frac{\mu}{2} (W_q^T + W_q). \end{aligned}$$

Moreover, the controller gain matrices are given by $K_q = P_2^{-1} R_q$, $q \in \mathcal{S}$ and the observer gain matrices are given by $L_q = P_2^{-1} W_q$, $q \in \mathcal{S}$.

V. NUMERICAL EXAMPLE

We now give an example to illustrate the effectiveness of the designed sub-predictor-based fuzzy controller (35) for the system as follows:

$$\begin{cases} y_t(x, t) = \gamma y_{xx}(x, t) + f(y(x, t)) + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r), \\ d_L y(0, t) + (1 - d_L) y_x(0, t) = 0, \\ d_R y(1, t) + (1 - d_R) y_x(1, t) = 0, \\ y(x, 0) = y_0(x), \end{cases} \quad (51)$$

where $0 < x < 1$, $\gamma = 0.2$, $r = 0.2$, $y_0(x) = 0.5 \sin(\frac{\pi}{2}x)$ and $f(y(x, t)) = 0.8(y(x, t) - 2y^3(x, t))$.

Different values of d_L and d_R represent different boundary conditions as follows:

$$\begin{cases} d_L = 0, d_R = 0 : \text{Neumann boundary conditions,} \\ d_L = 1, d_R = 1 : \text{Dirichlet boundary conditions,} \\ d_L = 1, d_R = 0 : \text{Mixed boundary conditions,} \\ d_L = 0, d_R = 1 : \text{Mixed boundary conditions.} \end{cases}$$

The open-loop system with mixed boundary conditions ($d_L = 1$, $d_R = 0$) is unstable (see Fig. 1).

The following IF-THEN fuzzy rules for system (51) are constructed:

Plant Rule 1:

IF $\kappa(x, t)$ is “big”, **THEN**

$$\begin{cases} y_t(x, t) = \gamma y_{xx}(x, t) + a_1 y(x, t) + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r), \\ d_L y(0, t) + (1 - d_L) y_x(0, t) = 0, \\ d_R y(1, t) + (1 - d_R) y_x(1, t) = 0, \\ y(x, 0) = y_0(x). \end{cases}$$

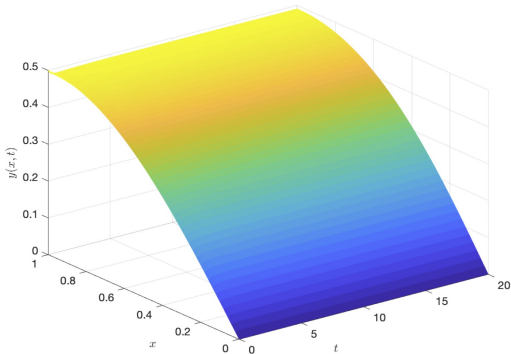


Fig. 1: The open-loop system with $d_L = 1$ and $d_R = 0$

Plant Rule 2:

IF $\kappa(x, t)$ is “small”, THEN

$$\begin{cases} y_t(x, t) = \gamma y_{xx}(x, t) + a_2 y(x, t) + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r), \\ d_L y(0, t) + (1 - d_L) y_x(0, t) = 0, \\ d_R y(1, t) + (1 - d_R) y_x(1, t) = 0, \\ y(x, 0) = y_0(x), \end{cases}$$

where $\kappa(x, t) = y^2(x, t)$, $v \triangleq \max_{y(\cdot, t)} y^2(\cdot, t)$, $a_1 = 0.8(1 - v)$ and $a_2 = 0.8$. From Fig. 1, it is clear that $v = 0.4998^2 = 0.25$, and thus, $a_1 = 0.6$.

Therefore,

$$\begin{cases} y_t(x, t) = \gamma y_{xx}(x, t) + \sum_{\alpha=1}^2 h_{\alpha}(\kappa(x, t)) a_{\alpha} y(x, t) \\ \quad + \sum_{j=1}^N \delta(x - \bar{x}_j) u_j(t - r), \\ d_L y(0, t) + (1 - d_L) y_x(0, t) = 0, \\ d_R y(1, t) + (1 - d_R) y_x(1, t) = 0, \\ y(x, 0) = y_0(x), \end{cases} \quad (52)$$

where the membership functions are described by

$$\begin{aligned} h_1(\kappa(x, t)) &= v^{-1} \kappa(x, t), \\ h_2(\kappa(x, t)) &= 1 - h_1(\kappa(x, t)). \end{aligned}$$

By Yalimp, LMI conditions are verified with $\Delta_u = 0.5$, $\delta = 0.2$, $\delta_0 = 0.15$, $\mu = 0.5$ in Theorems 1 and 2. For different values of r and M , the feasible solutions of LMI conditions are shown in Table I. From the simulation results, we find that the LMI conditions are always feasible for $\frac{r}{M} < 60.2$.

A finite difference method is utilized in numerical results to calculate the solution of the actuated system (52) under the control law

$$u_j(t) = - \sum_{\alpha=1}^2 K_{\alpha} \int_{\Omega_j} h_{\alpha}(\kappa(x, t)) \hat{y}^1(x, t) dx.$$

For different values of r and M , the control inputs $u_1(t)$ and $u_2(t)$ are shown in Fig. 2. Consider the case of $r = 0.2$ and $M = 2$ for system (52). The steps in time and space

TABLE I: Feasible solutions with different values of r and M

	$r = 0.2$ $M = 1$	$r = 0.2$ $M = 2$	$r = 0.2$ $M = 3$	$r = 1.2$ $M = 3$
L_1	0.7845	0.8282	0.8405	0.6625
L_2	0.7569	0.7998	0.8119	0.6365
K_1	1.9984	2.0569	2.0808	1.8767
K_2	2.2135	2.2676	2.2917	2.0941
P	1.7549	1.4511	1.3417	2.4749
P_1	10.8604	11.3573	11.5333	10.0166
P_2	12.5043	12.3629	12.4108	13.0940
P_3	4.8556	4.9442	4.9492	4.3732
P_4	5.7471	3.0131	1.9998	8.4484
P_5	1.6214	1.7621	1.8120	1.3531
W_1	9.8094	10.2388	10.4308	8.6754
W_2	9.4648	9.8883	10.0767	8.3337
R_1	24.9888	25.4288	25.8241	24.5733
R_2	27.6788	28.0339	28.4417	27.4205
λ	0.8872	0.9395	0.9812	0.8130
λ_1	2.5009	2.4726	2.4822	2.6188

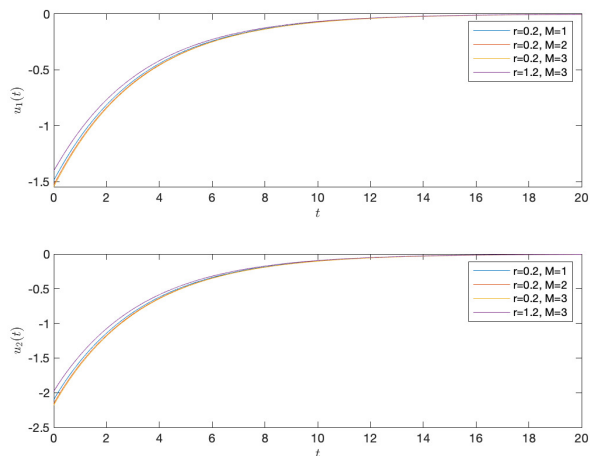


Fig. 2: Control inputs with different values of r and M

are selected as $dt = 0.001$ and $dx = 0.02$ respectively. As expected, the result shown in Fig. 3 demonstrates that the sub-predictor-based T-S fuzzy control law is effective to stabilize the system exponentially. Meanwhile, the observed state variables $\hat{y}^1(x, t - r)$, $\hat{y}^2(x, t - \frac{1}{2}r)$ and the errors $e^1(x, t)$, $e^2(x, t)$ have been shown in Fig. 4-Fig. 7. Theoretical results in previous sections are confirmed by the above simulations.

VI. CONCLUSION

This paper proposes a chain of sub-predictors and a sub-predictor-based fuzzy controller for the nonlinear one-dimensional reaction-diffusion equation. The delayed input and output of the system are compensated. The designed T-S fuzzy control law stabilizes the system successfully under different boundary cases. Especially, our results are also valid

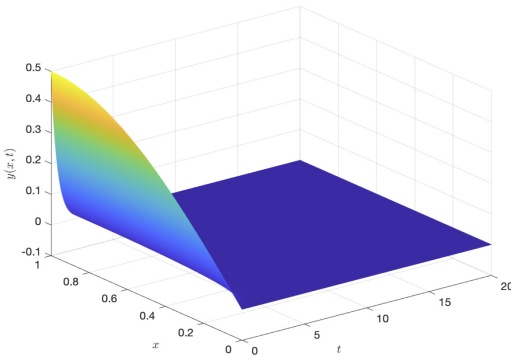
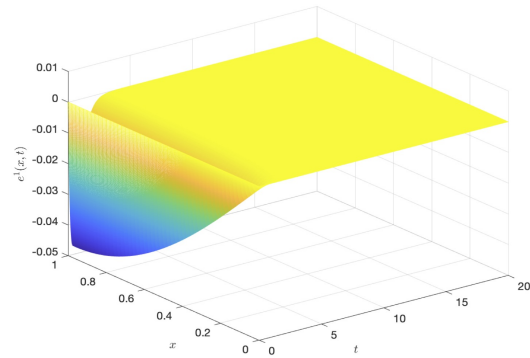
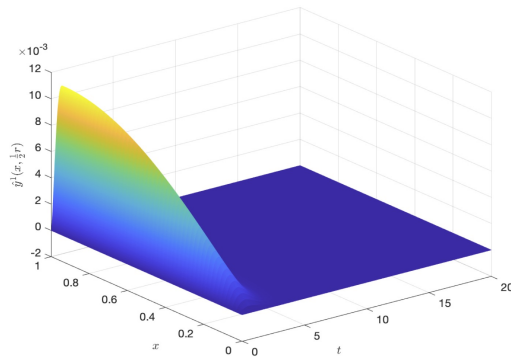
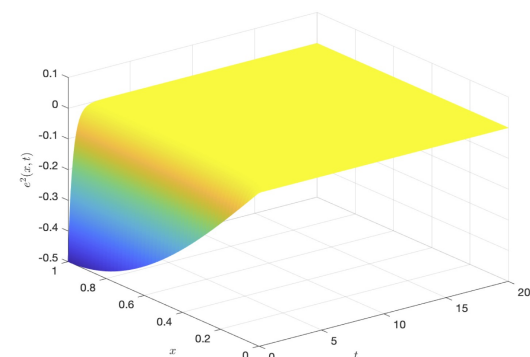
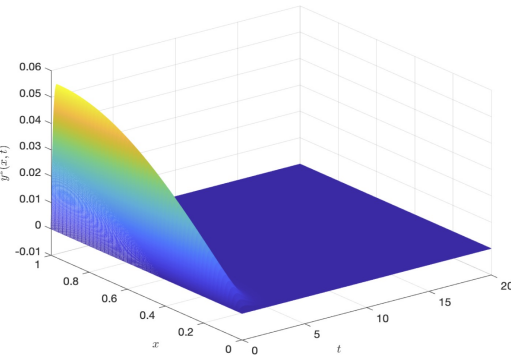


Fig. 3: The state of closed-loop system

Fig. 6: Error $e^1(x, t)$ Fig. 4: Estimation $\hat{y}^1(x, t - r)$ Fig. 7: Error $e^2(x, t)$ Fig. 5: Estimation $\hat{y}^2(x, t - \frac{1}{2}r)$

for a large delay. Based on this, the effectiveness of the presented results is confirmed with an example. Note that in the present work we focus on the study of sub-predictor fuzzy observer-based control design for the case of constant delay. As for the case of time-varying delay, it brings significant challenges, which will become our next step. For further research, one potential direction could be the extension to more complex systems like coupled ODE-PDE systems.

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