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Interval Type-2 Fuzzy Control for HMM-Based Multiagent Systems Via Dynamic Event-Triggered Scheme

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Abstract—**This paper investigates the interval type-2 (IT2) Takagi-Sugeno (T-S) fuzzy asynchronous controller design problem for nonlinear multiagent systems via a dynamic event-triggered scheme in the discrete-time context. Essentially different from most current literature, considering a realistic situation that the system mode and the anticipant controller mode are hardly to maintain synchronization at any time, the problem is characterized by means of hidden Markov model (HMM). The primary attention is focused on devising a feasible dynamic event-triggered strategy with novel threshold parameters to mitigate the communication burden efficiently. On this occasion, the information renewal of the controller is aperiodic. Furthermore, the nonlinear characteristics are effectually disposed through utilizing an unique IT2 T-S fuzzy model, which is with mismatched membership functions (MFs). As a result, the derived closed-loop fuzzy multiagent systems are accompanied by mismatched MFs and asynchronous modes. Whereafter, via solving the convex optimization problem, the desired controller gains are acquired. Eventually, the validity and practicability of the developed control scheme is illustrated by two examples.**

Index Terms—**Multiagent systems, distributed dynamic event-triggered mechanism, interval type-2 Takagi-Sugeno fuzzy control, hidden Markov model.**

I. Introduction

As is well known, multiagent systems (MASs) are composed by multiple agents, which reveal the characteristics of excellent autonomy, strong robustness, and collaboration ability [1]. In the last years, MASs have received great interest from researchers due to their promising applications in swarming, satellite formation flying, sensor networks, and other areas [2]–[4]. It is worth mentioning that MASs can accomplish some complex tasks through cooperative control among agents that cannot be accomplished by a single agent. As such, particular attention from academic communities has paid on the relevant issues of cooperative control of MASs, meantime, outstanding and profound results have been achieved, see also [5]–[8]. However, it is noticed that the above-mentioned literature is achieved under the continuous communication framework, which means that all sampled data are transmitted.

In practice, the ways of broadcasting data packets in works [5]–[8] are sometimes infeasible, because the microprocessor has limited resources. Additionally, considering the capacity of communication channels, releasing dispensable data into the communication network not only waste network resources, but also induce a certain extent of network load pressure [9]–[11], which may give rise to the so-called network congestion. As an effective tool, event-triggered mechanism (ETM) provides a feasible solution in tackling this problem [12], [13]. Under the framework of ETM, an event detector is introduced, and the current sampled data are unable to be transmitted unless the constructed error exceeds certain threshold [14], [15]. Attributed to this mechanism, the communication traffic can be prominently reduced while ensuring the system capability in the mass. Inspired by the pioneering theory, being a hot field of academic communities, the investigation of ETM of MASs is carried out diffusely [16]–[18]. However, for static event-triggered scheme, there undoubtedly exist difficulties in the selection of an appropriate threshold in the predesigned triggering condition. More recently, the authors proposed the ETM comprising dynamic threshold parameter (DTP) to prevent needless continuous monitoring and over-provision of network bandwidth. In this regard, applying this dynamic event-triggered mechanism (DETM) to MASs to alleviate the transmission burden is meaningful, which is the motivation of our current investigation.

One common feature of MASs is that it presents nonlinear characteristics in practical engineering, which challenges the application of theoretical results on MASs [19], [20]. It has been certified that T-S fuzzy model with exact MFs is a powerful tool of handling the intricate nonlinear part of systems [21]–[25]. Relying on T-S fuzzy model, nonlinear MASs can be commendably analyzed through using modified linear control theory. With this superiority, T-S fuzzy theory promotes the developments of MASs to a large extent, and corresponding study results have been achieved. Through constructing a DETM, the asynchronous control problem of MASs based on T-S fuzzy model was discussed in [26]. In [27], for polynomial fuzzy MASs subject to switching topologies, the dissipativity problem was further studied. Under the precondition of multiple dynamic leaders, contraposing to nonlinear MASs, the T-S fuzzy containment tracking control issue was explored by the authors in [28]. Nevertheless, it must be ad-

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mitted that considering the uncertainty of parameters, precisely obtaining the MFs is unrealistic [29]. To accommodate actual situations, the IT2 T-S fuzzy model was proposed, whose MFs are uncertain such that it is appropriate for complex nonlinear systems [30]–[32]. Therefore, extending IT2 T-S fuzzy model to nonlinear MASs deserves investigation.

As the occurrence of external disturbances or component failures in many practical engineering application of MASs, the structure and parameters of MASs may occur unpredictable random change. On this situation, Markov jump systems (MJSs) can be employed to model this dynamics. Despite correlative results about Markov jumping MASs related to ETM have been achieved, most of which are under an implicit assumption (the mode signals are completely available to the controller all the time). However, in actual scenarios, the asynchronization phenomenon usually exists between system modes and controller modes [33], [34]. This phenomenon can be described by hidden Markov model (HMM) [35]. Despite HMM exhibits conspicuous advantages, its application in MASs has not yet received enough attention, thus we are going to fill the gap in the paper.

Motivated by above statements, the IT2 T-S fuzzy control for HMM-based MASs under the framework of dynamic event-triggered is investigated in this paper. The primary contributions of this work are summarized as: In the paper, a novel DETM is designed in terms of discrete-time MASs, the threshold parameter of which is non-monotone and varies with system states. This paper reveals that DETM is more advantageously to deal with the constrained communication bandwidth issue. Due to the introduction of dynamic event-triggered control (DETC), i.e., discontinuous feedback control, the measured output of MASs may different from the signal received by the controller, which leads to the inconsistency of mode and premise variables of the membership functions (MFs). As a result, it creates additional difficulties when we design the fuzzy controller. Therefore, this paper adopts HMM and mismatched IT2 T-S fuzzy model, and this is more consistent with the actual situation. By virtue of HMM, the controller proposed can be extended to investigate the mode-independent, mode-dependent, and asynchronous case.

Graph theory: Graph $\mathbb{G}=\{\mathcal{Q},\mathcal{F},\mathcal{M}\}$ is used to stand the communication topology among agents. The set of n-odes is described by $\mathcal{Q}=\{1,2,\ldots,N\}$. $\mathcal{F}\subseteq\mathcal{Q}\times\mathcal{Q}$ and $\mathcal{M}=[a_{ij}]\in\mathbb{R}^{N\times N}$ signify the set of edges and the weighted adjacency matrix, respectively. $(i,j)\in\mathcal{F}$ means that the data information from node j is capable to be obtained by node i. In consequence, node j is called as the neighbor of node i. If $(i,j)\in\mathcal{F}$, $a_{ij}=1$, otherwise, $a_{ij}=0$. Laplacian matrix L linked to $\mathbb{G}$ is defined by $L=[l_{ij}]_{N\times N}$, with $l_{ii}=\sum\limits_{j=1}^{N}a_{ij}$, and $l_{ij}=-a_{ij}$, for $i\neq j$.

Notations: The basic notations are organized in the following table, and the enumerated notations will be employed next.

Notations	Explanations
$\mathcal{H}e\{\mathcal{J}\}$	$\mathcal{J}+\mathcal{J}^T$
$\mathbb{R}^{n_2}$	The n_2-dimensional Euclidean space
$\mathcal{A}>0\,(\geq 0)$	Matrix $\mathcal{A}$ is positive definite (semi-definite)
$\star$	The symmetric element in a matrix
$\otimes$	The Kronecker product
Pr	Probability of a random event
$\mathbb{E}$	The mathematical expectation operator
diag $\{\ldots\}$	The block-diagonal matrix

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Fuzzy Hidden-Markov Jump Multi-agent Systems

Naturally, we consider the discrete-time fuzzy MASs with N identical agents, and the mathematical model of the ith agent is depicted as

Plant Rule f_i: *IF* $\eta_{i1}(x(k))$ is F_{f_i1} and $\eta_{i2}(x(k))$ is F_{f_i2} … and $\eta_{iq}(x(k))$ is F_{f_iq}, *THEN*

$$x_i(k+1)=\mathcal{A}_{f_i}(s(k))x_i(k)+\mathcal{B}_{f_i}(s(k))u_i(k) \quad (1)$$

where $i=1,2,\ldots,N$. $\eta_i(x(k))=\left[\eta_{i1}(x(k)),\eta_{i2}(x(k)),\ldots,\eta_{iq}(x(k))\right]^T$ denotes the premise variable. $F_{f_ir}\,(a=1,2,\ldots,q)$ is the IT2 fuzzy set for the premise variables $\eta_{ia}(x(k))$. $x_i(k)\in\mathbb{R}^{n_1}$ denotes the system state. $u_i(k)\in\mathbb{R}^{n_2}$ means the control input. $\mathcal{A}_{f_i}(s(k))$ and $\mathcal{B}_{f_i}(s(k))$ represent the known matrices with suitable dimensions. $s(k)$ symbolizes a discrete-time homogeneous Markov chain that takes values in a pregiven set $\mathbb{S}_1\triangleq\{1,2,\cdots,m\}$. The transition probability matrix $\Pi\triangleq\{\pi_{cd}\}$ of $s(k)$ is subject to

$$\pi_{cd}=\Pr\{s(k+1)=d\,|s(k)=c\}\geq 0,\forall c,d\in\mathbb{S},k\in Z^+$$

satisfying $\sum\limits_{d=1}^{m}\pi_{cd}=1,\forall c\in\mathbb{S}_1$. For simplicity, we consider $\mathcal{A}_{f_ic}\triangleq\mathcal{A}_{f_i}(s(k))$, and $\mathcal{B}_{f_ic}\triangleq\mathcal{B}_{f_i}(s(k))$ for $\forall s(k)=c$ in the following expressions. Some interval sets are employed to denote the uncertain MFs, and the interval set of the f_ith rule is represented as the following form

$$O_{f_i}(\eta_i(x(k)))\triangleq[\underline{\rho}_{f_i}(\eta_i(x(k)))\ \ \overline{\rho}_{f_i}(\eta_i(x(k)))]$$

in which

$$\underline{\rho}_{f_i}(\eta_i(x(k))) \triangleq \prod_{\varepsilon=1}^{q}\underline{\mu}_{F_{f_i}\varepsilon}(\eta_\varepsilon(x(k)))$$
$$\overline{\rho}_{f_i}(\eta_i(x(k))) \triangleq \prod_{\varepsilon=1}^{q}\overline{\mu}_{F_{f_i}\varepsilon}(\eta_\varepsilon(x(k)))$$

satisfying

$$0 \leq \underline{\rho}_{f_i}(\eta_i(x(k)))\leq\overline{\rho}_{f_i}(\eta_i(x(k)))\leq 1$$
$$0 \leq \underline{\mu}_{F_{f_i}\varepsilon}(\eta_\varepsilon(x(k)))\leq\overline{\mu}_{F_{f_i}\varepsilon}(\eta_\varepsilon(x(k)))\leq 1$$

where $\bar{\mu}_{F_{f_i}\varepsilon}(\eta_\varepsilon(x(k)))$ is the lower grade of membership (LGM), and $\overline{\mu}_{F_{f_i}\varepsilon}(\eta_\varepsilon(x(k)))$ expresses the upper grade of membership (UGM). $\underline{\rho}_{f_i}(\eta_i(x(k)))$ indicates the lower membership function (LMF), while $\overline{\rho}_{f_i}(\eta_i(x(k)))$ denotes the upper membership function (UMF). Subsequently, the weighted model can be given by

$$x_i(k+1) = \sum_{f_i=1}^{q} \rho_{f_i}(\eta_i(x(k)))[\mathcal{A}_{f_i}(s(k))x_i(k) + \mathcal{B}_{f_i}(s(k))u_i(k)] \quad (2)$$

where

$$\rho_{f_i}(\eta_i(x(k))) \triangleq \frac{\tilde{\rho}_{f_i}(\eta_i(x(k)))}{\sum_{\tilde{f}_i=1}^{r} \tilde{\rho}_{\tilde{f}_i}(\eta_i(x(k)))}$$

$$\tilde{\rho}_{f_i}(\eta_i(x(k))) \triangleq \underline{\varsigma}_{f_i}(\eta_i(x(k)))\underline{\rho}_{f_i}(\eta_i(x(k))) + \overline{\varsigma}_{f_i}(\eta_i(x(k)))\overline{\rho}_{f_i}(\eta_i(x(k)))$$

meeting

$$\sum_{\tilde{f}_i=1}^{q} \rho_{\tilde{f}_i}(\eta_i(x(k))) = 1, 0 \le \rho_{f_i}(\eta_i(x(k))) \le 1$$
$$0 \le \underline{\varsigma}_{f_i}(\eta_i(x(k))) \le 1, 0 \le \overline{\varsigma}_{f_i}(\eta_i(x(k))) \le 1$$
$$\underline{\varsigma}_{f_i}(\eta_i(x(k))) + \overline{\varsigma}_{f_i}(\eta_i(x(k))) = 1$$

in which $\rho_{f_i}(\eta_i(x(k)))$ is considered as the IT2 membership function.

B. Distributed Dynamic Event-Triggered Mechanism

For mitigating communication burden and decreasing update frequencies of the controller, a new DETM is proposed. Sequence $\{k_l^i | l \in Z_{\ge 0}\}$ denotes the set of trigger instants of the ith agent. For $i \in N$, if the current event-triggered instant is k_l^i, then the next triggering instant k_{l+1}^i can be determined aperiodically by the following rule

$$\begin{aligned} k_{l+1}^i = & \inf\{k \in Z_+ | k \ge k_l^i, \text{ and } \\ & [x_i(k_l^i) - x_i(k)]_c^T \Upsilon [x_i(k_l^i) - x_i(k)] \\ \ge & \gamma_i[1 - \upsilon_i \tanh(e_i(k)^T e_i(k) - \theta_i)] \\ & \times [\delta_i^T(k_l^i) \Upsilon_c \delta_i(k_l^i)]\} \end{aligned} \quad (3)$$

where $\delta_i(k_l^i) \triangleq \sum_{j=1}^{N} a_{ij}[(x_i(k_l^i) - x_j(k_{l'}^j)) + \epsilon_i x_i(k_l^i)]$. a_{ij} represents the formation-keeping behavior. γ_i represents threshold scalar of the ith agent. υ_i means the change degree of γ_i. Moreover, ϵ_i is the station-keeping behavior. $\gamma_i[1 - \upsilon_i \tanh(e_i(k)^T e_i(k) - \theta_i)]$ symbols the real DTP of the ith agent. $e_i(k)$ is defined as $e_i(k) \triangleq x_i(k_l^i) - x_i(k)$, which denotes the measurement error. $k_{l'}^j$ describes the last event instant of agent j, with $k_{l'}^j \triangleq \arg\min_{l \in \mathbb{N}, k \ge k_l^j}\{k - k_l^j\}$, $x_j(k_{l'}^j) \triangleq x_j(k) + e_j(k)$.

Remark 1: The communication bandwidth is usually constrained, given the circumstances, data transmission through networks ineluctably encountering congestion. As a result, as one of the optimal choices to make more rational utilization of bandwidth resources, ETM has been attracted extensive attention of numerous researchers. Although the traditional static event-triggered mechanism can decrease the frequency of data transfer, it is quite difficult to ascertain a suitable value of threshold parameter. It should be noted that the threshold parameter needs to be modulated to adapt to the actual situation when the system state changes greatly or tends to be stable. In the paper, a DETM is designed, in which the threshold parameter varies with the state of the considered systems.

The key step of determining the expected DETM is to design an applicable event-triggering condition (ETC). Throughout the paper, let the trigger instant of the ith agent with the DETM condition as

$$\begin{aligned} & [x_i(k_l^i) - x_i(k)]^T \Phi_{s(t)} [x_i(k_l^i) - x_i(k)] \\ \le & \gamma_i[1 - \upsilon_i \tanh(e_i(k)^T e_i(k) - \theta_i)] \\ & \times [\delta_i^T(k_l^i) \Phi_{s(t)} \delta_i(k_l^i)] \end{aligned} \quad (4)$$

Remark 2: In order to better adapt to the actual situation, a novel DETM is employed for discrete time MASs, and a more applicable DTP $\gamma_i[1 - \upsilon_i \tanh(e_i(k)^T e_i(k) - \theta_i)]$ is designed. Obviously, as the basic threshold parameter, γ_i is associated with time-varying term $1 - \upsilon_i \tanh(e_i(k)^T e_i(k) - \theta_i)$. It is quite clear that the DTP $\gamma_i[1 - \upsilon_i \tanh(e_i(k)^T e_i(k) - \theta_i)]$ changes with the measurement error $e_i(k)$, meanwhile, $e_i(k)$ varies with the variation degree of system state. In the light of the tendency of function $\tanh(\bullet)$, it can be concluded that:

Case i: if $e_i(k) > \theta_i$, the state fluctuation of system is regarded to be large, while the value of real threshold parameter is on the contrary, such that more data packages can be transmitted.

Case ii: if $e_i(k) < \theta_i$, the state of system is stable by degrees. Consequently, threshold parameter will become bigger, and less data packets will be transmitted, that means more bandwidth resources are being economized.

C. Fuzzy Event-Triggered Feedback Controller

In this part, we intend to design a fuzzy state feedback controller for system (1).

Control Rule ω_i: *IF* $\eta_{i1}(x(k_l^i))$ is $F_{\omega_i 1}$ and $\eta_{i2}(x(k_l^i))$ is $F_{\omega_i 2}$ ... and $\eta_{iq}(x(k_l^i))$ is $F_{\omega_i q}$, *THEN*

$$u_i(k) = -K_{\omega_i}(\zeta(k))[\sum_{j=1}^{N} a_{ij}(x_i(k_l^i) - x_j(k_{l'}^j)) + \epsilon_i x_i(k_l^i)] \quad (5)$$

where $K_{\omega_i}(\zeta(k))$ are the controller gains, which can be determined later. Homogeneous Markov chain $\zeta(k)$ taking values in the pregiven set $\mathbb{S}_2$, $\mathbb{S}_2 \triangleq \{1, 2, \cdots, t\}$, and its transition probability matrix $\Lambda \triangleq \{\lambda_{c\kappa}\}$ is subject to

$$\lambda_{c\kappa} = \Pr\{\zeta(k+1) = \kappa | \zeta(k) = c\} \ge 0, \forall c, \kappa \in \mathbb{S}_2, k \in Z^+$$

and $\sum_{\kappa=1}^{t} \lambda_{c\kappa} = 1, \forall c \in \mathbb{S}_2$.

Remark 3: Noteworthy, directly obtaining the information of Markov parameter $s(k)$ is virtually impossible in some scenarios. Furthermore, the asynchronous phenomenon may be embodied in controller and system modes because of the consideration of discontinuous feedback control. However, one can observe it through using mode detector or signal $\zeta(k)$ in the paper. Relying on HMM, one obvious feature of (5) is reflected in: it covers different types of controller, including mode-dependent, mode-independent one, as well as asynchronous controller. Moreover, the framework of nonlinear HMM-based MASs under DETC is presented in Fig. 1.

Whereafter, the state feedback controller can be constructed

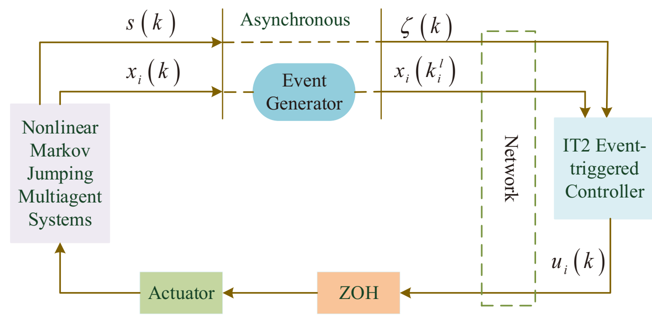

Fig. 1. The framework of HMM-based MASs under DETC.

as

$$u_i(k) = -\sum_{\omega_i=1}^{q} \rho_{\omega_i}(\eta_i(x(k_l^i)))[K_{\omega_i}(\zeta(k))\sum_{j=1}^{N} a_{ij}(x_i(k_l^i) - x_j(k_{l'}^j)) + \epsilon_i x_i(k_l^i)] \quad (6)$$

in which, $\rho_\omega(\eta_i(x(k_l^i)))$ denotes the IT2 grade of membership. In order to simplify the notation, we define $K_{\omega_i\kappa} \triangleq K_{\omega_i}(\zeta(k))$ for each $\zeta(k) = \kappa$.

Remark 4: Entirely distinguish from the current obtained results for MASs concerning with the event-triggered control issue, which only focus on the impact of event-triggered strategy in the process of designing the controller. Whereas, the constructed dynamic event-triggered controller (6) also demonstrates the following features:

1) The IT2 T-S fuzzy theory with better flexibility is integrated, which has imperfectly matched MFs.

2) The mode-based controller is taken into consideration, in which $\zeta(k)$ merely dependent on the current system mode $s(k)$.

In this way, the fuzzy closed-loop system can be described as the following form

$$\begin{aligned} x_i(k+1) &= \sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \rho_{f_i}(\eta_i(x(k)))\rho_{\omega_i}(\eta_i(x(k_l^i)))\{ \\ &\mathbb{A}_{f_ic}x_i(k) - \mathbb{B}_{f_ic}K_{\omega_i\kappa}[\sum_{j=1}^{N} a_{ij}(x_i(k_l^i) \\ &-x_j(k_{l'}^j)) + \epsilon_i x_i(k_l^i)]\} \\ &= \sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \rho_{f_i}(\eta_i(x(k)))\rho_{\omega_i}(\eta_i(x(k_l^i)))\{ \\ &\mathbb{A}_{f_ic}x_i(k) - \mathbb{B}_{f_ic}K_{\omega_i\kappa}[\sum_{j=1}^{N} a_{ij}(x_i(k) + e_i(k) \\ &-(x_j(k) + e_j(k))) + \epsilon_i(x_i(k) + e_i(k))]\} \quad (7) \end{aligned}$$

For brevity, let $\rho_{f_i} \triangleq \rho_{f_i}(\eta_i(x(k)))$, $h_{\omega_i} \triangleq \rho_{\omega_i}(\eta_i(x(k_l^i)))$ in the following. Thus, relying on Kronecker product, ones have

$$\begin{aligned} x(k+1) &= \sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \rho_{f_i}h_{\omega_i}\{\mathbb{A}_{f_ic}x(k) - \\ &(L+M)\otimes(\mathcal{B}_{f_ic}K_{\omega_i\kappa})(x(k)+e(k))\} \quad (8) \end{aligned}$$

where $\mathbb{A}_{f_is} \triangleq I_N \otimes \mathcal{A}_{f_is}$.

For further developing the theoretical results, we give the following useful lemma.

Definition 1 ([34]): The closed-loop system (8) is called to be stochastically stable (SS), if under any initial condition $x(0)$, and $s(0) \in \mathbb{S}_1$, $\zeta(0) \in \mathbb{S}_2$, such that

$$\mathbb{E}\{\sum_{k=0}^{\infty} \|x(k)\| \,|x(0), s(0) \in \mathbb{S}_1, \zeta(0) \in \mathbb{S}_2\} < \infty \quad (9)$$

Lemma 1 ([36]): For any matrices $R_{f_i\omega_is(t)}, E_{f_i\omega_is(t)}, F_{s(t)} > 0$, and $\rho_{f_i}, h_{\omega_i} \in [0,1]$, such that the following inequality instantly holds

$$\begin{aligned} &[\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \alpha_{f_i\omega_i}R_{f_i\omega_is(t)}]^T F_{s(t)}[\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \alpha_{f_i\omega_i}R_{f_i\omega_is(t)}] \\ \le\ & \frac{1}{2}\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \alpha_{f_i\omega_i}[R_{f_i\omega_is(t)}^T F_{s(t)}E_{f_i\omega_is(t)} \\ &+E_{f_i\omega_is(t)}^T F_{s(t)}R_{f_i\omega_is(t)}] \quad (10) \end{aligned}$$

where $\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \alpha_{f_i\omega_i} \triangleq \sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \rho_{f_i}h_{\omega_i}$.

III. Main Results

In this section, the attention is focused on exploring the IT2 fuzzy control based on the DTEM mechanism. For ensuring the stochastic stability of system (8), firstly, sufficient conditions are provided. For convenience and reasonable layout, we denote following notations

$$\begin{aligned} \varphi(k) &\triangleq [x^T(k), e^T(k)]^T \\ \Gamma(t) &\triangleq \mathrm{diag}\{\gamma_1[1-\upsilon_1\tanh(e_1^T(k)e_1(k)-\theta_1)], \ldots, \\ &\quad \gamma_N[1-\upsilon_N\tanh(e_N^T(k)e_N(k)-\theta_N)]\} \end{aligned}$$

A. Stabilization Analysis

In this subsection, in order to guarantee the stochastic stability of system, some stability criteria with less conservativeness are established in Theorem 1.

Theorem 1: For given scalars γ_i, υ_i, system (8) is SS, if there exist matrices $U_c > 0$, $Q_{c\kappa} > 0$, symmetric matrix $P_c > 0$, for $\forall c \in \mathbb{S}_1$, $\forall \kappa \in \mathbb{S}_2$, such that the following conditions hold

$$\begin{bmatrix} -U_c & \sqrt{\lambda_{c1}}U_1 & \sqrt{\lambda_{c2}}U_2 & \ldots & \sqrt{\lambda_{cn}}U_n \\ \star & -Q_{c1} & 0 & \ldots & 0 \\ \star & \star & -Q_{c2} & \ldots & 0 \\ \star & \star & \star & \ddots & 0 \\ \star & \star & \star & \star & -Q_{ct} \end{bmatrix} < 0 \quad (11)$$

$$\begin{bmatrix} \mathbf{U} & \boldsymbol{H}_c\mathbb{F}_{f_i\omega_i}^{(1)} & \boldsymbol{H}_c\mathbb{F}_{f_i\omega_i}^{(2)} \\ \star & \mathbb{F}_{f_i\omega_i}^{(3)} & \mathbb{F}_{f_i\omega_i}^{(4)} \\ \star & \star & \mathbb{F}_{f_i\omega_i}^{(5)} \end{bmatrix} < 0 \quad (12)$$

where

$$\begin{aligned} \mathbf{U} &= \mathrm{diag}\{-U_1, -U_2, \ldots, -U_m\} \\ \boldsymbol{H}_c &= \mathrm{diag}\{\sqrt{\pi_{c1}}I, \sqrt{\pi_{c2}}I, \ldots, \sqrt{\pi_{cm}}I\} \\ \mathbb{F}_{f_i\omega_i}^{(1)} &= \sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q} \alpha_{f_i\omega_i}[\mathbb{A}_{f_ic} - (L+M)\otimes(\mathcal{B}_{f_ic}K_{\omega_i\kappa})] \end{aligned}$$

$$\begin{aligned}\mathbb{F}^{(2)}_{f_i\omega_i} &= \sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\alpha_{f_i\omega_i}[-(L+M)\otimes(\mathcal{B}_{f_ic}K_{\omega_i\kappa})]\\ \mathbb{F}^{(3)}_{f_i\omega_i} &= (L+M)^T\Gamma(t)(L+M)\otimes\Upsilon_c - Q^{-1}_{c\kappa}\\ \mathbb{F}^{(4)}_{f_i\omega_i} &= (L+M)^T\Gamma(t)(L+M)\otimes\Upsilon_c\\ \mathbb{F}^{(5)}_{f_i\omega_i} &= (L+M)^T\Gamma(t)(L+M)\otimes\Upsilon_c - I_N\otimes\Upsilon_c\end{aligned}$$

Proof: Choosing the candidate Lyapunov function for closed-loop system (9) as follows

$$V(k,x(k),s(k)=c) = x^T(k)P_cx(k) \quad (13)$$

Subsequently, the following target inequality can be deduced by Lemma 1

$$\begin{aligned}&\mathbb{E}\{\Delta V(k)\}\\ =\ &\mathbb{E}\{V(k+1,x(k+1),s(k+1)=d|\,s(k)=c)\}\\ =\ &x^T(k+1)\mathcal{P}_cx(k+1) - x^T(k)P_cx(k)\\ =\ &[\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\alpha_{f_i\omega_i}\mathbb{A}_{f_ic}x(k)]^T\mathcal{P}_c[\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\alpha_{f_i\omega_i}\mathbb{A}_{f_ic}x(k)]\\ &+[\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\alpha_{f_i\omega_i}\mathbb{H}(x(k)+e(k))]^T\mathcal{P}_c\times\\ &[\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\alpha_{f_i\omega_i}\mathbb{H}(x(k)+e(k))] - x^T(k)P_cx(k)\\ \le\ &\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\alpha_{f_i\omega_i}(\mathbb{A}_{f_ic}x(k))^T\mathcal{P}_c(\mathbb{A}_{f_ic}x(k))\\ &+\sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\alpha_{f_i\omega_i}(\mathbb{H}(x(k)+e(k)))^T\mathcal{P}_c\times\\ &(\mathbb{H}(x(k)+e(k))) - x^T(k)P_cx(k) \quad (14)\end{aligned}$$

where $\mathbb{H}\triangleq-(L+M)\otimes(\mathcal{B}_{f_ic}K_{\omega_i\kappa})$, $\mathcal{P}_c\triangleq\sum_{d=1}^{m}\pi_{cd}P_d$, $\sum\limits_{f_i=1}^{q}\sum\limits_{\omega_i=1}^{q}\alpha_{f_i\omega_i}\triangleq\sum\limits_{f_i=1}^{q}\sum\limits_{\omega_i=1}^{q}\rho_{f_i}h_{\omega_i}$. For brevity, $\rho_{f_i}(x(k))$ and $h_{\omega_i}(x_i(k_l^i))$ is represented by ρ_{f_i} and h_{ω_i}.

In addition, the event-triggered condition straightforwardly insures

$$\begin{aligned}&e^T(k)\left(I_N\otimes\Phi_{s(t)}\right)e(k)\\ \le\ &[x(k)+e(k)]^T[(L+M)^T\Gamma(t)(L+M)\otimes\Upsilon_c]\\ &\times[x(k)+e(k)] \quad (15)\end{aligned}$$

Then, by defining $P_c\triangleq U_c^{-1}$, it can be elicited from (11) that $P_c > Q^{-1}_{c\kappa}$. Combining (15) with Schur complement, one can further deduce from (12) that

$$\mathbb{E}\{\Delta V(k)\} = \sum_{f_i=1}^{q}\sum_{\omega_i=1}^{q}\varphi^T(k)(\sum_{\kappa=1}^{t}\lambda_{c\kappa}\bar{\Omega}_{c\kappa})\varphi(k) < 0 \quad (16)$$

where

$$\bar{\Omega}_{c\kappa} = \begin{bmatrix} J_{c\kappa} & \mathbb{H}^T\mathbf{U}_c\mathbb{H}\\ \star & \mathbb{H}^T\mathbf{U}_c\mathbb{H}\end{bmatrix}$$

in which

$$\begin{aligned}J_{c\kappa} &\triangleq \mathbb{A}^T_{f_is}\mathbf{U}_c\mathcal{A}_{f_is}+\mathbb{H}^T\mathbf{U}_c\mathbb{H}-Q^{-1}_{c\kappa}\\ \mathbf{U}_c &\triangleq \sum_{d=1}^{m}\pi_{cd}U_c^{-1}\end{aligned}$$

Assuming that ϑ is the smallest eigenvalue of $-\sum_{\kappa=1}^{t}\lambda_{c\kappa}\bar{\Omega}_{c\kappa}$, obviously, it can be achieved that

$$\begin{aligned}\mathbb{E}\{V(\infty)-V(0)\} &= \mathbb{E}\{\sum_{k=0}^{\infty}V(k)\}\\ &\le \mathbb{E}\{\sum_{k=0}^{\infty}(-\vartheta x^T(k)x(k))\} \quad (17)\end{aligned}$$

Therefore

$$\begin{aligned}\mathbb{E}\{\sum_{k=0}^{\infty}x^T(k)x(k)\} &\le \frac{1}{\vartheta}\{\mathbb{E}\{V(0)\}-\mathbb{E}\{V(\infty)\}\}\\ &\le \frac{1}{\vartheta}\mathbb{E}\{V(0)\}\\ &\le 0 \quad (18)\end{aligned}$$

Hence, one can conclude from Definition 1 that the IT2 fuzzy MAS (8) is SS. This proof is completed. ■

B. Fuzzy Event-Triggered Controller Design

Theorem 2: For given scalars γ_i, υ_i, $\mu_1>0$, $\mu_2>0$, system (8) is SS, if there exist matrices $U_c>0$, $Q_{c\kappa}>0$ for $\forall c\in\mathbb{S}_1$, $\forall\kappa\in\mathbb{S}_2$, matrix $\mathbb{Y}$, symmetric matrix $P_c>0$, condition (11) and the following conditions hold for f_i, $\omega_i, r_i = 1,2,\ldots,q$, $f_i<\omega_i$

$$\Xi_{r_ir_i} < 0 \quad (19)$$

$$\Xi_{f_i\omega_i}+\mu_1\Xi_{\omega_if_i} < 0 \quad (20)$$

$$\Xi_{f_i\omega_i}+\mu_2\Xi_{\omega_if_i} < 0 \quad (21)$$

where

$$\begin{aligned}\Xi_{r_ir_i} &= \begin{bmatrix}\mathbf{U} & \boldsymbol{H}_c\bar{\mathbb{F}}^{(1)}_{r_ir_i} & \boldsymbol{H}_c\bar{\mathbb{F}}^{(2)}_{r_ir_i}\\ \star & \mathbb{F}^{(3)}_{f_i\omega_i} & \mathbb{F}^{(4)}_{f_i\omega_i}\\ \star & \star & \mathbb{F}^{(5)}_{f_i\omega_i}\end{bmatrix}\\ \Xi_{f_i\omega_i} &= \begin{bmatrix}\mathbf{U} & \boldsymbol{H}_c\bar{\mathbb{F}}^{(1)}_{f_i\omega_i} & \boldsymbol{H}_c\bar{\mathbb{F}}^{(2)}_{f_i\omega_i}\\ \star & \mathbb{F}^{(3)}_{f_i\omega_i} & \mathbb{F}^{(4)}_{f_i\omega_i}\\ \star & \star & \mathbb{F}^{(5)}_{f_i\omega_i}\end{bmatrix}\\ \Xi_{\omega_if_i} &= \begin{bmatrix}\mathbf{U} & \boldsymbol{H}_c\bar{\mathbb{F}}^{(1)}_{\omega_if_i} & \boldsymbol{H}_c\bar{\mathbb{F}}^{(2)}_{\omega_if_i}\\ \star & \mathbb{F}^{(3)}_{f_i\omega_i} & \mathbb{F}^{(4)}_{f_i\omega_i}\\ \star & \star & \mathbb{F}^{(5)}_{f_i\omega_i}\end{bmatrix}\end{aligned}$$

in which

$$\begin{aligned}\bar{\mathbb{F}}^{(1)}_{r_ir_i} &= \mathbb{A}_{r_ic}-(L+M)\otimes(\mathcal{B}_{r_ic}K_{r_i\kappa})\\ \bar{\mathbb{F}}^{(2)}_{r_ir_i} &= -(L+M)\otimes(\mathcal{B}_{r_ic}K_{r_i\kappa})\\ \bar{\mathbb{F}}^{(1)}_{f_i\omega_i} &= \mathbb{F}^{(1)}_{f_i\omega_i}, \bar{\mathbb{F}}^{(2)}_{f_i\omega_i} = \mathbb{F}^{(2)}_{f_i\omega_i}\\ \bar{\mathbb{F}}^{(1)}_{\omega_if_i} &= \mathbb{A}_{\omega_ic}-(L+M)\otimes(\mathcal{B}_{\omega_ic}K_{f_i\kappa})\\ \bar{\mathbb{F}}^{(2)}_{\omega_if_i} &= -(L+M)\otimes(\mathcal{B}_{\omega_ic}K_{f_i\kappa})\end{aligned}$$

Proof: Firstly, give the asynchronous constraints as follows

$$\begin{aligned}\left|\rho_{\omega_i}-h_{\omega_i}\right| &\le \chi_{\omega_i}, \forall\omega_i=1,\ldots,q\\ h_{\omega_i} &= \sigma_{\omega_i}\rho_{\omega_i} \quad (22)\end{aligned}$$

Additionally, by letting the lower and upper bounds of σ_{ω_i} as $\varpi_l^{\omega_i}$ and $\varpi_{\tilde{H}}^{\omega_i}$, respectively, it can be inferred that

$$\varpi_l^{\omega_i} = 1-\frac{\chi_{\omega_i}}{\rho_{f_i}} \le \sigma_{\omega_i} \le 1+\frac{\chi_{\omega_i}}{\rho_{f_i}} = \varpi_{\tilde{H}}^{\omega_i} \quad (23)$$

Subsequently, define

$$\xi_1 = \frac{\min\{\varpi_l^{\omega_i}\}}{\max\{\varpi_{\tilde{H}}^{\omega_i}\}}, \ \xi_2 = \frac{\max\{\varpi_l^{\omega_i}\}}{\min\{\varpi_{\tilde{H}}^{\omega_i}\}}$$

With that, one has

$$\xi_1 \leq \frac{\sigma_{f_i}}{\sigma_{\omega_i}} \leq \xi_2 \tag{24}$$

Then, it can be elicited from (24) that

$$\begin{aligned} &\sum_{f_i=1}^{q-1} \sum_{\omega_i > f_i}^{q} \sigma_{\omega_i}\rho_{f_i}\rho_{\omega_i}[\Xi_{f_i\omega_i} + \frac{\sigma_{f_i}}{\sigma_{\omega_i}}\Xi_{\omega_i f_i}] \\ + &\sum_{f_i=1}^{q} \sigma_{f_i}\rho_{f_i}\rho_{f_i}\Xi_{f_i f_i} \\ < \ & 0 \end{aligned} \tag{25}$$

which implies

$$\sum_{f_i=1}^{q} \sum_{\omega_i=1}^{q} \sigma_{\omega_i}\rho_{f_i}\rho_{\omega_i}[\Xi_{f_i\omega_i}] < 0 \tag{26}$$

■

Theorem 3: For given scalars γ_i, υ_i, $\mu_1 > 0$, $\mu_2 > 0$, system (8) is SS, if there exist matrices $U_c > 0$, $Q_{c\kappa} > 0$, matrix $\mathcal{Y}_\kappa$, symmetric matrix $P_c > 0$, for $\forall c \in \mathbb{S}_1$, $\forall \kappa \in \mathbb{S}_2$, condition (11) and the following inequalities hold for f_i, $\omega_i, r_i = 1, 2, \ldots, q$, $f_i < \omega_i$

$$\Theta_{r_i r_i} < 0 \tag{27}$$

$$\Theta_{f_i\omega_i} + \mu_1\Theta_{\omega_i f_i} < 0 \tag{28}$$

$$\Theta_{f_i\omega_i} + \mu_2\Theta_{\omega_i f_i} < 0 \tag{29}$$

where

$$\Theta_{r_i r_i} = \begin{bmatrix} \mathbf{U} & \boldsymbol{H}_c\tilde{\mathbb{F}}^{(1)}_{r_i r_i} & \boldsymbol{H}_c\tilde{\mathbb{F}}^{(2)}_{r_i r_i} \\ \star & \tilde{\mathbb{F}}^{(3)}_{r_i r_i} & \tilde{\mathbb{F}}^{(4)}_{r_i r_i} \\ \star & \star & \tilde{\mathbb{F}}^{(5)}_{r_i r_i} \end{bmatrix}$$

$$\Theta_{f_i\omega_i} = \begin{bmatrix} \mathbf{U} & \boldsymbol{H}_c\tilde{\mathbb{F}}^{(1)}_{f_i\omega_i} & \boldsymbol{H}_c\tilde{\mathbb{F}}^{(2)}_{f_i\omega_i} \\ \star & \tilde{\mathbb{F}}^{(3)}_{f_i\omega_i} & \tilde{\mathbb{F}}^{(4)}_{f_i\omega_i} \\ \star & \star & \tilde{\mathbb{F}}^{(5)}_{f_i\omega_i} \end{bmatrix}$$

$$\Theta_{\omega_i f_i} = \begin{bmatrix} \mathbf{U} & \boldsymbol{H}_c\tilde{\mathbb{F}}^{(1)}_{\omega_i f_i} & \boldsymbol{H}_c\tilde{\mathbb{F}}^{(2)}_{\omega_i f_i} \\ \star & \tilde{\mathbb{F}}^{(3)}_{\omega_i f_i} & \tilde{\mathbb{F}}^{(4)}_{\omega_i f_i} \\ \star & \star & \tilde{\mathbb{F}}^{(5)}_{\omega_i f_i} \end{bmatrix}$$

in which

$$\begin{aligned} \tilde{\mathbb{F}}^{(1)}_{r_i r_i} &= \mathbb{A}_{r_i c}\mathcal{Y}_\kappa - (L+M) \otimes (\mathcal{B}_{r_i c}\bar{K}_{r_i\kappa}) \\ \tilde{\mathbb{F}}^{(2)}_{r_i r_i} &= -(L+M) \otimes (\mathcal{B}_{r_i c}\bar{K}_{r_i\kappa}) \\ \tilde{\mathbb{F}}^{(3)}_{r_i r_i} &= (L+M)^T \Gamma(t)(L+M) \otimes \Phi_c + \mathcal{J}Q_{c\kappa}^{-1}\mathcal{J}^T \\ &\quad -\mathcal{H}e\{\mathcal{J}\mathcal{Y}_\kappa^T\} \\ \tilde{\mathbb{F}}^{(4)}_{r_i r_i} &= (L+M)^T \Gamma(t)(L+M) \otimes \Phi_c \\ \tilde{\mathbb{F}}^{(5)}_{r_i r_i} &= (L+M)^T \Gamma(t)(L+M) \otimes \Phi_c - \Phi_c \\ \tilde{\mathbb{F}}^{(1)}_{f_i\omega_i} &= \mathbb{A}_{f_i c}\mathcal{Y}_\kappa - (L+M) \otimes (\mathcal{B}_{f_i c}\bar{K}_{\omega_i\kappa}) \\ \tilde{\mathbb{F}}^{(2)}_{f_i\omega_i} &= -(L+M) \otimes (\mathcal{B}_{f_i c}\bar{K}_{\omega_i\kappa}) \\ \tilde{\mathbb{F}}^{(1)}_{\omega_i f_i} &= \mathbb{A}_{\omega_i c}\mathcal{Y}_\kappa - (L+M) \otimes (\mathcal{B}_{\omega_i c}\bar{K}_{f_i\kappa}) \\ \tilde{\mathbb{F}}^{(2)}_{r_i r_i} &= -(L+M) \otimes (\mathcal{B}_{\omega_i c}\bar{K}_{f_i\kappa}) \\ \tilde{\mathbb{F}}^{(3)}_{f_i\omega_i} &= \tilde{\mathbb{F}}^{(3)}_{\omega_i f_i} = \tilde{\mathbb{F}}^{(3)}_{r_i r_i} \\ \tilde{\mathbb{F}}^{(4)}_{f_i\omega_i} &= \tilde{\mathbb{F}}^{(4)}_{\omega_i f_i} = \tilde{\mathbb{F}}^{(4)}_{r_i r_i} \\ \tilde{\mathbb{F}}^{(5)}_{f_i\omega_i} &= \tilde{\mathbb{F}}^{(5)}_{\omega_i f_i} = \tilde{\mathbb{F}}^{(5)}_{r_i r_i} \end{aligned}$$

Moreover, the controller gain matrices are given by

$$K_{r_i\kappa} \triangleq \bar{K}_{r_i\kappa}Y_\kappa^{-1} \tag{30}$$

Proof: Defining matrix $\mathcal{Y}_\kappa \triangleq I_N \otimes Y_\kappa^{-1}$, imple congruent transformation (with regard to pre- and post-multiplying (27)−(29) by $\text{diag}\{\underbrace{I,\ldots I}_{2m}, \mathcal{Y}_\kappa^{-1}, \mathcal{Y}_\kappa^{-1}\}$ and its transposition.) Then, together with inequality

$$-\mathcal{Y}_\kappa Q_{c\kappa}^{-1}\mathcal{Y}_\kappa^T \leq \mathcal{J}Q_{c\kappa}^{-1}\mathcal{J}^T - \mathcal{H}e\{\mathcal{J}\mathcal{Y}_\kappa^T\}$$

(27)−(29) insures that (19)−(21) hold immediately. The proof is completed. ■

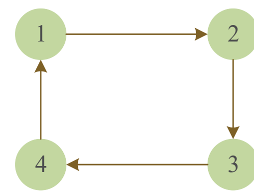

Fig. 2. One possible communication topology.

IV. Simulation

In this part, for IT2 fuzzy MASs, the validity and merit of the developed asynchronous event-triggered control strategy are demonstrated by two examples.

Example 1: The switching topology among agents $(N = 4)$ of the IT2 fuzzy multiagent systems with two modes is assumed in line with Fig.2. Accordingly, the Laplacian matrix is described as

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

In addition, the station keeping weighting constants is considered to be $b_i = 1$, for $i \in \{1, 2, 3, 4\}$.

The parameters of system (8) are given by

$$\begin{aligned} Mode\ 1: \ \mathcal{A}_{11} &= \begin{bmatrix} 1.1 & 0.01 \\ 0 & 1.03 \end{bmatrix}, \ \mathcal{B}_{11} = \text{diag}\{1, 1.1\} \\ \mathcal{A}_{12} &= \begin{bmatrix} 1.1 & 0.02 \\ 0.1 & 1.1 \end{bmatrix}, \ \mathcal{B}_{12} = \text{diag}\{1, 1\} \\ Mode\ 2: \ \mathcal{A}_{21} &= \begin{bmatrix} 1.05 & 0.2 \\ 0 & 0.8 \end{bmatrix}, \ \mathcal{B}_{21} = \text{diag}\{1, 0.9\} \\ \mathcal{A}_{22} &= \begin{bmatrix} 1.05 & 0.1 \\ 0.01 & 0.8 \end{bmatrix}, \ \mathcal{B}_{22} = \text{diag}\{1, 1\} \end{aligned}$$

and in the asynchronous case, transition probability matrices Π and Λ are selected as

$$\Pi = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix}, \ \Lambda = \begin{bmatrix} 0.65 & 0.35 \\ 0.15 & 0.85 \end{bmatrix}$$

Moreover, the LGM and UGM are chosen as

$$\begin{aligned} \underline{\rho}(\eta_1(x(k))) &\triangleq 1 - \frac{1}{9}x_i^2, \ \bar{\rho}(\eta_1(x(k))) \triangleq 1 - \frac{1}{27}x_i^2 \\ \underline{\rho}(\eta_2(x(k))) &\triangleq \frac{1}{27}x_i^2, \ \bar{\rho}(\eta_2(x(k))) \triangleq \frac{1}{9}x_i^2 \end{aligned}$$

and nonlinear weighting functions about the IT2 T-S fuzzy set are taken as

$$\underline{\varsigma}_{f_i}(\eta_1(x(k))) \triangleq \sin^2(x_i(k))$$
$$\overline{\varsigma}_{f_i}(\eta_1(x(k))) \triangleq 1-\underline{\varsigma}_{f_i}(\eta_1(x(k)))$$

Consider the saturation levels being $\mu_1=0.8$, $\mu_2=1.25$. Then, we set $\gamma_i=0.005$, $\upsilon_i=8$, $\epsilon_i=1$ $(i=1,2,3,4)$ in dynamic event-triggered condition (4). Furthermore, solving inequalities (11) and (27)–(29) in Theorem 3, the desired controller gains $K_{r_i\kappa}$ are computed as

$$K_{11} = \begin{bmatrix} 0.1727 & 0.0058 \\ 0.0064 & 0.1467 \end{bmatrix}$$
$$K_{12} = \begin{bmatrix} 0.1712 & 0.0064 \\ 0.0113 & 0.1541 \end{bmatrix}$$
$$K_{21} = \begin{bmatrix} 0.1597 & 0.0334 \\ 0.0056 & 0.1185 \end{bmatrix}$$
$$K_{22} = \begin{bmatrix} 0.1679 & 0.0261 \\ 0.0039 & 0.1183 \end{bmatrix}$$

Under the initial conditions of the isolated agents, $x_1=[1;0.8], x_2=[-1;1], x_3=[0.5;0.6], x_4=[-0.5;1.2]$, the state responses of the open-loop multiagent systems under DETM is depicted in Fig. 3. Based on the calculated $K_{r_i\kappa}$, the state responses of the closed-loop multiagent systems are presented in Fig. 4, from which one can obtained a fact that the controller design method is feasible. Moreover, the event-triggered instants of each agent are given in Fig. 5. From the simulation results, the triggering rates (TRs) of four agents are listed in Table I.

$$\text{TRs of agent } i = \frac{\text{The number of the transmitted data packages of } i\text{th agent}}{\text{The total number of sampled data packages of } i\text{th agent}}$$

TABLE I
TRS OF FOUR AGENTS UNDER DETC IN EXAMPLE 1

Agent	1	2	3	4
TRs	24%	26%	25%	28%

Example 2: (Mass-spring-damper system model) In this example, we consider a modified mass–spring–damper system borrowed from [37], which can be characterized by the following equation

$$\mathcal{M}_{ii}\ddot{y}_i + F_f + F_r = u_i(t), i=1,2,3,4 \quad (31)$$

where notations $\mathcal{M}_i$ and y_i symbol mass and the displacement of agent i, respectively. The spring friction force is defined as $F_f=\tau_i\dot{y}_i$ with $\tau_i>0$, and the spring restoring force $F_r=\tilde{k}_i(1+\nu^2y_i^2)y_i$. $u_i(t)$ stands the force input.

Thus, (31) can be described as

$$\mathcal{M}_i\ddot{y}_i+\tau_i\dot{y}_i(t)+\tilde{k}_ix_i+\tilde{k}_i\nu^2x_i^3=u_i(t), i=1,2,3,4.$$

Donate $x_i(t)=\begin{bmatrix} y_i^T(t) & \dot{y}_i^T(t)\end{bmatrix}^T$, $\psi(t)=\frac{-\tilde{k}_i-\tilde{k}_i\nu^2x_i^3(t)}{\mathcal{M}_i}$, $x(t)\in[-2,2]$, $\mathcal{M}_i=1$ kg, $\tilde{k}_i\in[5,8]$ N/m, $i\in\{1,2,3,4\}$. $\nu=0.3$ m^{-1}. Then, one has $\psi_{\max}=-5$ with $\tilde{k}_i=5$, $x(t)=0$. $\psi_{\min}=-10.88$ with $\tilde{k}_i=8$, $x(t)=\pm2$.

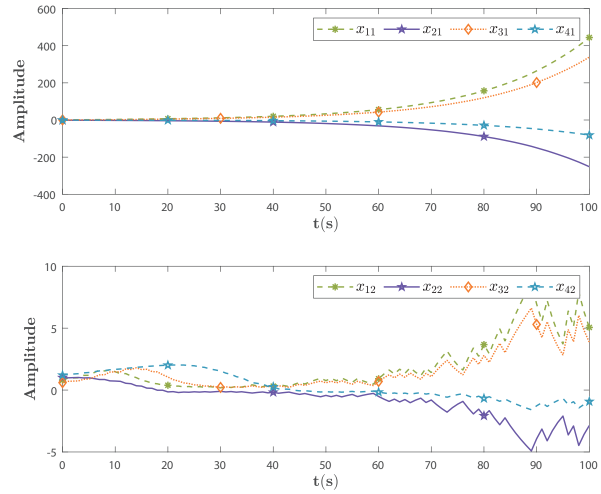

Fig. 3. State responses of the open-loop systems in Example 1.

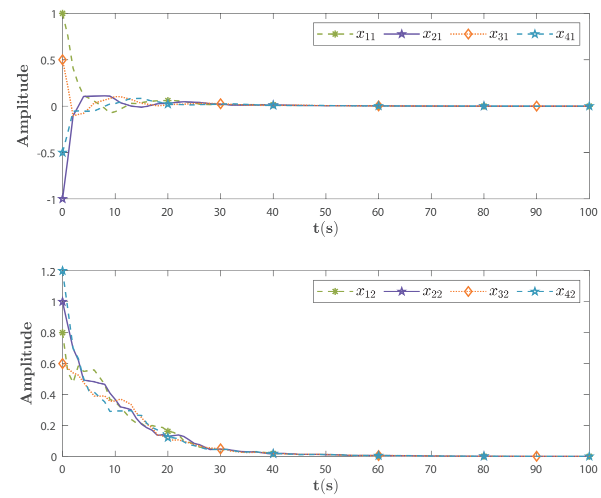

Fig. 4. State responses of the closed-loop systems in Example 1.

Similar to [38], we obtain the following LGM and UGM

$$\underline{\rho}(\eta_1(x(k))) \triangleq \frac{-\psi+\psi_{\max}}{\psi_{\max}-\psi_{\min}}, \tilde{k}_i=5$$
$$\overline{\rho}(\eta_1(x(k))) \triangleq \frac{-\psi+\psi_{\max}}{\psi_{\max}-\psi_{\min}}, \tilde{k}_i=8$$
$$\underline{\rho}(\eta_2(x(k))) \triangleq \frac{\psi-\psi_{\max}}{\psi_{\max}-\psi_{\min}}, \tilde{k}_i=8$$
$$\overline{\rho}(\eta_2(x(k))) \triangleq \frac{\psi-\psi_{\max}}{\psi_{\max}-\psi_{\min}}, \tilde{k}_i=5$$

and the following IT2 T-S fuzzy model

$$\dot{x}_i(t) = \sum_{f_i=1}^{q} g_{f_i}(\eta_i(x(t)))[\mathcal{A}_{f_i}(s(t))x_i(t) + \mathcal{B}_{f_i}(s(t))u_i(t)] \quad (32)$$

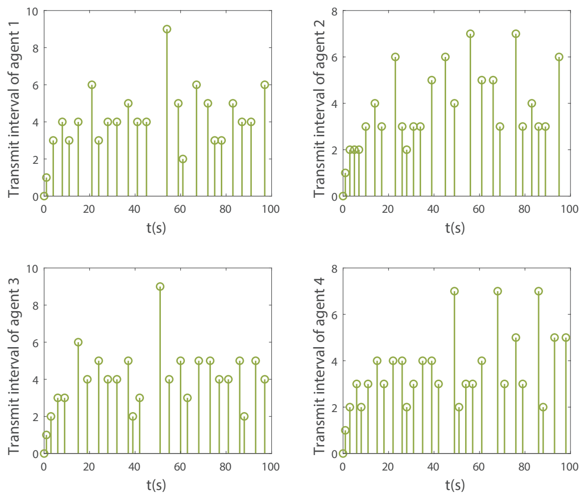

Fig. 5. The triggering intervals in Example 1.

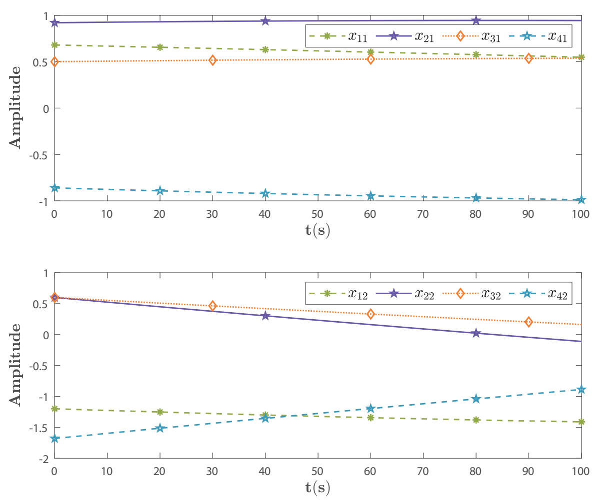

Fig. 6. State responses of the open-loop systems in Example 2.

where

$$\begin{aligned}\mathcal{A}_{1c} &= \begin{bmatrix} 0 & 1 \\ \psi_{\min} & -\frac{\tau_i}{\mathcal{M}_i} \end{bmatrix}, \ \mathcal{B}_{1c} = \begin{bmatrix} 0 \\ \frac{1}{\mathcal{M}_i} \end{bmatrix} \\ \mathcal{A}_{2c} &= \begin{bmatrix} 0 & 1 \\ \psi_{\max} & -\frac{\tau_i}{\mathcal{M}_i} \end{bmatrix}, \ \mathcal{B}_{2c} = \begin{bmatrix} 0 \\ \frac{1}{\mathcal{M}_i} \end{bmatrix}\end{aligned}$$

selecting $\tau_i = 2$ N·m/s for mode 1 and $\tau_i = 1.5$ N·m/s for mode 2.

Besides, nonlinear weighting functions are chosen as

$$\begin{aligned}\underline{\varsigma}_{f_i}(\eta_1(x(k))) &\triangleq 0.6\sin^2(x_i(k)) \\ \overline{\varsigma}_{f_i}(\eta_1(x(k))) &\triangleq 1 - \underline{\varsigma}_{f_i}(\eta_1(x(k)))\end{aligned}$$

The system parameters of discrete-time system model in the form of equation (32) can be acquired as

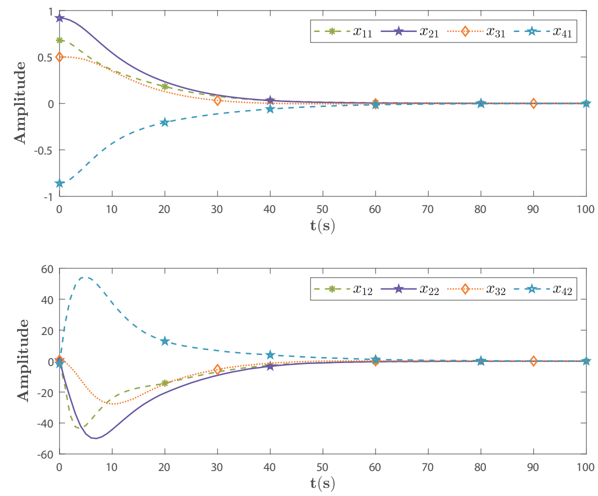

Fig. 7. State responses of the closed-loop systems in Example 2.

$$\begin{aligned}Mode\ 1: \ \mathcal{A}_{11} &= \begin{bmatrix} 1.0000 & 0.0010 \\ -0.0109 & 0.9980 \end{bmatrix} \\ \mathcal{A}_{12} &= \begin{bmatrix} 1.0000 & 0.0010 \\ -0.0109 & 0.9985 \end{bmatrix} \\ \mathcal{B}_{11} &= \mathcal{B}_{12} = 10^{-3} * \begin{bmatrix} 0.0005 \\ 0.9992 \end{bmatrix} \\ Mode\ 2: \ \mathcal{A}_{21} &= \begin{bmatrix} 1.0000 & 0.0010 \\ -0.0050 & 0.9980 \end{bmatrix} \\ \mathcal{A}_{22} &= \begin{bmatrix} 1.0000 & 0.0010 \\ -0.0050 & 0.9985 \end{bmatrix} \\ \mathcal{B}_{21} &= \mathcal{B}_{22} = 10^{-3} * \begin{bmatrix} 0.0005 \\ 0.9992 \end{bmatrix}\end{aligned}$$

By solving (11) and (27)–(29), one obtains the corresponding controller gains as

$$\begin{aligned}K_{11} &= 10^4 * \begin{bmatrix} 1.0325 & 0.0159 \end{bmatrix} \\ K_{12} &= 10^4 * \begin{bmatrix} 1.0670 & 0.0164 \end{bmatrix} \\ K_{21} &= 10^4 * \begin{bmatrix} 1.0858 & 0.0167 \end{bmatrix} \\ K_{22} &= 10^4 * \begin{bmatrix} 1.1065 & 0.0170 \end{bmatrix}\end{aligned}$$

After given the initial conditions of four agents, $x_1 = [0.68; -1.2], x_2 = [0.92; 0.6], x_3 = [0.5; 0.6], x_4 = [-0.86; 1.68]$, we can acquire that the state responses of the open-loop MASs under DETM as shown in Fig. 6. The state responses of the closed-loop MASs and event-triggered instants are drawn in Figs. 7 and 8, respectively. These figures demonstrate that the proposed DETM is beneficial for reducing needless communications. Analogously, the TRs of four agents are exhibited in Table II.

V. Conclusion

In this paper, the IT2 fuzzy asynchronous controller design problem has been explored for discrete-time Markov jumping MASs with constrained communication bandwidth. Under

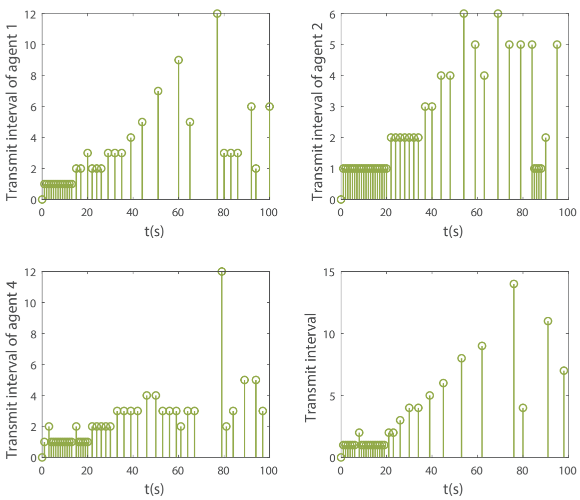

Fig. 8. The triggering intervals in Example 2.

TABLE II
TRS OF FOUR AGENTS UNDER DETC IN EXAMPLE 2

Agent	1	2	3	4
TRs	35%	43%	40%	34%

the opened network environments, the distributed dynamic event-triggered mechanism depended on the dynamic threshold parameter has been adopted to ulteriorly improve the communication efficiency. The IT2 T-S fuzzy model with mismatched MFs has been synthesized to depict the nonlinear characteristics of MASs. With the aid of introduced HMM, the phenomenon of incomplete accessibility of system modes in regard to the designed event-triggered controller modes has been fully considered. Besides, by virtue of stochastic analysis methods and other theory knowledge, the stochastically stable criteria and event-triggered controller design method have been deduced. Ultimately, two illustrative examples have been provided and the corresponding simulation results which can manifest the superiority of the designed controller have been given.

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