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Non-Lorentzian avatars of (1,0) theories

N. Lambert and T. Orchard

*Department of Mathematics, King's College London,
London, WC2R 2LS, U.K.*

E-mail: neil.lambert@kcl.ac.uk, tristan.orchard@kcl.ac.uk

ABSTRACT: We construct five-dimensional non-Lorentzian Lagrangian gauge field theories with an $SU(1,3)$ conformal symmetry and 12 (conformal) supersymmetries. Such theories are interesting in their own right but can arise from six-dimensional (1,0) superconformal field theories on a conformally compactified Minkowski spacetime. In the limit that the conformal compactification is removed the Lagrangians we find give field theory formulations of DLCQ constructions of six-dimensional (1,0) conformal field theories.

KEYWORDS: Conformal Field Theory, Extended Supersymmetry, M-Theory

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1 Introduction

Starting with [1], there is now known to be a wide variety of six-dimensional $(1, 0)$ superconformal field theories, for example see [2, 3]. As with their $(2, 0)$ cousins [4], these do not readily possess Lagrangian descriptions, although they typically reduce to five-dimensional super-Yang-Mills theories upon compactification on S^1 . In fact, a class of six-dimensional Lagrangians with $(1, 0)$ supersymmetry has been constructed in [5] (see also [6] for an on-shell construction).¹ However, we generically see that Lagrangian field theories of Lorentzian signature above four dimensions are non-renormalisable, or have unbounded energy from below. It is thus not clear what relationship such classical actions have with their strongly coupled quantum counterparts. In this paper we will construct a family of five-dimensional non-Lorentzian Lagrangian theories, with 12 (conformal) supersymmetries, and an $SU(1, 3)$ spacetime symmetry. We believe this provides a novel class of higher dimensional theories worthy of further exploration.

There is a growing interest in non-Lorentzian field theories from a variety of view points and applications, for some recent studies see [8–16]. Our own interest stems from a novel form of conformal compactification, which appears to be able to reproduce aspects of the original non-compact six-dimensional SCFT from a five-dimensional Lagrangian theory with a Kaluza-Klein-like tower [17–19]. In particular, the mode number of the tower is identified with the charge arising from the topological current

$$J = \frac{1}{8\pi^2} \star \text{tr}(F \wedge F). \quad (1.1)$$

¹And [7] for a $(2, 0)$ Lagrangian theory with $SU(2)$ gauge group.

Reduction of a six-dimensional Lorentzian field theory can lead to five-dimensional non-Lorentzian field theories, if the compact direction is taken to be null. This breaks the $SO(2, 6)$ conformal group in six-dimensions to an $SU(1, 3)$ symmetry. Thus the Lagrangians of this paper have the correct symmetries to be identified with the null conformal compactification of six-dimensional superconformal field theories. In the limit that the conformal compactification is removed, these Lagrangians have on-shell constraints that reduce the dynamics to motion on instanton moduli space. These constraints arise naturally from the imposition of a null isometry, and their realisation is achieved without adding additional unphysical degrees of freedom in the form of Lagrange multipliers. In fact, different components of the fields in six-dimensions act as Lagrange multipliers to other fields in five dimensions. In this way, in the limit where there is no conformal compactification, we recover the DLCQ prescriptions for six-dimensional superconformal field theories [20, 21]. It is therefore our hope that the Lagrangians presented here can be used to understanding six-dimensional (1,0) theories.

The rest of this paper is organised as follows. In section two we briefly discuss the conformal compactification of Minkowski space leading to so-called Ω -deformed Minkowski space. In section three we discuss (1, 0) conformal supermultiplets and their null reduction. In section four we present a class of non-Lorentzian gauge theories in 4+1 dimensions with 4(+8) (conformal) supersymmetries, and an $SU(1, 3)$ conformal symmetry. Section five contains our discussion and conclusions. We also include an appendix which gives in detail the relation of the six-dimensional spinors used here to those of eleven-dimensions.

2 Conformal field theories on Ω -deformed Minkowski space

We start by observing that the metric of six-dimensional Minkowski space can be written as

$$\begin{aligned} d\hat{s}_{\text{Minkowski}}^2 &= -2d\hat{x}^+d\hat{x}^- + \delta_{ij}d\hat{x}^i d\hat{x}^j \\ &= \frac{1}{\cos^2(x^+/2R)} d\hat{s}_{\Omega}^2, \end{aligned} \tag{2.1}$$

where

$$d\hat{s}_{\Omega}^2 = -2dx^+ \left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j \right) + \delta_{ij}dx^i dx^j. \tag{2.2}$$

Here $\Omega = -\star\Omega$ and $\Omega_{ik}\Omega_{kj} = -R^{-2}\delta_{ij}$. We refer to (2.2) as Ω -deformed Minkowski space. Clearly for $R \rightarrow \infty$ we recover ordinary Minkowski spacetime. Furthermore, (2.2) naturally arises as the boundary of AdS_7 in the parametrisation of [22].

The conformal transformation (2.1) maps the $\hat{x}^+ \in \mathbb{R}$ coordinate of Minkowski space into a finite range $x^+ \in (-\pi R, \pi R)$. Although the fields need not be periodic in x^+ , their restriction to a finite range is sufficient to capture the entire dynamics of the non-compact Minkowski spacetime. Thus a superconformal field theory on six-dimensional Minkowski space is equivalent the same theory on Ω -deformed Minkowski space, with $x^+ \in (-\pi R, \pi R)$. By expanding as a Fourier series in x^+ , the full non-compact six-dimensional theory can be realised by a discrete Kaluza-Klein tower of five-dimensional operators which depend on (x^-, x^i) .

It also follows that (2.2) admits the maximal number of conformal Killing spinors and Killing vectors. Although some will have explicit x^+ dependence and hence won't survive a reduction to the zero-Fourier mode sector. Such generators will not lead to symmetries of the five-dimensional Lagrangian constructed from the zero-modes, but could nevertheless still be present in the quantum theory. It was shown in [17] that the reduction to the zero-modes preserves, classically, 3/4 of the supersymmetries and conformal supersymmetries. In addition in [18] it was shown that the reduction preserves a $SU(1,3)$ conformal symmetry. The $SU(1,3)$ conformal symmetry imposes non-trivial restrictions on the n -point functions [19].

Furthermore, the Fourier mode number is naturally identified with the instanton number of the five-dimensional gauge theory. This raises the possibility of reconstructing correlation functions of the full six-dimensional theory from the five-dimensional Lagrangian gauge theory [19]. It is therefore natural to construct more general $(1,0)$ supersymmetric conformal field theories, obtained by reduction on Omega-deformed Minkowski space.

We can also consider the case $\Omega_{ij} = 0$. Here the metric is six-dimensional Minkowski space where the range of x^+ is the whole real line. Compactifying x^+ corresponds to the familiar DLCQ matrix model proposals for $(1,0)$ and $(2,0)$ theories [20, 21], where the dynamics is described by quantum mechanics on the moduli space of instantons.

3 Six-dimensional conformal multiplets and their reduction

There are known to be interacting superconformal field theories in six-dimensions with $(1,0)$ supersymmetry. However, it is not thought that a Lagrangian description for such theories exists. Nevertheless, if we allow ourselves to think in terms of fields there are several types of linear six-dimensional $(1,0)$ supermultiplets: vector-multiplets, linear-multiplets, scalar-multiplets, hyper-multiplets, tensor-multiplets and half-hyper-multiplets [23, 24] (see [25, 26] for a detailed classification).

In this paper we will not consider vector and linear multiplets, as their scaling dimensions are incompatible with a quadratic kinetic term, it could be interesting to include them in further work. In particular vectors in a vector-multiplet have canonical scaling dimension one, and play a central role in the construction of [5] (whereas the scalars in a linear multiplet have scaling dimension four). In our construction we will obtain five-dimensional gauge fields, and hence non-abelian interactions, through the dimensionally reduced tensor multiplet.

The $(1,0)$ superalgebra admits an $SU(2)$ R-symmetry and is generated by a spinor ϵ_α , which is chiral and satisfies a symplectic Majorana condition

$$\gamma_{012345}\epsilon_\alpha = \epsilon_\alpha \quad \epsilon^\alpha = (\epsilon_\alpha)^* = i\varepsilon^{\alpha\beta}C\gamma^0\epsilon_\beta. \tag{3.1}$$

Here $\alpha, \beta = 1, 2$ label the $SU(2)$ R-symmetry and C is the unitary charge conjugation matrix $C\gamma_\mu C^{-1} = -\gamma_\mu^T$, $C^T = C$ with $\mu = 0, 1, 2, \dots, 5$. We define $\bar{\psi} = i\psi^\dagger\gamma^0$ for any spinor ψ . We have also introduced a notation where complex conjugation raises (or lowers) the R-symmetry index (as well as any flavour index). Thus for example $\bar{\epsilon}^\alpha = i\epsilon_\alpha^\dagger\gamma^0$. In this section we will restrict our attention to free abelian fields in flat six-dimensional spacetime.

A hyper-multiplet consists of two complex scalars X^α , $\alpha = 1, 2$ and an anti-chiral fermion χ , $\gamma_{012345}\chi = -\chi$. In the free, abelian theory the six-dimensional supersymmetry transformations are

$$\begin{aligned}\delta X^\alpha &= -\bar{\epsilon}^\alpha \chi \\ \delta \chi &= 2\gamma^\mu \partial_\mu X^\alpha \epsilon_\alpha - 8X^\alpha \eta_\alpha.\end{aligned}\tag{3.2}$$

where ϵ_α is a conformal Killing spinor satisfying

$$\partial_\mu \epsilon_\alpha = \gamma_\mu \eta_\alpha.\tag{3.3}$$

Reducing a hyper-multiplet on a null direction or otherwise is relatively straight-forward, as one simply demands that the fields do not depend on the relevant null coordinate and the multiplet is otherwise unchanged.

We can combine two hyper-multiplets $(X^\alpha_{\dot{\beta}}, \chi_{\dot{\beta}})$, $\dot{\beta} = 1, 2$, if we simultaneously impose a reality conditions

$$\begin{aligned}X^\alpha_{\dot{\beta}} &= (X^\alpha_{\dot{\beta}})^* = \varepsilon_{\alpha\gamma} \varepsilon^{\dot{\beta}\dot{\delta}} X^{\gamma\dot{\delta}} \\ \chi_{\dot{\beta}} &= (\chi_{\dot{\beta}})^* = \varepsilon^{\dot{\beta}\dot{\delta}} \chi_{\dot{\delta}}.\end{aligned}\tag{3.4}$$

This leads to the so-called scalar multiplet of [23]. We will describe half-hyper-multiplets at the end of this section once we introduce a non-abelian structure.

Of particular interest is the tensor-multiplet, which consists of a real scalar ϕ , a self-dual and closed three-form $H_{\mu\nu\rho}$, and anti-chiral fermions λ_α : $\gamma_{012345}\lambda_\alpha = -\lambda_\alpha$. These are also subjected to the symplectic Majorana constraint

$$\lambda^\alpha = (\lambda_\alpha)^* = i\varepsilon^{\alpha\beta} C\gamma^0 \lambda_\beta.\tag{3.5}$$

The supersymmetry transformation are

$$\begin{aligned}\delta \phi &= -\bar{\epsilon}^\alpha \lambda_\alpha \\ \delta H_{\mu\nu\rho} &= 3\bar{\epsilon}^\alpha \gamma_{[\mu\nu} \partial_{\rho]} \lambda_\alpha - 3\bar{\eta}^\alpha \gamma_{\mu\nu\rho} \lambda_\alpha \\ \delta \lambda_\alpha &= \gamma^\mu \partial_\mu \phi \epsilon_\alpha + \frac{1}{2 \cdot 3!} H_{\mu\nu\lambda} \gamma^{\mu\nu\lambda} \epsilon_\alpha + 4\phi \eta_\alpha.\end{aligned}\tag{3.6}$$

where again $\partial_\mu \epsilon_\alpha = \gamma_\mu \eta_\alpha$. These only close on-shell and with the constraint $H = \star H$.

Difficulties arise when one wants to make an interacting theory, presumably based on allowing the fields to become non-abelian. For example there is no clear understanding of a non-abelian self-dual tensor arising as a field strength of some kind of non-abelian 2-form connection (for some recent developments see [27]).

One way to proceed is to reduce the theory on a spacelike circle parametrised by $x^5 \cong x^5 + 2\pi R_5$. In this case the hyper-multiplet is essentially unchanged. On the other hand, for a tensor multiplet, as a consequence of self-duality, the independent components of the three-form are encoded in $F = R_5 i_5 H$. One can then identify $F = dA$, where A is a five-dimensional one-form gauge field. Reduction then leads to a five-dimensional vector multiplet, which one can then make non-abelian. The resulting theories are well-studied as one

can indeed construct more-or-less traditional five-dimensional gauge theory Lagrangians. However, the problem is that such theories are power-counting non-renormalizable. As a result their relation to the original superconformal field theory is at best obscured.

Another approach is to reduce on a null circle; $x^+ \cong x^+ + 2\pi R_+$. In the simplest form this leads to the so-called DLCQ constructions. More recently, as discussed above, a variation of this has been studied where the x^+ direction is only conformally compactified [17] so that the non-compact six-dimensional theory can be reconstructed [19]. In these reductions one finds the following fields from the reduction of the three-form:

$$\begin{aligned} H_{ij+} &\sim F_{ij} \\ H_{ij-} &\sim G_{ij}, \end{aligned} \tag{3.7}$$

where $i, j = 1, 2, 3, 4$ and we use \sim as the exact relation depends upon the details of the null reduction [28]. Furthermore $F \sim -\star_4 F$ and $G = \star_4 G$ where \star_4 is the Hodge star in x^1, \dots, x^4 . One finds that the remaining components of H can be determined using six-dimensional self-duality from F and G . It was observed in [13, 28] that non-Lorentzian five-dimensional Lagrangians can be constructed and generalized to non-abelian interacting fields by taking $F = dA$, where $A = (A_-, A_i)$ is a five-dimensional gauge field and $G = \star_4 G$ off-shell. In the action G acts as a Lagrangian multiplier that imposes the anti-self-duality of F on-shell. Thus after a null reduction the tensor multiplet gives rise to the following fields:

$$(\phi, H_{\mu\nu\lambda}, \lambda_\alpha) \xrightarrow{\text{Null Reduction}} (\phi, A_-, A_i, G_{ij}^+, \lambda_\alpha), \tag{3.8}$$

where the +-superscript indicates that G_{ij}^+ is taken to be self-dual off-shell.

Our task in the next section is to construct interacting non-abelian Lagrangian gauge theories in five-dimensions that might arise from six-dimensional $(1, 0)$ theories made from hyper and tensor-multiplets. As discussed above, we expect these to have 3/4 of the supersymmetry and superconformal symmetry as well as an $SU(1, 3)$ conformal symmetry. We will see that this is indeed the case.

4 Actions

In this section we construct non-Lorentzian five-dimensional actions and show that they admit an $SU(1, 3)$ conformal symmetry, along with 4 supersymmetries and 8 conformal supersymmetries. Our approach is to take the action in [17] where the symmetries are appropriate to six-dimensional $(2, 0)$ supersymmetry and all the fields are in the adjoint representation. We then rewrite it in terms of $(1, 0)$ supermultiplets (see the appendix for this map). This allows us to generalise the resulting action to an arbitrary number of hyper-multiplets, in arbitrary representations of the gauge group.

To this end we consider matter content of a null reduced tensor multiplet $(\phi, A_-, A_i, G_{ij}^+, \lambda_\alpha)$, which takes values in the adjoint of some gauge group \mathcal{G} , and K hyper-multiplets (X_m^α, χ_m) $m = 1, \dots, K$, which can take values in a vector space \mathcal{V} that carries a representation $\Pi(\mathcal{G})$ of the gauge group \mathcal{G} . In our conventions the generators of this representation are Hermitian T_a , $a = 1, \dots, \dim(\mathcal{G})$ (for notational clarity we do not put an m -index

on \mathcal{V} and T_a but it is possible that each hyper-multiplet is in a different representation). Their complex conjugates (X_α^m, χ^m) therefore take values in the complex conjugate representation with generators $-T_a^*$. One can also think of just one hyper-multiplet but in a reducible representation of \mathcal{G} . We assume that the Lie-algebra has an invariant non-degenerate inner-product $(\ , \)$ and each representation has an invariant non-degenerate inner-product $\langle \ , \ \rangle$. We will assume that the latter is complex anti-linear in general, and we use a notation whereby the first entry is explicitly conjugated: e.g. on \mathbb{C}^N we write the inner-product as $\langle w^*, z \rangle = \sum_r w_r^\dagger z_r$.

Starting with the action constructed in [17] we are led to propose the following action

$$\begin{aligned}
 S = \frac{1}{g_{\text{YM}}^2} \int dx^- d^4x \left\{ \frac{1}{2}(F_{i-}, F_{i-}) + \frac{1}{2}(\mathcal{F}_{ij}, G_{ij}^+) - \frac{1}{2}(\nabla_i \phi, \nabla_i \phi) - \langle \nabla_i X_\alpha^m, \nabla_i X_\alpha^m \rangle \right. \\
 - \frac{1}{2}(\bar{\lambda}^\alpha, \gamma_+ D_- \lambda_\alpha) + \frac{1}{2}(\bar{\lambda}^\alpha, \gamma_i \nabla_i \lambda_\alpha) - \frac{1}{2}\langle \bar{\chi}^m, \gamma_+ D_- \chi_m \rangle + \frac{1}{2}\langle \bar{\chi}^m, \gamma_i \nabla_i \chi_m \rangle \\
 \left. - \frac{i}{2}(\bar{\lambda}^\alpha \gamma_+, [\phi, \lambda_\alpha]) + \frac{i}{2}\langle \bar{\chi}^m \gamma_+, \phi(\chi_m) \rangle + i\langle \bar{\chi}^m \gamma_+, \lambda_\alpha(X_\alpha^m) \rangle - i\langle \bar{\lambda}^\alpha(X_\alpha^m), \gamma_+ \chi_m \rangle \right\}.
 \end{aligned} \tag{4.1}$$

The covariant derivative is defined on a hyper-multiplet as

$$D_- X_\alpha^m = \partial_- X_\alpha^m - iA_-(X_\alpha^m) \quad D_i X_\alpha^m = \partial_i X_\alpha^m - iA_i(X_\alpha^m), \tag{4.2}$$

where $A_-(X_\alpha^m), A_i(X_\alpha^m)$ etc., are taken to act in the appropriate representation: e.g. $\phi(X_\alpha^m) = \phi^a T_a(X_\alpha^m)$. In addition we define

$$\begin{aligned}
 \mathcal{F}_{ij} &= F_{ij} - \frac{1}{2}\Omega_{[i|k} x^k F_{j]-} \\
 \nabla_i &= D_i - \frac{1}{2}\Omega_{ij} x^j D_- .
 \end{aligned} \tag{4.3}$$

Where, for the tensor multiplet, we have

$$D_- \phi = \partial_- \phi - i[A_-, \phi], \quad D_i \phi = \partial_i \phi - i[A_i, \phi], \tag{4.4}$$

and similarly for the other fields.

4.1 Supersymmetries

We note that the Ω -deformed Minkowski space (2.2) admits a maximal number of (conformal) Killing spinors

$$D_\mu \epsilon_\alpha = \gamma_\mu \eta_\alpha. \tag{4.5}$$

However only those that are independent of x^+ are expected to lead to symmetries of the five-dimensional action. Explicitly these Killing spinors are of three types [17]:

$$\begin{aligned}
 \epsilon_\alpha &= \zeta_{-\alpha} & \eta_\alpha &= 0 \\
 \epsilon_\alpha &= \zeta'_{+\alpha} + \frac{1}{2}x^i \Omega_{ij} \gamma_j \gamma_- \zeta'_{+\alpha} & \eta_\alpha &= \frac{1}{16} \Omega_{ij} \gamma_i \gamma_j \gamma_- \zeta'_{+\alpha} \\
 \epsilon_\alpha &= -\frac{1}{2}x^i \gamma_i \gamma_+ \zeta''_{-\alpha} - \frac{1}{4} \Omega_{ik} \gamma_{kj} x^i x^j \zeta''_{-\alpha} + x^- \zeta''_{-\alpha} & \eta_\alpha &= -\frac{1}{2} \gamma_+ \zeta''_{-\alpha} - \frac{1}{16} \Omega_{ij} \gamma_i \gamma_j x^k \gamma_k \zeta''_{-\alpha},
 \end{aligned} \tag{4.6}$$

where $\zeta_{-\alpha}, \zeta'_{+\alpha}, \zeta''_{-\alpha}$ are constant spinors. Here the \pm subscript refers to their γ_{-+} chirality. Thus each one has four real independent components.

The action (4.1) is indeed supersymmetric with respect to these spinors. In particular the transformations of the hyper-multiplets are

$$\begin{aligned} \delta X^\alpha_m &= -\bar{\epsilon}^\alpha \chi_m \\ \delta \chi_m &= -2\gamma_+ D_- X^\alpha_m \epsilon_\alpha + 2\nabla_i X^\alpha_m \gamma_i \epsilon_\alpha + 2i\phi(X^\alpha_m) \gamma_+ \epsilon_\alpha - 8X^\alpha_m \eta_\alpha, \end{aligned} \tag{4.7}$$

and those of the tensor multiplet are

$$\begin{aligned} \delta \phi &= -\bar{\epsilon}^\alpha \lambda_\alpha \\ \delta A_- &= -\bar{\epsilon}^\alpha \gamma_{+-} \lambda_\alpha \\ \delta A_i &= -\bar{\epsilon}^\alpha \left(\gamma_{+i} + \frac{1}{2} \Omega_{ij} x^j \gamma_{+-} \right) \lambda_\alpha \\ \delta G_{ij}^+ &= -\frac{1}{2} \bar{\epsilon}^\alpha \gamma_{+\gamma-} \gamma_{ij} D_- \lambda_\alpha + \frac{1}{2} \bar{\epsilon}^\alpha \gamma_k \gamma_{ij} \gamma_- \nabla_k \lambda_\alpha + \frac{i}{2} \bar{\epsilon}^\alpha \gamma_{+\gamma-} \gamma_{ij} [\phi, \lambda_\alpha] \\ &\quad - \frac{1}{2} \bar{\epsilon}^\alpha \gamma_{+\gamma-} \gamma_{ij} [[X^\alpha_m, \chi_m]] - \frac{1}{2} [[\bar{X}^m, X^\alpha_m]] \gamma_{-\gamma+} \gamma_{ij} \epsilon_\alpha - 3\bar{\eta}^\alpha \gamma_{-\gamma_{ij}} \lambda_\alpha \\ \delta \lambda_\alpha &= -F_{-i} \gamma_{+-} \gamma_i \epsilon_\alpha - \frac{1}{4} \mathcal{F}_{ij} \gamma_{-\gamma_{ij}} \epsilon_\alpha - \frac{1}{4} G_{ij} \gamma_{+\gamma_{ij}} \epsilon_\alpha - \gamma_+ D_- \phi \epsilon_\alpha + \gamma_i \nabla_i \phi \epsilon_\alpha \\ &\quad + [[X^\alpha_m, X^\beta_m]] \gamma_{+\epsilon\beta} - \epsilon_{\alpha\gamma} \epsilon^{\beta\delta} [[X_\delta^m, X^\gamma_m]] \gamma_{+\epsilon\beta} + 4\phi \eta_\alpha. \end{aligned} \tag{4.8}$$

Here we have introduced a map:

$$[[;]]: \mathcal{V}^* \times \mathcal{V} \rightarrow \text{Lie}(\mathcal{G}), \tag{4.9}$$

which is defined to satisfy, for any $X, Y \in \mathcal{V}$ and $\phi \in \text{Lie}(\mathcal{G})$,

$$([[X^*, Y]], \phi) = i \langle X^*, \phi(Y) \rangle. \tag{4.10}$$

In terms of components this tells us that

$$[[X^*; Y]]^a = i \langle X^*, T_b(Y) \rangle \kappa^{ab}, \tag{4.11}$$

where κ^{ab} is the inverse inner-product on $\text{Lie}(\mathcal{G})$ evaluated on the adjoint basis: $\kappa_{ab} = (T_a^{\text{adj}}, T_b^{\text{adj}})$. With our conventions

$$([[X^*; Y]]^a)^* = -[[Y^*; X]]^a. \tag{4.12}$$

The normalisation is chosen so that, in the special case that the fields are real and take values in the adjoint representation, $[[X, Y]] = -i[X, Y]$ and in this case the condition (4.10) just asserts the invariance of the inner-product under the adjoint action.

4.2 Conformal symmetries

Six-dimensional Minkowski space admits a family of 28 linearly independent conformal Killing vectors that obey

$$D_\mu k_\nu + D_\nu k_\mu = \omega g_{\mu\nu}. \tag{4.13}$$

A six-dimensional conformal field theory inherits an $SO(2,6)$ spacetime symmetry group. The same is true for the conformally equivalent spacetime (2.2). However, as was shown in [18], demanding that the fields are independent of x^+ reduces the $SO(2,6)$ conformal group to $SU(1,3)$. In terms of an AdS_7 dual description the $SO(2,6)$ symmetry group is broken to $SU(1,3)$ by a timelike reduction [22]. For the presently considered spacetime, one finds the general form for such a k^μ and ω can be divided into seven classes:

$$\begin{aligned}
 \text{type I} & \quad (b, 0, 0, 0, 0, 0) \\
 \text{type II} & \quad (0, c, 0, 0, 0, 0) \\
 \text{type III} & \quad \left(0, \frac{1}{2}c_i\Omega_{ij}x^j, c_i\right) \\
 \text{type IV} & \quad \left(0, 0, M_{ij}x^j\right) \\
 \text{type V} & \quad \left(0, \omega_1x^-, \frac{1}{2}\omega_1x_i\right) \\
 \text{type VI} & \quad \left(v_ix^i, \frac{1}{2}x^-v_i\Omega_{ij}x^j - \frac{1}{8}R^{-2}|x|^2x^k v_k, x^-v_i - \frac{1}{2}x^k v_k\Omega_{li}x^l + \frac{1}{2}v^k x^l\Omega_{kl}x^i - \frac{1}{4}|x|^2v^k\Omega_{ki}\right) \\
 \text{type VII} & \quad \left(\frac{1}{4}\omega_2|x|^2, \frac{1}{2}\omega_2(x^-)^2 - \frac{\omega_2}{32}R^{-2}|x|^4, -\frac{1}{8}\omega_2\Omega_{ki}|x|^2x_k + \frac{1}{2}\omega_2x_ix^-\right).
 \end{aligned} \tag{4.14}$$

where the conformal factor is given by

$$\omega = \omega_1 + v_i\Omega_{ij}x^j + \omega_2x^- . \tag{4.15}$$

Here $b, c, c_i, v_i, \omega_1, \omega_2$ are constants and M_{ij} is an anti-symmetric matrix that commutes with Ω_{ij} . In [18] it was shown types II–VII form a representation of $SU(1,3)$ that the (2,0) version of the action (4.1) admits these as spacetime symmetries. We now want to extend this result to the more general (1,0) theories.

Cases I–IV are rather obvious spacetime symmetries

- I – Translations in x^+ , which are trivial in our five-dimensional action.
- II – Translations in x^- .
- III – Translations in x^i , with a compensating shift in x^- .
- IV – Spatial rotations that preserve Ω .

Furthermore it is not hard to identify case V as the Liftshitz rescaling

$$x^- \longrightarrow \zeta x^-, \quad x^i \longrightarrow \zeta^{1/2}x^i, \tag{4.16}$$

with the fields in the tensor and hyper multiplets transforming as

$$\begin{aligned}
 \phi & \longrightarrow \zeta^{-1}\phi & X_m^\alpha & \longrightarrow \zeta^{-1}X_m^\alpha \\
 \lambda_{+\alpha} & \longrightarrow \zeta^{-3/2}\lambda_{+\alpha} & \chi_{+m} & \longrightarrow \zeta^{-3/2}\chi_{+m} \\
 \lambda_{-\alpha} & \longrightarrow \zeta^{-1}\lambda_{-\alpha} & \chi_{-m} & \longrightarrow \zeta^{-1}\chi_{-m} \\
 A_- & \longrightarrow \zeta^{-1}A_- & & \\
 A_i & \longrightarrow \zeta^{1/2}A_i & & \\
 G_{ij} & \longrightarrow \zeta^{-2}G_{ij} & &
 \end{aligned} \tag{4.17}$$

Here the subscript \pm on spinors indicates eigenvalue under γ_{-+} .

The remaining transformations are not so obviously symmetries, they correspond to special conformal transformations. For type VI the infinitesimal coordinate transformations are

$$\begin{aligned}\delta x^- &= \frac{1}{2}\Omega_{ij}v_i x^j x^- - \frac{1}{8}R^{-2}|x|^2 v_i x^i \\ \delta x^i &= \frac{1}{2}\Omega_{jk}v_j x^k x^i + v_i x^- + \frac{1}{2}\Omega_{ij}v_k x^j x^k + \frac{1}{4}|x|^2\Omega_{ij}v_j,\end{aligned}\tag{4.18}$$

which is found to be a symmetry of the action if the tensor multiplet transforms as

$$\begin{aligned}\delta\phi &= -\Omega_{ij}v_i x^j \phi \\ \delta\lambda_{+\alpha} &= -\frac{3}{2}\Omega_{ij}v_i x^j \lambda_{+\alpha} + \frac{1}{2}v_i \Gamma_+ \Gamma_i \lambda_{-\alpha} + \frac{1}{4}\Lambda_{ij}\Gamma_{ij}\lambda_{+\alpha} \\ \delta\lambda_{-\alpha} &= -\Omega_{ij}v_i x^j \lambda_{-\alpha} + \frac{1}{4}\Lambda_{ij}\Gamma_{ij}\lambda_{-\alpha} \\ \delta A_- &= -\frac{1}{2}\Omega_{ij}v_i x^j A_- - v_i A_i \\ \delta A_i &= -\frac{1}{2}\Omega_{jk}v_j x^k A_i + \frac{1}{2}\left(\Omega_{ij}v_k x^k + \Omega_{ik}v_k x^j - \Omega_{jk}(v_i x^k + v_k x^i)\right) A_j \\ &\quad + \frac{1}{8}\left(R^{-2}|x|^2 v_i + 2R^{-2}v_j x^j x^i + 4\Omega_{ij}v_j x^-\right) A_- \\ \delta G_{ij} &= -2\Omega_{kl}v_k x^l G_{ij} - \frac{1}{2}\left(\Lambda^{ki}G_{kj} - \Lambda^{kj}G_{ki} + \varepsilon_{ijkl}\Lambda^{mk}G_{ml}\right) \\ &\quad + v_i F_{-j} - v_j F_{-i} + \varepsilon_{ijkl}v_k F_{-l},\end{aligned}\tag{4.19}$$

and the hyper multiplets as

$$\begin{aligned}\delta X_m^\alpha &= -\Omega_{ij}v_i x^j X_m^\alpha \\ \delta\chi_{+m} &= -\frac{3}{2}\Omega_{ij}v_i x^j \chi_{+m} + \frac{1}{2}v_i \Gamma_+ \Gamma_i \chi_{-m} + \frac{1}{4}\Lambda_{ij}\Gamma_{ij}\chi_{+m} \\ \delta\chi_{-m} &= -\Omega_{ij}v_i x^j \chi_{-m} + \frac{1}{4}\Lambda_{ij}\Gamma_{ij}\chi_{-m}.\end{aligned}\tag{4.20}$$

We have defined

$$\Lambda_{ij} = \frac{1}{2}\left(\Omega_{ij}v_k x^k + \Omega_{ik}v_k x^j - \Omega_{jk}v_k x^i + \Omega_{ik}v_j x^k - \Omega_{jk}v_i x^k\right).\tag{4.21}$$

Note that in the limit that $R \rightarrow \infty$ we have $\Omega_{ij} \rightarrow 0$, and these transformations become Galilean boosts: $\delta x^- = 0, \delta x^i = v^i x^-$.

While for type VII, the infinitesimal coordinate transformations are

$$\begin{aligned}\delta x^- &= \frac{1}{2}\omega_2(x^-)^2 - \frac{1}{32}\omega_2 R^{-2}|x|^4 \\ \delta x^i &= \frac{1}{8}\omega_2\Omega_{ij}|x|^2 x^j + \frac{1}{2}\omega_2 x^i x^-, \end{aligned}\tag{4.22}$$

which we find to be a symmetry of the action if the corresponding transformations are

$$\begin{aligned}
\delta\phi &= -\omega_2 x^- \phi \\
\delta\lambda_{+\alpha} &= -\frac{3}{2}\omega_2 x^- \lambda_{+\alpha} + \frac{1}{4}\omega_2 x^i \Gamma_+ \Gamma_i \lambda_{-\alpha} + \frac{1}{4}\Lambda_{ij} \Gamma_{ij} \lambda_{+\alpha} \\
\delta\lambda_{-\alpha} &= -\omega_2 x^- \lambda_{-\alpha} + \frac{1}{4}\Lambda_{ij} \Gamma_{ij} \lambda_{-\alpha} \\
\delta A_- &= -\omega_2 x^- A_- - v_i A_i \\
\delta A_i &= -\frac{1}{2}\omega_2 x^- A_i + \frac{1}{8}\omega_2 R^{-2} |x|^2 x^i A_- + \frac{1}{8}\omega_2 \left(\Omega_{ij} |x|^2 - 2\Omega_{jk} x^i x^k \right) A_j \\
\delta G_{ij} &= -2\omega_2 x^- G_{ij} - \frac{1}{2} \left(\Lambda^{ki} G_{kj} - \Lambda^{kj} G_{ki} + \varepsilon_{ijkl} \Lambda^{mk} G_{ml} \right) \\
&\quad + \frac{1}{2}\omega_2 \left(x^i F_{-j} - x^j F_{-i} + \varepsilon_{ijkl} x^j F_{-l} \right),
\end{aligned} \tag{4.23}$$

and

$$\begin{aligned}
\delta X_m^\alpha &= -\omega_2 x^- X_m^\alpha \\
\delta\chi_{+m} &= -\frac{3}{2}\omega_2 x^- \chi_{+m} + \frac{1}{4}\omega_2 x^i \Gamma_+ \Gamma_i \chi_{-m} + \frac{1}{4}\Lambda_{ij} \Gamma_{ij} \chi_{+m} \\
\delta\chi_{-m} &= -\omega_2 x^- \chi_{-m} + \frac{1}{4}\Lambda_{ij} \Gamma_{ij} \chi_{-m}.
\end{aligned} \tag{4.24}$$

Where now Λ is given by

$$\Lambda^{ij} = \frac{1}{4}\omega_2 \left(\Omega_{ik} x^k x^j - \Omega_{jk} x^k x^i \right) + \frac{1}{8}\omega_2 \Omega_{ij} |x|^2. \tag{4.25}$$

4.3 Enhancement to (2, 0) and other variations

Let us now outline how a (2, 0) tensor multiplet, and corresponding action, can be obtained (see the appendix for additional details). Since we need a total of five scalars one might think that it is sufficient to consider one tensor and one hyper-multiplet. However this will not work as the R-symmetry remains only $SU(2)$, and not $SO(5)$, as is required. Instead one must take a (1, 0) tensor multiplet $(\phi, H_{\mu\nu\lambda}, \lambda_\alpha)$ along with two copies of the (1, 0) hyper-multiplet (X_m^α, χ_m) , where $m = 1, 2$ which we combine into the scalar multiplet using the reality condition (3.4). Crucially the representation of all multiplets must be the adjoint. The various reality conditions reduce the total number of scalars to five, (ϕ, X_m^α) and the total number of on-shell fermionic degrees of freedom to eight.

In addition to the original $SU(2)$ R-symmetry for the scalars, we find an $Sp(1) \cong SU(2)$ flavour symmetry. The resulting $SU(2) \times SU(2) \cong SO(4)$ then enhances to an $SO(5) \cong Usp(4)$ R-symmetry with the addition of ϕ from the (1, 0) tensor multiplet, this is the appropriate R-symmetry for (2, 0) supersymmetry. The fermions also combine to give ψ_r where r covers the range of m, α which is $1, \dots, 4$. In this way we can consider λ_α, χ_m as the Weyl spinors of the Dirac spinor ψ_r

$$\left. \begin{array}{l} (1, 0) \text{ tensor } (\phi, H, \lambda_\alpha) \\ (1, 0) \text{ scalar } (X_m^\alpha, \chi_m) \end{array} \right\} \longrightarrow (2, 0) \text{ tensor } (X^{rs}, \psi_r, H), \tag{4.26}$$

with the scalars now written as transforming in the $\mathbf{5}$ of $Usp(4) \cong SO(5)$. In this way the $(2, 0)$ model obtained in [17] arises from (4.1) by taking two hypermultiplets in the adjoint representation and imposing the additional reality condition (3.4). This relation is explored in more detail in the appendix, where it is performed explicitly, relating this theory to the natural M5 picture in M-theory.

More generally, if we consider a $2L$ -dimensional subset of hyper-multiplets in the same representation $\Pi(\mathcal{G})$ then we can impose a similar reality constraint. For this to make sense, we require that the representation is real in the sense that there exists a unitary matrix U such that

$$-T_a^* = UT_a U^{-1}. \tag{4.27}$$

This is trivially true for the adjoint representation, $U = I$. In the more general case we impose

$$X_\alpha^m = \varepsilon_{\alpha\beta} \omega^{mn} U(X^\beta_n) \quad \chi^m = \omega^{mn} U(\chi_n), \tag{4.28}$$

where ω^{mn} is an anti-symmetric matrix which squares to minus the identity. This reduces the flavour symmetry from $U(2L)$ to $Sp(L)$. However unlike the $(2, 0)$ case discussed above there will not be, in general, an enhancement of supersymmetry.

Lastly we mention half-hyper-multiplets. These can be defined whenever a hyper-multiplet take values in a pseudo real-representation of the gauge group. In this case the unitary matrix U in (4.27) satisfies

$$U^{-1} = -U^*, \tag{4.29}$$

or equivalently $U^T = -U$. This implies that the dimension of the representation must be even. This enables us to impose the conditions

$$X_\alpha = \varepsilon_{\alpha\beta} U(X^\beta) \quad \chi^* = Ci\gamma^0 U(\chi). \tag{4.30}$$

5 Discussion and conclusion

The main result of this paper was the construction of five-dimensional non-abelian gauge theories, given in (4.1), without Lorentz invariance but with an $SU(1, 3)$ spacetime conformal symmetry. In addition these actions admit 4 supersymmetries and 8 superconformal symmetries. These theories also have a topological $U(1)$ current

$$J_- = \frac{1}{32\pi^2} \varepsilon_{ijkl} \text{tr}(F_{ij} F_{kl}) \quad J_i = \frac{1}{16\pi^2} \varepsilon_{ijkl} \text{tr}(F_{-j} F_{kl}). \tag{5.1}$$

As is the case for the $(2, 0)$ theory [17] we expect that one can construct gravitational duals in the form of a $U(1)$ fibre reduction of AdS_7 [22] but now starting with the solutions obtained in [29, 30].

We hope these theories are of interest but our primary motivation is that they can be identified with Lorentz invariant six-dimensional $(1, 0)$ conformal field theories on conformally compactified Minkowski space (2.2). The Kaluza-Klein mode is then identified

with the charge arising from the topological U(1) current. Using the techniques developed in [19] we hope that these Lagrangian field theories can be used to describe six-dimensional Lorentzian conformal field theories. In contrast to other studies, our actions are based on a null reduction of six-dimensional hyper and tensor multiplets with no vector multiplets. It could be of interest to find more general theories by including other multiplets.

We can also take $R \rightarrow \infty$ which sets $\Omega_{ij} = 0$. In this case there is no conformal compactification and we are simply reducing over x^+ which we impose by hand to be compact with period $2\pi R_+$. This is the DLCQ description. Furthermore the type VI symmetry above becomes a boost. In our Lagrangians setting $\Omega_{ij} = 0$ means that $\mathcal{F}_{ij} = F_{ij}$ and the constraint imposed by G_{ij}^+ is simply that F_{ij} is anti-self-dual. Thus the dynamics is reduced to motion on instanton moduli space. This in line with a well-known prescription [20, 21]. However our models allow one to understand in a controlled way how various aspects of the six-dimensional theory are encoded in the resulting quantum mechanics in terms of additional parameters and fields, for example see [31].

Finally, let us recall that in the case of the (2, 0) theory the action can be obtained by a non-Lorentzian rescaling of five-dimensional maximally supersymmetric Yang-Mills theory [17, 32]. The Liftshitz scaling then arises as a symmetry of the fixed point. For $\Omega_{ij} \neq 0$ there is a dual AdS description where the Yang-Mills theory arises from M5-branes reduced on a timelike S^1 fibration of AdS₇. The SU(1, 3) conformal symmetry, including a Liftshitz scaling, emerges in the limit where the M5-brane is taken to the boundary of AdS₇ [17]. We expect analogous constructions to hold for the case of the (1, 0) theories we have constructed here.

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A Relation to Spin(1, 10)

Let us start with the 32×32 Γ -matrices of Spin(1, 10). In this appendix we use a hat to denote elements associated to Spin(1, 10) and $M = 0, 1, 2, \dots, 10$. These have a charge conjugation matrix \hat{C} :

$$\hat{C}^T = -\hat{C}, \quad (\hat{C}\hat{\Gamma}^M)^T = \hat{C}\hat{\Gamma}^M. \tag{A.1}$$

The spinors in eleven-dimensions satisfy a Majorana condition

$$\bar{\hat{\psi}} := \hat{\psi}^T \hat{C} = \hat{\psi}^\dagger i\hat{\Gamma}^0, \tag{A.2}$$

and similarly for the supersymmetry generator \hat{e} . We decompose the Γ and charge conjugation matrices as ($\mu = 0, 1, 2, \dots, 5$, $A = 6, 7, 8, 9$)

$$\begin{aligned}\hat{\Gamma}^\mu &= \gamma^\mu \otimes \mathbb{I}_4 \\ \hat{\Gamma}^A &= \gamma_* \otimes \rho^A \\ \hat{\Gamma}^{10} &= \gamma_* \otimes \rho_*.\end{aligned}\tag{A.3}$$

Here $\gamma_* = \gamma_0 \gamma_1 \dots \gamma_5$, $\rho_* = \rho_6 \rho_7 \rho_8 \rho_9$ and

$$\hat{C} = C \otimes C_4.\tag{A.4}$$

Applying this decomposition to eq. (A.1), we see the inherited conditions

$$\begin{aligned}C^T &= C, & (C\gamma^\mu)^T &= -C\gamma^\mu \\ C_4^T &= -C_4, & (C_4\rho^A)^T &= -C_4\rho^A.\end{aligned}\tag{A.5}$$

We can combine the first of each of these with the general unitarity of charge conjugation matrices to obtain

$$\begin{aligned}C^* &= C^{-1}, & C_4^* &= -C_4^{-1}, \\ \gamma^{\mu T} &= -C\gamma^\mu C^{-1}, & \rho^{AT} &= C_4\rho^A C_4^{-1}.\end{aligned}\tag{A.6}$$

Next we must expand the eleven-dimensional spinors in a basis $(\xi_+^\alpha, \xi_-^\alpha)$ of the internal spinor space $\text{Spin}(4)$:

$$\begin{aligned}\hat{\psi} &= \sum_\alpha \psi_\alpha \otimes \xi_+^\alpha + \sum_{\dot{\alpha}} \psi_{\dot{\alpha}} \otimes \xi_-^{\dot{\alpha}} \\ \hat{e} &= \sum_\alpha \epsilon_\alpha \otimes \xi_+^\alpha + \sum_{\dot{\alpha}} \epsilon_{\dot{\alpha}} \otimes \xi_-^{\dot{\alpha}}.\end{aligned}\tag{A.7}$$

Here we have split the basis according to their internal chirality $\rho_*\xi_+^\alpha = \xi_+^\alpha$, $\rho_*\xi_-^{\dot{\alpha}} = -\xi_-^{\dot{\alpha}}$ with $\alpha, \dot{\alpha} = 1, 2$.² We will also assume that these have been normalised to

$$\begin{aligned}(\xi_+^\beta)^\dagger \xi_+^\alpha &= \delta_\beta^\alpha \\ (\xi_-^{\dot{\beta}})^\dagger \xi_-^{\dot{\alpha}} &= \delta_{\dot{\beta}}^{\dot{\alpha}} \\ (\xi_-^{\dot{\beta}})^\dagger \xi_+^\alpha &= 0.\end{aligned}\tag{A.8}$$

The condition of $(2, 0)$ supersymmetry imposes the constraints $\hat{\Gamma}_{012345}\hat{\psi} = -\hat{\psi}$ and $\hat{\Gamma}_{012345}\hat{e} = \hat{e}$ which in turn imply

$$\begin{aligned}\gamma_*\psi_\alpha &= -\psi_\alpha & \gamma_*\psi_{\dot{\alpha}} &= -\psi_{\dot{\alpha}} \\ \gamma_*\epsilon_\alpha &= \epsilon_\alpha & \gamma_*\epsilon_{\dot{\alpha}} &= \epsilon_{\dot{\alpha}}.\end{aligned}\tag{A.9}$$

²In the main body of this paper a \pm subscript on a six-dimensional spinor refers to its chirality under γ_{05} .

In six-dimensions we do not have Majorana spinors, but symplectic Majorana-Weyl ones. Thus the decomposition of the eleven-dimensional Majorana condition (A.2) is solved by the conditions

$$\begin{aligned}(\psi_\alpha)^* &= \varepsilon^{\alpha\beta} C i \gamma^0 \psi_\beta \\ (\epsilon_\alpha)^* &= \varepsilon^{\alpha\beta} C i \gamma^0 \epsilon_\beta \\ (\xi_+^\alpha)^* &= \varepsilon_{\alpha\beta} C_4 \xi_+^\beta,\end{aligned}\tag{A.10}$$

and similarly for $\psi_{\dot{\alpha}}$, $\epsilon_{\dot{\alpha}}$ and $\xi_-^{\dot{\alpha}}$.³ Here $\varepsilon_{\alpha\beta} = \varepsilon^{\alpha\beta}$ is the Levi-Civita symbol.

Next we need to impose a further constraint to reduce to (1,0) supersymmetry. For this we project to

$$\hat{\Gamma}^{10} \hat{\epsilon} = \hat{\epsilon}.\tag{A.11}$$

We further identify $\hat{\lambda} = \frac{1}{2}(1 - \hat{\Gamma}^{10})\hat{\psi}$ and $\hat{\chi} = \frac{1}{2}(1 + \hat{\Gamma}^{10})\hat{\psi}$. Thus we must expand

$$\begin{aligned}\hat{\epsilon} &= \sum_\alpha \epsilon_\alpha \otimes \xi_+^\alpha \\ \hat{\lambda} &= \sum_\alpha \lambda_\alpha \otimes \xi_+^\alpha \\ \hat{\chi} &= \sum_{\dot{\alpha}} \chi_{\dot{\alpha}} \otimes \xi_-^{\dot{\alpha}}.\end{aligned}\tag{A.12}$$

As before we still have $\gamma_{012345} \lambda_\alpha = -\lambda_\alpha$, $\gamma_{012345} \chi_{\dot{\alpha}} = -\chi_{\dot{\alpha}}$ and $\gamma_{012345} \epsilon_\alpha = \epsilon_\alpha$.

In addition to the fermions $\hat{\psi}$ the (2,0) representation has five real scalars X^A , X^{10} and a self-dual tensor $H_{\mu\nu\lambda}$. The transformation rules are

$$\begin{aligned}\delta X^A &= \bar{\hat{\epsilon}} \hat{\Gamma}^A \hat{\psi} \\ \delta X^{10} &= \bar{\hat{\epsilon}} \hat{\Gamma}^{10} \hat{\psi} \\ \delta H_{\mu\nu\rho} &= 3 \bar{\hat{\epsilon}} \hat{\Gamma}_{[\mu\nu} \partial_{\rho]} \hat{\psi} \\ \delta \hat{\psi} &= \hat{\Gamma}^\mu \hat{\Gamma}^A \partial_\mu X^A \hat{\epsilon} + \hat{\Gamma}^\mu \hat{\Gamma}^{10} \partial_\mu X^{10} \hat{\epsilon} + \frac{1}{2 \cdot 3!} \hat{\Gamma}^{\mu\nu\rho} H_{\mu\nu\rho} \hat{\epsilon}.\end{aligned}\tag{A.13}$$

As can be seen, selecting a special γ matrix, γ_{10} splits the scalar fields $I \rightarrow (A, 10)$, $A \in \{6, \dots, 9\}$. Substituting in the above decomposition one readily sees that $(X^{10}, H_{\mu\nu\lambda}, \lambda_\alpha)$ form a (1,0) tensor multiplet, we then label $X^{10} = \phi$.

To untangle the remaining fields $(X^A, \chi_{\dot{\alpha}})$ we note that we can define

$$T^{A\beta}_{\dot{\alpha}} = (\xi_-^{\dot{\alpha}})^\dagger \rho^A \xi_+^\beta,\tag{A.14}$$

which satisfies

$$T^{A\beta}_{\dot{\alpha}} (T^{A\delta}_{\dot{\gamma}})^* = 2 \delta_{\dot{\alpha}}^{\dot{\gamma}} \delta_{\dot{\delta}}^\beta.\tag{A.15}$$

This allows us to define

$$\begin{aligned}X^{\beta}_{\dot{\alpha}} &= \frac{1}{2} X^A T^{A\beta}_{\dot{\alpha}} \\ X^A &= (T^{A\alpha}_{\dot{\beta}})^* X^\alpha_{\dot{\beta}},\end{aligned}\tag{A.16}$$

³In the main body of this paper we use a notation where complex conjugation raises/lowers α indices.

and the reality of X^A implies that

$$(X^\beta_{\dot{\alpha}})^* = \varepsilon_{\beta\alpha}\varepsilon^{\dot{\alpha}\dot{\beta}}X^\alpha_{\dot{\beta}}. \quad (\text{A.17})$$

We then recognise $(X^\beta_{\dot{\alpha}}, \chi_{\dot{\alpha}})$ as a doublet of hyper-multiplets $\dot{\alpha} = 1, 2$ subjected to the additional reality constraint (A.17).

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