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DOI:

[10.1109/TCYB.2020.3024754](https://doi.org/10.1109/TCYB.2020.3024754)

*Document Version*

Early version, also known as pre-print

[Link to publication record in King's Research Portal](#)

*Citation for published version (APA):*

Zeng, Y., Lam, H. K., Xiao, B., & Wu, L. (2020). I - I Control of Discrete-Time State-Delay Interval Type-2 Fuzzy Systems via Dynamic Output Feedback. *IEEE Transactions on Cybernetics*.  
<https://doi.org/10.1109/TCYB.2020.3024754>

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# $l_2 - l_\infty$ Control of Discrete-time State-delay Interval Type-2 Fuzzy Systems via Dynamic Output Feedback

Yi Zeng, Hak-Keung Lam, *Fellow, IEEE*, Bo Xiao, Ligang Wu, *Fellow, IEEE*

**Abstract**—This paper investigates the design of  $l_2-l_\infty$  dynamic output feedback (DOF) controller for interval type-2 (IT2) T-S fuzzy systems with state delay. For nonlinear systems, IT2 fuzzy model is an efficient modeling method which can better express uncertainties than (type-1) fuzzy model. In addition, state delay is also a general factor that affects the system performance. After analyzing the stability of the system, based on convex linearization and the projection theorem, this paper proposes a delay dependent output feedback controller design method. The IT2 membership functions of fuzzy controller are chosen to be different from those of the model so as to increase the freedom of controller selection. A membership-function-dependent (MFD) method based on staircase membership functions (MFs) is applied to relax the stability analysis results. Finally, numerical simulation example is given to illustrate the effectiveness of the results.

**Index Terms**—output feedback control, state-delay IT2 fuzzy system, fuzzy-model-based system, discrete-time system

## I. INTRODUCTION

As a high-efficient modeling method of nonlinear system, fuzzy model has received extensive attention from academia and industry. Since the (type-1) T-S fuzzy system was proposed [1], the research on fuzzy-model-based (FMB) system covers many fields and problems. Compared with non-fuzzy control methods, FMB control has demonstrated better performance in system modeling, system robustness and fault tolerance. [2], [3] focus on the dynamic control design of FMB systems, where the  $l_2 - l_\infty$  performance and Hankel-norm performance were considered respectively. [4] suggested an application of the fuzzy PID controllers in robotic teleoperation which has notable prospects in telemedicine. Both [5] and [6] designed fuzzy logic control schemes to stabilize the chaotic systems. [7] and [8] offered observer-based fuzzy control schemes, in which the former was specially applied to the mechanical exoskeleton and the latter was mainly based on a new interpolation functions in design. [9], [10] studied the control design problem of sampled-data fuzzy systems. Among them, [9] designed a tracking controller, and [10] was mainly

designed for the vehicle suspension systems. In [11], [12], the MFD stability condition was obtained by approximating the original MFs with staircase MFs. Based on the above results, [13] study the stability condition and stabilization problem for state-delay FMB systems. [14] proposed a further relaxed stability condition for FMB system. Because of the existence of time delay in all kinds of systems, time delay is often considered when studying fuzzy systems. Based on a delay partitioning method, [15] considered the time delay of the fuzzy sampled-data systems. Some results about filter design for state-delay fuzzy systems can be found in [16], [17].

In the research of fuzzy model, researchers try to improve the fuzzy model to enhance the expression efficiency of the FMB systems to the actual nonlinear system. Type-1 fuzzy set can represent the nonlinear information of the system effectively, but it shows limitations in dealing with the uncertainty existing in the system [18]. IT2 T-S fuzzy model is to use the interval MFs to enhance the expression of uncertainty in the systems. In view of the inevitable uncertainty in industrial systems, taking account of the modeling of uncertainty can improve the robustness and control performance of actual industrial systems. Lam investigated the stability condition of IT2 T-S FMB control system for the first time, in which an initiative work is done for IT2 fuzzy modeling and controller design [19]. Extended from [19], Lam proposed an imperfect premise matching (IPM) controller and obtained relaxed stability conditions, where the IPM concept was first proposed in [20] that the issue due to the inherently mismatched MFs between IT2 fuzzy model and controller can be handled. In order to further utilizing the information in the IT2 MFs, Lam in [18] proposed a piecewise linearization based MFD method to further support the stability condition analysis. By introducing a novel performance index, [21] studied the output and state control for IT2 T-S FMB systems. In addition to the controller design problem of IT2 T-S FMB systems, other related problems have been studied recently. For networked IT2 T-S fuzzy systems, [22], [23] studied the filter design in the consideration of data packet dropouts and random link failures. For discrete-time IT2 T-S FMB systems, [24] proposed a method to solve model reduction problem. [25]–[27] are pioneering works investigated the stability condition of polynomial FMB control systems. [28] compared comprehensively the membership-function-independent/dependent methods for type-1 and IT2 FMB control systems. To utilize the information of MFs in the analysis, various MFD analysis techniques are proposed by Lam [29], which can be

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categorized as 1) Global boundary information of MFs, 2) Local boundary information of MFs, 3) Approximate MFs that the approximation methods include staircase method, piecewise linear method, polynomial method and Taylor series method [29]. Given that approximate membership function is difficult to perfectly capture all the information of upper and lower bound MFs, and the computational complexity will increase with the increase of approximation degree, how to capture more information of MFs with reasonable computational demand is a point to be explored in this paper. Some notable related practical applications of IT2 fuzzy models in various types of systems are shown in [30]–[34], which will enlighten the theoretical applications studied in this paper. Among them, [30]–[32] explored the implement of IT2 fuzzy model in mobile robots systems. [33] improved the control performance of the generator-based wind turbines by using IT2 fuzzy model. [34] implemented the IT2 fuzzy framework to the control problem of power system.

Considering the uncertainty of MFs of IT2 FMB system and the complexity and instability of the system caused by state delay, for the past few years, the study of delayed IT2 T-S systems has been paid more and more attention. In [35], stability condition analysis and state feedback control synthesis of delayed IT2 FMB systems are carried out. [36] designed an optimal guaranteed cost sliding-mode controller for delayed IT2 T-S fuzzy systems. Some other noteworthy results can be found in [37]–[39]. Nevertheless, to the best of the authors' knowledge, there are few results about the design of dynamic  $\ell_2 - \ell_\infty$  output feedback controller for delayed IT2 FMB systems, and the research on it has theoretical significance and practical application prospect through the above discussion. In addition, there are still some aspects that can be further optimized and improved in the analysis of the IT2 FMB systems. Comparing to the problem of type-1 FMB systems, the challenges exist in the analysis of the IT2 MFs and the mismatch between the IT2 controller and the IT2 model. The challenges also exist in how to make more use of the information contained in the IT2 MFs and how to process the state-delay term, which are essential and will be discussed in this paper.

Motivated by the above mentioned, the  $\ell_2 - \ell_\infty$  dynamic output controller design of delayed IT2 T-S is studied in this research. First, by constructing the Lyapunov-Krasovskii function, the stability condition of the closed-loop delayed IT2 T-S fuzzy systems is analyzed. The matrices constructed in the condition are intended to avoid unnecessary amplification which increases conservativeness when designing DOF controller. Second, based on the above analysis, an LMI-based method of DOF controller design is proposed. Wirtinger-based inequality [40]–[42], of which Jensen inequality [43] is a corollary, is applied to deal with the time delay term in the delayed IT2 T-S fuzzy systems. Based on cone complement linearization (CCL) and convex linearization process, an effective algorithm with iterative procedure is designed to obtain parameters of the controller. Inspired by [11], [12] and considering the difference between the MFs of IT2 fuzzy systems and type-1 fuzzy systems, a MFD method based on the staircase upper and lower bound of IT2 MFs is designed.

Compared with [44], [45], the method in this paper captures more information in the MFs. The re-designed staircase MFD method in this paper has a good effect on reducing the conservativeness for the analysis for IT2 T-S fuzzy systems. Moreover, the differences between the approximation staircase membership functions and the upper and lower bound membership functions are considered in the parameters to further relax the stability analysis results. Compared with [29], [37], the MFs dependent method is improved for IT2 fuzzy sets to make the approximation upper and lower bound MFs to be flexibly selected in the practical implement.

The rest content of this paper consists of the following parts. Section II introduces the system modeling and the problem studied, as well as some definitions and lemmas needed in the main result section. Section III introduces the main results of this paper and the process of its analysis. Section IV shows the simulation example to verify the effectiveness of the obtained theorems. Section V summarizes the main content of this paper and shows the conclusion.

## II. PROBLEM FORMULATION AND PRELIMINARIES

### A. IT2 FMB system

In this paper, we express nonlinear systems with uncertainty for the following discrete-time state-delay IT2 T-S FMB system (1) with  $n_p$  rules:

Plant Rule  $i$ : IF  $f_1(\rho(k))$  is  $\tilde{W}_1^i$  AND ... AND  $f_{n_\alpha}(\rho(k))$  is  $\tilde{W}_{n_\alpha}^i$  THEN:

$$\begin{aligned} x(\kappa + 1) &= A_i x(\kappa) + A_{di} x(\kappa - d(\kappa)) + B_{1i} u(\kappa) + D_i \omega(\kappa), \\ y(\kappa) &= C_{yi} x(\kappa) + C_{ydi} x(\kappa), \\ z(\kappa) &= C_i x(\kappa) + C_{di} x(\kappa - d(\kappa)) + B_{2i} u(\kappa), \end{aligned} \quad (1)$$

where  $\tilde{W}_\alpha^i$  and  $f_\alpha(\rho(\kappa))$  stand for the IT2 fuzzy set and the premise variable;  $A_i, A_{di}, B_{1i}, B_{2i}, C_i, C_{yi}, C_{di}, C_{ydi}$ , and  $D_i$  are the matrices with appropriate dimensions;  $x(\kappa) \in \mathbb{R}^{n_x}$ ,  $y(\kappa) \in \mathbb{R}^{n_y}$ ,  $u(\kappa) \in \mathbb{R}^{n_u}$ ,  $\omega(\kappa) \in \mathbb{R}^{n_\omega}$  and  $z(\kappa) \in \mathbb{R}^{n_z}$  stand for the system state vector, the measured output, the control input vector, the disturbance input and the controlled output, respectively;  $\alpha = 1, 2, \dots, n_\alpha$ ;  $i = 1, 2, \dots, n_p$ ;  $n_\alpha$  is a positive integer; The following shows the firing strength of the  $i$ th fuzzy rule represented by interval sets:

$$W_i(\rho(\kappa)) = [\underline{w}_i(\rho(\kappa)), \bar{w}_i(\rho(\kappa))], i = 1, 2, \dots, n_p, \quad (2)$$

where

$$\begin{aligned} \underline{w}_i(\rho(\kappa)) &= \prod_{\alpha=1}^{n_\alpha} \underline{\mu}_{\tilde{W}_\alpha^i}(f_\alpha(\rho(\kappa))) \geq 0 \\ \bar{w}_i(\rho(\kappa)) &= \prod_{\alpha=1}^{n_\alpha} \bar{\mu}_{\tilde{W}_\alpha^i}(f_\alpha(\rho(\kappa))) \geq 0. \end{aligned}$$

$\underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\rho(\kappa))) \geq 0$  and  $\bar{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\rho(\kappa))) \geq 0$  stand for the lower and upper MFs, respectively.  $\bar{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\rho(\kappa))) \geq \underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\rho(\kappa)))$  for all  $i$ .  $\underline{w}_i(x(\kappa))$  and  $\bar{w}_i(x(\kappa))$  stand for the lower and upper grades of membership, respectively.

$\bar{w}_i(\rho(\kappa)) \geq \underline{w}_i(\rho(\kappa))$  for all  $i$ . A compact formulation of system (1) is shown as follows:

$$\begin{aligned} x(\kappa+1) &= \sum_{i=1}^{n_p} \tilde{w}_i(\rho(\kappa)) [A_i x(\kappa) + A_{di} x(\kappa - d(\kappa)) \\ &\quad + B_{1i} u(\kappa) + D_i \omega(\kappa)], \\ y(\kappa) &= \sum_{i=1}^{n_p} \tilde{w}_i(\rho(\kappa)) [C_{yi} x(\kappa) + C_{ydi} x(\kappa - d(\kappa))], \\ z(\kappa) &= \sum_{i=1}^{n_p} \tilde{w}_i(\rho(\kappa)) [C_i x(\kappa) + C_{di} x(\kappa - d(\kappa))] \end{aligned} \quad (3)$$

where

$$\begin{aligned} \tilde{w}_i(x(\kappa)) &= \frac{\underline{a}_i(\rho(\kappa)) \underline{w}_i(\rho(\kappa)) + \bar{a}_i(\rho(\kappa)) \bar{w}_i(\rho(\kappa))}{\sum_{k=1}^{n_p} (\underline{a}_k(\rho(\kappa)) \underline{w}_k(\rho(\kappa)) + \bar{a}_k(\rho(\kappa)) \bar{w}_k(\rho(\kappa)))}, \\ \forall i, \sum_{i=1}^{n_p} \tilde{w}_i(\rho(\kappa)) &= 1. \end{aligned}$$

$\underline{a}_i(\rho(\kappa))$  and  $\bar{a}_i(\rho(\kappa))$  are nonlinear functions which satisfy:

- (i)  $0 \leq \underline{a}_i(\rho(\kappa)) \leq 1$  and  $0 \leq \bar{a}_i(\rho(\kappa)) \leq 1$ .
- (ii)  $\underline{a}_i(\rho(\kappa)) + \bar{a}_i(\rho(\kappa)) = 1$ .

### B. IT2 Fuzzy DOF Controller

In this paper, we focus on the IT2 fuzzy DOF controller expressed by the following form.

$$\begin{aligned} \hat{x}(\kappa+1) &= \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} \tilde{w}_i(\rho(\kappa)) \tilde{m}_j(\varrho(\kappa)) [A_{cij} \hat{x}(\kappa) \\ &\quad + B_{cj} y(\kappa)], \\ u(\kappa) &= \sum_{j=1}^{n_c} \tilde{m}_j(\varrho(\kappa)) [C_{cj} \hat{x}(\kappa)], \end{aligned} \quad (4)$$

where  $\tilde{M}_\beta^j$  and  $g_\beta(\varrho(\kappa))$  stand for the IT2 fuzzy set and the premise variable, respectively.  $A_{cij}$ ,  $B_{cj}$  and  $C_{cj}$  stand for the control matrices with appropriate dimensions.  $\hat{x}(\kappa) \in \mathbb{R}^{n_x}$  stand for the controller state vector of the DOF controller;  $j = 1, 2, \dots, n_c$ ;  $\beta = 1, 2, \dots, n_\beta$ ;  $n_\beta$  is a positive integer; The following shows the firing strength of the  $j$ th fuzzy rule represented by interval sets:

$$M_j(\varrho(\kappa)) = [\underline{m}_j(\varrho(\kappa)), \bar{m}_j(\varrho(\kappa))], j = 1, 2, \dots, n_c \quad (5)$$

where

$$\begin{aligned} \underline{m}_j(\varrho(\kappa)) &= \prod_{\beta=1}^{n_\beta} \underline{\mu}_{\tilde{M}_\beta^j}(g_\beta(\varrho(\kappa))) \geq 0, \\ \bar{m}_j(\varrho(\kappa)) &= \prod_{\beta=1}^{n_\beta} \bar{\mu}_{\tilde{M}_\beta^j}(g_\beta(\varrho(\kappa))) \geq 0, \end{aligned}$$

$\underline{\mu}_{\tilde{M}_\beta^j}(g_\beta(\varrho(\kappa))) \geq 0$  and  $\bar{\mu}_{\tilde{M}_\beta^j}(g_\beta(\varrho(\kappa))) \geq 0$  stand for the lower and upper MFs, respectively.  $\bar{\mu}_{\tilde{M}_\beta^j}(g_\beta(\varrho(\kappa))) \geq \underline{\mu}_{\tilde{M}_\beta^j}(g_\beta(\varrho(\kappa)))$ .  $\underline{m}_j(\varrho(\kappa))$  and  $\bar{m}_j(\varrho(\kappa))$  stand for the lower

and upper grades of membership, respectively.  $\bar{m}_j(\varrho(\kappa)) \geq \underline{m}_j(\varrho(\kappa))$ .

$$\begin{aligned} \tilde{m}_j(\varrho(\kappa)) &= \frac{\underline{b}_j(\varrho(\kappa)) \underline{m}_j(\varrho(\kappa)) + \bar{b}_j(\varrho(\kappa)) \bar{m}_j(\varrho(\kappa))}{\sum_{k=1}^{n_c} (\underline{b}_k(\varrho(\kappa)) \underline{m}_k(\varrho(\kappa)) + \bar{b}_k(\varrho(\kappa)) \bar{m}_k(\varrho(\kappa)))} \\ &\geq 0, \forall j \text{ and } \sum_{j=1}^{n_c} \tilde{m}_j(\varrho(\kappa)) = 1. \end{aligned}$$

$\underline{b}_j(\varrho(\kappa))$  and  $\bar{b}_j(\varrho(\kappa))$  are users-determined functions which satisfy:

- (i)  $0 \leq \underline{b}_j(\varrho(\kappa)) \leq 1$  and  $0 \leq \bar{b}_j(\varrho(\kappa)) \leq 1$ .
- (ii)  $\underline{b}_j(\varrho(\kappa)) + \bar{b}_j(\varrho(\kappa)) = 1$ .

### C. Augmented Closed-loop System

The augmented system consisting of the IT2 FMB system with state delay and the controller is represented as follows:

$$\begin{aligned} \bar{x}(\kappa+1) &= \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} \tilde{w}_i(\rho(\kappa)) \tilde{m}_j(\varrho(\kappa)) [\bar{A}_{ij} \bar{x}(\kappa) \\ &\quad + \bar{A}_{dij} H \bar{x}(\kappa - d(\kappa)) + \bar{D}_i \omega(\kappa)], \\ z(\kappa) &= \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} \tilde{w}_i(\rho(\kappa)) \tilde{m}_j(\varrho(\kappa)) [\bar{C}_{ij} \bar{x}(\kappa) \\ &\quad + \bar{C}_{dij} \bar{x}(\kappa - d(\kappa))], \end{aligned} \quad (6)$$

where  $\bar{x} = [x^T(\kappa), \hat{x}^T(\kappa)]^T$ ,  $H = [I, 0]$ ,

$$\begin{aligned} \bar{A}_{ij} &= \begin{bmatrix} A_i & B_{1i} C_{cj} \\ B_{cj} C_{yi} & A_{cij} \end{bmatrix}, \bar{A}_{dij} = \begin{bmatrix} A_{di} \\ B_{cj} C_{ydi} \end{bmatrix} \\ \bar{C}_{ij} &= [C_i \quad B_{2i} C_{cj}], \bar{C}_{di} = [C_{di} \quad 0], \\ \bar{D}_i &= \begin{bmatrix} D_i \\ 0 \end{bmatrix}. \end{aligned}$$

From (3) and (4), we can obtain

$$\begin{aligned} \bar{x}(\kappa+1) &= \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}(\iota(\kappa)) [\bar{A}_i \bar{x}(\kappa) + \bar{A}_{di} \bar{x}(\kappa - d(\kappa)) \\ &\quad + \bar{D}_i \omega(\kappa)], \\ z(\kappa) &= \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}(\iota(\kappa)) [\bar{C}_i \bar{x}(\kappa) + \bar{C}_{di} \bar{x}(\kappa - d(\kappa))], \end{aligned} \quad (7)$$

where  $h_{ij}(\iota(\kappa)) = \tilde{w}_i(\rho(\kappa)) \tilde{m}_j(\varrho(\kappa))$ ,  $\sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}(\iota(\kappa)) = 1$  and  $\iota(\kappa) = [\rho^T(\kappa), \varrho^T(\kappa)]^T$ .

### D. Definitions

*Definition 1:* The closed-loop state-delay IT2 FMB control system (7) is said to be asymptotically stable under  $\omega(\kappa) = 0$  if the following condition satisfies:

$$\lim_{k \rightarrow \infty} |\bar{x}(\kappa)| = 0.$$

For an asymptotically stable closed-loop state-delay IT2 FMB control system (7), we have  $z \equiv z(\kappa) \in \ell_2[0, \infty)$  when  $\omega \equiv \omega(\kappa) \in \ell_2[0, \infty)$ .

*Definition 2:* [2] The closed-loop state-delay IT2 FMB control system (7) is said to be asymptotically stable with a  $\ell_2 - \ell_\infty$  performance  $\gamma > 0$  if it is asymptotically stable and under zero initial condition ( $\bar{x}(\kappa) = 0, \forall t \leq 0$ ) for all non-zero  $\omega \in \ell_2[0, \infty)$ ,

$$\sup_{\kappa} \sqrt{z^T(\kappa)z(\kappa)} < \gamma \sqrt{\sum_{k=0}^{\infty} \omega^T(\kappa)\omega(\kappa)}.$$

for all  $\omega \in \ell_2[0, \infty)$ .

*Lemma 1:* (Wirtinger inequality) [41], [42] For symmetric matrix  $M > 0$  with appropriate dimensions, integers  $a < b$ , then the inequality blow holds.

$$\begin{aligned} & -(b-a) \sum_{l=\kappa-b}^{\kappa-a-1} \delta^T(l)M\delta(s) \\ & \leq - \begin{bmatrix} \Lambda_0 \\ \Lambda_1 \end{bmatrix}^T \begin{bmatrix} M & 0 \\ 0 & 3M \end{bmatrix} \begin{bmatrix} \Lambda_0 \\ \Lambda_1 \end{bmatrix}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} \delta(\kappa) &= \bar{x}(\kappa+1) - \bar{x}(\kappa), \Lambda_0 = \bar{x}(\kappa-a) - \bar{x}(\kappa-b), \\ \Lambda_1 &= \bar{x}(\kappa-a) + \bar{x}(\kappa-b) - \frac{2}{b-a+1} \left[ \sum_{l=\kappa-b}^{\kappa-a} \bar{x}(l) \right]. \end{aligned}$$

### III. MAIN RESULTS

Based on the preliminaries above, this section will show the main results of this paper.

*Theorem 1:* In terms of the closed-loop state-delay IT2 T-S FMB control system in (7), it is asymptotically stable with a  $\ell_2 - \ell_\infty$  performance  $\gamma$  if there exist matrices  $P, Q, R_1, R_2, \Xi_{ij1}, \Xi_{ij2}, X_{ij1}, X_{ij2}, Y_{ij1}$  and  $Y_{ij2}$  satisfying the following inequalities for  $i = 1, 2, \dots, n_p; j = 1, 2, \dots, n_c$ :

$$\begin{aligned} & \Upsilon_{ij1} + \Xi_{ij1} - \bar{\Xi}_{ij1} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [(\hat{h}_{st}(\hat{k}\sigma_{\hat{k}})(-\bar{\Xi}_{st1}) + \Delta h_{stmin}) \\ & \times (-\bar{\Xi}_{st1}) + (\Delta h_{stmax} - \Delta h_{stmin})X_{ij1} + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st1} \\ & + \Delta \bar{h}_{stmin}\bar{\Xi}_{st1} + (\Delta \bar{h}_{stmax} - \Delta \bar{h}_{stmin})Y_{ij1}] < 0 \end{aligned} \quad (9)$$

$$\begin{aligned} & \Upsilon_{ij1} + \Xi_{ij2} - \bar{\Xi}_{ij2} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [(\hat{h}_{st}(\hat{k}\sigma_{\hat{k}})(-\bar{\Xi}_{st2}) + \Delta h_{stmin}) \\ & \times (-\bar{\Xi}_{st2}) + (\Delta h_{stmax} - \Delta h_{stmin})X_{ij2} + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st2} \\ & + \Delta \bar{h}_{stmin}\bar{\Xi}_{st2} + (\Delta \bar{h}_{stmax} - \Delta \bar{h}_{stmin})Y_{ij2}] < 0 \end{aligned} \quad (10)$$

$$-\bar{\Xi}_{st1} - X_{ij1} < 0, \bar{\Xi}_{st1} - Y_{ij1} < 0 \quad (11)$$

$$-\bar{\Xi}_{st2} - X_{ij2} < 0, \bar{\Xi}_{st2} - Y_{ij2} < 0 \quad (12)$$

$$P > 0, Q > 0, R_1 > 0, R_2 > 0, \bar{\Xi}_{ij1} \geq 0, \bar{\Xi}_{ij2} \geq 0, \quad (13)$$

$$\bar{\Xi}_{ij1} \geq 0, \bar{\Xi}_{ij2} \geq 0, X_{ij1} \geq 0, X_{ij2} \geq 0, Y_{ij1} \geq 0, Y_{ij2} \geq 0, \quad (14)$$

where  $\hat{h}_{ij}(\hat{k}\sigma_{\hat{k}})$  and  $\hat{\bar{h}}_{ij}(\hat{k}\sigma_{\hat{k}})$  are the staircase MFs that approximate to the lower and upper MFs  $\underline{h}_{ij}$  and  $\bar{h}_{ij}$ .  $\underline{h}_{ij}$  and  $\bar{h}_{ij}$  satisfy  $\underline{h}_{ij} \leq h_{ij}(\iota(\kappa)) \leq \bar{h}_{ij}$ , where  $\hat{k}$  and  $\sigma_{\hat{k}}$  denote the number and width of the stairs. Denote  $\Delta \underline{h}_{ij}(\hat{k}\sigma_{\hat{k}}) = \underline{h}_{ij} - \hat{h}_{ij}(\hat{k}\sigma_{\hat{k}})$  and  $\Delta \bar{h}_{ij}(\hat{k}\sigma_{\hat{k}}) = \bar{h}_{ij} - \hat{\bar{h}}_{ij}(\hat{k}\sigma_{\hat{k}})$ . Then  $\Delta \underline{h}_{ijmin} =$

$$\min(\Delta \underline{h}_{ij}(\hat{k}\sigma_{\hat{k}})), \Delta \underline{h}_{ijmax} = \max(\Delta \underline{h}_{ij}(\hat{k}\sigma_{\hat{k}})), \Delta \bar{h}_{ijmin} = \min(\Delta \bar{h}_{ij}(\hat{k}\sigma_{\hat{k}})) \text{ and } \Delta \bar{h}_{ijmax} = \max(\Delta \bar{h}_{ij}(\hat{k}\sigma_{\hat{k}})).$$

$$\Upsilon_1^{11} = -P + \underline{d}^2 R_1 + \bar{d}^2 R_2$$

$$\Upsilon_{ij1}^{12} = P\bar{A}_{ij} - \underline{d}^2 R_1 - \bar{d}^2 R_2$$

$$\begin{aligned} \Upsilon_1^{22} &= -P + (\bar{d} - \underline{d} + 1)H^T QH + \underline{d}^2 R_1 + \bar{d}^2 R_2 \\ &\quad - 4R_1 - 4R_2 \end{aligned}$$

$$\Omega_{ij2} = \begin{bmatrix} \bar{C}_{ij}^T \bar{C}_{ij} - \frac{1}{2}\gamma^2 P & \bar{C}_{ij}^T \bar{C}_{di} \\ * & \bar{C}_{di}^T \bar{C}_{di} - \frac{1}{2}\gamma^2 P \end{bmatrix}$$

*Proof 1:* First, choose a Lyapunov functional candidate as

$$V(\kappa) = V_1(\kappa) + V_2(\kappa) + V_3(\kappa) + V_4(\kappa).$$

$$V_1(\kappa) = \bar{x}^T(\kappa)P\bar{x}(\kappa),$$

$$V_2(\kappa) = \sum_{g=-\bar{d}+1}^{-\underline{d}+1} \sum_{l=\kappa-1+g}^{\kappa-1} \bar{x}^T(l)H^T QH\bar{x}(l),$$

$$\begin{aligned} V_3(\kappa) &= \underline{d} \sum_{g=-\underline{d}}^{-1} \sum_{l=\kappa+g}^{\kappa-1} \delta^T(l)R_1\delta(l) \\ &\quad + \bar{d} \sum_{g=-\bar{d}}^{-1} \sum_{l=\kappa+g}^{\kappa-1} \delta^T(l)R_2\delta(l), \end{aligned}$$

where  $\delta(\kappa) = \bar{x}(\kappa+1) - \bar{x}(\kappa)$ ,

Then calculate the difference of  $V_1(\kappa)$ ,  $V_2(\kappa)$  and  $V_3(\kappa)$ :

$$\Delta V_1(\kappa) = \bar{x}^T(\kappa+1)P\bar{x}(\kappa+1) - \bar{x}^T(\kappa)P\bar{x}(\kappa)$$

$$\begin{aligned} \Delta V_2(\kappa) &\leq \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}(\bar{x}(\kappa)) \\ &\quad \times [(\bar{d} - \underline{d} + 1)\bar{x}^T(\kappa)H^T QH\bar{x}(\kappa) \\ &\quad - \bar{x}^T(\kappa - d(\kappa))H^T QH\bar{x}(\kappa - d(\kappa))] \\ &= \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}(\bar{x}(\kappa)) \\ &\quad \times [(\bar{d} - \underline{d} + 1)\bar{x}^T(\kappa)H^T QH\bar{x}(\kappa) \\ &\quad - \bar{x}^T(\kappa - d(\kappa))Q\bar{x}(\kappa - d(\kappa))] \end{aligned}$$

Using discrete Wirtinger-based inequality in Lemma 1, we have

$$\begin{aligned} \Delta V_3(\kappa) &= \underline{d}^2 \delta^T(\kappa)R_1\delta(\kappa) - \underline{d} \sum_{l=\kappa-\underline{d}}^{\kappa-1} \delta^T(l)R_1\delta(l) \\ &\quad + \bar{d}^2 \delta^T(\kappa)R_2\delta(\kappa) - \bar{d} \sum_{l=\kappa-\bar{d}}^{\kappa-1} \delta^T(l)R_2\delta(l), \\ &\leq \underline{d}^2 \delta^T(\kappa)R_1\delta(\kappa) - \varsigma_1^T(\kappa)\Gamma_1\varsigma_1(\kappa) \\ &\quad + \bar{d}^2 \delta^T(\kappa)R_2\delta(\kappa) - \varsigma_2^T(\kappa)\Gamma_2\varsigma_2(\kappa) \end{aligned}$$

$$\Upsilon_{ij1} = \begin{bmatrix} \Upsilon_1^{11} & \Upsilon_{ij1}^{12} & P\bar{A}_{dij} & 0 & 0 & 0 & 0 & P\bar{D}_i \\ * & \Upsilon_1^{22} & 0 & -2R_1 & -2R_2 & 6R_1 & 6R_2 & 0 \\ * & * & -Q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -4R_1 & 0 & 6R_1 & 0 & 0 \\ * & * & * & * & -4R_2 & 0 & 6R_2 & 0 \\ * & * & * & * & * & -12R_1 & 0 & 0 \\ * & * & * & * & * & * & -12R_2 & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix},$$

where

$$\varsigma_1(\kappa) = \begin{bmatrix} \bar{x}^T(\kappa) & \bar{x}^T(\kappa - \underline{d}) & \frac{1}{\underline{d}+1} \sum_{l=\kappa-\underline{d}}^{\kappa} \bar{x}(l) \end{bmatrix}^T$$

$$\varsigma_2(\kappa) = \begin{bmatrix} \bar{x}^T(\kappa) & \bar{x}^T(\kappa - \bar{d}) & \frac{1}{\bar{d}+1} \sum_{l=\kappa-\bar{d}}^{\kappa} \bar{x}(l) \end{bmatrix}^T$$

$$\Gamma_1 = \begin{bmatrix} 4R_1 & 2R_1 & -6R_1 \\ * & 4R_1 & -6R_1 \\ * & * & 12R_1 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} 4R_2 & 2R_2 & -6R_2 \\ * & 4R_2 & -6R_2 \\ * & * & 12R_2 \end{bmatrix}$$

Notice that

$$0 = -2\bar{x}(\kappa + 1)P \left\{ \bar{x}(\kappa + 1) - \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}(\iota(\kappa)) \right. \\ \left. \times [\bar{A}_{ij}\bar{x}(\kappa) + \bar{A}_{dij}H\bar{x}(\kappa - d(\kappa)) + \bar{D}_i\omega(\kappa)] \right\}$$

Then we have

$$\begin{aligned} & \Delta V(\kappa) - \omega^T(\kappa)\omega(\kappa) \\ & \leq \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}\xi^T(\kappa) \left\{ \Upsilon_{ij1} + \Xi_{ij1} - \bar{\Xi}_{ij1} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [\underline{h}_{st} \right. \\ & \quad \left. \times (-\bar{\Xi}_{st1}) + \bar{h}_{st}\bar{\Xi}_{st1}] \right\} \xi(\kappa) \\ & = \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}\xi^T(\kappa) \left\{ \Upsilon_{ij1} + \Xi_{ij1} - \bar{\Xi}_{ij1} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [\hat{h}_{st}(\hat{k}\sigma_{\hat{k}}) \right. \\ & \quad \left. \times (-\bar{\Xi}_{st1}) + \Delta\underline{h}_{st}(-\bar{\Xi}_{st1}) + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st1} + \Delta\bar{h}_{st}\bar{\Xi}_{st1}] \right\} \xi(\kappa) \\ & = \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}\xi^T(\kappa) \left\{ \Upsilon_{ij1} + \Xi_{ij1} - \bar{\Xi}_{ij1} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [\hat{h}_{st}(\hat{k}\sigma_{\hat{k}}) \right. \\ & \quad \left. \times (-\bar{\Xi}_{st1}) + (\Delta\underline{h}_{st} - \Delta\underline{h}_{stmin} + \Delta\bar{h}_{stmin})(-\bar{\Xi}_{st1}) \right. \\ & \quad \left. + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st1} + (\Delta\bar{h}_{st} - \Delta\bar{h}_{stmin} + \Delta\underline{h}_{stmin})\bar{\Xi}_{st1}] \right\} \xi(\kappa) \\ & = \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}\xi^T(\kappa) \left\{ \Upsilon_{ij1} + \Xi_{ij1} - \bar{\Xi}_{ij1} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [\hat{h}_{st}(\hat{k}\sigma_{\hat{k}}) \right. \\ & \quad \left. \times (-\bar{\Xi}_{st1}) + (\Delta\underline{h}_{st} - \Delta\underline{h}_{stmin})(-\bar{\Xi}_{st1} + X_{ij1} - X_{ij1}) \right. \\ & \quad \left. + \Delta\underline{h}_{stmin}(-\bar{\Xi}_{st1}) + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st1} + (\Delta\bar{h}_{st} - \Delta\bar{h}_{stmin})(\bar{\Xi}_{st1} \right. \\ & \quad \left. + Y_{ij1} - Y_{ij1}) + \Delta\bar{h}_{stmin}\bar{\Xi}_{st1}] \right\} \xi(\kappa) \\ & \leq \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}\xi^T(\kappa) \left\{ \Upsilon_{ij1} + \Xi_{ij1} - \bar{\Xi}_{ij1} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [(\hat{h}_{st}(\hat{k}\sigma_{\hat{k}}) \right. \\ & \quad \left. \times (-\bar{\Xi}_{st1}) + (\Delta\underline{h}_{st} - \Delta\underline{h}_{stmin})(-\bar{\Xi}_{st1} - X_{ij1}) \right. \\ & \quad \left. + \Delta\underline{h}_{stmin}(-\bar{\Xi}_{st1}) + (\Delta\bar{h}_{stmax} - \Delta\bar{h}_{stmin})X_{ij1} \right. \\ & \quad \left. + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st1} + (\Delta\bar{h}_{st} - \Delta\bar{h}_{stmin})(\bar{\Xi}_{st1} - Y_{ij1}) \right. \\ & \quad \left. + \Delta\bar{h}_{stmin}\bar{\Xi}_{st1} + (\Delta\bar{h}_{stmax} - \Delta\bar{h}_{stmin})Y_{ij1}] \right\} \xi(\kappa) \end{aligned}$$

where  $\xi(\kappa) = [\bar{x}^T(\kappa + 1), \bar{x}^T(\kappa), \bar{x}^T(\kappa - d(\kappa)), \bar{x}^T(\kappa - \underline{d}), \bar{x}^T(\kappa - \bar{d}), \frac{1}{\underline{d}+1} \sum_{l=\kappa-\underline{d}}^{\kappa} \bar{x}(l), \frac{1}{\bar{d}+1} \sum_{l=\kappa-\bar{d}}^{\kappa} \bar{x}(l)]^T$ . (9) and (11) indicate

$$\Delta V(\bar{x}(\kappa)) < \omega^T(\kappa)\omega(\kappa). \quad (15)$$

Taking sum from 0 to  $T - 1$  of the both sides of (15), we obtain

$$V(T) - V(0) = \sum_{k=0}^{T-1} \Delta V(\kappa) < \sum_{k=0}^{T-1} \omega^T(\kappa)\omega(\kappa). \quad (16)$$

Noticing  $V(0) = 0$  (zero initial condition), then following inequality is obtained.

$$V(T) < \sum_{k=0}^{T-1} \omega^T(\kappa)\omega(\kappa) \quad (17)$$

Considering zero initial condition and (17), we obtain

$$\bar{x}^T(\kappa)P\bar{x}(\kappa) < V(\kappa) < \sum_{k=0}^{\infty} \omega^T(\kappa)\omega(\kappa), \quad (18)$$

$$\bar{x}^T(\kappa - d(\kappa))P\bar{x}(\kappa - d(\kappa)) < \sum_{k=0}^{\infty} \omega^T(\kappa)\omega(\kappa). \quad (19)$$

Next, similar to above process, we consider

$$\begin{aligned} & z^T(\kappa)z(\kappa) - \frac{1}{2}\gamma^2\bar{x}^T(\kappa)P\bar{x}(\kappa) \\ & - \frac{1}{2}\gamma^2\bar{x}^T(\kappa - d(\kappa))P\bar{x}(\kappa - d(\kappa)) \\ & \leq \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}(\bar{x}(\kappa)) [\bar{C}_{ij}\bar{x}(\kappa) + \bar{C}_{ijd}\bar{x}(\kappa - d(\kappa))]^T \\ & \quad \times [\bar{C}_{ij}\bar{x}(\kappa) + \bar{C}_{ijd}\bar{x}(\kappa - d(\kappa))] - \frac{1}{2}\gamma^2\bar{x}^T(\kappa)P\bar{x}(\kappa) \\ & \quad - \frac{1}{2}\gamma^2\bar{x}^T(\kappa - d(\kappa))P\bar{x}(\kappa - d(\kappa)) \\ & \leq \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} h_{ij}\xi^T(\kappa) \left\{ \Upsilon_{ij2} + \Xi_{ij2} - \bar{\Xi}_{ij2} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [(\hat{h}_{st}(\hat{k}\sigma_{\hat{k}}) \right. \\ & \quad \times (-\bar{\Xi}_{st2}) + (\Delta\hat{h}_{st} - \Delta\hat{h}_{stmin})(-\bar{\Xi}_{st2} - X_{ij2}) \\ & \quad + \Delta\hat{h}_{stmin}(-\bar{\Xi}_{st2}) + (\Delta\hat{h}_{stmax} - \Delta\hat{h}_{stmin})X_{ij2} \\ & \quad + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st2} + (\Delta\bar{h}_{st} - \Delta\bar{h}_{stmin})(\bar{\Xi}_{st2} - Y_{ij2}) \\ & \quad \left. + \Delta\bar{h}_{stmin}\bar{\Xi}_{st2} + (\Delta\bar{h}_{stmax} - \Delta\bar{h}_{stmin})Y_{ij2}] \right\} \xi(\kappa) \end{aligned}$$

where

$$\mu(\kappa) = \begin{bmatrix} \bar{x}^T(\kappa) & \bar{x}^T(\kappa - d(\kappa)) \end{bmatrix}^T. \quad (20)$$

Then, we have

$$\begin{aligned} z^T(\kappa)z(\kappa) & < \frac{1}{2}\gamma^2 \begin{bmatrix} \bar{x}(\kappa) \\ \bar{x}(\kappa - d(\kappa)) \end{bmatrix}^T \begin{bmatrix} P & 0 \\ * & P \end{bmatrix} \\ & \times \begin{bmatrix} \bar{x}(\kappa) \\ \bar{x}(\kappa - d(\kappa)) \end{bmatrix} < \gamma^2 \sum_{k=0}^{\infty} \omega^T(\kappa)\omega(\kappa). \end{aligned}$$

Taking the supremum of  $\sqrt{z^T(\kappa)z(\kappa)}$ , then we have

$$\sup_k \sqrt{z^T(\kappa)z(\kappa)} < \gamma \sqrt{\sum_{k=0}^{\infty} \omega^T(\kappa)\omega(\kappa)}.$$

The proof is completed.

*Theorem 2:* In terms of the state-delay IT2 T-S FMB control system (3), it is asymptotically stable with a  $\ell_2 - \ell_\infty$  performance  $\gamma$  if there exist matrices  $\mathcal{P}$ ,  $\mathcal{L}$ ,  $\mathcal{G}$ ,  $\mathcal{J}$ ,  $\mathcal{A}_{ij}$ ,  $\mathcal{B}_j$ ,

$\mathcal{C}_j$ ,  $\hat{Q}^g$ ,  $\hat{R}_1^g$ ,  $\hat{R}_2^g$ ,  $\hat{X}_{ij1}$ ,  $\hat{X}_{ij2}$ ,  $\hat{Y}_{ij1}$ ,  $\hat{Y}_{ij2}$  with  $i = 1, 2, \dots, r$ ,  $j = 1, 2, \dots, q$ ,  $g = 1, 2, 3$ , satisfying

$$\begin{aligned} & \tilde{\Upsilon}_{ij1} + \tilde{\Xi}_{ij1} - \bar{\Xi}_{ij1} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [(\hat{h}_{st}(\hat{k}\sigma_{\hat{k}})(-\bar{\Xi}_{st1}) + \Delta\hat{h}_{stmin} \\ & \quad \times (-\bar{\Xi}_{st1}) + (\Delta\hat{h}_{stmax} - \Delta\hat{h}_{stmin})\tilde{X}_{ij1} + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st1} \\ & \quad + \Delta\bar{h}_{stmin}\bar{\Xi}_{st1} + (\Delta\bar{h}_{stmax} - \Delta\bar{h}_{stmin})\tilde{Y}_{ij1}] < 0 \quad (21) \end{aligned}$$

$$\begin{aligned} & \tilde{\Upsilon}_{ij1} + \tilde{\Xi}_{ij2} - \bar{\Xi}_{ij2} + \sum_{s=1}^{n_p} \sum_{t=1}^{n_c} [(\hat{h}_{st}(\hat{k}\sigma_{\hat{k}})(-\bar{\Xi}_{st2}) + \Delta\hat{h}_{stmin} \\ & \quad \times (-\bar{\Xi}_{st2}) + (\Delta\hat{h}_{stmax} - \Delta\hat{h}_{stmin})\tilde{X}_{ij2} + \hat{h}_{st}(\hat{k}\sigma_{\hat{k}})\bar{\Xi}_{st2} \\ & \quad + \Delta\bar{h}_{stmin}\bar{\Xi}_{st2} + (\Delta\bar{h}_{stmax} - \Delta\bar{h}_{stmin})\tilde{Y}_{ij2}] < 0 \quad (22) \end{aligned}$$

$$-\bar{\Xi}_{st1} - \tilde{X}_{ij1} < 0, \bar{\Xi}_{st1} - \tilde{Y}_{ij1} < 0 \quad (23)$$

$$-\bar{\Xi}_{st2} - \tilde{X}_{ij2} < 0, \bar{\Xi}_{st2} - \tilde{Y}_{ij2} < 0 \quad (24)$$

$$\mathcal{P} > 0, \mathcal{L} > 0, \hat{R}_1 > 0, \hat{R}_2 > 0, \bar{\Xi}_{ij1} \geq 0, \bar{\Xi}_{ij2} \geq 0,$$

$$\tilde{\Xi}_{ij2} \geq 0, \tilde{\Xi}_{ij1} \geq 0, Q > 0, \mathcal{Q} > 0, \quad (25)$$

$$\begin{bmatrix} \mathcal{P} & I \\ * & \mathcal{L} \end{bmatrix} > 0, \quad (26)$$

$$Q\mathcal{Q} = I, \quad (27)$$

where

$$\tilde{\Upsilon}_1^{11} = - \begin{bmatrix} \mathcal{P} & I \\ * & \mathcal{L} \end{bmatrix} + \underline{d}^2 \begin{bmatrix} \tilde{R}_1^1 & \tilde{R}_1^2 \\ * & \tilde{R}_1^3 \end{bmatrix} + \bar{d}^2 \begin{bmatrix} \tilde{R}_2^1 & \tilde{R}_2^2 \\ * & \tilde{R}_2^3 \end{bmatrix},$$

$$\tilde{\Upsilon}_{ij1}^{12} = \begin{bmatrix} \mathcal{P}A_i + \mathcal{B}_jC_{yi} & A_{ij} \\ A_i & A_i\mathcal{L} + B_{1i}C_j \end{bmatrix} - \underline{d}^2 \begin{bmatrix} \tilde{R}_1^1 & \tilde{R}_1^2 \\ * & \tilde{R}_1^3 \end{bmatrix},$$

$$- \bar{d}^2 \begin{bmatrix} \tilde{R}_2^1 & \tilde{R}_2^2 \\ * & \tilde{R}_2^3 \end{bmatrix}, \tilde{\Upsilon}_{ij1}^{13} = \begin{bmatrix} \mathcal{P}A_{di} + \mathcal{B}_jC_{ydi} \\ A_{di} \end{bmatrix},$$

$$\tilde{\Upsilon}_{i1}^{18} = \begin{bmatrix} \mathcal{P}D_i \\ D_i \end{bmatrix}, \tilde{\Upsilon}_1^{22} = - \begin{bmatrix} \mathcal{P} & I \\ * & \mathcal{L} \end{bmatrix} + \underline{d}^2 \begin{bmatrix} \tilde{R}_1^1 & \tilde{R}_1^2 \\ * & \tilde{R}_1^3 \end{bmatrix}$$

$$+ \bar{d}^2 \begin{bmatrix} \tilde{R}_2^1 & \tilde{R}_2^2 \\ * & \tilde{R}_2^3 \end{bmatrix} - 4 \begin{bmatrix} \tilde{R}_1^1 & \tilde{R}_1^2 \\ * & \tilde{R}_1^3 \end{bmatrix} - 4 \begin{bmatrix} \tilde{R}_2^1 & \tilde{R}_2^2 \\ * & \tilde{R}_2^3 \end{bmatrix}$$

$$\tilde{\Upsilon}_{ij1}^{29} = \sqrt{\bar{d} - \underline{d} + 1} \begin{bmatrix} I \\ \mathcal{L} \end{bmatrix},$$

$$\tilde{R}_1 = \begin{bmatrix} \tilde{R}_1^1 & \tilde{R}_1^2 \\ * & \tilde{R}_1^3 \end{bmatrix}, \tilde{R}_2 = \begin{bmatrix} \tilde{R}_2^1 & \tilde{R}_2^2 \\ * & \tilde{R}_2^3 \end{bmatrix},$$

$$\tilde{\Upsilon}_{ij2} = \begin{bmatrix} -\frac{1}{2}\gamma^2 I & \tilde{\Upsilon}_{ij2}^{12} & \tilde{\Upsilon}_{i2}^{13} \\ * & \tilde{\Upsilon}_2^{22} & 0 \\ * & * & \tilde{\Upsilon}_2^{33} \end{bmatrix},$$

$$\tilde{\Upsilon}_{ij2}^{12} = [C_i \quad C_i\mathcal{L} + B_{2i}C_j], \tilde{\Upsilon}_{i2}^{13} = [C_{di} \quad C_{di}\mathcal{L}],$$

$$\tilde{\Upsilon}_2^{22} = - \begin{bmatrix} \mathcal{P} & I \\ * & \mathcal{L} \end{bmatrix}, \tilde{\Upsilon}_2^{33} = - \begin{bmatrix} \mathcal{P} & I \\ * & \mathcal{L} \end{bmatrix},$$

Then the  $\ell_2 - \ell_\infty$  DOF controller (4) are given by

$$A_{cj} = \mathcal{G}^{-1}(A_{ij} - \mathcal{P}A_i\mathcal{L} - \mathcal{G}B_{cj}C_{yi}\mathcal{L} - \mathcal{P}B_{1i}C_{cj}\mathcal{J}^T)\mathcal{J}^{-T}, \quad (28)$$

$$B_{cj} = \mathcal{G}^{-1}\mathcal{B}_j, \quad (29)$$

$$C_{cj} = C_j\mathcal{J}^{-T}, \quad (30)$$

where  $\mathcal{G}$  and  $\mathcal{J}$  are any nonsingular matrices satisfying

$$\mathcal{G}\mathcal{J}^T = I - \mathcal{P}\mathcal{L}. \quad (31)$$

$$\Upsilon_{ij1} = \begin{bmatrix} \tilde{\Upsilon}_1^{11} & \tilde{\Upsilon}_{ij1}^{12} & \tilde{\Upsilon}_{ij1}^{13} & 0 & 0 & 0 & 0 & \tilde{\Upsilon}_{ij1}^{18} & 0 \\ * & \tilde{\Upsilon}_1^{22} & 0 & -2\tilde{R}_1 & -2\tilde{R}_2 & 6\tilde{R}_1 & 6\tilde{R}_2 & 0 & \tilde{\Upsilon}_{ij1}^{29} \\ * & * & -Q & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -4\tilde{R}_1 & 0 & 6\tilde{R}_1 & 0 & 0 & 0 \\ * & * & * & * & -4\tilde{R}_2 & 0 & 6\tilde{R}_2 & 0 & 0 \\ * & * & * & * & * & -12\tilde{R}_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -12\tilde{R}_2 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -\mathcal{Q} \end{bmatrix},$$

*Proof 2:* Denote  $P^{-1} = L$ . Then re-write  $P$  and  $L$  as partitioned matrices:

$$P \triangleq \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}, L \triangleq \begin{bmatrix} L_1 & L_2 \\ * & L_3 \end{bmatrix}. \quad (32)$$

Define the following nonsingular matrices:

$$K_1 \triangleq \begin{bmatrix} P_1 & I \\ P_2^T & 0 \end{bmatrix}, K_2 \triangleq \begin{bmatrix} I & L_1 \\ 0 & L_2^T \end{bmatrix}. \quad (33)$$

(26) implies that  $I - \mathcal{P}\mathcal{L}$  is not singular. Then there exist  $\mathcal{G}$  and  $\mathcal{L}$  that (31) is satisfied. It is obtained that  $PK_2 = K_1$ ,  $LK_1 = K_2$ . Define

$$\begin{aligned} P_1 &\triangleq \mathcal{P}, P_2 \triangleq \mathcal{G}, L_1 \triangleq \mathcal{L}, L_2 \triangleq \mathcal{J}, \\ \mathcal{A}_{ij} &\triangleq \mathcal{P}\mathcal{A}_i\mathcal{L} + \mathcal{G}\mathcal{B}_{cj}\mathcal{C}_{yi}\mathcal{L} + \mathcal{P}\mathcal{B}_{1i}\mathcal{C}_{cj}\mathcal{J}^T + \mathcal{G}\mathcal{A}_{cij}\mathcal{J}^T, \\ \mathcal{B}_j &\triangleq \mathcal{G}\mathcal{B}_{cj}, \mathcal{C}_j \triangleq \mathcal{C}_{cj}\mathcal{J}^T. \end{aligned}$$

Perform Schur complement and congruence transformations to (21)-(24) by  $\text{diag}\{K_2^{-1}, K_2^{-1}, I, K_2^{-1}, K_2^{-1}, K_2^{-1}, K_2^{-1}, I, I\}$ ,  $\text{diag}\{I, K_2^{-1}, K_2^{-1}\}$ ,  $\text{diag}\{K_2^{-1}, K_2^{-1}, I, K_2^{-1}, K_2^{-1}, I, I\}$  and  $\text{diag}\{I, K_2^{-1}, K_2^{-1}\}$ , respectively, we can get (9)-(12). This completes the proof.

Notice that due to  $Q\mathcal{Q} = I$ , some conditions in Theorem 2 are not convex. For solving this, the following CCL process is introduced.

*Algorithm 1:*

- 1) Set  $\alpha$  and approximation MFs. Get a feasible set  $(\mathcal{P}^0, \mathcal{L}^0, \tilde{X}_{ij1}^0, \tilde{X}_{ij2}^0, \tilde{Y}_{ij1}^0, \tilde{Y}_{ij2}^0, Q^0, \mathcal{Q}^0, \tilde{R}_1^0, \tilde{R}_2^0, \mathcal{A}_{ij}^0, \mathcal{B}_j^0, \mathcal{C}_j^0, \tilde{\Xi}_{ij1}^0, \tilde{\Xi}_{ij2}^0, \tilde{\Xi}_{ij1}^0, \tilde{\Xi}_{ij2}^0)$  satisfying (21) to (26) and (34). Set the counter  $\mathcal{N} = 0$ .

$$\begin{bmatrix} Q^{\mathcal{N}} & I \\ I & \mathcal{Q}^{\mathcal{N}} \end{bmatrix} \geq 0. \quad (34)$$

- 2) Solve the following minimization problem

$$\min \text{tr}(Q\mathcal{Q}^{\mathcal{N}} + Q^{\mathcal{N}}\mathcal{Q})$$

subject to (21) to (26) and (34).

- 3) Replace the calculated matrices  $(\mathcal{P}, \mathcal{L}, \tilde{X}_{ij1}, \tilde{X}_{ij2}, \tilde{Y}_{ij1}, \tilde{Y}_{ij2}, Q, \mathcal{Q}, \tilde{R}_1^g, \tilde{R}_2^g, \mathcal{A}_{ij}, \mathcal{B}_j, \mathcal{C}_j, \tilde{\Xi}_{ij1}, \tilde{\Xi}_{ij2}, \tilde{\Xi}_{ij1}, \tilde{\Xi}_{ij2})$  into (21). If conditions (21) is satisfied and for a sufficiently enough  $\delta > 0$ :

$$|\text{tr}(Q\mathcal{Q}^{\mathcal{N}} + Q^{\mathcal{N}}\mathcal{Q}) - 2n| < \delta, \quad (35)$$

Thus the feasible solutions are found. EXIT.

- 4) If  $\mathcal{N} > \mathcal{M}$  where  $\mathcal{M}$  is the pre-chosen maximum value of iterations by users, EXIT.

- 5) Make  $(\mathcal{P}^{\mathcal{N}}, \mathcal{L}^{\mathcal{N}}, \tilde{X}_{ij1}^{\mathcal{N}}, \tilde{X}_{ij2}^{\mathcal{N}}, \tilde{Y}_{ij1}^{\mathcal{N}}, \tilde{Y}_{ij2}^{\mathcal{N}}, Q^{\mathcal{N}}, \mathcal{Q}^{\mathcal{N}}, \tilde{R}_1^{g\mathcal{N}}, \tilde{R}_2^{g\mathcal{N}}, \mathcal{A}_{ij}^{\mathcal{N}}, \mathcal{B}_j^{\mathcal{N}}, \mathcal{C}_j^{\mathcal{N}}, \tilde{\Xi}_{ij1}^{\mathcal{N}}, \tilde{\Xi}_{ij2}^{\mathcal{N}}, \tilde{\Xi}_{ij1}^{\mathcal{N}}, \tilde{\Xi}_{ij2}^{\mathcal{N}})$  equal to  $(\mathcal{P}, \mathcal{L}, \tilde{X}_{ij1}, \tilde{X}_{ij2}, \tilde{Y}_{ij1}, \tilde{Y}_{ij2}, Q, \mathcal{Q}, \tilde{R}_1^g, \tilde{R}_2^g, \mathcal{A}_{ij}, \mathcal{B}_j, \mathcal{C}_j, \tilde{\Xi}_{ij1}, \tilde{\Xi}_{ij2}, \tilde{\Xi}_{ij1}, \tilde{\Xi}_{ij2})$ ,  $\mathcal{N} = \mathcal{N} + 1$ , and back to Step 2.

#### IV. SIMULATION EXAMPLE

This section will show a simulation example to verify the effectiveness of the obtained theorems. Consider the following state-delay IT2 T-S FMB control system in the form of (3):

$$\begin{aligned} A_1 &= \begin{bmatrix} 1 & 0.3 \\ 0.4 & 0.5 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \\ C_{y1} &= [1 \quad 0.2], C_{yd1} = [0.05 \quad 0.05], D_1 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \\ C_1 &= [1 \quad 0.2], C_{d1} = [0.01 \quad 0.01], B_{21} = 0.2, \\ A_2 &= \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 1 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \\ C_{y2} &= [0.3 \quad 0.1], C_{yd2} = [0.03 \quad 0.03], D_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\ C_2 &= [0.7 \quad 0.4], C_{d2} = [0.01 \quad 0.1], B_{22} = 0.2. \end{aligned}$$

The MFs of (3) are shown in the Table I with  $\kappa \in [0.15, 0.3]$ ,  $\lambda \in [0, 0.05]$ .  $\underline{d} = 3$ ,  $\bar{d} = 4$ . The related lower and upper MFs are demonstrated in Table II.

TABLE I  
THE IT2 MFs OF SYSTEM (3)

The IT2 MFs	
$\tilde{w}_1(x_1) = \begin{cases} \kappa & x_1 \leq -2 \\ -0.5(\kappa - \lambda)(x_1 + \lambda) & -2 < x_1 \leq 0 \\ 0.5(\kappa - \lambda)(x_1 + \lambda) & 0 < x_1 \leq 2 \\ \kappa & 2 < x_1 \end{cases}$	
$\tilde{w}_2(x_1) = 1 - \tilde{w}_1(x_1)$	

Choose the initial state  $x_1(0) = -1$ ,  $x_2(0) = -3$  and give the disturbance input:

$$\omega(\kappa) = \begin{cases} 2e^{-0.1k} & 0 < k \leq 40 \\ 0 & k > 40 \end{cases}$$

The open-loop state responses and the output signal of system (3) is shown graphically in Fig. 1, which shows that

TABLE II  
THE LOWER AND UPPER MFs OF SYSTEM(3)

The lower MFs	
$\underline{w}_1(x_1) = \begin{cases} 0.15 & x_1 \leq -2 \\ -0.075x_1 & -2 < x_1 \leq 0 \\ 0.075x_1 & 0 < x_1 \leq 2 \\ 0 & 2 < x_1 \end{cases}$	
$\underline{w}_2(x_1) = 1 - \bar{w}_1(x_1)$	
The upper MFs	
$\bar{w}_1(x_1) = \begin{cases} 0.3 & x_1 \leq -2 \\ -0.125x_1 + 0.05 & -2 < x_1 \leq 0 \\ 0.125x_1 + 0.05 & 0 < x_1 \leq 2 \\ 0.05 & 2 < x_1 \end{cases}$	
$\bar{w}_2(x_1) = 1 - \underline{w}_1(x_1)$	

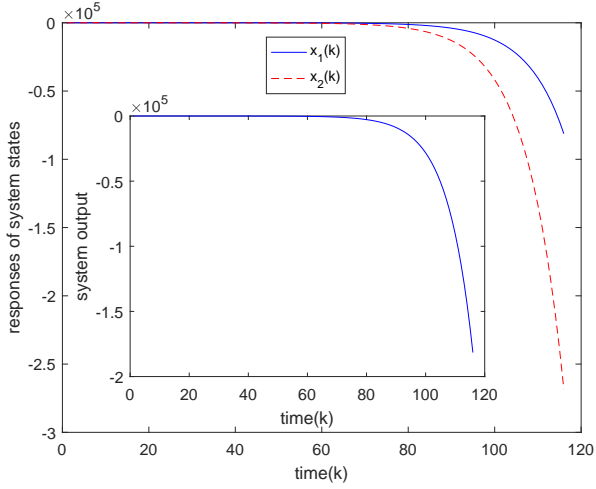


Fig. 1. Open-loop state and output signals of system (3).

the system is open-loop unstable. Then, our goal is to design a DOF controller in the form of (4) for the system (3). The IT2 MFs of the controller are shown in Table. III.

TABLE III  
THE LOWER AND UPPER MFs OF SYSTEM(4)

The lower MFs	
$\underline{m}_1(x_1) = 0.95 - 0.95e^{-\frac{x_1^2}{2}}$	$\underline{m}_2(x_1) = 1 - \bar{m}_1(x_1)$
The upper MFs	
$\bar{m}_1(x_1) = 1 - 0.95e^{-\frac{x_1^2}{0.7}}$	$\bar{m}_2(x_1) = 1 - \underline{m}_1(x_1)$

Then, we apply Theorem 2 to get the output feedback controller. Denote  $\underline{h}_{ij}(x_1) = \underline{w}_i(x_1)\underline{m}_j(x_1)$  and  $\bar{h}_{ij}(x_1) = \bar{w}_i(x_1)\bar{m}_j(x_1)$ . When the number of stairs of the staircase MFs is 5 and 10, respectively. By applying Theorem 2, we get the following results.

Case 1: when the number of stairs of the MFs is 5, the minimum  $\ell_2 - \ell_\infty$  performance is  $\gamma = 0.78$ . The controller

parameters are listed as follows.

$$\begin{aligned} A_{c11} &= \begin{bmatrix} -1.3311 & -1.6743 \\ -1.3820 & -0.4602 \end{bmatrix} \\ A_{c12} &= \begin{bmatrix} -1.3364 & -1.7062 \\ -1.3997 & -0.4710 \end{bmatrix}, \\ A_{c21} &= \begin{bmatrix} -0.1351 & -0.3914 \\ -0.3138 & 0.5952 \end{bmatrix} \\ A_{c22} &= \begin{bmatrix} -0.1350 & -0.3956 \\ -0.3207 & 0.5828 \end{bmatrix}, \\ B_{c1} &= \begin{bmatrix} 1.1458 \\ 1.9655 \end{bmatrix}, B_{c2} = \begin{bmatrix} 1.1383 \\ 1.9947 \end{bmatrix}, \\ C_{c1} &= \begin{bmatrix} -1.0483 & -1.1715 \end{bmatrix} \\ C_{c2} &= \begin{bmatrix} -1.0537 & -1.1924 \end{bmatrix}, \end{aligned}$$

Case 2: when the number of stairs of the MFs is 10, the minimum  $\ell_2 - \ell_\infty$  performance is  $\gamma = 0.66$ . The controller parameters listed are as follows.

$$\begin{aligned} A_{c11} &= \begin{bmatrix} -1.6589 & -1.8853 \\ -1.4080 & -0.4959 \end{bmatrix}, \\ A_{c12} &= \begin{bmatrix} -1.6638 & -1.9191 \\ -1.3740 & -0.4863 \end{bmatrix}, \\ A_{c21} &= \begin{bmatrix} -0.2001 & -0.4319 \\ -0.3619 & 0.5579 \end{bmatrix}, \\ A_{c22} &= \begin{bmatrix} -0.1988 & -0.4363 \\ -0.3511 & 0.5645 \end{bmatrix}, \\ B_{c1} &= \begin{bmatrix} 1.4533 \\ 2.3983 \end{bmatrix}, B_{c2} = \begin{bmatrix} 1.4458 \\ 2.3407 \end{bmatrix}, \\ C_{c1} &= \begin{bmatrix} -1.0162 & -1.0662 \end{bmatrix} \\ C_{c2} &= \begin{bmatrix} -1.0157 & -1.0814 \end{bmatrix}, \end{aligned}$$

To demonstrate the simulation results graphically, taking the case of  $\hat{k} = 10$  as an example, Fig. 2 shows its upper and lower MFs and its corresponding approximate staircase MFs. Fig. 3 shows the corresponding responses of system states.

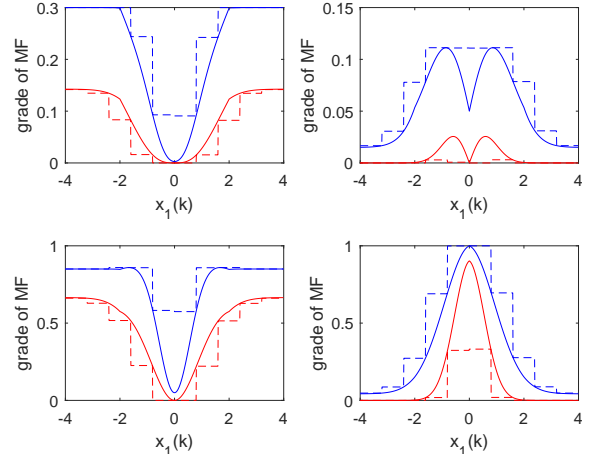


Fig. 2. The lower and upper MFs and its staircase MFs approximation.

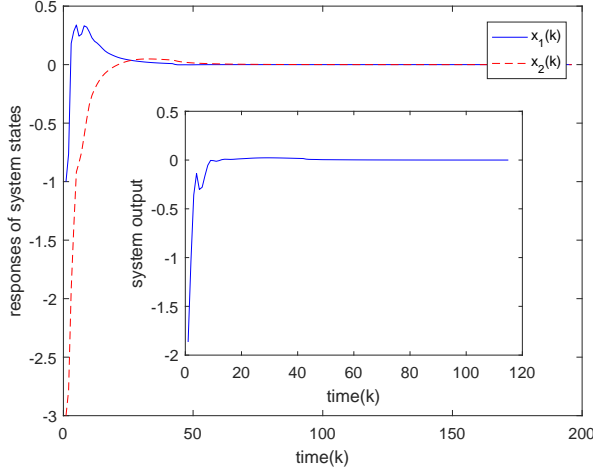


Fig. 3. The system state and output signals under the controller (4).

Simulation results show that the system is successfully stabilized by using Theorem 2. Besides, the increase of approximation degree (which is reflected by the number of staircases) of approximation MFs to the upper and lower bound MFs can improve the  $\ell_2 - \ell_\infty$  performance of IT2 fuzzy controller, which is reasonable. To obtain the minimum  $\gamma$  in Algorithm 1, one can process a linear search of  $\gamma$ . The value of  $\mathcal{M}$  can generally be taken as a relatively large value, but this will also increase the computational burden. From Theorem 2, we can easily get a membership-functions-independent (MFI) condition: Consider state-delay IT2 FMB control system (3), it is asymptotically stable with a  $\ell_2 - \ell_\infty$  performance index  $\gamma$  if there exist matrices  $\mathcal{P} > 0$ ,  $\mathcal{L} > 0$ ,  $\mathcal{G}$ ,  $\mathcal{J}$ ,  $\mathcal{A}_{ij}$ ,  $\mathcal{B}_j$ ,  $\mathcal{C}_j$ ,  $\mathcal{Q} > 0$ ,  $\hat{R}_1^g$ ,  $\hat{R}_2^g$  with  $i = 1, 2, \dots, n_p$ ,  $j = 1, 2, \dots, n_c$ ,  $g = 1, 2, 3$ , satisfying  $\hat{\Upsilon}_{ij1} < 0$ ,  $\hat{\Upsilon}_{ij2} < 0$ ,  $\mathcal{Q}\mathcal{Q} = I$ . Through the CCL process similar to Algorithm 1, the controller can be obtained same as (28)-(31). The controller parameters obtained by corresponding MFI approach are shown as follows:

$$\begin{aligned} A_{c11} &= A_{c12} \begin{bmatrix} -1.0070 & -1.1899 \\ -1.2180 & -0.3320 \end{bmatrix} \\ A_{c21} &= A_{c22} \begin{bmatrix} -0.0896 & -0.3014 \\ -0.2167 & 0.7609 \end{bmatrix} \\ B_{c1} &= B_{c2} \begin{bmatrix} 1.1668 \\ 1.4949 \end{bmatrix}, \\ C_{c1} &= C_{c2} \begin{bmatrix} -0.8757 & -0.9776 \end{bmatrix}. \end{aligned}$$

Fig. 4 shows the controlled system states obtained by membership-functions-independent condition. Table IV compares the minimum performance indexes  $\gamma$  achieved by several methods.

TABLE IV  
OBTAINED MINIMUM  $\gamma$  BY DIFFERENT  $\hat{k}$  OR APPROACHES

Theorem 2 with $\hat{k} = 10$	Theorem 2 with $\hat{k} = 5$	membership-independent approach
$\gamma_{min} = 0.66$	$\gamma_{min} = 0.78$	$\gamma_{min} = 1.02$

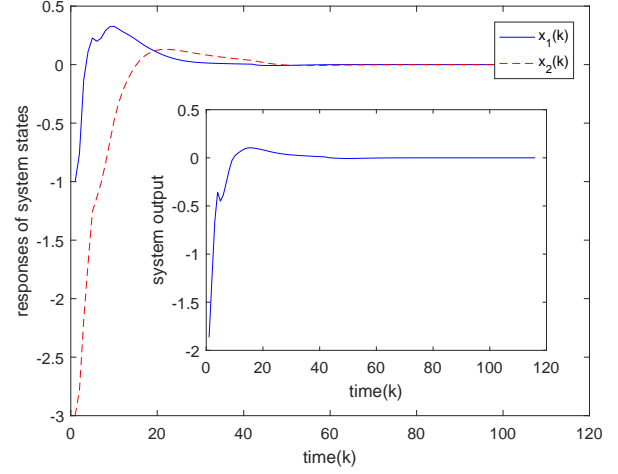


Fig. 4. The system state and output signals under the controller (4) by membership-functions-independent condition.

## V. CONCLUSION

This paper focus on the design of  $\ell_2 - \ell_\infty$  DOF controller for state-delay nonlinear systems via IT2 FMB systems. Though the analysis for the augmented closed-loop state-delay IT2 FMB control system, the LMI-based delay-dependent stability condition is obtained. The analysis of time delay term benefits from the application of Wirtinger-based inequality. Benefit by convex linearization method and projection theorem, a method of DOF controller design satisfying  $\ell_2 - \ell_\infty$  performance is obtained. A MFD method based on staircase MFs is introduced to relax the condition. Simulation results demonstrate that the controller designed in this paper can stabilize the system successfully and the MFD technique can achieve a lower  $\ell_2 - \ell_\infty$  performance index.

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