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Magnetic Monopoles from Global Monopoles in the presence of a Kalb-Ramond Field

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Abstract

A classical solution for electromagnetic monopoles induced by gravitational (global) monopoles in the presence of a (four-dimensional) Kalb-Ramond axion field is found. The magnetic charge of such a solution is induced by a non-zero Kalb-Ramond field strength, prevalent in string theory. Bounds from the current run of the LHC experiments are used to constrain the parameters of the model. Because the production mechanism depends on the details of the model and its ultraviolet completion, such bounds, presently, are only indicative.

I. INTRODUCTION

The existence of magnetic monopoles has been a key question for nearly a century. The current experiments at the LHC, including MoEDAL (which is designed specifically to search for magnetic monopoles and other highly ionising particles), have started to provide interesting new bounds for the mass [1] of such messengers of new physics. Consequently it is timely to examine new ways that monopoles may manifest themselves and the possibility for their detection by MoEDAL and other LHC experiments. We show the novel and surprising possibility that, in four dimensional spacetime, gravitation in the presence of Maxwell and Kalb-Ramond axion fields (the latter being the dual of the field strength of a spin-one antisymmetric tensor field in the massless gravitational multiplet of string theories [2, 3]) can lead to a magnetic monopole with strength determined by the Kalb-Ramond charge.

In the 1873 work of Maxwell, magnetic monopoles did not appear in the magnetic Gauss' law since Nature had electric monopoles but no magnetic monopoles. This asymmetry has puzzled physicists as far back as P. Curie. Indeed, the formulation of electromagnetism in terms of a non-singular 4-vector potential A_μ requires the magnetic induction \vec{B} to be divergence free and no monopole is allowed. Dirac [4] showed that a monopole is possible with a *singular* gauge potential. He considered the magnetic field from a solenoid in the limit of an arbitrarily thin semi-infinite solenoid. In this limit the vanishingly small solenoid became the Dirac string. Such a string cannot be detected through the Aharonov-Bohm effect once the magnetic charge g and the electric charge e satisfy (in natural units) $eg = \frac{n}{2}$ where n is a positive integer. The end of the solenoid becomes the monopole. The energy of the monopole is not finite.

A major paradigm shift in the theory of monopoles was initiated independently by 't Hooft and Polyakov [5]. They considered a model due to Georgi and Glashow [6] which is a field theory with spontaneously broken gauge symmetry. The non-Abelian gauge group of the Georgi-Glashow model is $SU(2)$. This gauge symmetry is spontaneously broken down to the $U(1)$ gauge group of electromagnetism by using a scalar field in the adjoint representation. Monopole solutions with finite energy and quantised magnetic charges were found.

The paradigm of large extra dimensions [7] implied the possibility of a lower Planck scale for gravitational physics, even as low as a few TeV, and so, in principle, gravitational effects

may become observable at the LHC (which is already in its RUN II phase, operating at collision energies of 13 TeV in the centre of mass frame). In particular micro-black holes could be produced and decay rapidly. So far, however, there is no current evidence to support this scenario. Nevertheless, from a theoretical point of view, and for future collider or cosmic searches, a relatively low Planck scale opens up a plethora of possibilities, *e.g.* in the field of black holes and other space-time defects such as self-gravitating magnetic monopoles.

The magnetic monopoles discussed by 'tHooft and Polyakov arise in gauge theories in the absence of gravity. Global gravitational (non-magnetic) monopoles have been found as classical solutions of a coupled system of gravity and a self-interacting scalar field in the adjoint representation of a global $O(3)$ group, but in the absence of a gauge field [8]. The global monopole is a solution for the gravitational field similar to a Schwarzschild black hole with an asymptotic space-time which is Minkowski but with a deficit angle. A necessary condition for a gravitational monopole to behave also as a *magnetic* monopole configuration is to couple covariantly a local $U(1)$ gauge field strength to gravity. Calculations show that this is not a sufficient condition to determine whether a *magnetic* monopole is induced. Since in our model the scalar field is not related to electro-weak symmetry breaking, on phenomenological grounds the symmetry breaking parameter can be chosen to allow a monopole with a mass accessible to the LHC, without the need to invoke large extra dimensions.

In some (closed) string theories [3], a 2-form gauge field, the spin-1 Kalb-Ramond (KR) gauge field, appears in the massless spectrum. It is well known that, for the bosonic *gravitational* part of low-energy string effective actions, the Kalb-Ramond field strength, which the string effective actions depend upon, on account of the Kalb-Ramond gauge invariance, can be thought of as providing a source of torsion [2]. Recently [9], in string-inspired effective theories, we have considered some cosmological implications of a dual formulation of a time-dependent four-dimensional Kalb-Ramond field, in connection with the generation of matter-antimatter asymmetry in the Universe. In four space-time dimensions, the dual of the Kalb-Ramond field strength is a pseudoscalar axion-like field. This formulation will be used here.

We will investigate the role of *static* configurations of the (dual of the) Kalb-Ramond field strength in inducing monopole solutions with a non-trivial magnetic charge. Our effective field theory contains the gravitational metric tensor, a triplet of scalar fields in the adjoint representation of the $O(3)$ group (necessary for the the spontaneous breaking of the

$O(3)$ symmetry), a local $U(1)$ 2-form, the electromagnetic field strength, and a (static) 3-form, the Kalb-Ramond field strength. It is also *necessary* to introduce into the model an additional $O(3)$ -singlet scalar field, which is stabilised to a constant value. In the context of string theory, this is the dilaton (spin zero part of the gravitational massless string multiplet), and in principle its stabilisation could be guaranteed by an appropriate (string-loop induced) dilaton potential. However, one may imagine phenomenological scenarios independent of string theory, in which this extra scalar is ultraheavy, is stabilised by its own potential, and is coupled only gravitationally to the other scalar and gauge fields of the model. For this model there is a solution whereby the magnetic charge of the monopole is determined by the strength of the Kalb-Ramond field ¹. As we shall show below, within the context of string theory, it is the dilaton equation of motion that provides the link between the electromagnetic and the Kalb-Ramond field strengths. This link leads to the connection between the magnetic and the “Kalb-Ramond torsion” charges.

Since our treatment is inspired by both the 't Hooft-Polyakov (HP) monopole solution [5] and the (self-gravitating) non-magnetic global monopole of [8], we will briefly review the main features of these solutions in Sec. II. This will be followed in Sec. III by an introduction of the Lagrangian for our model, a derivation of the coupled classical equations of the model, and an asymptotic analysis of the equations of the model for small and large (radial) distances from the monopole centre. We shall demonstrate analytically the existence of magnetic monopole solutions in these two regimes; we estimate the monopole mass, which agrees in order of magnitude with the non-magnetic global monopole of [8]. The concluding Section IV discusses the phenomenology of the magnetic monopole solution, and makes some conjectural remarks on the possibility of its production and detection at the LHC.

¹ It has been shown in [10], that the structure of the global (non-magnetic) monopole [8] remains intact in the presence of the Kalb-Ramond field. Our model differs by the inclusion of a $U(1)$ -gauge field, antisymmetric tensor degrees of freedom and a singlet scalar field; the non-trivial Kalb-Ramond field strength determines the magnetic charge of the monopole. In simple string-theory sigma models with just lowest-order graviton and antisymmetric tensor fields, the Kalb-Ramond field strength can be absorbed as torsion inside a generalised scalar curvature. However in the presence of other fields this is not the case; so, in four space-time dimensions, we consider the Kalb-Ramond field to be a massless axion-like field, and the gravitational part of our Lagrangian is kept torsion free.

II. THE 'T HOOFT-POLYAKOV AND GLOBAL MONOPOLE SOLUTIONS

We will review the basic features of the HP [5] and global monopole [8] solutions, which are relevant for our model. We commence with the original HP monopole within the context of an $SU(2)$ spontaneously broken gauge theory with adjoint ‘‘Higgs’’ triplet fields. Such solutions can be generalised to Grand Unified Theory (GUT) groups, such as $SU(5)$, leading to realistic particle phenomenology, and providing GUT monopoles with masses near the GUT scale ($\sim 10^{14} - 10^{15}$ GeV). Such cosmic monopoles are expected to have been diluted by inflation.

A. The ‘t Hooft-Polyakov $SU(2)$ Monopole

The fields in the HP $SU(2)$ -gauge-theory model [5] are a scalar field $\phi^a(t, \vec{x})$ and gauge field $A_\mu^a(t, \vec{x})$ where $a (= 1, 2, 3)$ is a $SU(2)$ index. The Lagrangian density $\mathcal{L}(t, \vec{x})$ is

$$\mathcal{L}(t, \vec{x}) = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - \frac{1}{4}\lambda(\phi^a\phi^a - \eta^2)^2. \quad (2.1)$$

The field tensor $F_{\mu\nu}^a$ is

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c \quad (2.2)$$

where ϵ^{abc} is the anti-symmetric Levi-Civita symbol; $D_\mu\phi^a$ is defined by

$$D_\mu\phi^a = \partial_\mu\phi^a + g\epsilon^{abc}A_\mu^b\phi^c \quad (2.3)$$

is the covariant derivative. The model parameters are $g, \lambda > 0$ and η . (The covariant derivative of $F_{\mu\nu}^a$ is defined in an analogous fashion.) The equations of motion that follow from \mathcal{L} are

$$D_\mu F^{a\mu\nu} = g\epsilon^{abc}(D^\nu\phi^b)\phi^c \quad (2.4)$$

and

$$D_\mu D^\mu\phi^a = -\lambda(\phi^b\phi^b)\phi^a + \lambda\eta^2\phi^a. \quad (2.5)$$

The ansatz used for a static solution of (2.4) and (2.5) (in the gauge $A_0^a(\vec{x}) = 0$) is

$$\phi^a(\vec{x}) = \delta_{ia}\left(\frac{x^i}{r}\right)F(r) \quad (2.6)$$

and

$$A_i^a(\vec{x}) = \epsilon_{aij}\left(\frac{x^j}{r}\right)W(r) \quad (2.7)$$

where $a, i, j = 1, 2, 3$ and $r = |\vec{x}|$. Furthermore the boundary conditions adopted are

$$F(r) \rightarrow \eta \text{ and } W(r) \rightarrow 1/gr \quad (2.8)$$

as $r \rightarrow \infty$. 't Hooft and Polyakov found that

$$grW(r) = 1 - \frac{rg\eta}{\sinh(g\eta r)} \text{ and } grF(r) \sim \frac{rg\eta}{\tanh(g\eta r)} - 1. \quad (2.9)$$

The electromagnetic field tensor $f_{\mu\nu}$ is defined to be [5]

$$f_{\mu\nu} = \hat{\phi}^a F_{\mu\nu}^a - \frac{1}{g} \epsilon^{abc} \hat{\phi}^a D_\mu \hat{\phi}^b D_\nu \hat{\phi}^c \quad (2.10)$$

where $\hat{\phi}^a = \phi^a / |\vec{\phi}|$ and $|\vec{\phi}| = (\sum_{a=1}^3 \phi^a \phi^a)^{1/2}$. The magnetic induction, determined by $B_k = \frac{1}{2} \epsilon_{kij} f_{ij}$, has an asymptotic behaviour

$$\vec{B}(\vec{x}) \rightarrow \vec{x}/gr^3 \quad (2.11)$$

as $r \rightarrow \infty$ which corresponds to a magnetic monopole of strength $1/g$. Moreover, as $r \rightarrow \infty$, where $\phi^a \rightarrow \eta \frac{x^a}{r}$ (2.8), one can show that [11]

$$\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial^\nu f^{\rho\sigma} = \frac{1}{2g} \epsilon_{\mu\nu\rho\sigma} \epsilon_{abc} \partial^\nu \hat{\phi}^a \partial^\rho \hat{\phi}^b \partial^\sigma \hat{\phi}^c \equiv \frac{k_\mu}{g} \quad (2.12)$$

where k_μ is a topological current. The topological charge $Q = \int d^3x k_0$ is quantised to be an integer n and the monopole charge is n/g . The HP monopole has $n = 1$. It should be noted that $f_{\mu\nu}$ does not satisfy the Bianchi identity ².

² To be precise, following [11] one can construct a version $f_{\mu\nu}^{\text{reg}}$ of the 't Hooft electromagnetic tensor which, unlike (2.10), is not singular at the zeros of the Higgs triplet, and is finite everywhere,

$$F_{\mu\nu}^a = f_{\mu\nu}^{\text{reg}} \frac{\phi^a}{\eta} \Rightarrow f_{\mu\nu}^{\text{reg}} = \frac{\phi^a}{\eta} F_{\mu\nu}^a = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + \frac{1}{\eta^3 g} \epsilon_{abc} \phi^a \partial_\mu \phi^b \partial_\nu \phi^c, \quad \phi^a \phi^a = \eta^2. \quad (2.13)$$

The presence of the vector potential \mathcal{A}_μ stems from the fact [11] that a general solution of the equation $D_\mu \phi^a = 0$ for $\phi^a \phi^a = \eta^2$ reads: $A_\mu^a = \frac{1}{\eta^2 g} \epsilon_{abc} \phi^b \partial_\mu \phi^c + \frac{1}{\eta} \phi^a \mathcal{A}_\mu$, with \mathcal{A}_μ an arbitrary four vector, which can be identified with the electromagnetic potential A_μ^{em} , since for $\phi^a \phi^a = \eta^2$, the solution yields $\mathcal{A}_\mu = A_\mu^{\text{em}} \frac{\phi^a}{\eta}$. Upon substituting the above ansatz for A_μ^a in the expression for the non-Abelian ($SU(2)$ group) field strength $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c$, one obtains the structure (2.13) for $f_{\mu\nu}^{\text{reg}}$. Both definitions, (2.10) and (2.13), coincide at the spatial boundary $r \rightarrow \infty$. In the topologically trivial sector, where the ϕ -dependent terms on the right-hand-side of the definition of $f_{\mu\nu}^{\text{reg}}$ in (2.13) vanish, one obtains the standard expression for the ‘‘electromagnetic’’ field strength $f_{\mu\nu}^{\text{reg}}$ in terms of regular gauge potentials and the Bianchi identity is satisfied. However, in the presence of monopoles, one obtains a violation of the Bianchi identity for the dual of $f_{\mu\nu}^{\text{reg}}$, as given in (2.12) above. Equivalently, the latter result may be understood as a consequence of the fact that, in the presence of monopoles, the electromagnetic tensor can be formally expressed in terms of singular potentials at the monopole centre, $r \rightarrow 0$, using a construction outlined by Halpern [12], which we shall adopt in this work.

B. The (self-gravitating) $O(3)$ Global Monopole Solution

The scalar fields in the $O(3)$ global monopole solution of Barriola and Vilenkin [8] (BV) also form a triplet χ^a , $a = 1, 2, 3$ which parametrise the spontaneous breaking of a global $O(3)$ symmetry down to a global $U(1)$, by means of an appropriate potential, in which the scalar field triplet acquires a non-trivial vacuum expectation value η . Moreover the model was embedded into Einstein gravity. The Lagrangian of the model is given by

$$L = (-g)^{1/2} \left\{ \frac{1}{2} \partial_\mu \chi^a \partial^\mu \chi^a - \frac{\lambda}{4} (\chi^a \chi^a - \eta^2)^2 - R \right\} \quad (2.14)$$

where $g_{\mu\nu}$ is the (four space-time dimensional) metric tensor, $g = \det(g_{\mu\nu})$ its determinant and R is the Ricci scalar for $g_{\mu\nu}$ ³.

As a result of the Goldstone theorem, such monopoles have massless Goldstone fields associated with them, which have energy densities that scale like $1/r^2$ with the radial distance from the monopole core. This results in a linear divergence of the monopole total energy density (that is mass), which is a characteristic feature of such solutions, in a way similar to the linearly divergent energy of a cosmic string. In the original work of [8] only estimates of the total monopole mass have been given by considering the solution in the exterior of the monopole core, whose size in flat space time has been estimated to be of order $\delta \sim \lambda^{-1/2} \eta^{-1}$, leading to a heuristic mass estimate of order $M_{\text{core}} \sim \delta^3 \lambda \eta^4 = \lambda^{-1} \eta$. The presence of the monopole curves the space-time exterior, and these estimates have to be rethought. However, the main argument of [8] was that gravitational effects are weak for $\eta \ll M_{\text{P}}$, the Planck mass; this is certainly the case of interest for η of order of a few TeV, the case of relevance to new physics searches at LHC. In this sense, BV argued that the flat space-time estimates for the core mass might still be valid, as an order of magnitude estimate. Outside the monopole core, BV used approximate asymptotic analysis of the Einstein equations ,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}^x \quad (2.15)$$

where $T_{\mu\nu}^x$ is the matter stress tensor derived from the Lagrangian (2.14), and the equations of motion for the scalar fields χ^a , $a = 1, 2, 3$. The scalar field configuration for a global

³ Our conventions and definitions [13, 14] throughout this work are: $(+, -, -, -)$ for the signature of the metric, the Riemann tensor is defined as $R^\lambda{}_{\mu\nu\sigma} = \partial_\nu \Gamma^\lambda{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\sigma} \Gamma^\lambda{}_{\rho\nu} - (\nu \leftrightarrow \sigma)$, and the Ricci tensor and scalar are given by $R_{\nu\alpha} = R^\lambda{}_{\nu\lambda\alpha}$ and $R = g^{\mu\nu} R_{\mu\nu}$ respectively.

monopole is [8]

$$\chi^a = \eta f(r) \frac{x^a}{r}, \quad a = 1, 2, 3 \quad (2.16)$$

where x^a are spatial Cartesian coordinates, $r = \sqrt{x^a x^a}$ is the radial distance, and $f(r) \rightarrow 1$ for $r \gg \delta$. So at such large distances, the amplitude squared of the scalar field triplet approaches the square of the vacuum expectation value η , $\chi^a \chi^a \rightarrow \eta^2$. The reader should note the similarity between the expression (2.16) and corresponding one for the HP monopole (2.6).

As a result of the symmetry breaking, the space-time, for $r \gg \delta$, differs from the standard Schwarzschild metric corresponding to a massive object with mass M_{core} (assuming that all the mass of the monopole is concentrated in the core's interior):

$$ds^2 = \left(1 - 8\pi G_N \eta^2 - \frac{2G_N M_{\text{core}}}{r}\right) dt^2 - \frac{dr^2}{1 - 8\pi G_N \eta^2 - \frac{2G_N M_{\text{core}}}{r}} + r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad r \gg \delta, \quad (2.17)$$

where (r, θ, ϕ) are spherical polar coordinates. The Schwarzschild metric is obtained in the unbroken phase ($\eta \rightarrow 0$). In the asymptotic limit $r \rightarrow \infty$, upon appropriate rescaling of the time $t \rightarrow (1 - 8\pi G_N \eta^2)^{-1/2} t'$, and radial coordinate r , $r \rightarrow (1 - 8\pi G_N \eta^2)^{1/2} r'$, the space-time (2.17) becomes a Minkowski space-time with a conical *deficit solid angle* $\Delta\Omega = 8\pi G_N \eta^2$:

$$ds^2 = dt'^2 - dr'^2 - \left(1 - 8\pi G_N \eta^2\right) r'^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad r \gg \delta. \quad (2.18)$$

The space-time (2.18) is not flat, since the scalar curvature behaves as $R \propto 16\pi G_N \eta^2 / r^2$. The presence of such a monopole-induced deficit solid angle, can have important physical consequences for scattering processes in such space-times: the scattering amplitude in the forward direction is very large [15] in angular regions of order of the deficit angle (or equivalently the squared ratio of the monopole mass to the Planck mass).

After the initial work of [8], a debate has taken place regarding the stability of the configuration [16], which is still ongoing; we shall comment on this debate briefly at the end of our article. Subsequent to the work of [8] more detailed analysis of the gravitational back reaction effects of such defects has been performed, by requiring a matching of the solutions of the non-linear coupled system of gravitational and matter equations at the core radius; thus the core size is determined dynamically, rather than heuristically from flat space arguments as in the work of [8]. Indeed, in [17], the core radius $r_c = 2\lambda^{-1/2}\eta^{-1}$ for

the self-gravitating solution was found by matching an exterior Schwarzschild-like metric

$$ds^2 = \left(1 - 8\pi G_N \eta^2 - \frac{2 G_N M}{r}\right) dt^2 - \left(1 - 8\pi G_N \eta^2 - \frac{2 G_N M}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 ,$$

to an interior local de Sitter metric

$$ds^2 = \left(1 - \mathcal{H}^2 r^2\right) dt^2 - \left(1 - \mathcal{H}^2 r^2\right)^{-1} dr^2 - r^2 d\Omega^2$$

where M denotes the monopole mass and $\mathcal{H}^2 = \frac{8\pi G_N \lambda \eta^4}{12}$ the de Sitter parameter. Here η denotes a quantity with dimension of mass. Unfortunately such a matching yields a negative mass for the monopole, $M \sim -6\pi\lambda^{-1/2}\eta < 0$ ⁴. The interpretation of this sign in [17] is based on the repulsive nature of gravity induced by the vacuum-energy H^2 provided by the global monopole. Moreover it has been argued [17] that this interpretation is consistent with the monopole being an entity with complicated structure rather than an elementary particle-like excitation. Such a construction with negative mass would not be of relevance to collider physics⁵.

As compared to the model for the global monopole, our model (see the next section III) includes additional fields, which allow for a positive mass solution, albeit from a ‘bag model’ standpoint. Our model contains the (non-gauged) scalar field triplet χ^a of the global monopole model, an Abelian $U(1)$ gauge field of electromagnetism with Maxwell tensor $f_{\mu\nu}$, an extra $O(3)$ singlet scalar field and the tensor $H_{\mu\nu\rho}$ (the field strength of a 2-form $B_{\mu\nu}$, the antisymmetric tensor (Kalb-Ramond) field of spin 1). The Maxwell tensor $f_{\mu\nu}$ has a structure similar to (2.10); however, in our case, as we shall discuss later, the first term on the right-hand-side of (2.10) is absent, since we do not have $SU(2)$ gauge fields. The second term will involve the scalar triplet field χ^a , as well as the $O(3)$ singlet scalar field (either a

⁴ The motivation for using such a matching comes from the observation that, at the origin ($r \rightarrow 0$), the Higgs potential for the scalars leads to a cosmological constant $\propto \eta^4$, since any “matter” scalar fields go to zero. However, if a black hole or other geometric singularity is present as $r \rightarrow 0$, as in our case of the induced Reissner-Nordström geometry due to the antisymmetric tensor and electromagnetic fields (to be discussed below), the space-time is different for small r ($r \rightarrow 0$). The argument leading to negative mass would then not hold.

⁵ A classification of the space-times arising from a self-gravitating global monopole solution of the type considered in [8] and in [17], *i.e.* in field theories with only a triplet of Higgs-type scalar fields and Ricci scalar curvature, has been given in [18], where it was argued that, upon requiring *regularity at the centre* of the monopole, but otherwise independently of the shape of the Higgs potential, the metric can contain at most one horizon, and, in case there is an horizon, the global space-time structure is that of a de Sitter space-time.

constant dilaton or an ultraheavy scalar), stabilised to a constant value (e.g. the minimum value of a scalar potential).

III. THE MODEL AND ITS BACKGROUND

In this section we discuss our model for the magnetic monopole and the analytic form of its asymptotic solutions, for large and small distances from the monopole core. We will first describe the Lagrangian of the model, which may be viewed either as purely phenomenological or as inspired by the bosonic sector of closed string theories upon compactification to four large target-space-time dimensions. The Kalb-Ramond antisymmetric tensor field strength will determine the magnetic charge of the monopole solution [2]. In four dimensions the Kalb-Ramond field is equivalent to a massless pseudoscalar (gravitational axion-like) field $b(x)$ [19].

A. A Model for a Self-gravitating Global Monopole with Kalb-Ramond Torsion

Our model is given by the effective 4-dimensional Lagrangian density L involving the graviton $g_{\mu\nu}$, the antisymmetric Kalb-Ramond field $B_{\mu\nu}$, the electromagnetic field tensor $f_{\mu\nu}$, a real scalar field Φ , whose origin and importance will be discussed in detail below, and the triplet Higgs-like scalar χ^a . The latter is associated with the spontaneous breaking of a global $O(3)$ group down to a global $O(2)$. The Goldstone theorem implies the existence of massless Goldstone Bosons in such a case, which will be neutral under the Standard Model group. As we shall discuss later, our monopole solutions are expected [8] to lose energy and annihilate (with their antimonopoles) through such Goldstone radiation. The Lagrangian density reads:

$$L = (-g)^{1/2} \left\{ \frac{1}{2} \partial_\mu \chi^a \partial^\mu \chi^a - \frac{\lambda}{4} (\chi^a \chi^a - \eta^2)^2 - R + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - \frac{1}{12} e^{-2\gamma\Phi} H_{\rho\mu\nu} H^{\rho\mu\nu} - \frac{1}{4} e^{-\gamma\Phi} f_{\mu\nu} f^{\mu\nu} \right\} \quad (3.1)$$

where γ is a real constant, $g = \det(g_{\mu\nu})$, R is the Ricci scalar for $g_{\mu\nu}$, and the antisymmetric tensor field strength $H_{\rho\mu\nu} = \partial_{[\rho} B_{\mu\nu]}$, where the brackets $[\dots]$ denote total antisymmetrization of the respective indices. The quantity $\eta > 0$ plays the role of the vacuum expectation value of the Higgs-field in the symmetry broken phase. We shall assume that a singular gauge

field A_μ (up to a gauge transformation) may be associated with $f_{\mu\nu}$, on using a construction outlined by Halpern [12].

In the case of string-inspired models [2, 3], the constant $\gamma = 1$. In such a case Φ is the dilaton field of the massless string multiplet, and $V(\Phi)$ is a dilaton potential, possibly generated by string loops - the dilaton potential is absent at tree-level in string theory. In addition to string theory, we shall also consider another version of the model, in which $\gamma = 0$. In such a case the field Φ may be a superheavy real scalar field that is stabilised by its potential $V(\Phi)$ to be some constant value. We also assume that, once the scalar field (or dilaton) is stabilised, its potential vanishes (similar to the case of a Higgs-like potential), so there are no contributions to the stress tensor. We shall see that the presence of the extra scalar degree of freedom in either case is essential for the association of the ‘‘Kalb-Ramond torsion charge’’ with the magnetic charge of the monopole. Notice that in our model the χ^a -matter in the Einstein frame is assumed to be decoupled from the dilaton ⁶ or heavy scalar.

Let us first proceed with the $\gamma = 1$ (string) case. The Lagrangian (3.1) is in the Einstein frame [3, 19], where the Einstein-Hilbert curvature term R in the action is canonically normalised. Leaving aside, for the moment, the dilaton equation of motion, the equations of motion for the remaining fields are deduced from (3.1):

$$g^{\nu\beta}\chi_{,\nu\beta}^a + \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\nu\beta}\right)\chi_{,\beta}^a = -\lambda\left(\chi^b\chi^b - \eta^2\right)\chi^a, \quad (3.2)$$

⁶ In the context of string theory effective actions, this can be achieved as follows: one starts, in the σ -model-frame for the scalar triple effective action, which has the form:

$$\int d^4x e^{-\Phi} \sqrt{-G^S} \left[\dots + \frac{1}{2} \partial^\mu \chi^a \partial_\mu \chi^a - \frac{1}{4} \tilde{\lambda}(\Phi) \lambda \left(\chi^a \chi^a - \eta^2 \right)^2 \right],$$

where \dots denote the rest of the fields, $\tilde{\lambda}(\Phi)$ is an appropriate function of the dilaton, to be determined, and $e^{\Phi/2}$ is the string coupling in our normalisation. The above form of the action is a standard one in a tree-level string theory model, propagating on a closed spherical world sheet, with the overall factor $e^{-\Phi}$ indicating precisely the appropriate power of the (inverse) string coupling pertinent to this genus two world-sheet surface; G^S denotes the determinant of the σ -model-frame metric of the space-time which is related to the Einstein-frame metric, $g_{\mu\nu}$, by $G_{\mu\nu}^S = e^\Phi g_{\mu\nu}$. Passing to the Einstein frame and choosing the function $\tilde{\lambda}(\Phi) = e^{-\Phi}$, defines the scalar sector self-interaction in such a way that the self-coupling is strong for weak string couplings, we obtain the decoupling of the scalar-triplet- χ sector from the dilaton in (3.1).

$$\nabla_\kappa \left(e^{-2\gamma\Phi} H^{\kappa\beta\gamma} \right) = 0, \quad (3.3)$$

$$\nabla_\lambda \left(e^{-\gamma\Phi} f^{\lambda\kappa} \right) = 0, \quad (3.4)$$

and

$$G_{\mu\nu} = g_N \Theta_{\mu\nu} \quad (3.5)$$

where $G_{\mu\nu}$ is the Einstein tensor, $\Theta_{\mu\nu}$ is the energy-momentum tensor and

$$g_N = 8\pi G_N, \quad (3.6)$$

where $G_N = 1/M_P^2$ is Newton's constant, with M_P the Planck mass. These equations are supplemented with the Bianchi identity for the Kalb-Ramond field strength, stemming from its definition:

$$\epsilon^{\mu\nu\lambda\rho} \partial_\rho H_{\mu\nu\lambda} = 0. \quad (3.7)$$

Furthermore in 4-dimensions the Kalb-Ramond field strength is dual to a pseudoscalar (“axion”-like) field b ⁷:

$$H_{\mu\nu\lambda} = e^{2\Phi} \epsilon_{\mu\nu\lambda}{}^\sigma \partial_\sigma b, \quad (3.8)$$

where

$$\tilde{\epsilon}_{\mu\nu\rho\sigma} = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma}, \quad (3.9)$$

is the flat space-time Levi-Civita symbol, $\epsilon_{\mu\nu\rho\sigma}$ is the covariant Levi-Civita tensor density, with $\tilde{\epsilon}_{\mu\nu\rho\sigma} \tilde{\epsilon}_{0123} = +1$ etc. (and also $\epsilon^{\mu\nu\rho\sigma} = \sqrt{-g} \tilde{\epsilon}^{\mu\nu\rho\sigma}$). The form (3.8) for the field strength satisfies (3.3) automatically, taking into account that the gravitational covariant derivative is defined in terms of the usual symmetric Christoffel symbol.

It is important to note that in our approach we shall concentrate on the *dual theory*, where the physical degree of freedom for the Kalb-Ramond field is the axion $b(x)$, defined

⁷ In string theories [3], the field strength $H_{\mu\nu\rho}$, in the presence of gauge fields A_μ , is no longer given only by the curl of $B_{\mu\nu}$ but contains additional parts proportional to the Chern-Simons three form $A \wedge F$. Such terms lead to higher derivative terms in the string effective action, and are ignored in our model. Their inclusion for Abelian gauge fields could lead to additional interesting electromagnetic effects [20], which, however, are not of interest to us here.

in (3.8)⁸. In the context of the dual theory, when one considers the dilaton equations of motion from the Lagrangian (3.1) with $\gamma = 1$, one has to take into account the non-trivial variation $\delta H_{\mu\nu\rho}/\delta\Phi = 2H_{\mu\nu\rho} = 2e^{2\Phi}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma b$. With this in mind, it is then straightforward to see that the dilaton equation of motion obtained from the Lagrangian (3.1) implies:

$$e^{2\Phi}\partial_\mu b\partial^\mu b + \frac{1}{4}e^{-\Phi}f_{\mu\nu}f^{\mu\nu} - \frac{\delta V(\Phi)}{\delta\Phi} + \mathcal{O}(\partial\Phi) = 0, \quad (3.11)$$

where we did not write explicitly the terms involving $\partial_\mu\Phi$, since we will be interested in situations in which the dilaton is stabilised to a constant value $\Phi = \Phi_0$, which may occur at the minimum of its potential when

$$\left.\frac{\partial V(\Phi)}{\partial\Phi}\right|_{\Phi=\Phi_0} = 0, \quad \text{with } V(\Phi_0) = 0. \quad (3.12)$$

Thus, for a constant dilaton, the case of interest, the dilaton equation (3.11) implies a constraint on the Kalb-Ramond and Maxwell field strengths. This constraint will be at the heart of our considerations later on in the article, when we link the Kalb-Ramond torsion charge with the magnetic charge of the electromagnetic monopole.

In the limiting non-stringy case $\gamma = 0$, we may ensure the stabilization of the heavy scalar field to a constant value $\Phi = \Phi_0$ by imposing again (3.12); however in this limiting case the

⁸ In a Feynman path-integral formulation, the dual theory for the Lagrangian (3.1) (in a Minkowski-signature space-time) is obtained [9, 21] by implementing the Bianchi constraint (3.7) via a path-integral δ -function, represented using a Lagrange multiplier field $b(x)$. The dual theory is obtained on integrating out the field $H_{\mu\nu\rho}$ in the path-integral. In the language of differential forms, the latter constraint reads $\mathbf{d}^*\mathbf{S} = 0$, where $\mathbf{S} = *\mathbf{H}$ is the dual form of the Kalb-Ramond field strength \mathbf{H} : $S_d = \frac{1}{3!}\epsilon^{abc}{}_d H_{abc}$. Imposing this constraint on the full quantum theory, is equivalent to imposing an exact conservation of the ‘‘Kalb-Ramond-torsion charge’’ $Q = \int *\mathbf{S} = 0$. We then have in a path integral:

$$\begin{aligned} \mathcal{Z} &\propto \int D\mathbf{S} \exp\left[i\int \frac{3e^{-2\Phi}}{4g_N}\mathbf{S} \wedge *\mathbf{S}\right] \delta(\mathbf{d}^*\mathbf{S}) = \int D\mathbf{S} D b \exp\left[i\int \frac{3e^{-2\Phi}}{4g_N}\mathbf{S} \wedge *\mathbf{S} + \left(\frac{3}{2g_N}\right)^{1/2} b \mathbf{d}^*\mathbf{S}\right] \\ &\propto \int D b \exp\left[-i\int \frac{1}{2}e^{2\Phi} \mathbf{d}b \wedge *\mathbf{d}b\right], \end{aligned} \quad (3.10)$$

where the various proportionality factors represent appropriate normalisations of the various forms, and we work with a dimensionful field $b(x)$ with mass-dimension one; above we wrote explicitly only the part of the quantum path integral of the Lagrangian (3.1) that involves the (dual of the) Kalb-Ramond field \mathbf{S} , which is relevant for our discussion here. The ‘‘non-propagating’’ \mathbf{S} field has been integrated out completely, on implementing the Bianchi constraint (3.7), and on partially integrating the second term in the argument of the exponential in the middle equation of (3.10). We note the change in sign of the kinetic term of b as compared with that of \mathbf{S} , and the different scalings with the dilaton Φ between these two terms. This results in a path-integral over the pertinent Lagrange multiplier field b and leads to the equations of motion (3.11).

constraint (3.11) between the Kalb-Ramond field strength and the Maxwell tensor is not imposed. Nevertheless, even in this case, we shall see that an appropriate modification of the Maxwell tensor in the spirit of (2.10), involving the $H_{\mu\nu\rho}$ field and the heavy scalar Φ , can be constructed which remarkably still solves Maxwell's equation (3.4) for $\Phi = \Phi_0 = \text{constant}$. We next proceed to solving the equations (3.2), (3.3), (3.4) and (3.11) (with the condition (3.12)).

B. Solution of the Model Equations: Ansätze

We will consider static solutions of the equations (3.2), (3.3), (3.4), (3.5), and (3.7) by making the ansätze

$$g_{\mu\nu} = \begin{pmatrix} B(r) & & & \\ & -A(r) & & \\ & & -r^2 & \\ & & & -r^2 \sin^2 \theta \end{pmatrix} \quad (3.13)$$

and

$$f_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2r \sin \theta W(r) \\ 0 & 0 & -2r \sin \theta W(r) & 0 \end{pmatrix}. \quad (3.14)$$

This ansatz for $f_{\mu\nu}$ is compatible with the ansatz for $f_{\mu\nu}$ in the HP solution and satisfies (3.4). The associated magnetic field has only a radial component, which in contravariant form reads:

$$\mathcal{B}^r = \epsilon^{r\theta\phi} f_{\theta\phi} = \frac{1}{\sqrt{-g}} \eta^{r\theta\phi} f_{\theta\phi} = \frac{2}{\sqrt{AB}} \frac{W(r)}{r}, \quad (3.15)$$

where $\eta^{r\theta\phi} = +1$, *etc* is the three-space totally antisymmetric symbol, and we took into account Eq. (3.13). The electric field is zero.

The ansatz for the scalar field is

$$\chi^a = \eta f(r) \frac{x^a}{r}. \quad (3.16)$$

The ansätze in (3.13) and (3.16) are those for the gravitational monopole. Since in addition we have the electromagnetic and Kalb-Ramond tensors, we can investigate whether the gravitational monopole induces a magnetic monopole from the enlarged set of equations.

On using the ansätze in the Einstein equation (3.5), we obtain

$$\begin{aligned} \frac{-A(r) + A^2(r) + rA'(r)}{g_N A^2(r)} &= 2W^2(r) + \frac{1}{4}b'^2(r) \frac{r^2}{A(r)} + \frac{\eta^2}{2} \left(2f^2(r) + \frac{f'^2(r)r^2}{A(r)} \right) \\ &+ \frac{\lambda}{4}\eta^4 (f^2(r) - 1)^2 r^2, \end{aligned} \quad (3.17)$$

$$\begin{aligned} \frac{B(r) - A(r)B(r) + rB'(r)}{g_N B(r)} &= -2A(r)W^2(r) + \frac{1}{4}b'^2(r)r^2 + \frac{1}{2}\eta^2 r^2 f'^2(r) \\ &- \eta^2 A(r)f^2(r) - \frac{\lambda}{4}\eta^4 A(r)(f^2(r) - 1)^2 r^2, \end{aligned} \quad (3.18)$$

and

$$\begin{aligned} &\frac{r}{4g_N} \left(2\frac{A'(r)}{A(r)} + \frac{rB'^2(r)}{B^2(r)} + \frac{rA'(r)B'(r)}{A(r)B(r)} - 2\left(\frac{B'(r)}{B(r)} + \frac{rB''(r)}{B(r)}\right) \right) \\ &= -2W^2(r)A(r) + \frac{1}{4}r^2b'^2(r) + \frac{r^2\eta^2}{2}f'^2(r) + \frac{\lambda\eta^4}{4}A(r)r^2(f^2(r) - 1)^2, \end{aligned} \quad (3.19)$$

where prime indicates derivative with respect to r . Furthermore (3.2) leads to

$$\frac{f''(r)}{A(r)} - \frac{1}{2A(r)} \left(\frac{A'(r)}{A(r)} - \frac{B'(r)}{B(r)} - \frac{4}{r} \right) f'(r) - \frac{2f(r)}{r^2} = \lambda\eta^2 (f^2(r) - 1) f(r). \quad (3.20)$$

The last remaining equation, derived from (3.7), is

$$\frac{d}{dr} \left(\sqrt{\frac{B(r)}{A(r)}} r^2 \frac{db}{dr} \right) = 0. \quad (3.21)$$

Its solution is

$$b'(r) = \frac{\varsigma}{r^2} \sqrt{\frac{A(r)}{B(r)}} \quad (3.22)$$

where ς is a constant of integration which measures the strength of the Kalb-Ramond field strength.

It is necessary to be aware of units of variables and so we recast the equations in terms of dimensionless variables:

$$W \rightarrow \frac{W}{\sqrt{g_N}}, \quad r \rightarrow \sqrt{g_N} r, \quad b \rightarrow \frac{b}{\sqrt{g_N}}, \quad \eta \rightarrow \frac{\eta}{\sqrt{g_N}}. \quad (3.23)$$

The equations satisfied by these rescaled variables are the same as (3.17), (3.18) and (3.19) but with g_N replaced by 1.

C. Analytical Solution of the Model equations: Asymptotic analysis

Equations (3.17), (3.18), (3.19), and (3.20) will be solved in two asymptotic regions, the near-field ($r \rightarrow 0$) and far-field ($r \rightarrow \infty$). The existence of the full interpolating solution then is assumed and based on continuity in space. Approximate interpolating solutions will be discussed in a subsequent paper. In both regions, *to leading order*, we will require

$$B(r) \simeq A^{-1}(r) , \quad (3.24)$$

which is certainly required for the far field (Newtonian) limit. However, as we shall show, the presence of a non-trivial antisymmetric tensor field strength (3.22) and of the scalar triplet field with non-trivial vacuum expectation value η , imply (next-to-leading order) modifications in (3.24), which are *crucial* for the consistency of the solutions. In particular, as we shall discuss below, for the small r ($r \rightarrow 0$) region, we find

$$B(r) A(r) = 1 + \mathcal{O}(r^2) , \quad r \rightarrow 0 , \quad (3.25)$$

while for the large $r \rightarrow \infty$ region we have:

$$B(r) A(r) = 1 + \mathcal{O}\left(\frac{1}{r^2}\right) , \quad r \rightarrow \infty . \quad (3.26)$$

Working in units with $g_N = 1$, the necessity of such deviations in both regions can be seen by manipulating eqs. (3.17) and (3.18) to rewrite them as:

$$\begin{aligned} 1 - \frac{1}{A} + \frac{rA'(r)}{A^2(r)} &= 2W^2(r) + \frac{1}{4}b'^2(r) \frac{r^2}{A(r)} + \frac{\eta^2}{2} \left(2f^2(r) + \frac{f'^2(r)r^2}{A(r)} \right) \\ &+ \frac{\lambda}{4}\eta^4 (f^2(r) - 1)^2 r^2 \end{aligned} \quad (3.27)$$

and

$$\begin{aligned} 1 - \frac{1}{A} - \frac{rB'(r)}{AB} &= 2W^2(r) - \frac{1}{4}b'^2(r) \frac{r^2}{A(r)} + \frac{\eta^2}{2} \left(2f^2(r) - \frac{f'^2(r)r^2}{A(r)} \right) \\ &+ \frac{\lambda}{4}\eta^4 (f^2(r) - 1)^2 r^2. \end{aligned} \quad (3.28)$$

If (3.24) were to hold exactly, then one would have $\frac{A'}{A} = -\frac{B'}{B}$, which would make the left hand sides of (3.27) and (3.28) identical and, on subtracting the equations, it would yield

$$0 = \frac{r^2}{A} \left(\frac{1}{2}(b')^2 + \eta^2 (f')^2 \right). \quad (3.29)$$

As we shall discuss below, for small $r \rightarrow 0$ one has $B \sim p_0/r^2$, $p_0 > 0$ a constant, and $f' = f_0 = \text{constant}$ (cf. (3.34)); if (3.29) had been valid then we would have $\varsigma = f_0 = 0$. For large r , the ansatz we take for the function $f(r)$ (cf. (3.61),(3.63) for $r \rightarrow \infty$) makes the contributions of the f' -terms in (3.27) and (3.28) subleading in the region $r \rightarrow \infty$, as compared to the rest of the terms. Upon ignoring such terms then, and subtracting the latter two equations leads to $\varsigma = 0$.

The leading order assumption (3.24) is used in many non-trivial black-hole solutions, e.g. for the Reisser-Nordström (RN) black hole solution in the presence of electromagnetic fields (corresponding to magnetic charges in our case) where the metric is [22]:

$$ds^2 = \Delta dt^2 - \Delta^{-1} dr^2 - r^2 d\Omega^2 \quad (3.30)$$

with $d\Omega^2$ the metric on a 2-sphere, $\Delta = 1 - \frac{2G_N M}{r} + \frac{G_N \mu^2}{r^2}$, μ the magnetic charge and M the mass of the black hole. Consequently the assumption of $B(r) = A^{-1}(r)$ for $r \rightarrow 0$ or $r \rightarrow \infty$, which we adopt in this work, is a relevant one.⁹ The RN black hole is not singular

⁹ In [23], an exact spherically symmetric solution in a Kalb-Ramond background, but in the absence of scalar and gauge fields, has been found with a naked singularity, which - in our notation - amounts to $B = 1$ and $A = \frac{1}{1 - \frac{\alpha}{r^2}}$, that is

$$AB \simeq -\frac{r^2}{\alpha}, \quad \text{as } r \rightarrow 0, \quad (3.31)$$

whilst $A \rightarrow 1, B = 1$ as $r \rightarrow \infty$. This solution corresponds to a special case in which one of the integration constants $c_1 = 0$ (cf. [23]). To understand this we notice that, on subtracting (3.27) and (3.28), we obtain

$$\frac{(AB)'}{AB} = r \left(\frac{1}{2}(b')^2 + \eta^2 (f')^2 \right). \quad (3.32)$$

The right-hand side of this equation depends on the form of A and B (cf. (3.20) and (3.22)). The case of [23], for which the behaviour (3.31) is valid near the origin, does *not* have scalar or gauge fields; hence, in such a case, $f = 0$ exactly in (3.32). Eq. (3.22) is still valid in that case, as it follows from the Bianchi identity (3.7) for static b fields. Setting $B = 1$ and $f = 0$ in (3.32), and using (3.22), we easily arrive at an equation for $A(r)$:

$$\frac{d}{dr} \left(-1/A \right) = \frac{\varsigma^2}{2r^3} \Rightarrow A = \frac{1}{1 + \frac{\varsigma^2}{4r^2}}, \quad (3.33)$$

upon imposing asymptotic Minkowski flatness for the metric as $r \rightarrow \infty$. The rest of the gravitational equations are also satisfied with this ansatz for the metric. This solution is the exact solution of [23] with $\alpha = -\varsigma^2/4$ (For imaginary ς the solution corresponds to a wormhole [23] with a throat radius $\varsigma/2$). The alert reader should notice that there is a sign difference in the Kalb-Ramond field strength term between the action of [23] and ours (3.1), which leads in our case to the naked singularity being associated with a positive energy *real* Kalb-Ramond pseudoscalar field b , whilst the wormhole solution corresponds to a negative energy *purely imaginary* b . In this respect, our formalism and results agree with those in

at the horizons, the apparent singularities being co-ordinate artefacts.¹⁰ In our case, in view of the corrections (3.25), (3.26), we obtain deformed RN-type solutions, but the shielding of the curvature singularities at $r = 0$ by horizons (i.e. absence of naked singularities) holds for sufficiently large mass compared to the charge. (In our case of small masses the naked singularities can still be shielded; see discussion in section III D, following Eq. (3.75)).

Because of the use of scaled *dimensionless* variables, when r is of $O(1)$ the physical r is order of the Planck length. At this scale, the classical equations that we have restricted ourselves to here, cannot be expected to be valid because of quantum gravity corrections; hence, in principle, in order to be able to estimate the magnetic energy of our monopole, one should put the effective Planck length as a lower distance cut-off. However, if the classical equations determining our monopole mass are to be trusted, then any estimate on the mass based on them should be independent of the short distance cut-off. As we shall see in section III D, this is precisely the case for the monopole mass which depends only on the infrared (large distance) regime, with vanishing contributions from the $r \rightarrow 0$ regime (on estimating the leading asymptotic behaviour for $r \rightarrow 0$ and $r \rightarrow \infty$ of the stress tensor of the theory (*cf.* (3.68) below) by using expressions obtained from our classical solutions).

the literature on real (Kalb-Ramond and ordinary) axion fields [2, 3, 14, 24–26], see discussion below). However, in the presence of gauge and scalar fields it can be shown that the solution (3.33) with $B = 1$ no longer leads to a valid solution in the presence of non-trivial electromagnetic fields if $f = f_0 r$ as $r \rightarrow 0$ for the scalars is adopted, as in our work. (NB the scalar-field behaviour for the HP monopole [5] is similar.) This follows immediately from (3.20), which in the limit $r \rightarrow 0$ yields $\varsigma \rightarrow 0$. In this article, we concentrate on modified solutions which, both near the origin and asymptotically, exhibit the approximate behaviour (3.24) (specifically (3.25) for $r \rightarrow 0$, (3.26) for $r \rightarrow \infty$); the modified solution perturbs the RN solution by terms proportional to the Kalb-Ramond torsion charge ς . In this case, it can be readily seen, that, on using (3.41) with $\epsilon(r) = O(r^2)$, $r \rightarrow 0$, Eq. (3.32) yields (3.45). It is possible that other solutions are found, with different behaviour of the scalar and gauge fields, in which the Kalb-Ramond field behaves as in [23]. To understand and classify in general the solutions near $r = 0$ would be facilitated by a numerical study, which we postpone for the future.

¹⁰ In Maxwell-Einstein systems the effects of ordinary axion fields (which differ from those associated with our three form $H_{\mu\nu\rho}$) have been discussed in [24], with the conclusion that the axion charge, which we identify with $-\varsigma$ (see discussion below, Eq. (3.67)), contributes to the charge-terms in a metric pertaining to a RN black hole; in string theories with antisymmetric tensor fields present, there are rotating black hole solutions of Kerr-Newmann-Reissner-Nordstrom type [25]; charged non-rotating black hole solutions are present in string-inspired models with dilaton, gauge and Kalb-Ramond axion fields present, but without scalar triplet fields $\vec{\chi}$, associated with the global monopole [26].

1. *Small r analysis*

Asymptotically, for small r , let us write $B(r) \sim \frac{p(r)}{r^2}$ and assume that

$$f(r) \sim f_0 r, \quad (3.34)$$

(which is consistent with the scalar field equation of motion (3.20) in the limit $r \rightarrow 0$ and is similar to the r dependence found in the construction of the HP monopole [5]). From (3.17) and (3.19) we deduce that

$$1 - \frac{p''(r)}{2} = \frac{1}{2} \lambda \eta^4 r^2 (f_0^2 r^2 - 1)^2 + \frac{\zeta^2}{2p(r)} + \eta^2 f_0^2 (r^2 + p(r)). \quad (3.35)$$

We cannot solve this equation without approximation; on the right-hand side of (3.35), in the denominator of the term proportional to ζ^2 , we consider $p(r)$ to be approximately a non-zero constant p_0 which leads to the equation

$$1 - \frac{p''(r)}{2} = \frac{1}{2} \lambda \eta^4 r^2 (f_0^2 r^2 - 1)^2 + \frac{\zeta^2}{2p_0} + \eta^2 f_0^2 (r^2 + p(r)). \quad (3.36)$$

The general solution of (3.36) is

$$\begin{aligned} p(r) &= c_2 \sin(\sqrt{2} f_0 \eta r) + c_1 \cos(\sqrt{2} f_0 \eta r) + \frac{\mathcal{Z}}{2f_0^4 \eta^4 p_0}, \\ \mathcal{Z} &= \eta^2 (-\zeta^2 f_0^2 - 90 f_0^2 \lambda p_0 r^2 + 4 f_0^2 p_0 + 12 \lambda p_0) \\ &\quad + \eta^4 (15 f_0^4 \lambda p_0 r^4 - 12 f_0^2 \lambda p_0 r^2 - 2 f_0^4 p_0 r^2 + \lambda p_0) \\ &\quad + \eta^6 (f_0^6 \lambda p_0 (-r^6) + 2 f_0^4 \lambda p_0 r^4 - f_0^2 \lambda p_0 r^2) + 90 \lambda p_0 \end{aligned} \quad (3.37)$$

where c_1 and c_2 are constants of integration. Since η is small (on assuming that the symmetry breaking scale is much smaller than the Planck scale), $p(r)$ is well approximated by a constant near $r = 0$. Hence¹¹, upon making *the leading order approximation* $B(r) \simeq A^{-1}(r)$, we find

$$B(r) = \frac{p_0}{r^2}, \quad \text{for } r \rightarrow 0. \quad (3.40)$$

¹¹ In fact, on assuming (3.24) it is easily seen that (3.20) can be written as:

$$\frac{d}{dr} (f' r^2 B) = 2f + \lambda \eta^2 (f^2 - 1) f r^2 \quad (3.38)$$

which can be readily integrated to yield

$$B = \frac{c_0}{f' r^2} + \frac{1}{f' r^2} \int d\tilde{r} [2f(\tilde{r}) + \lambda \eta^2 (f(\tilde{r})^2 - 1) f(\tilde{r}) \tilde{r}^2], \quad (3.39)$$

where c_0 is an integration constant. Upon assuming $f(r) \sim f_0 r$ for $r \rightarrow 0$, the small r behaviour deduced from (3.39), is consistent with (3.40), obtained from our small r analysis, upon fixing the constants

We keep this expression for $B(r)$ and now proceed to find the next-to-leading order corrections in the product AB which are induced by the presence of the antisymmetric tensor and the non-trivial vacuum expectation value of the scalar fields (i.e. the global monopole). Let us assume that, for small $r \rightarrow 0$:

$$A(r) B(r) = 1 + \epsilon(r) \quad (3.41)$$

where $\epsilon(r) \rightarrow 0$ as $r \rightarrow 0$. This implies

$$\frac{A'}{A} = -\frac{B'}{B} + \frac{\epsilon'(r)}{1 + \epsilon(r)}. \quad (3.42)$$

Upon substituting (3.40), (3.34) and (3.42) into the Einstein equations (3.27) and (3.28), and subtracting them, we obtain, for small r , to leading order,

$$\frac{r \epsilon'}{1 + \epsilon} \frac{1}{A} = \frac{\zeta^2}{2p_0} + \frac{f_0^2 \eta^2 p_0}{1 + \epsilon}. \quad (3.43)$$

Upon assuming $\epsilon(r) = \mathcal{O}(r^2)$, we seek consistent solutions in the region $r \rightarrow 0$. In this case, the denominator of the second term on the right hand side of (3.43) can be approximated by unity, which on account of (3.41), yields

$$\frac{\epsilon'}{(1 + \epsilon)^2} = -\frac{d}{dr} \left(\frac{1}{1 + \epsilon} \right) = \left(\frac{\zeta^2}{2p_0^2} + f_0^2 \eta^2 \right) r \quad (3.44)$$

which can be integrated to give

$$\epsilon(r) = \frac{1}{1 - \left(\frac{\zeta^2}{4p_0^2} + \frac{\eta^2 f_0^2}{2} \right) r^2} - 1 = \left(\frac{\zeta^2}{4p_0^2} + \frac{\eta^2 f_0^2}{2} \right) r^2 + \dots, \quad r \rightarrow 0, \quad (3.45)$$

upon imposing the requirement that $\epsilon(0) = 0$.

Although we have used so far the leading order approximation (3.40) for B (as $r \rightarrow 0$), when dealing with eq. (3.27) we should make use of the complete Reissner-Nordstrom expression

$$B = 1 - \frac{2\mathcal{M}}{r} + \frac{p_0}{r^2} \quad (3.46)$$

$p_0 = c_0/f_0$. However, for large r , although (3.24) is assumed, and one might have surmised that (3.39) would still be valid, this is not the case: $f \simeq 1 - \frac{\alpha_1}{r^2}$, with α_1 a constant, which implies that as $r \rightarrow \infty$, $f' \rightarrow 0$ (and that $f' r^2 \sim r^{-1}$). Hence the derivation of (3.39) entails an implicit division by zero, which is an inappropriate operation.

with \mathcal{M} the monopole mass. Indeed, upon using (3.46), (3.41), (3.42) and (3.44), as $r \rightarrow 0$, we obtain:

$$\begin{aligned} 1 - \frac{B}{1+\epsilon} - r \frac{B'}{1+\epsilon} + \frac{r \epsilon'}{(1+\epsilon)^2} &\simeq 1 - B + B \epsilon - r B' + r B' \epsilon + r \epsilon' B (1 - 2\epsilon) + \dots \\ &= \frac{p_0}{r^2} + \frac{\zeta^2}{4p_0} + f_0^2 \eta^2 p_0 + \dots = 2W^2 + \frac{\zeta^2}{4p_0} + f_0^2 \eta^2 p_0, \end{aligned} \quad (3.47)$$

where the \dots indicate subleading terms that go to zero as $r \rightarrow 0$. \mathcal{M} is undetermined since the mass terms cancel altogether; the ζ^2 and $\eta^2 f_0^2$ dependent terms also cancel, leaving to leading order (as $r \rightarrow 0$) the relation

$$W^2(r) \sim \frac{p_0}{2r^2}, \quad r \ll 1. \quad (3.48)$$

One can easily see that the above results, (3.41) and (3.44)-(3.48), are also consistent with the third Einstein equation (3.19) in the region $r \rightarrow 0$. Thus, the constant $p_0 > 0$ cannot be determined in this asymptotic analysis, and the only relation that emerges is (3.48), which was to be expected from the Reissner-Nordstrom character of the metric.

However, as we shall presently see, the ‘‘charge’’ distortion part of the space-time metric function B (3.46) (*i.e.* the term p_0/r^2) comes exclusively from the torsion field:

$$p_0 \propto \zeta^2, \quad (3.49)$$

so that the magnetic charge $g \propto \zeta$ (*cf.* (3.15) and (3.76), (3.77) below). (The normalization factors will be fixed, as we shall see, once we embed the model in a string theory framework.) To this end, for the case $\gamma = 0$, we define $f_{\mu\nu}$, which is a solution of (3.4), as follows:

$$f_{\mu\nu} = -H_{abc} \phi^a \partial_\mu \phi^b \partial_\nu \phi^c. \quad (3.50)$$

The fields ϕ^a , $a = 0, 1, 2, 3$, is a tetrad of the scalar fields, with $\phi^0 = \text{constant}$, and ϕ^a ($a = 1, 2, 3$) identified with the triplet χ^a (3.16), normalised in such a way that ϕ^a which maps the $\text{SO}(3)$ internal group onto the sphere S^3 of three-space. The fourth member ϕ^0 of the tetrad is identified with (some function of) the superheavy scalar field Φ appearing in the Lagrangian (3.1), which is assumed to be stabilised to a constant value at an appropriate minimum of its potential (3.12). Using (3.8), (3.9), and the fact that the triplet (3.16) defines a S^3 -spatial coordinate set $(\eta f(r), \theta, \phi)$ which maps the $\text{SO}(3)$ internal space to the three space, one has for the non-trivial $\theta\phi$ component of the electromagnetic tensor (3.50):

$$f_{\theta\phi} = \frac{\phi^0}{A} b' r^2 \sin\theta = \phi^0 \zeta \sin\theta. \quad (3.51)$$

(In the last equality we have made use of the exact solution (3.22), a consequence of the Bianchi identity for $H_{\mu\nu\rho}$.) Comparing with (3.14) we thus obtain

$$W(r) = \phi_0 \frac{\varsigma}{2r} \quad (3.52)$$

for all r . This is consistent since the solution of the Maxwell's equations for the electromagnetic field (3.4) is independent of the explicit form of the function $W(r)$. Eq (3.52) is also consistent with (3.49). In view of (3.52) and (3.15), the induced magnetic monopole flux is controlled by the Kalb-Ramond field strength parameter ς , *i.e.* a vanishing or constant Kalb-Ramond axion field leads to the absence of the magnetic monopole.

We now remark that in the context of the string-inspired low energy Lagrangian (3.1) with $\gamma = 1$, we may arrive at a similar conclusion and moreover we can fix the proportionality coefficient ϕ_0 in (3.52). Indeed, to this end we first note that, on using (3.12), (3.22), as well as the fact that the spatial part of the Maxwell tensor $\frac{1}{4} f_{ij} f^{ij} = \frac{1}{2} (\mathcal{B}^r)^2 A$, with \mathcal{B}^r the radial (and, in our case, the only non-trivial) component of the magnetic field (3.15), the dilaton equation (3.11) yields the constraint (for constant dilaton $\Phi = \Phi_0$, which without loss of generality we can take it to be $\Phi_0 = 0$)

$$\frac{\varsigma^2}{r^4} \frac{1}{B} = \frac{1}{2} (\mathcal{B}^r)^2 A \Rightarrow \mathcal{B}^r = \sqrt{2} \frac{1}{\sqrt{AB}} \frac{\varsigma}{r^2}, \quad (3.53)$$

The reader should notice that equation (3.53) is valid for all r .

On taking into account (3.25), *i.e.* that to leading order as $r \rightarrow 0$ one has $AB \sim 1$, we see that (3.53) implies a singularity structure (as $r \rightarrow 0$) for the radial component of the magnetic field of magnetic monopole type [4],

$$\mathcal{B}^r \simeq \sqrt{2} \frac{\varsigma}{r^2} = \frac{g}{r^2}, \quad (3.54)$$

with a magnetic charge g being given by

$$g = \sqrt{2} \varsigma, \quad (3.55)$$

thus fixing the proportionality coefficient in (3.49) to:

$$p_0 = 2\varsigma^2. \quad (3.56)$$

Thus, from (3.48) we have

$$W(r) = \frac{\varsigma}{r}. \quad (3.57)$$

We will now examine (3.20) to investigate the consistency of our assumption that $f(r) \sim f_0 r$. On substituting the expression (3.40) for $B(r)$ in a linearised form of (3.20), we obtain

$$f''(r) = \frac{2 - \lambda\eta^2 r^2}{p_0} f(r). \quad (3.58)$$

These equations can be solved in terms of parabolic cylinder functions which are analytic in the neighbourhood of $r = 0$. A solution exists which (for small r) is proportional to $r + \frac{r^3}{3p_0}$ and so we have consistent ansätze.

2. Large r analysis:

For large r , since we expect the Newtonian limit to hold, we will consider the ansätze

$$A(r)B(r) = 1 + \frac{\epsilon_0}{r^2}, \quad \epsilon_0 \in \mathbb{R}, \quad \text{and} \quad B(r) \sim 1 + \beta_1 + \frac{\beta_2}{r} + \frac{\beta_3}{r^2}, \quad r \rightarrow \infty, \quad (3.59)$$

which imply

$$\frac{A'}{A} = -\frac{B'}{B} - \frac{2\epsilon_0}{r^3}, \quad r \rightarrow \infty. \quad (3.60)$$

Also the asymptotic ansatz

$$f(r) = 1 - \frac{\alpha_1}{r^2} + \delta(r), \quad (3.61)$$

where α_1 is a constant, solves the scalar field equation (3.2). From (3.20), on considering the leading order in $\frac{1}{r}$, we find $\alpha_1 = \frac{1}{\lambda\eta^2}$. From (3.17) and the leading behaviour in $\frac{1}{r}$ we have the requirement that $\beta_1 = -\eta^2$ which gives the deficit angle already noted by the authors of ref. [8] (using a different argument). This also matches the leading behaviour in (3.18). Ignoring, for the moment, the $1/r^2$ corrections on the right-hand-side of the AB product in (3.59), we observe that the equation linear in $\delta(r)$ that is derived from (3.20) is

$$(1 - \eta^2) \frac{d^2}{dr^2} \delta(r) + \frac{2}{r} (1 - \eta^2) \frac{d}{dr} \delta(r) - 2\lambda\eta^2 \delta(r) = 0. \quad (3.62)$$

This has a decaying solution

$$\delta(r) = \frac{\exp\left(-\eta\sqrt{\frac{2\lambda}{1-\eta^2}}r\right)}{r}, \quad (3.63)$$

and so $\delta(r)$ in (3.61) is exponentially small and can be ignored.

On subtracting (3.28) from (3.27), and taking into account (3.59) and (3.60), we readily obtain, to leading order in $r \rightarrow \infty$:

$$\epsilon_0 = -\frac{\zeta^2}{4(1-\eta^2)^2} . \quad (3.64)$$

Upon adding (3.27) and (3.28) we can determine W^2 in the large r region:

$$W^2(r) \simeq \frac{1}{2r^2} \left(\beta_3 + \frac{1}{\lambda} \right) . \quad (3.65)$$

From large r analysis, this solution, together with (3.64), is also consistent with the third Einstein equation (3.19).

The asymptotic analysis for $r \rightarrow \infty$ does not determine the constant β_3 . However, from (3.57), which is valid for all r , we are led to identify

$$\beta_3 + \frac{1}{\lambda} = 2\zeta^2 , \quad (3.66)$$

where on the right-hand-side we have used (3.56).

We remark that, in view of (3.22), and that asymptotically (for $r \rightarrow \infty$) we have $AB \simeq 1$, $B = 1 - \eta^2$ (*cf.* (3.59), (3.64)), the leading behaviour, for asymptotically large ($r \rightarrow \infty$), of the radial component of the magnetic field is still given by (3.54). On the other hand, the asymptotic behaviour for $r \rightarrow \infty$ of the axion field b is

$$b(r) \simeq -\frac{\zeta}{r} + \dots , \quad r \rightarrow \infty , \quad (3.67)$$

where we used the shift symmetry in the action (3.1), $b \rightarrow b + c_0$, with c_0 a constant, to impose the boundary condition $b(\infty) = 0$. Thus $-\zeta$ plays the rôle of an ‘axion’ charge. In this sense the fact that ζ^2 contributes to the “charge” term in the metric function B (3.59), is consistent with the findings of [24]. However, our model and monopole solution are quite different from those of [24]. Moreover, in our case, asymptotically for large r , there is the deficit η^2 in the metric function B due to the presence of the global monopole.

Secondly, for very large coupling $\lambda \rightarrow \infty$, which is of phenomenological relevance as it enforces the scalar triplet field to take on its classical vacuum expectation value, the $1/\lambda$ terms on the left-hand-side of (3.66) can be ignored. In this case, the metric function B is given by the Reissner-Nordstrom form (3.30) for *both* small and large r .

Finally, we remark that the $1/r$ -term in B in (3.59) corresponds to the contributions from the monopole mass, and has a coefficient β_2 which cannot be determined in our asymptotic

analysis up to $O(1/r^3)$. From the expected large r asymptotic RN form (3.30) of the metric tensor, one can identify $\beta_2 = -2\mathcal{M}$, where \mathcal{M} is the monopole mass [8]. An estimate of the monopole mass is given in the next subsection.

D. An estimate of the magnetic monopole mass

To make an estimate of the monopole mass, we shall use the analytic form of the solution in the two asymptotic regimes of small and large (radial) distances from the monopole centre. The monopole mass is concentrated in the core region whose size we will estimate following arguments similar to those in ref. [8]. The total energy (i.e. (rest) mass \mathcal{M}) is given by the integral over three space of the time-time component of the stress energy tensor:

$$\mathcal{M} = \int \sqrt{-g} d^3x \left[\frac{2W^2}{B r^2} + \frac{(b')^2}{4BA} + \eta^2 \left(\frac{f^2}{B r^2} + \frac{(f')^2}{2BA} \right) + \frac{\lambda \eta^4}{4B} (f^2 - 1)^2 \right]. \quad (3.68)$$

From the metric (3.13), and the property (3.24), we observe that the integration measure $\sqrt{-g} d^3x = r^2 \sin\theta dr d\theta d\phi$ in spherical polar coordinates assumes its flat space-time form. Taking into account the small- r form of the various functions appearing in (3.68), we obtain that, for $r \rightarrow 0$, the corresponding contributions to the integral are vanishing to leading order. However, for $r \rightarrow \infty$, there is a linearly divergent contribution in r coming from the third term of the integrand on the right-hand-side of (3.68), which is the dominant contribution to the integral. We have assumed that the interpolating functions of the various terms are non-singular in the non-asymptotic regions. Using a spatial infrared cutoff L , we estimate the mass of the monopole to be

$$\mathcal{M} \sim 4\pi \frac{\eta^2}{1 - \eta^2} \int_0^L dr \sim 4\pi \eta^2 L, \quad (3.69)$$

for $\eta \ll 1$ (or in terms of the dimensionful quantities (3.23) $\eta \ll M_{\text{Pl}}$, where M_{Pl} is the reduced Planck mass); this estimate is consistent phenomenologically (see below). Physically, and following the logic of ref. [8], which discusses self-gravitating global monopoles in the absence of both electromagnetic fields and (Kalb-Ramond) torsion, we may assume that the mass of the monopole is concentrated in its core, whose size is L , and outside this the scalar field configuration approaches its constant vacuum expectation value, that is $f \sim 1$.

It has been estimated in [8] that the core size is of order $\lambda^{-1/2} \eta^{-1}$ in flat space. If we replace L by the core size in (3.69), then we obtain $\mathcal{M} \sim 4\pi \lambda^{-1/2} \eta$ (the same order for the

mass of the monopole given in [8]). For small $\eta (\ll 1)$, gravity is expected not to change significantly the structure of the monopole at small distances.

However, in our case we see from (3.61) that the above estimate for the core size is not correct, in the sense that at such distances $f \simeq 0$ and the approximation that $f \simeq 1$ at large r is not valid. For a consistent picture, L must be such that $L \gg \sqrt{\alpha_1} = \lambda^{-1/2} \eta^{-1}$, so that $f \simeq 1$. It is sufficient to take the size of the core to be

$$L = \xi \lambda^{-1/2} \eta^{-1} \quad (3.70)$$

with $\xi = O(10)$ say.

In such a case, from (3.69) we obtain the following order of magnitude estimate of the monopole mass

$$\mathcal{M} \sim 4\pi \xi \lambda^{-1/2} \eta, \quad \xi = \mathcal{O}(10) \quad (3.71)$$

with λ and η phenomenological parameters to be constrained by experiment. We will attempt to estimate the order of the parameter ξ by *assuming* that the main contribution to the integral (3.68) comes from a thin shell of radius $R \sim L \gg 1$, and of thickness $\Delta L = (1 - \alpha)L$, $0 < \alpha < 1$; we then find for $\eta \ll 1$ and large λ that

$$\begin{aligned} \mathcal{M} &\sim \int_{\text{shell thickness } (1-\alpha)L} \sqrt{-g} d^3x \left[\frac{2W^2}{B r^2} + \frac{(b')^2}{4BA} + \eta^2 \left(\frac{f^2}{B r^2} + \frac{(f')^2}{2BA} \right) + \frac{\lambda \eta^4}{4B} (f^2 - 1)^2 \right] \\ &\simeq \frac{1}{\alpha} (1 - \alpha) \left(9\pi \zeta^2 + \frac{4\pi}{\lambda} \right) \frac{1}{L} + 4\pi \eta^2 (1 - \alpha) L \end{aligned} \quad (3.72)$$

which we assume here. In arriving at the above result we have used (3.57), (3.22), and the asymptotic behaviour for large r : $f^2 - 1 \simeq -\frac{2}{\lambda \eta^2 r^2}$ and $AB \simeq 1$. For $\lambda \gg 1$ assumed here, which ensures that the scalar fields χ^a approach their vacuum expectation values, the right-hand-side of (3.72) is practically independent of the coupling λ ¹². The core radius L_c then is estimated by minimizing \mathcal{M} as given in (3.72) with respect to L , which yields

$$L_c = \frac{3}{2} \sqrt{\frac{1}{\alpha}} \frac{|\zeta|}{\eta}. \quad (3.73)$$

¹² In fact, the $1/\lambda$ corrections on the right hand side of (3.72) are absent if one defines the shell radius L as the one signifying a region of space outside which one substitutes the value $f = 1$ for the scalar field configuration (i.e. the fields are replaced by their vacuum expectation values). The difference in the estimate of the core size from the approach based on (3.72) is then negligible for $\lambda \gg 1$.

Comparing with (3.70), this estimate yields $\xi = \frac{3}{2}\sqrt{\frac{\lambda}{\alpha}}$, which can be arranged to be of $O(10)$. This yields an estimate of the monopole mass

$$\mathcal{M} \sim 12\pi \sqrt{\frac{1}{\alpha}} (1 - \alpha) |\zeta| \eta, > 0. \quad (3.74)$$

The important point to notice is that the mass is proportional to the Kalb-Ramond field strength (“torsion charge”) ζ and independent of λ (in leading order for large λ). Within our phenomenological effective theory, ξ (equivalently α) cannot be completely determined without a full interpolating solution.

Before proceeding further we would like to make some comments regarding the nature of the mass in (3.74) which is *positive*, in contrast to the discussion in [17]; thus our monopole is an ordinary particle and can be produced at a collider such as the LHC. This case has to be contrasted with the solution of [17] which, as mentioned in section II, is associated with a matching (at the core radius) of an exterior Schwarzschild-like metric to an interior local de Sitter metric. Such a construction leads to a *negative* mass for the monopole, as we have discussed, which is not of relevance to collider physics, although there may be some cosmological interest.

However in our case, with $\zeta \neq 0$, the space-time at the origin $r \rightarrow 0$ (3.40) is *not* of de Sitter type but rather of Reissner-Nordström type (3.30) with B scaling like $1/r^2$ (3.40), owing to ζ being non-zero (see (3.22)). In our analysis we assumed that the entirety of the mass of the monopole is enclosed inside the core radius L_c (3.73), which implies a sort of ‘bag’ model. It is important to notice that in the limit $\zeta \rightarrow 0$, the core radius (3.73) $L_c \rightarrow 0$; hence, at least from our current asymptotic analysis, that leads to the above results, there seems to be no smooth limit connecting *our* global-monopole solution (with Kalb-Ramond axion charge ζ and positive mass proportional to the absolute value $|\zeta|$), to the solution of [17] with negative mass. Hence, it seems that our solution here represents a novel kind of a global monopole with Kalb-Ramond axion charge $\zeta \neq 0$. However, given that the $r \rightarrow 0$ behaviour of the mass function (3.68) is regular at the origin, one needs a full numerical solution connecting the $r \rightarrow 0$ and $r \rightarrow \infty$ regimes, before definite conclusions are made. In this sense, one cannot exclude the possibility that, in the case of a sufficiently small ζ (compared to other dimensionless parameters in the model, such as η (in units of Planck mass)), so that for all practical purposes $\zeta \rightarrow 0$, one recovers the negative mass instability of [17] and the corresponding de-Sitter space-time inside the horizon of that solution [17].

If this were the case, then one would face the possibility of having a critical minimum value of ζ for our bag model with positive mass to represent the charged self-gravitating global monopole. These are important issues that we postpone for future work. However, for our string-inspired Kalb-Ramond charged monopole solution, we should remark that, for $\zeta \neq 0$, the Dirac quantisation condition (to be discussed in the next section) points towards large values of ζ ; hence a small $\zeta \neq 0$ regime is excluded on such grounds.

Since we have assumed that $\eta \ll 1$, the mass of the monopole is much smaller than the Planck scale. In such a case there are no Reissner-Nordström horizons, defined by the vanishing of the metric function B , i.e.

$$1 - \frac{2\mathcal{M}}{r} + \frac{2\zeta^2}{r^2} = 0, \Rightarrow r^{(\pm)} = \mathcal{M} \left(1 \pm \sqrt{1 - \frac{2\zeta^2}{\mathcal{M}^2}} \right); \quad (3.75)$$

existence of horizons would require that $\mathcal{M} \geq \sqrt{2}\zeta$ which is incompatible with (3.74), since $\alpha = \mathcal{O}(1)$ and $\eta \ll M_P$. This means that a black hole and the corresponding horizons cannot form in our case, but the naked singularity at $r \rightarrow 0$ is still shielded inside the core radius, which essentially separates an outer region in space, where the scalar field is locked into its vacuum expectation value, from an inner region where symmetry breaking is not complete, and in fact the field f vanishes at the centre $r = 0$. On the other hand, the asymptotic form (3.59) for large but finite r is of Reissner-Nordström type (3.30). As we have already mentioned, the $1/r$ -term corresponds to the contributions from the monopole mass, and has a coefficient β_2 which cannot be determined in our asymptotic analysis up to $\mathcal{O}(1/r^3)$, but could be related to the monopole mass. These are crucial features of the solution ensuring a positive mass (3.71) for the monopole. The $1/r^2$ deviations (3.59), (3.66) from the Schwarzschild form occur due to the Kalb-Ramond axion-like field and the interactions of the Higgs field. This evasion of Birkhoff's theorem is permitted because in the exterior of the monopole core the space-time is not that of the vacuum, being characterised by non-zero gauge and axion fields that contribute to the magnetic charge contribution of the Reissner-Nordström solution (3.30).

E. Magnetic Charge Quantization and Discrete Kalb-Ramond field strength

An important property, of our magnetic monopole solution is the quantization of its magnetic charge. (We shall restrict ourselves to the case of very strong self interactions

among the scalar fields $\lambda \rightarrow \infty$.) As we have discussed above, from the form of $W(r)$ given in (3.57), we deduce the following form of the (radial) magnetic field (3.15) for $r \rightarrow \infty$ (where $AB \sim 1$ to leading order):

$$\mathbf{B} = \sqrt{2} \zeta \frac{\mathbf{r}}{r^3} . \quad (3.76)$$

This has the same ‘‘Coulomb-like’’ form $\mathbf{B} = g \frac{\mathbf{r}}{r^3}$ of the standard Dirac monopole magnetic field [4] with ‘‘magnetic charge’’ g given by (*cf.* (3.55)):

$$g = \sqrt{2} \zeta . \quad (3.77)$$

The magnetic charge is proportional to the Kalb-Ramond field strength. Since the latter can be positive or negative, the charge can be positive or negative, which implies the existence of both monopoles and antimonopoles.

Topological quantization of the magnetic charge (ζ in our case) found in the standard ’t Hooft-Polyakov monopole solution [5], does not follow from the H -dependent modification of the electromagnetic tensor (3.50). This is because the latter is four-dimensional in the internal space: there is a tetrad of scalar fields ϕ^a , $a = 0, 1, 2, 3$ (3.50) which does not provide a mapping of an internal $SO(3)$ sphere onto a spatial S^2 sphere. This is consistent with the fact, that the ansatz (3.50), which was considered for the case $\gamma = 0$ in the action (3.1), has an arbitrary constant normalisation factor, which, in contrast to the case $\gamma \neq 0$, can only be fixed if one requires matching with the solutions for the $\gamma \neq 0$ case; in particular $\gamma = 1$ corresponds to string theory effective actions, as discussed in detail in section III A. This constant factor would accompany the coefficient ζ in the magnetic charge (3.77), and, although Dirac’s quantization condition for the latter would occur, this would have no implications for a discretisation of ζ .

The quantization of ζ can *only* come in the $\gamma = 1$ string-inspired case from the standard Dirac argument [4], which considers the gauge transformations of the quantum relativistic wavefunction ψ (for $r \rightarrow \infty$) of an electron field (with electric charge e) in the presence of the Dirac-string singular vector potential $\mathbf{A}(\mathbf{r})$ for the monopole magnetic field \mathbf{B} (with $\mathbf{B} = \nabla \times \mathbf{A}$). Explicitly, requiring the single-valuedness of the wave function under the appropriate (singular) gauge transformations, yields the Dirac quantization rule

$$|ge| = \frac{n}{2} , \quad n \in \mathbf{Z}^+ \cup \{0\} . \quad (3.78)$$

From (3.77), we obtain the discretization of the Kalb-Ramond field strength

$$\varsigma e = \frac{n}{2\sqrt{2}}, \quad n \in \mathbf{Z}^+ \cup \{0\}. \quad (3.79)$$

The discreteness of the Kalb-Ramond field strength in the presence of a magnetic monopole is a novel feature of our solution ¹³.

We should also note that the antisymmetric-tensor monopole solutions of [27], differ from ours in several ways. Firstly, those monopoles are considered in D -dimensional Minkowski space time ($D = n + 3$, $n \geq 2$ a positive integer). Secondly, as a result of appropriately fixing the antisymmetric tensor gauge symmetry, and taking the B -field to be time-independent, the static field strength $H_{\mu_1 \dots \mu_{n+1}} = \partial_{[\mu_1} B_{\mu_2 \dots \mu_{n+1}]}$ can be considered as a form in a $D - 1$ Euclidean space, which can be patched appropriately in the presence of an antisymmetric-tensor monopole singularity at the spatial origin, leading to the quantization of the H -charge. By contrast, in our solution, the non-zero components of the Kalb-Ramond field strength read, for the asymptotic regions $r \rightarrow 0$ or $r \rightarrow \infty$:

$$H_{0\theta\phi} = \epsilon_{0\theta\phi r} \partial^r b \sim \varsigma \sin \theta, \quad (3.80)$$

corresponding to a *time-dependent* B -field in those regions of the form:

$$B_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varsigma t \sin \theta \\ 0 & 0 & -\varsigma t \sin \theta & 0 \end{pmatrix} \quad (3.81)$$

with t the time. The field strength (3.80) is regular as $r \rightarrow 0$, in contrast to the solutions of ref. [27], where the H -charge quantization was a consequence of monopoles in the antisymmetric tensor field, with singular behaviour of the H -field strength as $r \rightarrow 0$.

IV. DISCUSSION: CONSTRAINING THE MODEL BY EXPERIMENT

We have outlined how Kalb-Ramond axion fields could generate a magnetic monopole. There is spontaneous symmetry breaking of a global internal symmetry which is essential for

¹³ We should point out that discrete Kalb-Ramond field strengths may also arise in certain bosonic σ -models where the target space time is a group manifold [19], and the corresponding field strength is proportional to the level of the associated Kac-Moody algebra. Such models perhaps might provide a framework for embedding our solution in an ultraviolet-complete model of string theory.

producing a self-gravitating global monopole with a deficit angle. In the limit $\lambda \rightarrow \infty$ the scalar fields have no propagating degrees of freedom and are confined to classical values that interpolate between the two expectation values that minimize the potential, that is between zero and η . Without non-zero Kalb-Ramond field strengths (“torsion”), we showed that no magnetic monopole charge is induced by the global monopole (in the limit of large λ but there are $\mathcal{O}(1/\lambda)$ contributions of course). Because non-singular abelian gauge fields are incompatible with magnetic monopoles, we have worked directly with the electromagnetic field tensor and not required the abelian Bianchi identity. Nevertheless, formally, we can use a singular vector potential to represent the magnetic field, which has the Coulomb-like form of the standard Dirac monopole. Following the argument of Dirac the latter is quantised in terms of the fundamental electric charge.

The existence of the magnetic charge (3.77) implies high ionization, while the smallness of the monopole mass (3.71) (as compared to the Planck scale) makes our magnetic monopole model falsifiable in the current round of the LHC [1]. The scalars χ^a (3.16) in the model do *not* represent the Standard Model Higgs, but elementary defects (or composites of heavy fermions) associated with a spontaneous breaking of $O(3)$ symmetry [8]; the phenomenological parameters λ and the vacuum expectation value η , as well as the core size parameter ξ can be constrained by experiments, if one accepts the loose definition of the core in (3.70) which leads to (3.71); the core dimension is some large distance (compared to the Planck length) such that, for distances larger than this, the solution for the scalar field configuration is $f \simeq 1$ (up to terms of $\mathcal{O}(\xi^{-1}\lambda^{-1/2}\eta^{-1})$).

It is important to note that large λ self-interaction couplings affect significantly the probability for producing monopole-antimonopole pairs [8],

$$\mathcal{P} \propto e^{-(\text{const})\mathcal{M}^2/F} \propto e^{-\text{const}'/\lambda}, \quad (4.1)$$

where $F \sim \eta^2$ is the attractive force between a monopole and an antimonopole due to the linear divergent energy. In fact this can be understood by estimating the form factor in the cross-section for such processes. Indeed, from (3.70) and (3.71) one may estimate the ratio of the core size $R_{\text{core}} = L$ to the Compton wavelength $\lambda_{\text{Compt}} = 1/\mathcal{M}$ of the monopole as

$$\frac{R_{\text{core}}}{\lambda_{\text{Compt}}} \sim 4\pi \xi^2 \lambda^{-1} = \frac{\mathcal{M}^2}{4\pi \eta^2}. \quad (4.2)$$

For large monopoles $R_{\text{core}} > \lambda_{\text{Compt}}$, and therefore weak self-interaction couplings $\lambda \ll 4\pi\xi^2 = \mathcal{O}(10^3)$ for $\xi = \mathcal{O}(10)$, a semi-classical situation is reached where the form factor has

an exponential suppression [28] $e^{-4R_{\text{core}}/\lambda_{\text{Compt}}}$. This is in agreement with the estimate in [8] for the production probability \mathcal{P} of monopole-antimonopole pairs (4.1). Consequently, for small $\lambda < 1$ the production probability \mathcal{P} appears to be negligibly small. For large $\lambda \gg 1$, however, which is the case of interest here, \mathcal{P} is expected to be large, and thus of relevance to collider (including LHC) phenomenology (although strong coupling can complicate analytical calculations). On the other hand, once a monopole/antimonopole pair is produced, energy losses to Goldstone fields associated with the breaking of the $O(3)$ symmetry, at a rate of order η^2 [8] are probably expected, which must be taken into account when considering the relevant phenomenology. We also have in our model photons and Kalb-Ramond fields which couple to the monopoles gravitationally and complicate the situation.

In the present article we note that from the current bounds on the (scalar) monopole mass at the LHC [1], we obtain from (3.74) for the lowest magnetic charge:

$$6\pi\sqrt{\frac{1}{2\alpha}}(1-\alpha)\eta \geq 420 \text{ GeV} \quad (\text{for spin } 0, \varsigma = \frac{1}{2\sqrt{2}}, \lambda \gg 1, \eta \ll M_P, \alpha = \mathcal{O}(1)), \quad (4.3)$$

with higher bounds for higher magnetic charges. However we should exercise great caution in applying the above limits to our model. These bounds have been derived based on perturbative Drell-Yan processes [1], from the decay of virtual photons into monopole-antimonopole pairs. In the presence of strong magnetic charges, such bounds are not strictly valid. Moreover, in our model, the production mechanism of the global monopoles from standard model particle collisions needs to be carefully evaluated. In an effective field theory framework [9, 21], the derivative of the Kalb-Ramond axion field $\partial_\mu b$ couples to the axial fermion current $\psi_i \gamma^\mu \gamma^5 \psi_i$, where i runs over fermion species of the Standard Model (SM), but the scalars couple only gravitationally to the SM fields (unless they are composites of heavy fermions which may couple to SM fields through loops). Hence the actual production mechanism of the global monopole solution at colliders, such as the LHC, and their detection, needs to be examined carefully. We stress once again that the monopole production is certainly expected to be strongly suppressed unless λ is large, which is our case.

Before closing we would like to make two important additional remarks. Given that our monopole mass is of order TeV, and thus much smaller than the Planck mass M_P , gravitational collapse to a black hole is not expected. Indeed, it is known in general [29] that to form an Abelian black hole of Arnowitt-Deser-Misner (ADM) mass M_{BH} and magnetic

charge g , one needs to satisfy the condition

$$M_{\text{BH}} \geq \frac{g}{\sqrt{4\pi G_N}} .$$

Since in our monopole case $\mathcal{M} \sim \eta g$ (*cf.* (3.74)), in order to have a collapse one would need $\eta \geq \frac{M_P}{2\sqrt{\pi}}$, which is not the case, as η is assumed in our model to be of order TeV.

Moreover, we did not comment here on the stability of our monopole configurations. Although topologically non-trivial, indicating stability on generic grounds, there is nevertheless an ongoing debate [16] on the stability of global monopoles of [8], which may be subject to a sort of angular collapse. As stressed by Achúcarro and Urrestilla in [16], in the case of a global monopole, the energy barrier between the monopole and the vacuum is finite, despite the existence of a conserved topological charge and this feature is independent of the details of the scalar potential. But the issue of decay of such configurations, e.g. due to thermal fluctuation instabilities, remained inconclusive. Several extensions of the original model have been suggested in the literature [16]. The extension may involve more scalars, perhaps, for instance, gauged ones in addition to the global χ^a fields [30]. We shall not discuss such issues further here.

In our case, the presence of the Kalb-Ramond antisymmetric tensor and gauge fields, which leads to real magnetic monopoles, makes the model different from others in the literature. The stability of the monopoles deserves further studies. Even if there are instabilities, the collider production of unstable monopoles would lead to novel experimental signatures from the decay of such objects to Goldstone bosons and other particles. There are also, of course, many other interesting questions which we have mentioned and hope to address in the future.

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