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Static Output-Feedback Control for Interval Type-2 Discrete-Time Fuzzy Systems

Yabin Gao, Hongyi Li, Mohammed Chadli and Hak-Keung Lam

Abstract

This paper investigates the problem of reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control for discrete-time interval type-2 (IT2) fuzzy-model-based (FMB) systems via static output-feedback (SOF) control method. The number of fuzzy rules and the membership functions for the SOF controller are different from those for the plant. A sufficient criterion of reliable stability with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance is derived for the closed-loop system with sensor failure. The SOF controller is designed for two different cases (known sensor failure case and unknown sensor failure case) to guarantee the reliable stability with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance. Moreover, a novel criteria are presented to obtain the optimal \mathcal{H}_2 performance for the closed-loop system. Finally, an example is used to verify the effectiveness of the proposed design scheme.

Keywords: Reliable Control; Interval Type-2 Fuzzy Model; Static Output-Feedback Control.

I. INTRODUCTION

It is well known that complex nonlinearity exists in some physical systems and processes. Recently, there has been significant interesting on the modeling, stability analysis and controller synthesis problems

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for nonlinear systems [1]–[14]. Sensor and actuator failures often occur in the practical systems due to the sensors or actuators aging, zero shift, electromagnetic interference and so on, sensor failures, which may cause intolerable system performance [15]. Consequently, reliable control or reliable control method [15], [16] is introduced to tolerate the failures of actuators and sensors, and further maintain the stability and performances of systems. Over the past few decades, the reliable control problem for nonlinear systems has drawn considerable attention and many results have been developed [13], [16]–[26]. To mention a few, the work in [18] addressed the reliable mixed $\mathcal{L}_2/\mathcal{H}_\infty$ control for the T–S fuzzy model via static output feedback (SOF) control approach. The authors in [19] designed the reliable fuzzy \mathcal{H}_∞ controller for active suspension systems with actuator delay and actuator fault to improve the suspension performance. The authors in [13] investigated the problem of reliable \mathcal{H}_∞ control for discrete-time Takagi–Sugeno (T–S) fuzzy systems with distributed delay and actuator faults. The robust reliable guaranteed cost control for T–S fuzzy systems with interval time-varying delay was considered in [20]. In most of the existing literature on reliable control of nonlinear systems, the T–S fuzzy model approach [27], [28] is general in system modeling [12], [29]–[33]. However, this kind of T–S fuzzy model is based on the type-1 fuzzy set [34], which can effectively capture the system nonlinearities via an interpolation method [27], but cannot handle the parameter uncertainties entirely. As the uncertain information hidden in membership functions cannot be fully utilized under the parallel distributed compensation (PDC) design technique in stability analysis, it may lead to conservativeness.

In the past few years, the investigation in [35], [36] showed that the type-2 fuzzy set [37] is competent to capture the uncertainties specifically, and several kinds of uncertainties might be presented in a system. Recently, some results on type-2 fuzzy logic systems have been reported in [38]–[46]. More recently, the internal type-2 (IT2) fuzzy-model-based (FMB) control systems have been developed [42], [47], [48]. An IT2 T–S fuzzy model was proposed to describe the T–S fuzzy systems with uncertain membership functions in [42], and it was proved that the IT2 fuzzy state feedback controller can obtain less conservative results than the usual type-1 PDC fuzzy state feedback controller. It is worth mentioning that the work in [48] designed a novel IT2 fuzzy controller, in which the membership functions and number of rules can be freely chosen and different from those of the IT2 T–S fuzzy model. It should be pointed out that the controller design results are only obtained for continuous-time IT2 T–S fuzzy systems and there lack some modeling, stability analysis and control design results for discrete-time IT2 T–S fuzzy systems. Furthermore, there are few results on SOF control for IT2 T–S fuzzy systems, which motivates this study.

This paper focuses on designing a novel reliable SOF controller for discrete-time IT2 FMB systems with

mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance. Firstly, the discrete-time IT2 FMB systems with sensor failure and the IT2 fuzzy controller under imperfect premise matching are constructed for control design objective. The mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance index is established. Secondly, a sufficient condition of reliable stability is derived by applying the Lyapunov stability theory. Based on the condition, the desired IT2 fuzzy SOF controller is designed under the sensor failure known case and unknown case, respectively. Finally, a practical example is provided to demonstrate the effectiveness of the proposed results. The main contributions of this paper are summarized as follows: 1) a discrete-time IT2 FMB control system with sensor failure under imperfect premise matching is modeled to represent nonlinear systems; 2) the reliable IT2 fuzzy SOF controller with imperfectly matched membership functions is designed to guarantee the stability of the discrete-time IT2 FMB control system; 3) the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance is considered in the control design process.

The rest of this paper is organized as follows. Section II mainly represents the IT2 FMB discrete-time control system with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance and the transformed system is provided in Section III. The main results of stability analysis and reliable IT2 fuzzy SOF controller design are proposed in Section IV and Section V gives a numerical example to show the effectiveness of the proposed design scheme. Section VI concludes this paper.

Notation: The notation used throughout the paper is fairly standard. $\mathcal{L}_2[0, \infty)$ denotes the space of square-integrable vector functions over $[0, \infty)$. For $X \in \mathbb{R}^{n \times n}$, the notation $X > 0$ (respectively, $X \geq 0$) stands for the matrix X is real symmetric positive definite (respectively, positive semi-definite). The symbol “*” is used to denote the transposed elements in the symmetric positions of a matrix. The superscripts “ -1 ” and “ T ” stands for the matrix inverse and transpose, respectively. A block diagonal matrix is denoted by the shorthand $\text{diag}\{\cdot\cdot\cdot\}$. If not explicitly stated, for algebraic operations, identity matrices of appropriate dimensions will be denoted by “ I ”, and all matrices are assumed to have compatible dimensions.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Preliminaries

Firstly, we introduce the IT2 fuzzy sets for further characterizing the membership functions in the fuzzy model systems of discrete form. Considering the premise variable of the plant, which is represented by p -rules T-S fuzzy model, let M_α^i denotes an IT2 fuzzy set of i -th rule for $i = 1, 2, \dots, p$ and $\alpha = 1, 2, \dots, \Theta$ (Θ is a positive integer); define $f_\alpha(x(k))$ the measurable premise variable, where $x(k)$ is the system state variable with k the sampling time of discrete systems. Then the firing strength of

the i -th rule corresponds to the interval sets $\Phi_i(x(k)) = [\underline{\phi}_i(x(k)), \bar{\phi}_i(x(k))]$ where $\underline{\phi}_i(x(k)) = \prod_{\alpha=1}^{\Theta} \underline{\mu}_{M_{\alpha}^i}(f_{\alpha}(x(k))) \geq 0$, $\bar{\phi}_i(x(k)) = \prod_{\alpha=1}^{\Theta} \bar{\mu}_{M_{\alpha}^i}(f_{\alpha}(x(k))) \geq 0$, $0 \leq \underline{\phi}_i(x(k)) \leq \bar{\phi}_i(x(k)) \leq 1$, $0 \leq \underline{\mu}_{M_{\alpha}^i}(f_{\alpha}(x(k))) \leq \bar{\mu}_{M_{\alpha}^i}(f_{\alpha}(x(k))) \leq 1$; $\underline{\mu}_{M_{\alpha}^i}(f_{\alpha}(x(k)))$ and $\bar{\mu}_{M_{\alpha}^i}(f_{\alpha}(x(k)))$ are the lower and upper membership functions, respectively. $\underline{\phi}_i(x(k))$ and $\bar{\phi}_i(x(k))$ are the lower and upper grade of membership, respectively.

Then, considering the premise variable of the controller with c fuzzy rules, which is under imperfect premise matching. Let N_{β}^j denote an IT2 fuzzy set of j -th rule for $j = 1, 2, \dots, c$ and $\beta = 1, 2, \dots, \Omega$ (Ω is a positive integer); define $g_{\beta}(x(k))$ the measurable premise variable. Then the firing strength of the j -th rule corresponds to the interval sets $\Psi_j(x(k)) = [\underline{\psi}_j(x(k)), \bar{\psi}_j(x(k))]$, where $\underline{\psi}_j(x(k)) = \prod_{\beta=1}^{\Omega} \underline{\mu}_{N_{\beta}^j}(g_{\beta}(x(k))) \geq 0$, $\bar{\psi}_j(x(k)) = \prod_{\beta=1}^{\Omega} \bar{\mu}_{N_{\beta}^j}(g_{\beta}(x(k))) \geq 0$, $0 \leq \underline{\psi}_j(x(k)) \leq \bar{\psi}_j(x(k)) \leq 1$, $0 \leq \underline{\mu}_{N_{\beta}^j}(g_{\beta}(x(k))) \leq \bar{\mu}_{N_{\beta}^j}(g_{\beta}(x(k))) \leq 1$; $\underline{\mu}_{N_{\beta}^j}(g_{\beta}(x(k)))$ and $\bar{\mu}_{N_{\beta}^j}(g_{\beta}(x(k)))$ are the lower and upper membership functions, respectively; $\underline{\psi}_j(x(k))$ and $\bar{\psi}_j(x(k))$ are the lower and upper grade of membership, respectively.

B. Problem Formulation

Based on the IT2 fuzzy sets introduced above, a p -rule discrete-time IT2 T-S fuzzy model [48] for describing a nonlinear plant is of the following form:

Plant Rule i : IF $f_1(x(k))$ is M_1^i, \dots , and $f_{\alpha}(x(k))$ is M_{α}^i, \dots , and $f_{\Theta}(x(k))$ is M_{Θ}^i , THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) + B_{wi} w(k), \\ y(k) = C_i x(k) + D_{wi} w(k), \\ z(k) = E_i x(k) + G_i u(k) + G_{wi} w(k), \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ denotes the system state variable; $y(k) \in \mathbb{R}^s$ denotes the measured output; $z(k) \in \mathbb{R}^m$ denotes the controlled output; $u(k) \in \mathbb{R}^q$ is the control input; $w(k) \in \mathbb{R}^r$ is assumed to be an exogenous disturbance belonging to $\mathcal{L}_2[0, \infty)$. The vector-valued initial function is defined as $\chi(k)$. $A_i, B_i, B_{wi}, C_i, D_{wi}, E_i, G_i$ and G_{wi} are known appropriate dimensioned system matrices. Utilizing the bounds of the membership function from Preliminaries, the discrete-time IT2 T-S fuzzy system in (1) can be formulated as:

$$\begin{cases} x(k+1) = \sum_{i=1}^p \phi_i(x(k)) (A_i x(k) + B_i u(k) + B_{wi} w(k)), \\ y(k) = \sum_{i=1}^p \phi_i(x(k)) (C_i x(k) + D_{wi} w(k)), \\ z(k) = \sum_{i=1}^p \phi_i(x(k)) (E_i x(k) + G_i u(k) + G_{wi} w(k)), \end{cases} \quad (2)$$

where for $i = 1, 2, \dots, p$,

$$\phi_i(x(k)) = \underline{\alpha}_i(x(k)) \underline{\phi}_i(x(k)) + \bar{\alpha}_i(x(k)) \bar{\phi}_i(x(k)) \geq 0,$$

and $\{\underline{\alpha}_i(x(k)), \bar{\alpha}_i(x(k))\} \in [0, 1]$, $\underline{\alpha}_i(x(k)) + \bar{\alpha}_i(x(k)) = 1$. ($\underline{\alpha}_i(x(k))$ and $\bar{\alpha}_i(x(k))$) denote existent nonlinear weighting functions that are not necessary to be known in real applications); $\phi_i(x(k))$ is the grade of membership of the embedded membership function.

The IT2 fuzzy SOF controller with c rules for the system (2) is of the following form:

Controller Rule j : IF $g_1(x(k))$ is N_1^j, \dots , and $g_\beta(x(k))$ is N_β^j, \dots , and $g_\Omega(x(k))$ is N_Ω^j , THEN

$$u(k) = K_j y^F(k), \quad (3)$$

where $K_j \in \mathbb{R}^{q \times s}$ is the feedback gain matrix to be determined. The IT2 T-S fuzzy controller from (3) can be defined as:

$$u(k) = \sum_{j=1}^c \psi_j(x(k)) K_j y^F(k), \quad (4)$$

where for $j = 1, 2, \dots, c$,

$$\psi_j(x(k)) = \frac{\underline{\beta}_j(x(k)) \underline{\psi}_j(x(k)) + \bar{\beta}_j(x(k)) \bar{\psi}_j(x(k))}{\sum_{\kappa=1}^c (\underline{\beta}_\kappa(x(k)) \underline{\psi}_\kappa(x(k)) + \bar{\beta}_\kappa(x(k)) \bar{\psi}_\kappa(x(k)))} \geq 0,$$

in which $\underline{\beta}_j(x(k))$ and $\bar{\beta}_j(x(k))$ are predefined functions satisfying $\{\underline{\beta}_j(x(k)), \bar{\beta}_j(x(k))\} \in [0, 1]$, $\underline{\beta}_j(x(k)) + \bar{\beta}_j(x(k)) = 1$; $\psi_j(x(k))$ is the grade of membership of the embedded membership function.

From the details in (2) and (4), we have

$$\sum_{i=1}^p \phi_i(x(k)) = \sum_{j=1}^c \psi_j(x(k)) = \sum_{i=1}^p \phi_i(x(k)) \sum_{j=1}^c \psi_j(x(k)) = 1.$$

We adopt the following model of sensor failure from [49]

$$y^F(k) = \zeta y(k), \quad (5)$$

where $\zeta = \text{diag}\{\zeta_1, \zeta_2, \dots, \zeta_s\}$ and $0 \leq \underline{\zeta}_\ell \leq \zeta_\ell \leq \bar{\zeta}_\ell \leq 1$ ($\ell = 1, 2, \dots, s$). The variables ζ_ℓ quantify the failures of the sensor.

Remark 1: In the above model of sensor failure, there exist three cases of the feedback signal in sensor: When $\underline{\zeta}_\ell = 1$, it corresponds to the normal case $y^F(k) = y(k)$. When $\bar{\zeta}_\ell = 0$, it covers the outage case [50]. When $\underline{\zeta}_\ell \neq 0$ and $\bar{\zeta}_\ell \neq 1$, it corresponds to the partial failure case.

In order to design the reliable controller, let $\underline{\zeta} = \text{diag}\{\underline{\zeta}_1, \underline{\zeta}_2, \dots, \underline{\zeta}_s\}$, $\bar{\zeta} = \text{diag}\{\bar{\zeta}_1, \bar{\zeta}_2, \dots, \bar{\zeta}_s\}$, and

$$\hat{\lambda} = \text{diag}\{\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_s\}, \quad \check{\lambda} = \text{diag}\{\check{\lambda}_1, \check{\lambda}_2, \dots, \check{\lambda}_s\}, \quad \bar{\lambda} = \text{diag}\{\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_s\},$$

where $\hat{\lambda}_\iota = (\underline{\zeta}_\iota + \bar{\zeta}_\iota) / 2$, $\check{\lambda}_\iota = (\bar{\zeta}_\iota - \underline{\zeta}_\iota) / 2$. Thus, one can obtain

$$\zeta = \hat{\lambda} + \bar{\lambda}, \quad |\bar{\lambda}| \leq \check{\lambda}. \quad (6)$$

Hence, it follows from (2), (4) and (5) that the closed-loop IT2 FMB control system is represented as:

$$\begin{cases} x(k+1) &= \sum_{i=1}^p \sum_{j=1}^c \phi_i(x(k)) \psi_j(x(k)) [\bar{A}_{ij}x(k) + \bar{B}_{wij}w(k)], \\ z(k) &= \sum_{i=1}^p \sum_{j=1}^c \phi_i(x(k)) \psi_j(x(k)) [\bar{E}_{ij}x(k) + \bar{G}_{wij}w(k)], \end{cases} \quad (7)$$

where

$$\bar{A}_{ij} \triangleq A_i + B_i K_j \zeta C_i, \quad \bar{B}_{wij} \triangleq B_{wi} + B_i K_j \zeta D_{wi}, \quad (8)$$

$$\bar{E}_{ij} \triangleq E_i + G_i K_j \zeta C_i, \quad \bar{G}_{wij} \triangleq G_{wi} + G_i K_j \zeta D_{wi}. \quad (9)$$

In addition, to consider the performances of the system in (7), we introduce the following definitions.

Definition 1: Considering the disturbance-free system ($w(k) \equiv 0$) in (7), the corresponding \mathcal{H}_2 performance cost function is defined as

$$J_2 = \sum_{k=0}^{\infty} z^T(k) z(k). \quad (10)$$

Definition 2: Considering the system with disturbance input in (7), if the output $z(k)$ of system (7) and a prescribed level of disturbance attenuation $\gamma > 0$ under the zero initial condition satisfy

$$\|z\|_2 < \gamma \|w\|_2, \quad \forall 0 \neq w \in \mathcal{L}_2[0, \infty), \quad (11)$$

in which

$$\|z\|_2 = \sqrt{\sum_{k=0}^{\infty} z^T(k) z(k)},$$

then system (7) is said to be with γ -disturbance attenuation.

Definition 3: The IT2 fuzzy controller in (4) is said to be a reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ fuzzy SOF controller for IT2 FMB system (2) if the closed-loop system (2) is reliable stable and satisfies the definitions in (10) and (11).

In this work, we consider two cases of sensor failure matrix ζ , namely, the known failure and the unknown failure. The primary aim of this study is to design a fuzzy SOF controller in the form of (4) under the two cases of ζ such that the closed-loop system with sensor failure in (7) is asymptotically stable and has the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance for all $\iota = 1, 2, \dots, s$.

III. SYSTEM TRANSFORMATION

This section mainly processes the system transformation from the closed-loop system in (7) for the control design objective. The footprint of uncertainty (FOU) [42] and the state space of interest in model system are both considered for system transformation.

To deal with the parameter uncertainties in closed-loop system (7), we use the reconstructed membership functions expressed by the lower and upper membership functions to transform the system model for further analysis. Moreover, the state space of interest is considered for less conservativeness. Concretely, according to [48], the state space of interest and the FOU are both divided for the further stability analysis of the IT2 FMB control system in (7).

- 1) The state space Θ is partitioned θ connected sub-state spaces denoted as Θ_τ ($\tau = 1, 2, \dots, \theta$), such that $\Theta = \bigcup_{\tau=1}^{\theta} \Theta_\tau$.
- 2) The FOU is divided into $\vartheta + 1$ sub-FOUs. For $v = 1, 2, \dots, \vartheta + 1$, the lower and upper membership functions in the v -th sub-FOU are defined as follows for $\forall i, j, v, \tau$:

$$\underline{h}_{ijv}(x(k)) = \sum_{\tau=1}^{\theta} \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{a=1}^n \underline{\epsilon}_{ij i_1 i_2 \cdots i_n v \tau} \varrho_{ai_a v \tau}(x_a(k)), \quad (12)$$

$$\bar{h}_{ijv}(x(k)) = \sum_{\tau=1}^{\theta} \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{a=1}^n \bar{\epsilon}_{ij i_1 i_2 \cdots i_n v \tau} \varrho_{ai_a v \tau}(x_a(k)), \quad (13)$$

where $\underline{\epsilon}_{ij i_1 i_2 \cdots i_n v \tau}$ and $\bar{\epsilon}_{ij i_1 i_2 \cdots i_n v \tau}$ are constant scalars to be designed, and $0 \leq \underline{\epsilon}_{ij i_1 i_2 \cdots i_n v \tau} \leq \bar{\epsilon}_{ij i_1 i_2 \cdots i_n v \tau} \leq 1$. For $x(k) \in \Theta_\tau$, $\tau = 1, 2, \dots, \theta$, and $a, b = 1, 2, \dots, n$, it holds that $0 \leq \varrho_{ai_b v \tau}(x_a(k)) \leq 1$ and $\varrho_{a1 v \tau}(x_a(k)) + \varrho_{a2 v \tau}(x_a(k)) = 1$ ($i_a, i_b = 1, 2$); and $\varrho_{ai_b v \tau}(x_a(k)) = 0$ if else. Then, it follows that for $v = 1, 2, \dots, \vartheta + 1$,

$$\sum_{\tau=1}^{\theta} \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{a=1}^n \varrho_{ai_a v \tau}(x_a(k)) = 1. \quad (14)$$

Hence, for the stability analysis of the considered system in next section, we rewrite the IT2 fuzzy system in (7) as follows:

$$\begin{cases} x(k+1) &= \sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) [\bar{A}_{ij}x(k) + \bar{B}_{wij}w(k)], \\ z(k) &= \sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) [\bar{E}_{ij}x(k) + \bar{G}_{wij}w(k)], \end{cases} \quad (15)$$

where

$$\begin{aligned} h_{ij}(x(k)) &\triangleq \phi_i(x(k)) \psi_j(x(k)) \\ &= \sum_{v=1}^{\vartheta+1} \sigma_{ijv}(x(k)) \left(\underline{\rho}_{ijv}(x(k)) \underline{h}_{ijv}(x(k)) + \bar{\rho}_{ijv}(x(k)) \bar{h}_{ijv}(x(k)) \right), \end{aligned} \quad (16)$$

and $\sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) = 1$; the two functions $0 \leq \underline{\rho}_{ijv}(x(k)) \leq \bar{\rho}_{ijv}(x(k)) \leq 1$ satisfying that $\underline{\rho}_{ijv}(x(k)) + \bar{\rho}_{ijv}(x(k)) = 1$, and not necessarily being known; and $\sigma_{ijv}(x(k)) = 1$ if the membership function $h_{ijv}(x(k))$ is within the v -th sub-FOU, otherwise, $\sigma_{ijv}(x(k)) = 0$.

Based on the transformed system in (15), the stability analysis and controller synthesis can be in progress without the implementation of the IT2 T-S fuzzy model (2). Moreover, we give the property

$$\sum_{i=1}^p \phi_i(x(k)) = \sum_{j=1}^c \psi_j(x(k)) = \sum_{i=1}^p \sum_{j=1}^c \phi_i(x(k)) \psi_j(x(k)) = \sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) = 1 \quad (17)$$

for further processing.

In next section, the reliable control design scheme for nonlinear systems based on the transformed system (15) is provided, which means the reliable stability with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance can be achieved for the closed-loop system in (7) from the scheme.

The following lemmas are introduced for developing our main results.

Lemma 1: For any real matrices $\mathcal{X}_{ij}, \mathcal{Y}_{ij}$ for $1 \leq i \leq p$ and $1 \leq j \leq c$, and matrix $\mathcal{Z} > 0$ with an appropriate dimension, the following linear matrix inequality (LMI) holds:

$$\left[\sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) \mathcal{X}_{ij} \right]^T Q \left[\sum_{\kappa=1}^p \sum_{\ell=1}^c h_{\kappa\ell}(x(k)) \mathcal{Y}_{\kappa\ell} \right] \leq \frac{1}{2} \sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) (\mathcal{X}_{ij}^T Q \mathcal{X}_{ij} + \mathcal{Y}_{ij}^T Q \mathcal{Y}_{ij}),$$

where $h_{ij}(x(k))$ are defined in (16) and satisfying (17).

Proof: For matrices \mathcal{X}, \mathcal{Y} , and $\mathcal{Z} > 0$, based on the well-known upper bound

$$2\mathcal{X}^T \mathcal{Z} \mathcal{Y} \leq \inf_{\mathcal{Z} > 0} \{ \mathcal{X}^T \mathcal{Z} \mathcal{X} + \mathcal{Y}^T \mathcal{Z} \mathcal{Y} \},$$

it is easily obtained that

$$\begin{aligned} & 2 \left[\sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) \mathcal{X}_{ij} \right]^T \mathcal{Z} \left[\sum_{\kappa=1}^p \sum_{\ell=1}^c h_{\kappa\ell}(x(k)) \mathcal{Y}_{\kappa\ell} \right] \\ & \leq \sum_{i=1}^p \sum_{j=1}^c \sum_{\kappa=1}^p \sum_{\ell=1}^c h_{ij}(x(k)) h_{\kappa\ell}(x(k)) (\mathcal{X}_{ij}^T \mathcal{Z} \mathcal{X}_{ij} + \mathcal{Y}_{\kappa\ell}^T \mathcal{Z} \mathcal{Y}_{\kappa\ell}) \\ & = \sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) \mathcal{X}_{ij}^T \mathcal{Z} \mathcal{X}_{ij} + \sum_{\kappa=1}^p \sum_{\ell=1}^c h_{\kappa\ell}(x(k)) \mathcal{Y}_{\kappa\ell}^T \mathcal{Z} \mathcal{Y}_{\kappa\ell} \\ & = \sum_{i=1}^p \sum_{j=1}^c h_{ij}(x(k)) (\mathcal{X}_{ij}^T \mathcal{Z} \mathcal{X}_{ij} + \mathcal{Y}_{ij}^T \mathcal{Z} \mathcal{Y}_{ij}). \end{aligned}$$

This completes the proof. ■

Lemma 2: [51] Given any matrices \mathcal{X}, \mathcal{Y} and $\mathcal{Z} > 0$ with appropriate dimensions, then, the following inequality holds $\mathcal{X}^T \mathcal{Y} + \mathcal{Y}^T \mathcal{X} \leq \mathcal{X}^T \mathcal{Z} \mathcal{X} + \mathcal{Y}^T \mathcal{Z}^{-1} \mathcal{Y}$.

IV. MAIN RESULTS

In this section, the reliable stability analysis and controller design under known sensor failure and unknown sensor failure are presented for IT2 FMB systems with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance. By applying Lyapunov stability theory, a sufficient criterion of reliable stability is derived for system (15). Based on the criterion, the two reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ fuzzy SOF controllers are designed.

A. Stability Analysis

On the basis of the transformed system in (15), considering the \mathcal{H}_2 performance in (10) and \mathcal{H}_∞ performance in (11), a sufficient condition of reliable stability with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance is given for the closed-loop system (15) in the following theorem.

Theorem 1: Considering the system with sensor failure in (15), for a given scalar $\gamma > 0$, system (15) is reliable stable and has \mathcal{H}_∞ performance index γ , if there exist symmetric matrices $P > 0$, $R_{ijv} > 0$, $X_{ijv} > 0$, $Y_{ijv} > 0$, $U_{ijv} > 0$, $V_{ijv} > 0$, ($i = 1, 2, \dots, p$, $j = 1, 2, \dots, c$, $v = 1, 2, \dots, \vartheta + 1$), and S with appropriate dimensions satisfying the following inequalities for $i = 1, 2, \dots, p$, $j = 1, 2, \dots, c$, $v = 1, 2, \dots, \vartheta + 1$, $\tau = 1, 2, \dots, \theta$:

$$\forall i_1, i_2, \dots, i_n, v, \tau, \sum_{i=1}^p \sum_{j=1}^c \Xi_{ij} - S < 0, \quad (18)$$

$$\forall i, j, v, \Sigma_{2ij} + R_{ijv} + S > 0, \quad (19)$$

where

$$\begin{aligned} \Xi_{ij} &= \bar{\epsilon}_{ij i_1 i_2 \dots i_n v \tau} \Sigma_{1ij} + (\bar{\epsilon}_{ij i_1 i_2 \dots i_n v \tau} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n v \tau}) R_{ijv} + \bar{\epsilon}_{ij i_1 i_2 \dots i_n v \tau} S, \\ \Sigma_{1ij} &= \begin{bmatrix} \Sigma_{11ij} & \bar{A}_{ij}^T (P + U_{ijv}) \bar{B}_{wij} + \bar{E}_{ij}^T (I + V_{ijv}) \bar{G}_{wij} \\ * & \bar{B}_{wij}^T (P + U_{ijv}) \bar{B}_{wij} + \bar{G}_{wij}^T (I + V_{ijv}) \bar{G}_{wij} - \gamma^2 I \end{bmatrix}, \\ \Sigma_{2ij} &= \begin{bmatrix} \Sigma_{21ij} & \bar{A}_{ij}^T (P - X_{ijv}) \bar{B}_{wij} + \bar{E}_{ij}^T (I - Y_{ijv}) \bar{G}_{wij} \\ * & \bar{B}_{wij}^T (P - X_{ijv}) \bar{B}_{wij} + \bar{G}_{wij}^T (I - Y_{ijv}) \bar{G}_{wij} - \gamma^2 I \end{bmatrix}, \\ \Sigma_{11ij} &= -P + \bar{A}_{ij}^T (P + U_{ijv}) \bar{A}_{ij} + \bar{E}_{ij}^T (I + V_{ijv}) \bar{E}_{ij}, \\ \Sigma_{21ij} &= -P + \bar{A}_{ij}^T (P - X_{ijv}) \bar{A}_{ij} + \bar{E}_{ij}^T (I - Y_{ijv}) \bar{E}_{ij}. \end{aligned}$$

If the above conditions have a feasible solution, then the bound of \mathcal{H}_2 performance cost function in (10) is determined by

$$J_2^* = x_0^T P x_0, \quad (20)$$

where x_0 is the initial state.

Proof: Based on the closed-loop system with disturbance input in (15), considering the Lyapunov function $V(x(k)) = x^T(k)Px(k)$, applying Lemma 2 and introducing some slack matrices (S is an arbitrary symmetric matrix, and symmetric matrices $R_{ijv} > 0$, $U_{ijv} > 0$, and $X_{ijv} > 0$ with appropriate dimensions) based on the S -procedure [52], we have

$$\begin{aligned}
\Delta V(x(k)) &\leq \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \left(\underline{\rho}_{ijv} \underline{h}_{ijv} + \bar{\rho}_{ijv} \bar{h}_{ijv} \right) \eta^T(k) \Sigma_{0ij} \eta(k) \\
&\quad + \left[\sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \left(\underline{\rho}_{ijv} \underline{h}_{ijv} + \bar{\rho}_{ijv} \bar{h}_{ijv} \right) - 1 \right] \eta^T(k) S \eta(k) \\
&\quad - \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \left(1 - \underline{\rho}_{ijv} \right) \left(\underline{h}_{ijv} - \bar{h}_{ijv} \right) \eta^T(k) R_{ijv} \eta(k) \\
&\quad + \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \bar{h}_{ijv} x^T(k+1) U_{ijv} x(k+1) \\
&\quad - \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \underline{\rho}_{ijv} \left(\underline{h}_{ijv} - \bar{h}_{ijv} \right) x^T(k+1) X_{ijv} x(k+1) \\
&= \eta^T(k) \left\{ \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \left[\bar{h}_{ijv} \tilde{\Sigma}_{1ijv} - \left(\underline{h}_{ijv} - \bar{h}_{ijv} \right) R_{ijv} + \bar{h}_{ijv} S \right] - S \right\} \eta(k) \\
&\quad + \eta^T(k) \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \underline{\rho}_{ijv} \left(\underline{h}_{ijv} - \bar{h}_{ijv} \right) \left(\tilde{\Sigma}_{2ijv} + R_{ijv} + S \right) \eta(k), \tag{21}
\end{aligned}$$

where $\eta(k) = \begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix}^T$, and

$$\begin{aligned}
\Sigma_{0ij} &= \begin{bmatrix} -P + \bar{A}_{ij}^T P \bar{A}_{ij} & \bar{A}_{ij}^T P \bar{B}_{wij} \\ * & \bar{B}_{wij}^T P \bar{B}_{wij} \end{bmatrix}, \\
\tilde{\Sigma}_{1ij} &= \begin{bmatrix} -P + \bar{A}_{ij}^T (P + U_{ijv}) \bar{A}_{ij} & \bar{A}_{ij}^T (P + U_{ijv}) \bar{B}_{wij} \\ * & \bar{B}_{wij}^T (P + U_{ijv}) \bar{B}_{wij} \end{bmatrix}, \\
\tilde{\Sigma}_{2ij} &= \begin{bmatrix} -P + \bar{A}_{ij}^T (P - X_{ijv}) \bar{A}_{ij} & \bar{A}_{ij}^T (P - X_{ijv}) \bar{B}_{wij} \\ * & \bar{B}_{wij}^T (P - X_{ijv}) \bar{B}_{wij} \end{bmatrix}.
\end{aligned}$$

Firstly, considering the \mathcal{H}_∞ performance in (11) and introducing some slack matrices (symmetric matrices $V_{ijv} > 0$ and $Y_{ijv} > 0$ with appropriate dimensions), under the zero initial condition, we have:

$$J_\infty = \sum_{k=0}^{\infty} [z^T(k) z(k) - \gamma^2 w^T(k) w(k)] \quad (22)$$

$$\leq J_\infty + V(x(\infty)) - V(x(0))$$

$$= \sum_{k=0}^{\infty} [z^T(k) z(k) - \gamma^2 w^T(k) w(k) + \Delta V(x(k))] \quad (23)$$

$$\begin{aligned} &\leq \sum_{k=0}^{\infty} \left\{ \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} (\underline{\rho}_{ijv} \underline{h}_{ijv} + \bar{\rho}_{ijv} \bar{h}_{ijv}) \eta^T(k) \begin{bmatrix} \bar{E}_{ij}^T E_{ij} & \bar{E}_{ij}^T \bar{G}_{wij} \\ * & \bar{G}_{wij}^T \bar{G}_{wij} \end{bmatrix} \eta(k) + \Delta V(x(k)) \right. \\ &\quad \left. + \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \bar{h}_{ijv} z^T(k) V_{ijv} z(k) - \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \underline{\rho}_{ijv} (\underline{h}_{ijv} - \bar{h}_{ijv}) z^T(k) Y_{ijv} z(k) \right\} \\ &= \sum_{k=0}^{\infty} \eta^T(k) \left\{ \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} [\bar{h}_{ijv} \Sigma_{1ijv} - (\underline{h}_{ijv} - \bar{h}_{ijv}) R_{ijv} + \bar{h}_{ijv} S] - S \right\} \eta(k) \\ &\quad + \sum_{k=0}^{\infty} \eta^T(k) \sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} \underline{\rho}_{ijv} (\underline{h}_{ijv} - \bar{h}_{ijv}) (\Sigma_{2ijv} + R_{ijv} + S) \eta(k). \quad (24) \end{aligned}$$

Obviously, $J_\infty < 0$ in (24) can be obtained from the following two sets of inequalities: $\Xi_{2ijv} + R_{ijv} + Q > 0$ (which is guaranteed by the condition in (19)), and

$$\sum_{i=1}^p \sum_{j=1}^c \sum_{v=1}^{\vartheta+1} \sigma_{ijv} (x(k)) [\bar{h}_{ijv} \Sigma_{1ijv} - (\underline{h}_{ijv} - \bar{h}_{ijv}) R_{ijv} + \bar{h}_{ijv} S] - S < 0. \quad (25)$$

Noticed that only one $\sigma_{ijv}(x(k)) = 1$ for each fixed value of i and j at any time instant and $\sum_{v=1}^{\vartheta+1} \sigma_{ijv}(x(k)) = 1$, the set of inequalities in (25) is satisfied by

$$\sum_{i=1}^p \sum_{j=1}^c [\bar{h}_{ijv} \Sigma_{1ijv} - (\underline{h}_{ijv} - \bar{h}_{ijv}) R_{ijv} + \bar{h}_{ijv} S] - S < 0. \quad (26)$$

Considering \underline{h}_{ijv} in (12), \bar{h}_{ijv} in (13), and the equalities in (14), we express the following set of inequalities, which is equivalent to the set of inequalities in (26),

$$\sum_{\tau=1}^{\theta} \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{a=1}^n \varrho_{ai_a v \tau} (x_a(k)) \left(\sum_{i=1}^p \sum_{j=1}^c \Xi_{ij} - S \right) < 0. \quad (27)$$

Thus, the set of inequalities in (27) is satisfied by the condition in (18). Hence, $\|z\|_2 < \gamma \|w\|_2$ as $J_\infty < 0$, which means for all nonzero $w = w(k) \in \mathcal{L}_2[0, \infty)$, the conditions in Theorem 1 can guarantee that the system in (15) is asymptotically stable with an \mathcal{H}_∞ performance index γ .

Moreover, under disturbance-free cases, it can be easily obtained $\Delta V(x(k)) < 0$ from (21), which means the system in (15) is asymptotically stable. Then, considering the \mathcal{H}_2 performance cost function in (10) and the inequality in (23), we have

$$J_2 = \sum_{k=0}^{\infty} z^T(k) z(k) \leq - \sum_{k=0}^{\infty} \Delta V(x(k)) = V(x(0)) - V(x(\infty)) \leq V(x(0)) = x^T(0) P x(0) = J_2^*.$$

The proof is completed. ■

B. Reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ IT2 fuzzy controller design

In this subsection, the reliable IT2 fuzzy controller is designed based on the criterion in Theorem 1. The failure parameter of the sensor is considered with two cases, in which the sensor failure parameter matrix is known or unknown. The controller design results of the two cases are given in the following two parts.

1) *Reliable controller design under known sensor failure parameter:* Firstly, assume that the sensor failure parameter matrix is known, the reliable mixed IT2 fuzzy controller is designed in the following theorem.

Theorem 2: Considering the system with sensor failure in (15), for a given sensor failure diagonal matrix ζ and a scalar $\gamma > 0$, system (15) is reliable stable and has \mathcal{H}_∞ performance index γ , if there exist symmetric matrices $P > 0$, $R_{1ijv} > 0$, $R_{2ijv} > 0$, $R_{3ijv} > 0$, $X_{ijv} > 0$, $Y_{ijv} > 0$, $U_{ijv} > 0$, $V_{ijv} > 0$, ($i = 1, 2, \dots, p$, $j = 1, 2, \dots, c$, $v = 1, 2, \dots, \vartheta + 1$), S_1 , S_3 , and arbitrary matrix S_2 with appropriate dimensions, such that the following LMIs hold for $i = 1, 2, \dots, p$, $j = 1, 2, \dots, c$, $v = 1, 2, \dots, \vartheta + 1$, $\tau = 1, 2, \dots, \theta$:

$$\forall i, j, i_1, i_2, \dots, i_n, v, \tau, \begin{bmatrix} \hat{\Gamma} & \hat{\epsilon}\bar{\Pi}_1 \\ * & \Lambda_1 \end{bmatrix} < 0, \quad (28)$$

$$\forall i, j, v, \begin{bmatrix} \check{\Gamma} & \bar{\Pi}_1 \\ * & \Lambda_2 \end{bmatrix} > 0, \quad (29)$$

$$\forall i, j, v, P - X_{ijv} < 0, \quad (30)$$

$$\forall i, j, v, I - Y_{ijv} < 0, \quad (31)$$

where

$$\begin{aligned}\hat{\Gamma} &\triangleq \begin{bmatrix} \hat{\Gamma}_1 & \tilde{\epsilon}R_{2ijv} + \check{\epsilon}S_2 \\ * & \hat{\Gamma}_2 \end{bmatrix}, \check{\Gamma} \triangleq \begin{bmatrix} \check{\Gamma}_1 & R_{1ijv} + S_1 \\ * & \check{\Gamma}_2 \end{bmatrix}, \bar{\Pi}_1 \triangleq \begin{bmatrix} \bar{A}_{ij}^T & \bar{E}_{ij}^T \\ \bar{B}_{wij}^T & \bar{G}_{wij}^T \end{bmatrix}, \\ \Lambda_1 &\triangleq \text{diag}\{U_{ijv} + P - 2I, U_{ijv} - I\}, \Lambda_2 \triangleq \text{diag}\{X_{ijv} - P - 2I, Y_{ijv} - 3I\}, \\ \hat{\Gamma}_1 &\triangleq -\hat{\epsilon}^2P + \tilde{\epsilon}R_{1ijv} + \check{\epsilon}S_1, \hat{\Gamma}_2 \triangleq -\hat{\epsilon}^2\gamma^2I + \tilde{\epsilon}R_{3ijv} + \check{\epsilon}S_3, \\ \check{\Gamma}_1 &\triangleq -P + R_{1ijv} + S_1, \check{\Gamma}_2 \triangleq -\gamma^2I + R_{2ijv} + S_2, \\ \tilde{\epsilon} &\triangleq \bar{\epsilon}_{ij i_1 i_2 \dots i_n v \tau} - \underline{\epsilon}_{ij i_1 i_2 \dots i_n v \tau}, \check{\epsilon} \triangleq \bar{\epsilon}_{ij i_1 i_2 \dots i_n v \tau} - \frac{1}{pc}, \hat{\epsilon} \triangleq \sqrt{\bar{\epsilon}_{ij i_1 i_2 \dots i_n v \tau}}.\end{aligned}$$

and \bar{A}_{ij} , \bar{B}_{wij} , \bar{E}_{ij} and \bar{G}_{wij} are defined in (8) and (9). If the above conditions have a feasible solution, then the matrices K_j for the desired controller in the form of (3) can be obtained from the solution. Moreover, the \mathcal{H}_2 performance cost function bound is determined by

$$J_2^* = x_0^T P x_0.$$

Proof: For $U_{ijv} + P > 0$, the inequality $(U_{ijv} + P - I)^T (U_{ijv} + P)^{-1} (U_{ijv} + P - I) \geq 0$ holds.

Thus, we have

$$-(U_{ijv} + P)^{-1} \leq U_{ijv} + P - 2I. \quad (32)$$

Similarly, for $V_{ijv} + I > 0$, we have

$$-(V_{ijv} + I)^{-1} \leq V_{ijv} - 3I. \quad (33)$$

Thus, from the condition in (28), the following set of inequalities holds

$$\begin{bmatrix} \hat{\Gamma}_1 & \tilde{\epsilon}R_{2ijv} + \check{\epsilon}S_2 & \hat{\epsilon}\bar{A}_{ij}^T & \hat{\epsilon}\bar{E}_{ij}^T \\ * & \hat{\Gamma}_2 & \hat{\epsilon}\bar{B}_{wij}^T & \hat{\epsilon}\bar{G}_{wij}^T \\ * & * & -(U_{ijv} + P)^{-1} & 0 \\ * & * & * & -(V_{ijv} + I)^{-1} \end{bmatrix} < 0, \quad \forall i_1, i_2, \dots, i_n, i, j, v, \tau. \quad (34)$$

Let the following matrices:

$$R_{ijv} = \begin{bmatrix} R_{1ijv} & R_{2ijv} \\ * & R_{3ijv} \end{bmatrix}, S = \begin{bmatrix} S_1 & S_2 \\ * & S_3 \end{bmatrix}. \quad (35)$$

Then, based on the set of inequalities in (34), according to the Schur complement, one can obtain that the following set of inequalities holds:

$$\Xi_{ij} - \frac{1}{pc}S < 0, \quad \forall i_1, i_2, \dots, i_n, i, j, v, \tau$$

which can derive the condition (18) in Theorem 1. Similar to the above proof, it can be seen that the condition (29) together with the conditions in (30) and (31) can guarantee the condition (18) in Theorem 1. The proof is completed. \blacksquare

Remark 2: From Theorem 2, under the known sensor failure case, the existence condition of desired controller in the form of (3) is provided, which can guarantee the reliable stability for the closed-loop system in (15) with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance. However, in some practical applications, the sensor failure is often unknown, which may destroy the stability of the system unpredictably. Therefore, it is necessary to design a reliable controller to tolerate the unknown sensor failure in the process. In the following part, the desired controller is designed under the sensor failure unknown case.

2) *Reliable controller design under unknown sensor failure parameter:* Assuming that the sensor failure parameter matrix is unknown, based on Theorem 2, the reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ IT2 controller is designed in the following theorem.

Theorem 3: Considering the system with unknown sensor failure in (15), for a given scalar $\gamma > 0$, system (15) is reliable stable and has \mathcal{H}_∞ performance index γ , if there exist symmetric matrices $P > 0$, $R_{1ijv} > 0$, $R_{2ijv} > 0$, $R_{3ijv} > 0$, $X_{ijv} > 0$, $Y_{ijv} > 0$, $U_{ijv} > 0$, $V_{ijv} > 0$, ($i = 1, 2, \dots, p$, $j = 1, 2, \dots, c$, $v = 1, 2, \dots, \vartheta + 1$), S_1 , S_3 , and arbitrary matrix S_2 with appropriate dimensions, and scalar $\delta > 0$ satisfying the following condition:

$$\forall i, j, i_1, i_2, \dots, i_n, v, \tau, \begin{bmatrix} \hat{\Gamma} & \hat{\epsilon}\tilde{\Pi}_1 & \hat{\epsilon}\Pi_2 \\ * & \Lambda_1 & \Pi_3 \\ * & * & -\Lambda_0 \end{bmatrix} < 0, \quad (36)$$

$$\forall i, j, v, \begin{bmatrix} \check{\Gamma} & \check{\Pi}_1 & \Pi_2 \\ * & \Lambda_2 & -\Pi_3 \\ * & * & \Lambda_0 \end{bmatrix} > 0, \quad (37)$$

$$\forall i, j, v, P - X_{ijv} < 0, \quad (38)$$

$$\forall i, j, v, I - Y_{ijv} < 0, \quad (39)$$

where

$$\tilde{\Pi}_1 = \begin{bmatrix} \tilde{A}_{ij}^T & \tilde{E}_{ij}^T \\ \tilde{B}_{wij}^T & \tilde{G}_{wij}^T \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} \delta C_i^T & \delta C_i^T & 0 & 0 \\ \delta D_{wi}^T & \delta D_{wi}^T & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}\Pi_3 &= \begin{bmatrix} 0 & 0 & B_i K_j \check{\lambda} & 0 \\ 0 & 0 & 0 & G_i K_j \check{\lambda} \end{bmatrix}, \quad \Lambda_0 = \text{diag} \{ \delta I, \delta I, \delta I, \delta I \}, \\ \tilde{A}_{ij} &\triangleq A_i + B_i K_j \hat{\lambda} C_i, \quad \tilde{B}_{wij} \triangleq B_{wi} + B_i K_j \hat{\lambda} D_{wi}, \\ \tilde{E}_{ij} &\triangleq E_i + G_i K_j \hat{\lambda} C_i, \quad \tilde{G}_{wij} \triangleq G_{wi} + G_i K_j \hat{\lambda} D_{wi}.\end{aligned}$$

and $\hat{\Gamma}$, $\check{\Gamma}$, Λ_1 and Λ_2 are defined in Theorem 2. If the above conditions have a feasible solution, then the control gain matrices K_j in the form of (3) can be obtained from the solution. Moreover, the \mathcal{H}_2 performance cost function bound is determined by

$$J_2^* = x^T(0) P x(0).$$

Proof: For the condition in (36), given a scalar $\delta > 0$, by Lemma 2 one can obtain that

$$\begin{aligned}\begin{bmatrix} \hat{\Gamma} & \hat{\Pi}_1 \\ * & \Lambda_1 \end{bmatrix} &= \begin{bmatrix} \hat{\Gamma} & \check{\Pi}_1 \\ * & \Lambda_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \hat{\epsilon} (B_i K_j \bar{\lambda} C_i)^T & \hat{\epsilon} (G_i K_j \bar{\lambda} C_i)^T \\ * & 0 & \hat{\epsilon} (B_i K_j \bar{\lambda} D_{wi})^T & \hat{\epsilon} (G_i K_j \bar{\lambda} D_{wi})^T \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \\ &= \begin{bmatrix} \hat{\Gamma} & \check{\Pi}_1 \\ * & \Lambda_1 \end{bmatrix} + \begin{bmatrix} \hat{\epsilon} C_i^T & \hat{\epsilon} C_i^T \\ \hat{\epsilon} D_{wi}^T & \hat{\epsilon} D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} \bar{\lambda} & 0 \\ 0 & \bar{\lambda} \end{bmatrix}^T \begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix} \right) \\ &\quad + \left(\begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix}^T \begin{bmatrix} \bar{\lambda} & 0 \\ 0 & \bar{\lambda} \end{bmatrix} \right) \begin{bmatrix} \hat{\epsilon} C_i^T & \hat{\epsilon} C_i^T \\ \hat{\epsilon} D_{wi}^T & \hat{\epsilon} D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \\ &\leq \begin{bmatrix} \hat{\Gamma} & \check{\Pi}_1 \\ * & \Lambda_1 \end{bmatrix} + \delta \begin{bmatrix} \hat{\epsilon} C_i^T & \hat{\epsilon} C_i^T \\ \hat{\epsilon} D_{wi}^T & \hat{\epsilon} D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\epsilon} C_i^T & \hat{\epsilon} C_i^T \\ \hat{\epsilon} D_{wi}^T & \hat{\epsilon} D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T\end{aligned}\tag{40}$$

$$\begin{aligned}
& +\delta^{-1} \left(\begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix}^T \begin{bmatrix} \check{\lambda} & 0 \\ 0 & \check{\lambda} \end{bmatrix} \right) \\
& \times \left(\begin{bmatrix} \check{\lambda} & 0 \\ 0 & \check{\lambda} \end{bmatrix}^T \begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix} \right). \tag{41}
\end{aligned}$$

Thus, according to the Schur complement, it can be obtained that (34) holds from (36), which satisfies (28) in Theorem 2. Also, by the same approach applying to (29), one can obtain that (37) satisfies (29) in Theorem 2.

$$\begin{aligned}
\begin{bmatrix} \check{\Gamma} & \check{\Pi}_1 \\ * & \Lambda_2 \end{bmatrix} &= \begin{bmatrix} \check{\Gamma} & \check{\Pi}_1 \\ * & \Lambda_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & (B_i K_j \bar{\lambda} C_i)^T & (G_i K_j \bar{\lambda} C_i)^T \\ * & 0 & (B_i K_j \bar{\lambda} D_{wi})^T & (G_i K_j \bar{\lambda} D_{wi})^T \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \\
&= \begin{bmatrix} \check{\Gamma} & \check{\Pi}_1 \\ * & \Lambda_2 \end{bmatrix} - \begin{bmatrix} C_i^T & C_i^T \\ D_{wi}^T & D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} -\bar{\lambda} & 0 \\ 0 & -\bar{\lambda} \end{bmatrix}^T \begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix} \right) \\
&\quad - \left(\begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix}^T \begin{bmatrix} -\bar{\lambda} & 0 \\ 0 & -\bar{\lambda} \end{bmatrix} \right) \begin{bmatrix} C_i^T & C_i^T \\ D_{wi}^T & D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \\
&\geq \begin{bmatrix} \check{\Gamma} & \check{\Pi}_1 \\ * & \Lambda_2 \end{bmatrix} - \delta \begin{bmatrix} C_i^T & C_i^T \\ D_{wi}^T & D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} C_i^T & C_i^T \\ D_{wi}^T & D_{wi}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \\
&\quad -\delta^{-1} \left(\begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix}^T \begin{bmatrix} -\check{\lambda} & 0 \\ 0 & -\check{\lambda} \end{bmatrix} \right) \\
&\quad \times \left(\begin{bmatrix} -\check{\lambda} & 0 \\ 0 & -\check{\lambda} \end{bmatrix}^T \begin{bmatrix} 0 & 0 & (B_i K_j)^T & 0 \\ 0 & 0 & 0 & (G_i K_j)^T \end{bmatrix} \right).
\end{aligned}$$

Therefore, all the conditions in Theorem 3 are satisfied by the criteria in Theorem 2. This complete the proof. \blacksquare

Remark 3: Theorem 2 and Theorem 3 provide the sufficient conditions for the existence of the reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ fuzzy IT2 controller in the form of (4), respectively. When LMIs (28)–(29) or (36)–(37) are feasible, each \mathcal{H}_2 performance cost function is bounded by J_2^* . Actually, the upper bound of cost function (20) depends on the initial state x_0 . In [18], x_0 is assumed to be a zero mean random variable satisfying $\mathcal{E}\{x_0 x_0^T\} = I$ to remove the dependence. According to this assumption, the cost bound (20) turns to $J_2 \triangleq \mathcal{E}\{J_2\} \leq \mathcal{E}\{x_0^T P x_0\} = \text{trace}\{P\} = J_2^*$. In this paper, we let $\mathcal{X}_0 \mathcal{X}_0^T = x^T(0)x(0)$ to remove the dependence, and an optimal \mathcal{H}_2 performance cost function bound is described, which results in the following corollary.

Corollary 1: Consider the closed-loop system in (15) associated with \mathcal{H}_2 performance cost function in (10). Suppose that the optimization problem

$$\min \tilde{J}_2^* = \text{trace}(\mathcal{Z}_0) \quad (42)$$

subject to (28) and (29) (or (36) and (37)), and

$$\begin{bmatrix} -\mathcal{Z}_0 & \mathcal{X}_0 P \\ * & -P \end{bmatrix} < 0, \quad (43)$$

has a feasible solution, where $\text{trace}(\cdot)$ denotes the trace of a matrix, symmetric matrix $\mathcal{Z}_0 > 0$, then the IT2 fuzzy controller in (4) is an optimal reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controller, which guarantees the minimization of the \mathcal{H}_2 performance cost function bound (20) for system (15), where $\mathcal{X}_0 \mathcal{X}_0^T = x_0^T x_0$.

Proof: Since (28) and (29) (or (36) and (37)) have been given the proof in Theorem 2 (respectively, in Theorem 3), we just proof (43) in the following. Recalling $\text{trace}(P_1 P_2) = \text{trace}(P_2 P_1)$, from (43), one can obtain that $\mathcal{X}_0 P P^{-1} (\mathcal{X}_0 P)^T = \mathcal{X}_0^T P \mathcal{X}_0 < \mathcal{Z}_0$, thus,

$$x_0^T P x_0 = \text{trace}(x_0^T P x_0) = \text{trace}(P x_0^T x_0) = \text{trace}(P \mathcal{X}_0 \mathcal{X}_0^T) < \text{trace}(\mathcal{Z}_0).$$

It follows from (20) that $J_2^* < \tilde{J}_2^*$. Then, the minimization of \tilde{J}_2^* implies the minimization of the \mathcal{H}_2 performance cost function bound for the system in (15). This completes the proof. ■

V. SIMULATION RESULTS

In this section, we provide the simulation results from a numerical example to verify the effectiveness of the control design scheme. Firstly, the desired SOF controller design method under sensor failure known case is used to testify the availability for the reliable mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance of the system via Theorem 2. Then, considering a disturbance-free system with unknown sensor failure, the desired \mathcal{H}_2 performance controller is obtained via Corollary 1.

We give an IT2 fuzzy system of discrete-time form representing a nonlinear system, which is with two uncertain parameters a and b . For simplicity, a 3-rule IT2 fuzzy model is employed to describe the nonlinear system with a sampling period $T = 0.1$ s as follows:

Plant Rule i : IF $a(x_1(k))$ is M_1^i , THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) + B_{wi} w(k), \\ z(k) = E_i x(k) + G_i u(k) + G_{wi} w(k), \\ y(k) = C_i x(k) + D_{wi} w(k), \end{cases} \quad (44)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.18 & -a_{\min} \\ 0.08 & -a_{\min} - 0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.18 & -a_{\text{avg}} \\ 0.08 & -a_{\text{avg}} - 0.5 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.18 & -a_{\max} \\ 0.08 & -a_{\max} - 0.5 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 2a_{\min} - 0.05 & 0.13 \end{bmatrix}^T, \quad B_2 = \begin{bmatrix} 2a_{\text{avg}} - 0.05 & 0.26 \end{bmatrix}^T, \quad B_3 = \begin{bmatrix} 2a_{\max} - 0.05 & 0.16 \end{bmatrix}^T, \\ B_{w1} &= \begin{bmatrix} -0.13 & 0.4 \end{bmatrix}^T, \quad B_{w2} = \begin{bmatrix} -0.11 & 0.2 \end{bmatrix}^T, \quad B_{w3} = \begin{bmatrix} -0.12 & 0.1 \end{bmatrix}^T, \\ E_1 &= \begin{bmatrix} -0.214 & 0.128 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.120 & 0.120 \end{bmatrix}, \quad E_3 = \begin{bmatrix} -0.214 & 0.128 \end{bmatrix}, \\ G_1 &= -0.214, \quad G_2 = -0.120, \quad G_3 = -0.214, \quad G_{w1} = G_{w2} = G_{w3} = 0.01, \\ C_1 &= \begin{bmatrix} -0.03 & 0.020 \\ -0.01 & b_{\max} \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.02 & 0.018 \\ -0.01 & b_{\text{avg}} \end{bmatrix}, \quad C_3 = \begin{bmatrix} -0.01 & 0.012 \\ -0.01 & b_{\min} \end{bmatrix}, \\ D_{w1} &= D_{w2} = D_{w3} = \begin{bmatrix} 0.01 & -0.02 \end{bmatrix}^T. \end{aligned}$$

Assuming that $x_1 \in [-80, 80]$, the uncertain parameters a and b satisfy $a_{\min} = 0.1 \leq a(x_1) \leq a_{\max} = 0.2$ and $b_{\min} = 0.012 \leq b(x_1) \leq b_{\max} = 0.025$, respectively. Thus, $a_{\text{avg}} = (a_{\min} + a_{\max})/2$ and $b_{\text{avg}} = (b_{\min} + b_{\max})/2$. The lower and upper membership functions of the plant and the SOF controller are defined in Table I while $\phi_i(x(k))$ the grades of membership of the embedded membership functions are determined by the weighting functions chosen as

$$\begin{aligned} \underline{\alpha}_1(x_1) &= \frac{1}{2} \sin^2(x_1), \quad \underline{\alpha}_3(x_1) = \frac{1}{2} \sin^2(x_1), \\ \underline{\alpha}_2(x_1) &= \frac{1}{\bar{\phi}_2(x_1) - \underline{\phi}_2(x_1)} \left[\bar{\phi}_1(x_1) - \underline{\alpha}_1(x_1) (\bar{\phi}_1(x_1) - \underline{\phi}_1(x_1) +) \right. \\ &\quad \left. + \bar{\phi}_3(x_1) - \underline{\alpha}_3(x_1) (\bar{\phi}_3(x_1) - \underline{\phi}_3(x_1) +) + \bar{\phi}_2(x_1) - 1 \right], \end{aligned}$$

and $\bar{\alpha}_i(x_1) = 1 - \underline{\alpha}_i(x_1)$ where $\bar{\phi}_i(x_1)$ and $\underline{\phi}_i(x_1)$ are defined in Preliminaries; besides, we choose $\underline{\beta}_j(x_1) = \bar{\beta}_j(x_1) = 0.5$ to determine the real membership functions of the plant and the controller, respectively.

TABLE I
LOWER AND UPPER MEMBERSHIP FUNCTIONS OF THE PLANT AND THE CONTROLLER

Lower membership functions of the pant	Upper membership functions of the pant
$\underline{\mu}_{M_1^1}(x_1) = 0.8 - 0.8/\exp(-\frac{x_1+80}{15})$	$\bar{\mu}_{M_1^1}(x_1) = 1 - 1/\exp(-\frac{x_1+80}{15})$
$\underline{\mu}_{M_1^3}(x_1) = 0.8/\exp(-\frac{x_1-80}{15})$	$\bar{\mu}_{M_1^3}(x_1) = 1/\exp(-\frac{x_1-80}{15})$
$\underline{\mu}_{M_1^2}(x_1) = 1 - \bar{\mu}_{M_1^1}(x_1) - \bar{\mu}_{M_1^3}(x_1)$	$\bar{\mu}_{M_1^2}(x_1) = 1 - \underline{\mu}_{M_1^1}(x_1) - \underline{\mu}_{M_1^3}(x_1)$
Lower membership functions of the controller	Upper membership functions of the controller
$\underline{\mu}_{N_1^1}(x_1) = \exp(-\frac{x_1^2}{4000})$	$\bar{\mu}_{N_1^1}(x_1) = \underline{\mu}_{N_1^1}(x_1)$
$\underline{\mu}_{N_1^2}(x_1) = \underline{\mu}_{N_1^1}(x_1)$	$\bar{\mu}_{N_1^2}(x_1) = \underline{\mu}_{N_1^2}(x_1)$

Considering the computational burden, we use only one sub-FOU (i.e., $v = 1$) and divide the state x_1 into 100 equal-size sub-states (i.e., $\tau = 1, 2, \dots, 100$), from which the upper and lower bounds of τ -th state $x_1^{v,\tau}$ in the FOU v are defined as $\underline{x}_1^{v,\tau} = 1.6(\tau - 51)$, $\bar{x}_1^{v,\tau} = 1.6(\tau - 50)$. Then the constant scalars in the form of (12) and (13) are determined by

$$\begin{aligned}\epsilon_{ij11\tau} &= \underline{\phi}_i(\underline{x}_1^{v,\tau}) \underline{\psi}_j(\underline{x}_1^{v,\tau}), \quad \epsilon_{ij21\tau} = \underline{\phi}_i(\bar{x}_1^{v,\tau}) \underline{\psi}_j(\bar{x}_1^{v,\tau}), \\ \bar{\epsilon}_{ij11\tau} &= \bar{\phi}_i(\underline{x}_1^{v,\tau}) \bar{\psi}_j(\underline{x}_1^{v,\tau}), \quad \bar{\epsilon}_{ij21\tau} = \bar{\phi}_i(\bar{x}_1^{v,\tau}) \bar{\psi}_j(\bar{x}_1^{v,\tau}).\end{aligned}$$

Moreover, the lower and upper membership functions \underline{h}_{ij1} and \bar{h}_{ij1} are defined by choosing $\varrho_{111\tau}(x_1) = 1 - (x_1 - \underline{x}_1^{v,\tau}) / (\bar{x}_1^{v,\tau} - \underline{x}_1^{v,\tau})$ and $\varrho_{121\tau}(x_1) = 1 - \varrho_{111\tau}(x_1)$, respectively. The state response of the open-loop system in (44) based on the parameters above is plotted in Fig. 1, which shows that this system is not stable. Fig. 2 depicts the output trajectory, which implies the the open-loop system is unstable.

Firstly, in order to make a comparison between the sensor failure (known) case and sensor normal case, we choose the sensor failure matrix $\zeta = \text{diag}\{0.2, 0.3\}$ and the normal one $\zeta = \text{diag}\{1, 1\}$, respectively, and give an \mathcal{H}_∞ performance index $\gamma = 0.20$. Then, based on Theorem 2, the feasible solutions for controller gain matrices for system (44) with disturbance input are obtained as follows:

1) Sensor failure case

$$K_1 = \begin{bmatrix} 162.6078 & 103.7725 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 172.2039 & 103.9296 \end{bmatrix}; \quad (45)$$

2) Sensor normal case

$$K_1 = \begin{bmatrix} 31.9047 & 32.3183 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 34.7427 & 30.7439 \end{bmatrix}. \quad (46)$$

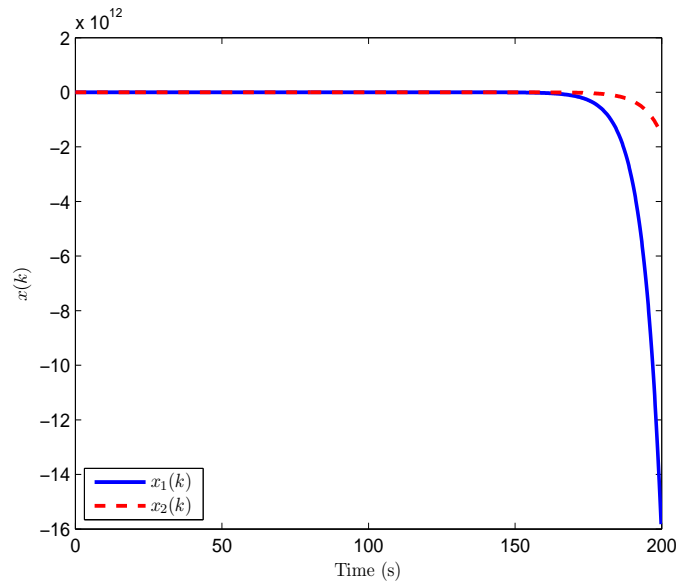


Fig. 1. States response of the open-loop system.

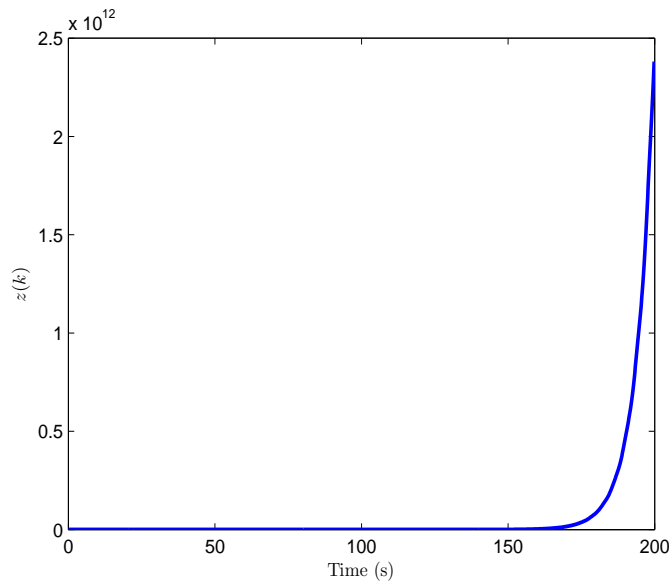


Fig. 2. Output of the open-loop system.

Next, based on the controller gains in (45) and (46), we analyze the stability of the plant under zero initial state $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. Considering the disturbance input $w(k) = 1/(2^k + 1)$, Fig. 3 and Fig. 4 show the state responses of the closed-loop system under the two cases. It can be seen that the states of two cases are both stable while the states in sensor failure case are slightly worse than that in normal one. The outputs of the closed-loop system are depicted in Fig. 5 under the two cases. Fig. 6 plots the control forces to the plant under the two cases. These figures illustrate that the nonlinear system in (44) with sensor failure can be controlled subject to uncertainties a and b under zero initial condition, and the failure in the sensor can be completely tolerated. The effectiveness of the proposed design method is confirmed.

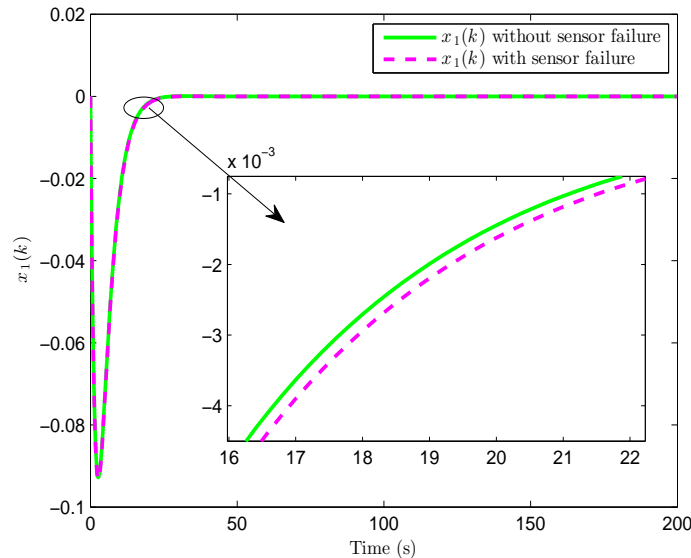


Fig. 3. State x_1 response of the closed-loop system.

Secondly, to further analyze the \mathcal{H}_2 performance of the closed-loop system, we use Corollary 1 to obtain the desired IT2 fuzzy SOF controller and guarantee an optimal \mathcal{H}_2 performance cost function bound. Assume that the sensor failure is unknown between the bounds of $\underline{\zeta} = \text{diag}\{0.65, 0.72\}$ and $\bar{\zeta} = \text{diag}\{0.72, 0.85\}$. Considering the disturbance-free system in (44) under initial state $x(0) = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}^T$, and giving another \mathcal{H}_∞ performance index $\gamma = 1.0$, by solving the convex optimization problem in (42), we obtain the fuzzy SOF controller matrices

$$K_1 = \begin{bmatrix} 40.2662 & 74.1886 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 6.7712 & 1.9683 \end{bmatrix}, \quad (47)$$

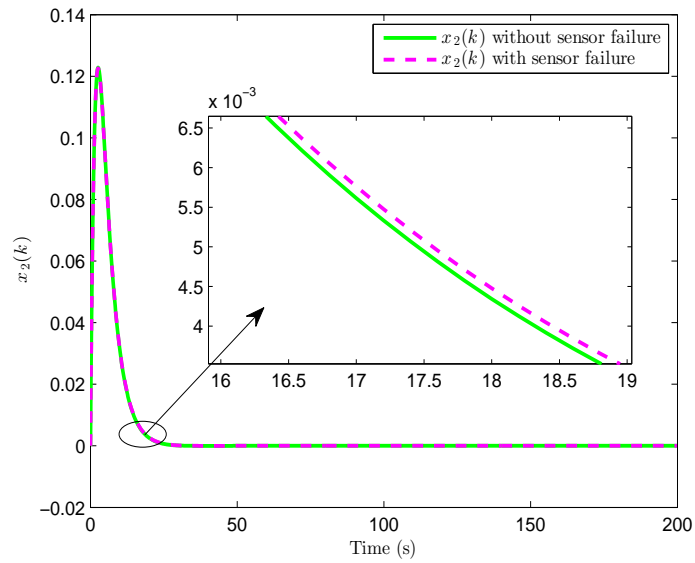


Fig. 4. State x_2 response for the closed-loop system.

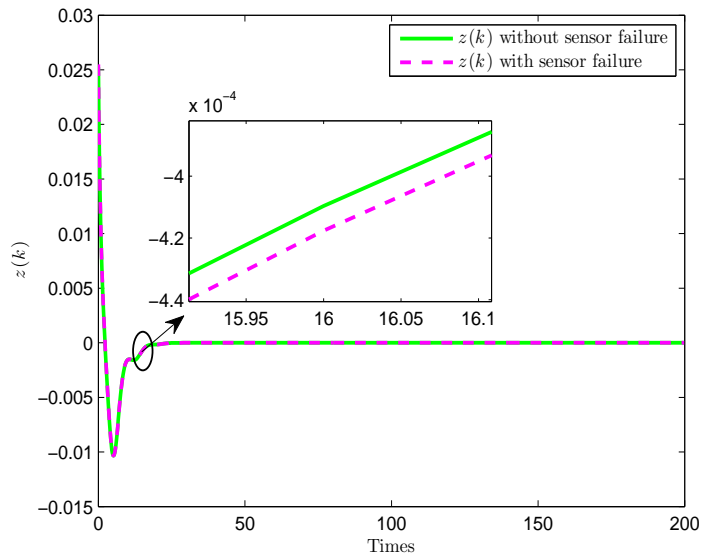


Fig. 5. Output of the closed-loop system.

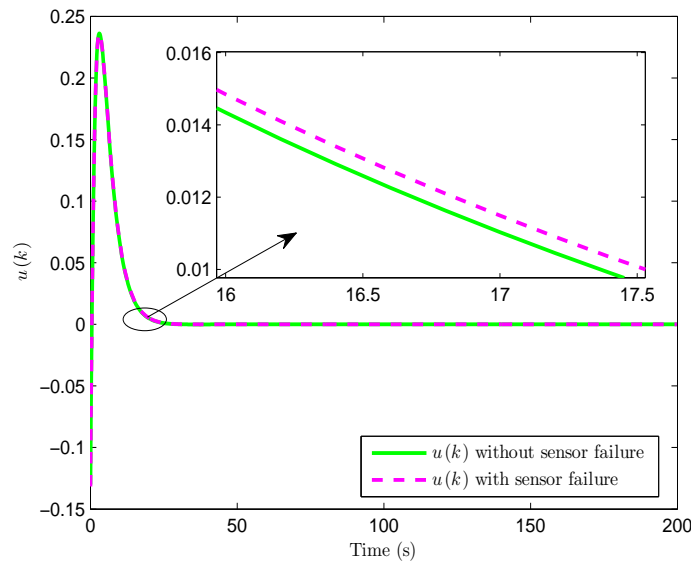


Fig. 6. Control input for the closed-loop system.

and the minimum \mathcal{H}_2 performance cost function bound $\tilde{J}_2^* = 50.0$. Besides, the actual \mathcal{H}_2 performance cost function is $J_2 = \sum_{k=0}^{200} z^T(k) z(k) = 40.6686$, which satisfies $J_2 < \tilde{J}_2^*$. This also verifies the effectiveness of the proposed design scheme. Furthermore, to observe the stability of the sensor failure system in (44), we assume the real sensor failure matrix $\zeta = \text{diag}\{0.68, 0.84\}$ satisfying the assumed bound above. Based on the controller matrices in (47), the simulation results are obtained in Figs. 7–8. Fig. 7 shows the state responses and Fig. 8 depicts the corresponding control input of the closed-loop system with sensor failure. Figs. 7–8 also present that the controlled system in (44) is reliable stability under the sensor failure unknown case.

VI. CONCLUSION

In this paper, the problem of reliable control for discrete-time IT2 FMB systems with sensor failure has been solved. The mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance has been considered. The number of fuzzy rules and the membership functions for the SOF controller are different from those for the plant. A sufficient criterion of reliable stability with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance has been given for the closed-loop system with sensor failure. The constraints of the SOF controller parameters have been provided for sensor failure known case and sensor failure unknown case, which can guarantee the reliable stability of the plant with mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance. Furthermore, the criteria of optimal $\mathcal{H}_2/\mathcal{H}_\infty$ performance for the closed-

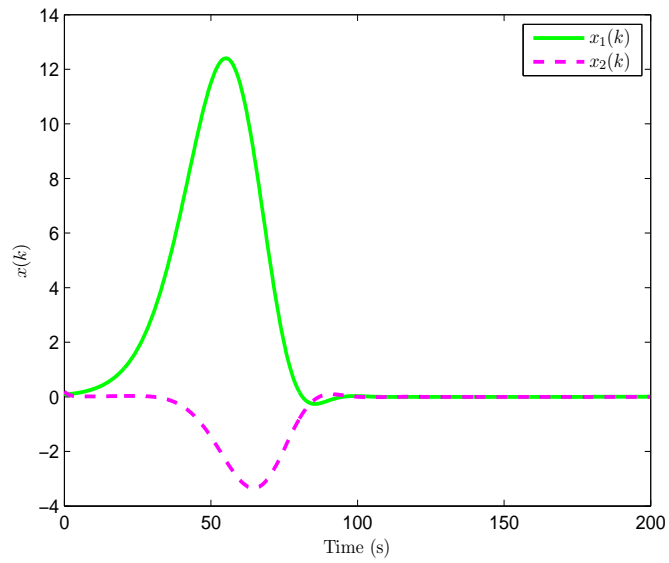


Fig. 7. State responses for the closed-loop system.

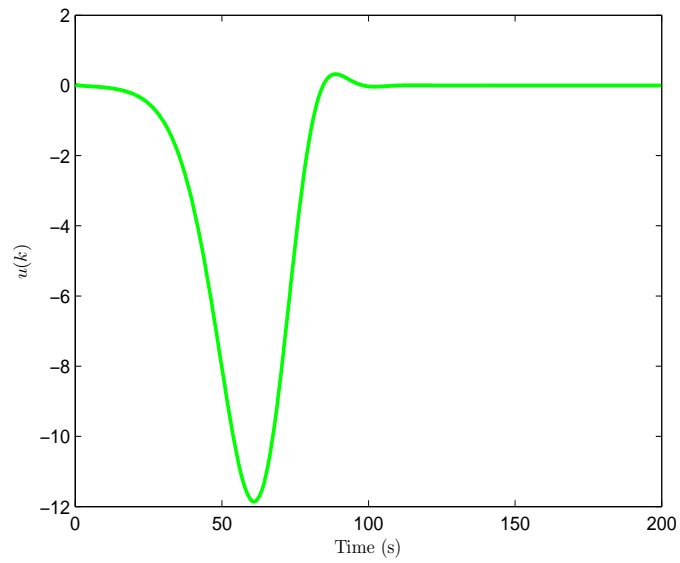


Fig. 8. Control input for the closed-loop system.

loop system are proposed. A numerical example has been employed to verified the effectiveness of the proposed results. In future work, the dynamic output-feedback control for IT2 FMB systems will be investigated by considering possible faults or errors occurring in the IT2 FMB systems.

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