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# Robust Chance-Constrained Distributed Beamforming for Multicell Interference Networks

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**Abstract**—We propose a robust coordinated game theoretic approach that distributively minimizes the aggregate downlink transmit power in a multicell interference network in the presence of imperfect channel state information (CSI). The optimization is constrained to satisfying the signal-to-interference-plus-noise-ratio (SINR) requirements at individual user terminals within certain predefined SINR outage probabilities. This problem is numerically intractable due to the cross-link coupling effect among a cluster of base stations (BSs) as well as the robust constraints that involve the second order statistical CSI estimation error. By employing cumulative distribution function of standard normal distribution, Lemma 2 and semidefinite relaxation technique, we first convert the original problem to a linear matrix inequality form. Then, we introduce an iterative subgradient algorithm that decomposes the multicell-wise general problem into a set of parallel subproblems at individual BSs to find the global optimality. We show that the proposed design coordinates intercell interference among the BSs with a light inter-BS communication overhead. Simulation results demonstrate the advantages of the proposed chance-constrained distributed design in terms of power efficiency and achievable robustness trade-off.

## I. INTRODUCTION

In recent years, coordinated transmission has shown its significant advantage in terms of intercell interference (ICI) mitigation and system performance improvement [1]. The coordinated transmission can be classified into two categories in accordance with the level of cooperation: joint transmission (JT) and coordinated scheduling/beamforming (CS/CB). In JT, the base stations (BSs) require all the channel state information (CSI) and user's data via capacity-limited backhaul links. Whereas in the latter category, each BS only coordinates the transmission to its intra-cell user terminals (UTs) using local CSI and a strict CS across cells to mitigate ICI, especially for the cell-edge UTs [2]. More recently, coordinated transmission based on coordinated game theoretic concept is proposed [3], [4], [5], where only the key intercell coupling parameters are shared among BSs iteratively to optimize their transmission strategies. [3] introduces a distributed iterative algorithm using subgradient method for coordinated quality of service (QoS) beamforming design via limited signaling among BSs. However, the problem is solved in a non-robust multicast manner. On the other hand, future wireless network is expected to support enormous increasing mobile traffic with different level of QoS for different UTs. Meantime, the acquisition of accurate CSI at transmitters, e.g., BSs, is of crucial practical importance to enable the efficient downlink coordi-

nated transmission. However, the QoS control in the practical multiuser system is limited by the channel uncertainties [6] since it may lead to the imperfect CSI at BSs. Therefore, beamforming designs based on the assumption of perfect CSI may no longer guarantee the SINR requirements at UTs. Assuming that the uncertainty region of CSI perturbations is bounded, the authors in [7] and [8] investigate the robust sum power minimization problem subject to worst-case signal-to-interference-plus-noise-ratio (SINR) constraints at UTs in a distributed manner in downlink multicell network. However, the assumption that CSI error is bounded within a known region is, in general, conservative since it may require higher transmit power to count for the worst-case QoS. Another decentralized approach to a robust power minimization problem is proposed in [11] where a SINR outage threshold is assigned to the QoS constraints and the problem is solved by employing the Bernstein-type inequality method. Moreover, the authors in [9] propose an iterative algorithm to distributively minimize a linear combination of total transmit power and weighted ICI in a coordinated network using statistical CSI. Nevertheless, the author take no consideration of any estimation error, which may result in unpredictable results in practice. Taken CSI uncertainties into account, the authors in [10] introduce an outage-constrained distributed robust beamforming scheme to jointly coordinate the total transmit power and ICI pricing in multicell networks using second order statistical CSI under the assumption that the total ICI can be accurately estimated by the UTs and then updated to the local BS.

In this paper, we introduce a probabilistic constrained robust transmission strategy that distributively minimizes the total transmit power across the multiple cells while optimally accounts for the coupling intercell effects. The problem is formulated as a sum-power minimization problem under individual UTs' target SINR constraints at a set of predetermined outage levels in the presence of CSI uncertainties. By using semidefinite relaxation (SDR) technique [12] and Lemma 2, we first reformulate the numerically intractable problem to a semidefinite programming (SDP) form with linear matrix inequality (LMI) constraints. Then, the multicell-wise general problem is decomposed into a set of equivalent parallel subproblems at individual BSs. Finally, by applying an iterative projected subgradient method, the optimality across the involved BSs is achieved with a light inter-BS communications overhead.

The rest of this paper is organized as follows. Section II presents the system model and optimization problem formula-

tion. In Section III, the original problem is reformulated as a probabilistic constrained stochastic optimization problem and then transformed into an numerically tractable SDP form with LMI constraints. Then, an algorithm is proposed for decoupling the problem into a distributed manner using projected subgradient method, followed by the backhaul signalling load analysis of the proposed design. Simulation results are analyzed in Section IV. Finally, Section V concludes the paper.

Notations: Throughout the paper,  $w$ ,  $\mathbf{w}$ ,  $\mathbf{W}$ , respectively, present a scalar  $w$ , a vector  $\mathbf{w}$  and a matrix  $\mathbf{W}$ . The notations  $(\cdot)^H$ ,  $\text{tr}(\cdot)$ ,  $\text{Pr}(\cdot)$  and  $[\cdot]_{mn}$  denote the complex conjugate transpose operators, the trace operators, the probability operator and the  $mn$ -th element of a matrix, respectively.  $\mathbf{W} \succeq 0$  indicates that  $\mathbf{W}$  is a positive semidefinite matrix. The notations  $\text{vec}(\mathbf{W})$  and  $\text{diag}(\mathbf{w})$  respectively, represent the vector obtained by stacking the column vectors of  $\mathbf{W}$  and the diagonal matrix with vector  $\mathbf{w}$  on its main diagonal.  $\mathbb{N}(\cdot)$ ,  $\mathbb{CN}(\cdot)$  and  $\mathbb{E}(\cdot)$  stand for real and complex Gaussian random variables and expectation value, respectively.  $\mathbf{0}_{m \times n}$ ,  $\mathbf{1}_k$  and  $\mathbf{I}_n$  represent, respectively, an all-zero  $m$ -by- $n$  matrix, a column vector with a one at the  $k$ -th entry and zeros elsewhere and a  $n$ -by- $n$  identity matrix.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a downlink multicell network with a coordinated cluster of  $N$  cells. Each cell consists of a BS equipped with an array of  $M$  antenna elements transmitting to  $K$  active single-antenna UTs over a shared bandwidth. Let the set of indexes for the BSs and the UTs be denoted as  $\mathcal{N} = \{1, \dots, N\}$  and  $\mathcal{K} = \{1, \dots, K\}$ , respectively. Let  $\text{BS}_i$ ,  $i \in \mathcal{N}$  indicate the BS in the  $i$ -th cell, and  $\text{UT}_{ik}$ ,  $k \in \mathcal{K}$  represent the  $k$ -th UT in the  $i$ -th cell. Then, the signal received by  $\text{UT}_{ik}$  is given by

$$z_{ik} = \mathbf{h}_{ik}^H \mathbf{w}_{ik} s_{ik} + \sum_{\substack{n \neq k, \\ n \in \mathcal{K}}} \mathbf{h}_{in}^H \mathbf{w}_{in} s_{in} \quad (1)$$

$$+ \sum_{\substack{j \neq i, \\ j \in \mathcal{N}}} \sum_{m \in \mathcal{K}} \mathbf{h}_{jik}^H \mathbf{w}_{jm} s_{jm} + n_{ik},$$

where  $s_{ik}$  indicates the data symbol for  $\text{UT}_{ik}$ ,  $\mathbf{w}_{ik} \in \mathbb{C}^{M \times 1}$  and  $\mathbf{h}_{ijk} \in \mathbb{C}^{M \times 1}$  denote the beamforming vector for  $\text{UT}_{ik}$  and the channel vector from  $\text{BS}_i$  to  $\text{UT}_{jk}$ , respectively. Note that the terms in the right hand side of (1), respectively, represent the desired signal, the total intra-cell interference, the aggregate ICI and the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise at  $\text{UT}_{ik}$ , e.g.,  $n_{ik} \sim \mathbb{CN}(0, \sigma_n^2)$ .

Let  $\hat{\mathbf{C}}_{ijk} = \mathbb{E}(\hat{\mathbf{h}}_{ijk} \hat{\mathbf{h}}_{ijk}^H) \in \mathbb{C}^{M \times M}$  denote the estimated channel covariance matrix of  $\text{UT}_{jk}$ , as seen by the  $i$ -th BS. Also let  $\Delta_{ijk} \in \mathbb{C}^{M \times M}$  represent the corresponding error matrix, where the  $cd$ -th entry of  $\Delta_{ijk}$  is distributed as  $[\Delta_{ijk}]_{cd} \sim \mathbb{CN}(0, \sigma_{cd}^2)$ . Then, we can model the true channel covariance matrix  $\mathbf{C}_{ijk}$  as

$$\mathbf{C}_{ijk} = \hat{\mathbf{C}}_{ijk} + \Delta_{ijk} \quad \forall i, j, k,$$

Assuming  $\mathbb{E}(|s_{ik}|^2) = 1$ , we can formulate the SINR at  $\text{UT}_{ik}$

as

$$\text{SINR}_{ik} = \frac{\mathbf{w}_{ik}^H \mathbf{C}_{ik} \mathbf{w}_{ik}}{\sum_{\substack{n \neq k, \\ n \in \mathcal{K}}} \mathbf{w}_{in}^H \mathbf{C}_{in} \mathbf{w}_{in} + \sum_{\substack{j \neq i, \\ j \in \mathcal{N}}} \sum_{m \in \mathcal{K}} \mathbf{w}_{jm}^H \mathbf{C}_{jm} \mathbf{w}_{jm} + \sigma_n^2}. \quad (2)$$

We consider a robust problem of minimizing the total transmit power in a multicell network under the constraints of satisfying the SINR requirements at individual UTs in the presence of channel estimation errors, as

$$\min_{\mathbf{w}_{ik}, \forall i, k} \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} \|\mathbf{w}_{ik}\|^2 \quad (3)$$

$$\text{s.t.} \quad \text{SINR}_{ik} \geq \gamma_{ik}, \quad \forall i, k,$$

where  $\gamma_{ik}$  is the target SINR requested by  $\text{UT}_{ik}$ .

In order to account for the coupling effects among the multiple cells, we begin by introducing slack variables  $\{p_{ijk}\}_{i,j,k} \in \mathbb{R}$  and reformulating the problem in (3) as

$$\min_{\mathbf{w}_{ik}, \forall i, k} \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} \|\mathbf{w}_{ik}\|^2 \quad (4)$$

$$\text{s.t.} \quad \frac{\mathbf{w}_{ik}^H (\hat{\mathbf{C}}_{iik} + \Delta_{iik}) \mathbf{w}_{ik}}{\sum_{\substack{n \neq k, \\ n \in \mathcal{K}}} \mathbf{w}_{in}^H (\hat{\mathbf{C}}_{iik} + \Delta_{iik}) \mathbf{w}_{in} + \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{lik} + \sigma_n^2} \geq \gamma_{ik}, \quad \forall i, k,$$

$$p_{ijk} \geq \sum_{m \in \mathcal{K}} \mathbf{w}_{im}^H (\hat{\mathbf{C}}_{ijk} + \Delta_{ijk}) \mathbf{w}_{im}, \quad \forall i, j \neq i, k,$$

where  $p_{ijk}$  indicates the ICI from  $\text{BS}_i$  to  $\text{UT}_{jk}$ .

## III. OUTAGE BASED DISTRIBUTED OPTIMIZATION

### A. Chance-constrained Optimization of problem in (4)

In the sequel, we reformulate the problem in (4) into a chance-constrained optimization problem, as

$$\min_{\mathbf{w}_{ik}, \forall i, k} \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} \|\mathbf{w}_{ik}\|^2 \quad (5)$$

$$\text{s.t.} \quad \Pr \left( \frac{\mathbf{w}_{ik}^H (\hat{\mathbf{C}}_{iik} + \Delta_{iik}) \mathbf{w}_{ik}}{\sum_{\substack{n \neq k, \\ n \in \mathcal{K}}} \mathbf{w}_{in}^H (\hat{\mathbf{C}}_{iik} + \Delta_{iik}) \mathbf{w}_{in} + \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{lik} + \sigma_n^2} \geq \gamma_{ik} \right) \geq 1 - \rho_{ik}, \quad \forall i, k,$$

$$\Pr \left( \sum_{m \in \mathcal{K}} \mathbf{w}_{im}^H (\hat{\mathbf{C}}_{ijk} + \Delta_{ijk}) \mathbf{w}_{im} \leq p_{ijk} \right) \geq 1 - \rho_{ik},$$

where  $\rho_{ik} \in (0, 1)$  is the maximum SINR outage probability and  $1 - \rho_{ik}$  indicates that the individual UTs is guaranteed to achieve its target SINR with probability of  $1 - \rho_{ik}$  at the least. The problem in (5) is NP-hard since the inclusion of CSI uncertainties in probabilistic constraints naturally lead to an infinite number of convex sets. Therefore, following the similar principles as in [10], we equivalently transform the

probabilistic constraints of the problems in (5) into a tractable form through the following Lemma.

**Lemma 1.** Let  $\Delta \in \mathbb{C}^{M \times M}$  be a Hermitian random matrix with each ZMCSCG element being characterized as  $[\Delta]_{cd} \sim \mathbb{CN}(0, \sigma_{cd}^2)$ . Then, for any Hermitian matrix  $\mathbf{L}$ ,  $\mathbf{L} \in \mathbb{C}^{M \times M}$ ,

$$\begin{aligned} \text{tr}(\mathbf{L}\Delta) &\sim \mathbb{N}(0, \|\mathcal{D}_\Delta \text{vec}(\mathbf{L})\|^2), \\ \text{tr}(\mathbf{L}\Delta) &= \|\mathcal{D}_\Delta \text{vec}(\mathbf{L})\|U, \quad U \sim \mathbb{N}(0, 1), \end{aligned}$$

where  $\mathcal{D}_\Delta = \text{diag}(\text{vec}(\Sigma_\Delta^H))$  and  $\Sigma_\Delta$  denotes a real-valued  $M \times M$  matrix with each entry  $[\Sigma_\Delta]_{cd} = \sigma_{cd}$ .

*Proof:* See [10].  $\blacksquare$

Let the rank-one positive semidefinite matrix be defined as  $\mathbf{W}_{ik} = \mathbf{w}_{ik}\mathbf{w}_{ik}^H$ . Also let the first and the second set of constraints in (5) be rewritten as follows

$$\Pr \left( \begin{aligned} \text{tr}(-\mathbf{B}_{ik}\Delta_{iik}) &\leq \text{tr}(\mathbf{B}_{ik}\hat{\mathbf{C}}_{iik}) - \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{lik} - \sigma_n^2 \\ &\geq 1 - \rho_{ik}, \end{aligned} \right) \quad (6)$$

$$\Pr \left( \begin{aligned} \text{tr}(\mathbf{Q}_{ijk}\Delta_{ijk}) &\leq p_{ijk} - \text{tr}(\hat{\mathbf{C}}_{ijk}\mathbf{Q}_{ijk}) \\ &\geq 1 - \rho_{ik}, \end{aligned} \right) \quad (7)$$

where  $\mathbf{B}_{ik} = \gamma_{ik}^{-1}\mathbf{W}_{ik} - \sum_{n \neq k, n \in \mathcal{K}} \mathbf{W}_{in}$  and  $\mathbf{Q}_{ijk} = \sum_{m \in \mathcal{K}} \mathbf{W}_{im}$ . By applying Lemma 1 and the cumulative distribution function (CDF) of a standard normal distribution, e.g.,  $\phi(u) = \Pr(U \leq u) = \frac{1}{2}[1 + \text{erf}(\frac{u}{\sqrt{2}})]$ , where  $U \sim \mathbb{N}(0, 1)$ , the first and the second constraints in (6) and (7), respectively, can be expressed as follows

$$\begin{aligned} &\Pr \left( \begin{aligned} \text{tr}(-\mathbf{B}_{ik}\Delta_{iik}) &\leq \text{tr}(\mathbf{B}_{ik}\hat{\mathbf{C}}_{iik}) - \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{lik} - \sigma_n^2 \end{aligned} \right) \\ &= \Pr \left( U \leq \frac{\text{tr}(\mathbf{B}_{ik}\hat{\mathbf{C}}_{iik}) - \sum_{l \in \mathcal{N}, l \neq i} p_{lik} - \sigma_n^2}{\|\mathcal{D}_{\Delta_{iik}} \text{vec}(-\mathbf{B}_{ik})\|} \right) \\ &= \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\text{tr}(\mathbf{B}_{ik}\hat{\mathbf{C}}_{iik}) - \sum_{l \in \mathcal{N}, l \neq i} p_{lik} - \sigma_n^2}{\sqrt{2}\|\mathcal{D}_{\Delta_{iik}} \text{vec}(-\mathbf{B}_{ik})\|} \right) \right] \\ &\geq 1 - \rho_{ik}, \end{aligned} \quad (8)$$

$$\begin{aligned} &\Pr \left( \text{tr}(\mathbf{Q}_{ijk}\Delta_{ijk}) \leq p_{ijk} - \text{tr}(\hat{\mathbf{C}}_{ijk}\mathbf{Q}_{ijk}) \right) \\ &= \Pr \left( U \leq \frac{p_{ijk} - \text{tr}(\hat{\mathbf{C}}_{ijk}\mathbf{Q}_{ijk})}{\|\mathcal{D}_{\Delta_{ijk}} \text{vec}(\mathbf{Q}_{ijk})\|} \right) \\ &= \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{p_{ijk} - \text{tr}(\hat{\mathbf{C}}_{ijk}\mathbf{Q}_{ijk})}{\sqrt{2}\|\mathcal{D}_{\Delta_{ijk}} \text{vec}(\mathbf{Q}_{ijk})\|} \right) \right] \geq 1 - \rho_{ik}, \end{aligned} \quad (9)$$

which are equivalent to the following expressions, respectively,

$$\Theta \geq \sqrt{2}\text{erf}^{-1}(1 - 2\rho_{ik})\|\mathcal{D}_{\Delta_{iik}} \text{vec}(-\mathbf{B}_{ik})\|, \quad (10)$$

$$\Upsilon \geq \sqrt{2}\text{erf}^{-1}(1 - 2\rho_{ik})\|\mathcal{D}_{\Delta_{ijk}} \text{vec}(\mathbf{Q}_{ijk})\|, \quad (11)$$

where  $\Theta = \text{tr}(\mathbf{B}_{ik}\hat{\mathbf{C}}_{iik}) - \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{lik} - \sigma_n^2$  and  $\Upsilon = p_{ijk} - \text{tr}(\hat{\mathbf{C}}_{ijk}\mathbf{Q}_{ijk})$ .

**Lemma 2.** The following second order cone constraint on  $x$

$$\|Ax + b\| \leq e^T x + d$$

is equivalent to the following LMI form [13]

$$\begin{bmatrix} (e^T x + d)\mathbf{I} & Ax + b \\ (Ax + b)^T & e^T x + d \end{bmatrix} \succeq 0,$$

Finally, by applying Lemma 2 to (10) and (11), the problem in (5) can be reformulated as LMI forms,

$$\begin{aligned} &\min_{\mathbf{W}_{ik}, \forall i, k} \sum_{i \in \mathcal{N}} f_i(\mathbf{W}_{ik}, \mathbf{p}_i) \triangleq \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} \text{tr}(\mathbf{W}_{ik}) \quad (12) \\ &\text{s.t.} \quad \begin{bmatrix} \frac{\Theta}{\sqrt{2}\text{erf}^{-1}(1-2\rho_{ik})}\mathbf{I}_{M^2} & \mathcal{D}_{\Delta_{iik}} \text{vec}(-\mathbf{B}_{ik}) \\ \text{vec}^H(-\mathbf{B}_{ik})\mathcal{D}_{\Delta_{iik}} & \frac{\Theta}{\sqrt{2}\text{erf}^{-1}(1-2\rho_{ik})} \end{bmatrix} \succeq 0, \\ &\quad \begin{bmatrix} \frac{\Upsilon}{\sqrt{2}\text{erf}^{-1}(1-2\rho_{ik})}\mathbf{I}_{M^2} & \mathcal{D}_{\Delta_{ijk}} \text{vec}(\mathbf{Q}_{ijk}) \\ \text{vec}^H(\mathbf{Q}_{ijk})\mathcal{D}_{\Delta_{ijk}} & \frac{\Upsilon}{\sqrt{2}\text{erf}^{-1}(1-2\rho_{ik})} \end{bmatrix} \succeq 0, \\ &\quad \mathbf{W}_{ik} \succeq 0, \\ &\quad \text{rank}(\mathbf{W}_{ik}) = 1, \end{aligned}$$

where  $\mathbf{p}_i \in \mathbb{R}^{NK \times 1}, \forall i, j \neq i$  is a real-valued vector that contains the local intercell coupling variables at the  $i$ -th BS,

$$\mathbf{p}_i = \left[ \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{li1}, \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{li2}, \dots, \sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{liK} \left| p_{ij1}, p_{ij2}, \dots, p_{iNK} \right]^T. \quad (13)$$

The function  $f_i(\mathbf{W}_{ik}, \mathbf{p}_i) = \sum_{k \in \mathcal{K}} \text{tr}(\mathbf{W}_{ik})$  in (12) indicates the dependence of  $f_i$  on  $\mathbf{p}_i$ . Moreover, the non-convex rank-one constraint  $\text{rank}(\mathbf{W}_{ik}) = 1$  can be relaxed using SDR.

### B. Distributed Optimization of problem in (12)

Let the global intercell coupling variables  $\mathbf{p} \in \mathbb{R}^{(N(N-1)+1)K \times 1}$  be defined as

$$\mathbf{p} = \left[ p_{121}, p_{122}, \dots, p_{12K}, \dots, p_{N11}, \dots, p_{NN-1K} \left| \mathbf{0}_{K \times 1}^T \right]^T. \quad (14)$$

In the sequel, we use a direction matrix  $\mathbf{X}_i$  to extract  $\mathbf{p}_i$  from  $\mathbf{p}$ , i.e.,  $\mathbf{p}_i = \mathbf{X}_i \mathbf{p}$ , so that the individual BSs can locally design the multicell-wise optimum beams towards its local users in a distributed manner. Let us define  $\mathbf{X}_i = [\mathbf{A}_i^T \ \mathbf{B}_i^T]^T \in \{0, 1\}^{NK \times (N(N-1)+1)K}$ , where  $\mathbf{A}_i \in \{0, 1\}^{K \times (N(N-1)+1)K}$  and  $\mathbf{B}_i \in \{0, 1\}^{(N-1)K \times (N(N-1)+1)K}$ . The  $i$ -th BS constructs  $\mathbf{A}_i$  and  $\mathbf{B}_i$  by rotating each one of the rows of matrices  $\hat{\mathbf{A}}$  and  $\hat{\mathbf{B}}$ , respectively,  $(i-1)NK$  and  $(i-1)(N-1)K$  times anticlockwise, where

$$\hat{\mathbf{A}} = \left[ \mathbf{0}_{K \times (N-1)K} \left| \overbrace{\mathbf{I}_{K \times (N-1)K}}^{(N-1)} \right| \mathbf{0}_{K \times K} \right]$$

$$\hat{\mathbf{B}} = \left[ \mathbf{I}_{(N-1)K \times (N-1)K} \left| \mathbf{0}_{(N-1)K \times ((N-1)^2+1)K} \right. \right]$$

Then the  $i$ -th BS extracts the entries of  $\mathbf{p}_i$ , as

$$\sum_{\substack{l \neq i, \\ l \in \mathcal{N}}} p_{lik} = \mathbf{1}_k^T \mathbf{X}_i \mathbf{p}, \quad \forall k, \quad (15)$$

$$p_{ijk} = \mathbf{1}_q^T \mathbf{X}_i \mathbf{p}, \quad \forall j \neq i, k, \quad (16)$$

where  $q = k + jK$  if  $j < i$  or  $q = k + (j - 1)K$  if  $j > i$ .

Consequently, for any given  $\mathbf{p}$ , we can decompose the problem in (12) into  $N$  sub-problems at each BS  $i$ , as

$$\begin{aligned} \min_{\mathbf{W}_{ik}, \forall k} \quad & f_i(\mathbf{W}_{ik}, \mathbf{p}_i) = \sum_{k \in \mathcal{K}} \text{tr}(\mathbf{W}_{ik}) \\ \text{s.t.} \quad & \mathbf{T}_{ik} = \mathbf{T}'_{ik} - (\mathbf{1}_k^T \mathbf{X}_i \mathbf{p}) \mathbf{I}_{(M^2+1)} \succeq 0, \\ & \mathbf{T}_{ijk} = \mathbf{T}'_{ijk} + (\mathbf{1}_q^T \mathbf{X}_i \mathbf{p}) \mathbf{I}_{(M^2+1)} \succeq 0, \\ & \mathbf{W}_{ik} \succeq 0, \end{aligned} \quad (17)$$

where

$$\begin{aligned} \mathbf{T}'_{ik} &= \begin{bmatrix} \frac{\text{tr}(\mathbf{B}_{ik} \hat{\mathbf{C}}_{iik}) - \sigma_n^2}{\sqrt{2\text{erf}^{-1}(1-2\rho_{ik})}} \mathbf{I}_{M^2} & \mathcal{D}_{\Delta_{iik}} \text{vec}(-\mathbf{B}_{ik}) \\ \text{vec}^H(-\mathbf{B}_{ik}) \mathcal{D}_{\Delta_{iik}} & \frac{\text{tr}(\mathbf{B}_{ik} \hat{\mathbf{C}}_{iik}) - \sigma_n^2}{\sqrt{2\text{erf}^{-1}(1-2\rho_{ik})}} \end{bmatrix}, \\ \mathbf{T}'_{ijk} &= \begin{bmatrix} \frac{-\text{tr}(\hat{\mathbf{C}}_{ijk} \mathbf{Q}_{ijk})}{\sqrt{2\text{erf}^{-1}(1-2\rho_{ik})}} \mathbf{I}_{M^2} & \mathcal{D}_{\Delta_{ijk}} \text{vec}(\mathbf{Q}_{ijk}) \\ \text{vec}^H(\mathbf{Q}_{ijk}) \mathcal{D}_{\Delta_{ijk}} & \frac{-\text{tr}(\hat{\mathbf{C}}_{ijk} \mathbf{Q}_{ijk})}{\sqrt{2\text{erf}^{-1}(1-2\rho_{ik})}} \end{bmatrix}. \end{aligned}$$

Since the optimal solution  $\mathbf{w}_{ik}^*$  is obtained as a function of  $\mathbf{p}$ , we introduce an algorithm to iteratively coordinates  $\mathbf{p}$  and  $\mathbf{w}_{ik}, \forall i, k$ , at their globally optimal settings of  $\mathbf{p}^*$  and  $\mathbf{w}_{ik}^*$ , respectively, to minimize the total power consumption in the multicell network. The Lagrangian of the  $i$ -th subproblem in (17) can be expressed as

$$L_i = \sum_{k \in \mathcal{K}} \text{tr}(\mathbf{W}_{ik}) - \sum_{k \in \mathcal{K}} \text{tr}(\lambda_{ik} \mathbf{T}_{ik}) - \sum_{\substack{l \neq i, k \in \mathcal{K} \\ l \in \mathcal{N}}} \text{tr}(\lambda_{ijk} \mathbf{T}_{ijk}), \quad (18)$$

where  $\lambda_{ik}, \lambda_{ijk} \in \mathbb{H}^{(M^2+1) \times (M^2+1)}$  are the Lagrange multipliers. Since the problem in (17) is convex and satisfies the Slater's condition, strong duality holds [12] and the dual function is given by

$$\begin{aligned} \ell_i(\mathbf{p}) &= \inf_{\mathbf{W}_{ik} \succeq 0} L_i = \Xi \left( \{\lambda_{ik}\}_k, \{\lambda_{ijk}\}_{j \neq i, k} \right) \\ &+ \left( \sum_{k \in \mathcal{K}} \text{tr}(\lambda_{ik} \mathbf{I}) \mathbf{1}_k^T - \sum_{\substack{l \neq i, k \in \mathcal{K} \\ l \in \mathcal{N}}} \text{tr}(\lambda_{ijk} \mathbf{I}) \mathbf{1}_q^T \right) \mathbf{X}_i \mathbf{p}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} \Xi \left( \{\lambda_{ik}\}_k, \{\lambda_{ijk}\}_{j \neq i, k} \right) &= \inf_{\mathbf{W}_{ik} \succeq 0} \sum_{k \in \mathcal{K}} \text{tr}(\mathbf{W}_{ik}) \\ &- \sum_{k \in \mathcal{K}} \text{tr}(\lambda_{ik} \mathbf{T}'_{ik}) - \sum_{\substack{l \neq i, k \in \mathcal{K} \\ l \in \mathcal{N}}} \text{tr}(\lambda_{ijk} \mathbf{T}'_{ijk}). \end{aligned}$$

Then we can write

$$\begin{aligned} f_i^*(\mathbf{W}_{ik}^*, \mathbf{p}_i) &= f_i^*(\mathbf{p}_i) = \ell_i^*(\mathbf{p}) \\ &= \mathbf{g}_i \mathbf{p} + \Xi \left( \{\lambda_{ik}^*\}_k, \{\lambda_{ijk}^*\}_{j \neq i, k} \right), \end{aligned} \quad (20)$$

where

$$\mathbf{g}_i = \left( \sum_{k \in \mathcal{K}} \text{tr}(\lambda_{ik}^* \mathbf{I}) \mathbf{1}_k^T - \sum_{\substack{l \neq i, k \in \mathcal{K} \\ l \in \mathcal{N}}} \text{tr}(\lambda_{ijk}^* \mathbf{I}) \mathbf{1}_q^T \right) \mathbf{X}_i. \quad (21)$$

It can be easily concluded from (20) that for any given  $\hat{\mathbf{p}}$ ,

$$\ell_i^*(\hat{\mathbf{p}}) \geq \ell_i^*(\mathbf{p}) + \mathbf{g}_i(\hat{\mathbf{p}} - \mathbf{p}). \quad (22)$$

Therefore,  $\mathbf{g}_i \in \mathbb{R}^{1 \times (N(N-1)+1)K}$  is the subgradient vector of  $\ell_i^*(\mathbf{p})$  and  $f_i^*(\mathbf{p}_i)$  obtained for the  $i$ -th subproblem. Following the similar steps of analysis as for the  $i$ -th subproblem in (17), one can easily calculate the global subgradient of  $\sum_{i \in \mathcal{N}} f_i^*(\mathbf{p}_i)$

obtained for the general problem in (12) at a given value of  $\mathbf{p}$ , as

$$\begin{aligned} \mathbf{g} &= \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} \text{tr}(\lambda_{ik}^* \mathbf{I}) \mathbf{1}_k^T \mathbf{X}_i - \sum_{i \in \mathcal{N}} \sum_{\substack{l \neq i, k \in \mathcal{K} \\ l \in \mathcal{N}}} \text{tr}(\lambda_{ijk}^* \mathbf{I}) \mathbf{1}_q^T \mathbf{X}_i \\ &= \sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{K}} \text{tr}(\lambda_{ik}^* \mathbf{I}) \mathbf{1}_k^T - \sum_{\substack{l \neq i, k \in \mathcal{K} \\ l \in \mathcal{N}}} \text{tr}(\lambda_{ijk}^* \mathbf{I}) \mathbf{1}_q^T \right) \mathbf{X}_i \\ &= \sum_{i \in \mathcal{N}} \mathbf{g}_i. \end{aligned} \quad (23)$$

Then, by sharing the subgradient vector  $\mathbf{g}_i$  with other BSs via inter-BS communications, each BS  $i$  can compute the global subgradient  $\mathbf{g}$  locally and updates the global intercell coupling vector  $\mathbf{p}$  as below

$$\mathbf{p}^{[t+1]} = \left[ \mathbf{p}^{[t]} - \frac{\alpha \mathbf{g}^{[t]T}}{\sqrt{t} \|\mathbf{g}^{[t]}\|} \right]^+, \quad (24)$$

where  $[\cdot]^+$  indicates the projection onto nonnegative orthant,  $t$  represents the iteration index and  $\alpha > 0$  is the step size. The steps of adjusting  $\mathbf{p}$  and solving the problem in (3) are summarized in Algorithm 1. Each BS  $i$  individually solves its own subproblem (17), obtains the subgradient vector  $\mathbf{g}_i$  in accordance with (23) and exchanges it with other BSs via inter-BS communications. Then, each BS  $i$  calculates the global subgradient  $\mathbf{g}$  locally and updates the global coupling vector  $\mathbf{p}$  according to the projected subgradient method.

### C. Backhaul Signalling Load Analysis

For the  $i$ -th BS, the only information that need to be exchanged with the other  $N - 1$  BSs in each iteration is the subgradient  $\mathbf{g}_i$  that contains  $NK$  non-zero entries, i.e.,  $\text{tr}(\lambda_{ik}^* \mathbf{I}), \forall k$  and  $\text{tr}(\lambda_{ijk}^* \mathbf{I}), \forall k, j \neq i$ . The resulting inter-BS communication overhead is  $O(NK(N-1))$  and thus, the total signaling overhead among all the BSs is  $O(\xi N^2 K(N-1))$ , where  $\xi$  is the total iteration number of Algorithm 1. However, for the CS/CB design in [17] that requires full CSI exchange, the information that need to be sent at each BS is  $O(NK(N-1))$  of  $M \times 1$  complex-valued CSI vectors. The total signaling overhead is then  $O(4MN^2 K(N-1))$ . The ratio of backhaul signaling load for the proposed strategy

**Algorithm 1** Distributed Algorithm for Solving (12) at individual BSs

- 1: **Initialize:**  $t = 0$  and  $\mathbf{p}(0) \in \mathbb{R}^{K(N(N-1)+1) \times 1}$ ;
- 2: **repeat** at each BS $_i$
- 3: **while** the solutions to (17) is not converged **do**
- 4:   Solve the subproblem  $i$  in (17);
- 5:   Calculate the local subgradient  $\mathbf{g}_i$  using (21);
- 6:   Exchange  $\mathbf{g}_i$  with other BSs;
- 7:   Upon obtaining subgradient vector  $\mathbf{g}_i$  from all other BSs, compute the global subgradient as  $\mathbf{g} = \sum_{i \in \mathcal{N}} \mathbf{g}_i$ ;
- 8:   Update the global variable  $\mathbf{p}$  according to (24);
- 9:   increment the iteration index  $t = t + 1$ ;
- 10: **end while**
- 11: **if**  $\mathbf{W}_{ik}^*$  is rank-one **then**
- 12:   The optimal  $\mathbf{w}_{ik}$  is the eigenvector of  $\mathbf{W}_{ik}^*$ ;
- 13: **else**
- 14:   Apply the standard Gaussian randomization method [16] to approximate rank-one  $\mathbf{w}_{ik}$  solutions;
- 15: **end if**
- 16: **return**  $\{\mathbf{w}_{ik}\}_{i,k}$ .

over CS/CB design in [17] can be expressed as  $\varphi = \frac{\xi}{4M}$ . As will be evident in the simulation results section, Algorithm 1 always converges only within several iterations. Therefore, for future cellular network that employs an increase number of antenna elements per BS, the proposed strategy requires lighter inter-BS communication overhead as compared to the CS/CB design that requires full CSI exchange.

#### IV. SIMULATION RESULTS

TABLE I: Simulation parameters

Parameter	value
Number of cells ( $N$ )	3
Number of users per cell ( $K$ )	2
Number of antennas per BS ( $M$ )	8
Spacing between two adjacent antenna elements $\delta$	$\lambda/2$
Array antenna gain	15 dBi
Noise power spectral density (all users)	-174 dBm/Hz
Noise figure at user receiver	5 dB
Distance between two adjacent BSs	500 m
Path loss model over a distance of $\ell$ m	$34.53 + 38 \log_{10}(\ell)$
Angular offset standard deviation $\sigma$	$2^\circ$
Log-normal shadowing standard deviation $\sigma_s$	10 dB

We consider 3 adjacent cells, each cell consists of a BS equipped with 8 antennas. Two users are randomly generated in the vicinity of the boundaries in each cell to account for the worst ICI effect. Similar to [14], we modeled the  $(m, n)$ -th element of the channel covariance matrix  $\mathbf{R}_{ijk} \in \mathbb{C}^{M \times M}$  as  $[\mathbf{R}_{ijk}]_{mn} = e^{j \frac{2\pi\delta}{\lambda} [(n-m)\sin\theta_{ijk}]} e^{-2[\frac{\pi\delta\sigma}{\lambda}(n-m)\cos\theta_{ijk}]^2}$ ,  $m, n \in [1, M]$ , where  $\theta_{ijk}$  is the angle of departure for UT $_{jk}$  with respect to the broadside of the antenna of BS $_i$ ,  $\lambda$  is the carrier wavelength. Besides, to take consideration of path loss, shadowing and fading, we scaled the channel covariance

matrix  $\mathbf{R}_{ijk}$  and its corresponding random error matrix  $\Delta_{ijk}$  by  $G_a L_{ijk} \sigma_F^2 e^{-0.5 \frac{(\sigma_s \ln 10)^2}{100}}$ , where  $L_{ijk}$  represents the path loss between BS $_i$  and UT $_{jk}$ ,  $\sigma_F^2$  is the variance of the complex Gaussian fading coefficient. Other important parameters are presented in Table 1. Equal SINR targets  $\gamma_{ik}$  and equal SINR outage probability  $\rho_{ik}$  are assumed for all UTs in different cells. We further assume that each entry of error matrix  $\Delta_{ijk}$  has the same variance  $\sigma_{cd}^2 = \sigma_e^2$ . All of the system designs in this paper are simulated and averaged via CVX [15].

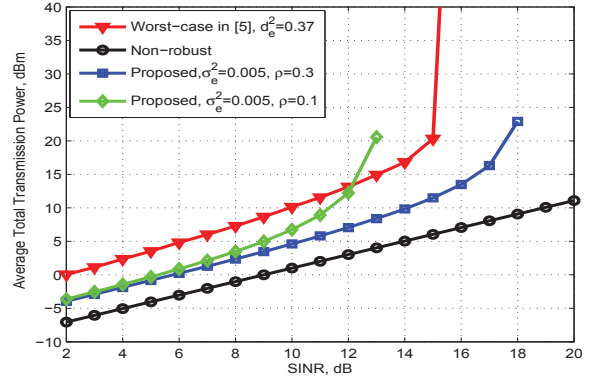


Fig. 1: Comparison of total transmit power versus various SINR targets.

The performance comparison of the proposed strategy with different SINR outage levels against worst-case bounded error design in [5] in terms of total transmit power is presented in Fig. 1. For fair comparison,  $d_e = \sqrt{\frac{\sigma_e^2 \psi^{-1} (1 - \rho_{ik})}{\chi_{2M}^2 M^2}}$  is employed to calculate the radius of bounded error in [5] corresponding to  $\rho_{ik} = 0.1$  and  $\sigma_e^2 = 0.005$ . It can be observed that the proposed strategy performs overwhelmingly better than the worst-case design in [5] in terms of providing better power efficiency up to medium SINR operational range. However, since the proposed strategy adopts statistical CSI, it has worse SINR operational range for high SINR requirements as compared with design in [5] for instantaneous CSI. One can also conclude that for a given CSI uncertainty variance, the total transmit power consumption increases as we decrease the outage probability  $\rho_{ik}$ . The performance gap can be interpreted that the higher level of robustness against CSI uncertainties comes with the cost of increment in total transmit power.

Fig. 2 compares the SINR satisfaction ratio at  $\gamma = 10$  dB target SINR of the proposed distributed robust design and its centralized non-robust counterpart. The SINR satisfaction ratio is defined as the achieved SINR over the target SINR of UT $_{ik}$ .

$$\eta_{ik} = \frac{\mathbf{w}_{ik}^H \mathbf{C}_{iik} \mathbf{w}_{ik}}{\gamma_{ik} \left( \sum_{\substack{n \neq k, \\ n \in \mathcal{K}}} \mathbf{w}_{in}^H \mathbf{C}_{iik} \mathbf{w}_{in} + \sum_{\substack{j \neq i, m \in \mathcal{K} \\ j \in \mathcal{N}}} \mathbf{w}_{jm}^H \mathbf{C}_{jik} \mathbf{w}_{jm} + \sigma_n^2 \right)},$$

where  $\eta_{ik} \geq 1$  indicates that the SINR requirement is satisfied. One can observe that by setting outage probability  $\rho_{ik} = 0.3$ ,

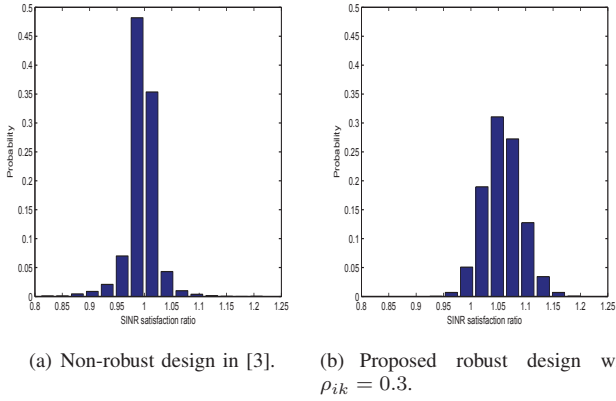


Fig. 2: Histograms of SINR satisfaction ratio for  $\gamma = 10$  dB.

the majority of SINR constraints is satisfied and only a small portion of SINR satisfaction ratios falls below 1. However, since the non-robust design provides no tolerance to any level of uncertainties, the achieved SINR fails to satisfy the target SINR for approximately 50 percent of the cases. In comparison with Fig. 1, the performance gap between robust and non-robust designs can be interpreted as the price for guaranteeing the QoS of UTs with certain outage probabilities, i.e., robustness to the imperfect CSI.

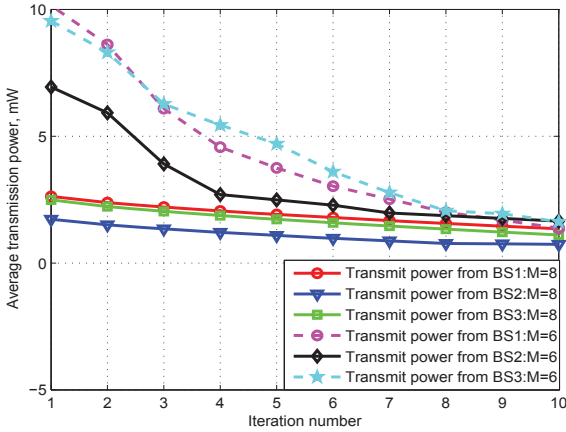


Fig. 3: Convergence behaviour of Algorithm 1 at  $\gamma = 10$  dB target SINR for  $M = 6, 8$  antenna elements per BS.

Fig. 3 demonstrates the convergence behaviour of the proposed Algorithm 1 with  $\sigma_e^2 = 0.005$  and  $\rho_{ik} = 0.3$  at  $\gamma = 10$  dB target SINR for  $M = 6, 8$  number of antenna elements per BS. It can be observed from the figure that as we increase the number of antenna elements per BS, the convergence speed increases while the power variation range between the initial and the final iterations decrease. The reason for that is with higher number of antenna elements per BS, extra degree and more accurate coordination can be provided by the BSs.

## V. CONCLUSION

In this paper, we propose a probabilistic constrained distributed robust coordinated transmission strategy in downlink multicell interference networks. The problem is formulated as a sum-power minimization problem under the constraints of satisfying SINR requirements at individual UTs with different outage probabilities. We first reformulate the NP-complete problem as a SDP form with LMI constraints based on CDF of standard normal distribution and Lemma 2. Then, we decompose the original problem into parallel subproblems at each BS and apply the projected subgradient iterations to coordinate the cross-link interference across the BSs with light backhaul signaling load. Simulation results confirm the advantages of the proposed design in terms of achieving power efficiency as compared to worst-case bounded error design and achievable robustness against CSI uncertainties as compared with non-robust design in a multicell scenario.

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