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Filtering of Interval Type-2 Fuzzy Systems With Intermittent Measurements

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Abstract—**In this paper, the problem of fuzzy filter design is investigated for a class of nonlinear networked systems on the basis of the interval type-2 fuzzy set theory. In the design process, two vital factors, intermittent data packet dropouts and quantization, are taken into consideration. The parameter uncertainties are handled effectively by the interval type-2 membership functions determined by lower and upper membership functions and relative weighting functions. A novel fuzzy filter is designed to guarantee the error system to be asymptotically stable with H_∞ performance. Moreover, the filter does not need to share the same membership functions and number of fuzzy rules as those of the plant. Finally, practical examples are used to validate the effectiveness of the proposed method.**

Index Terms—**Interval type-2 model; Fuzzy filter; Nonlinear networked system; Data packet dropouts.**

I. Introduction

IN modern industrial systems, the physical plants, sensors, filters and actuators are usually located in different geographical places, which require the signal to be transmitted from one place to another. The traditional communication via point-to-point cables can not satisfy the requirement. Thus, the network media is introduced into the system to connect these components, which gives rise to the networked systems [1], [2]. Networked systems have brought obvious advantages, such as low cost, simple installation and maintenance, and high reliability [3]. The authors in [4] have highlighted that increasing attention has been received in modeling, analysis and synthesis of networked systems. Recently, some stability and stabilization results concerning networked systems were reported in [5]–[11]; the filtering problem was studied in [12]–[15].

For unavailable system states, the filter and observer play a vital role in estimating them. The authors in [16] proposed a filtering design method to estimate state variables. In [17]–[20], the observer was used to handle the unmeasurable systems states with different systems requirements. However, nonlinearities often exist in the complex plants [21]–[23], which makes it difficult to design them in nonlinear systems. The Takagi-Sugeno (T-S) fuzzy-model-based approach [24] can cope with nonlinearities existing in the application systems, which "blends" every local linear system through "IF-THEN" rules [25]–[33]. In networked environment, considerable filter design results with the T-S fuzzy model have been reported. The H_∞ filtering problem for a new class of discrete-time nonlinear networked systems with mixed random delays and packet dropouts was investigated in [34]. The authors in [25] studied the distributed finite-horizon filtering problem for a class of time-varying systems over lossy sensor networks. The problem of H_∞ filtering for a class of nonlinear discrete-time systems with measurement quantization and packet dropouts was investigated in [16]. In [35], the filter design problem for fault detection with missing measurement was researched. The authors in [14] proposed a novel approach for the filtering problem of T-S fuzzy systems in a network environment. Both the delays of the premise variables and the measurements were considered.

However, it is worth noticing that the aforementioned results concerning networked systems were achieved through the T-S fuzzy model-based approach on the basis of the type-1 fuzzy set theory, which can deal with nonlinearities perfectly. There also exist parameter uncertainties in the application systems, which can not be handled by the type-1 fuzzy set theory effectively. And the accuracy of system modeling will be degraded if parameter uncertainties are not fully taken into account. The authors in [36] proposed an interval type-2 (IT2) fuzzy model to model the physical plants with parameter uncertainties. In view of its merits, researchers have been inspired to investigate it and many results concerning IT2 fuzzy systems were reported in [37]–[40]. The authors in [41] provided an IT2 T-S fuzzy model approach to model the plant subject to parameter uncertainties and achieved the stability condition. Additionally, it has been proved that the IT2 T-S fuzzy model performs better than the type-1 one when the parameter uncertainty exists in the system. In [42], [43], the designed controller did not need to share the same premise variables, membership functions and fuzzy rules as those of the plant, which enhance the flexibility of controller design. The filter design problem was studied in [44], in which the authors obtained the desired condition to guarantee the error system to be stochastically stable as well as the guaranteed H_∞ performance. However, the results on IT2 T-S fuzzy model are related to traditional control systems and few ones are concerned with nonlinear networked systems. Therefore, it is challenging to investigate the filter design problem for a class of nonlinear networked systems subject to parameter

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uncertainties on the basis of IT2 T-S fuzzy model.

This paper focuses on the problem of filter design for the networked nonlinear systems with parameter uncertainties in the framework of IT2 T-S fuzzy model. In the process of modeling the physical plant, the logarithmic quantizer and Bernoulli stochastic process are used to handle data quantization and intermittent data missing, respectively. The main contributions of this paper are summarized as follows. 1). The system considered in this paper is modeled by the IT2 T-S fuzzy model, in which lower and upper membership functions with weighting functions are utilized to capture the parameter uncertainty. 2). The footprint of uncertainty (FOU) [42], [43] is considered in order to take into account more information of uncertainties. 3). The filter design is independent of the premise variables, membership functions and number of fuzzy rules of the system in order to enhance the flexibility of the filter design, that is, the filter does not need to share the same ones as those of the system. Furthermore, the filter to be designed guarantees the resulting error system preserves the stochastic stability as well as the prescribed H_∞ performance. Finally, illustrative examples are provided to illustrate the effectiveness of the method proposed in the paper.

The remainder of the paper is organized as follows. Section II describes the considered problem. The main results are presented in Section III. Illustrative examples are provided to verify the usefulness of the proposed method in Section IV. Finally, Section V concludes the paper.

Notation: The notation utilized in this paper is quite standard. The superscript "T" and "-1" denote matrix transposition and matrix inverse, respectively. The identity matrix and zero matrix with compatible dimensions are represented by I and 0, respectively. The notation $P > 0\,(\geq 0)$ suggests that P is positive definite (semi-definite) with the real symmetric structure. The notation $\|A\|$ indicates the norm of matrix A defined by $\|A\| = \sqrt{\mathrm{tr}\,(A^TA)}$. $l_2[0,\infty)$ means the space of square-integrable vector functions over $[0,\infty)$; R^n describes the n-dimensional Euclidean space, and $\|\cdot\|_2$ shows the usual $l_2[0,\infty)$ norm. In complex matrix, the symbol $(*)$ is utilized to represent a symmetric term, and we utilize diag$\{...\}$ to describe the matrix with block diagonal structure. Furthermore, $E\{x|y\}$ and $E\{x\}$ signify expectation of x conditional on y and expectation of x, respectively. Prob$(\cdot)$ and $\lambda_{\min}(A)$ represent the occurrence probability of the event "$\cdot$" and the the minimum eigenvalue of the matrix A, respectively. Matrices in this paper without dimensions explicitly stated, we assume they have compatible dimensions.

II. Problem Formulation

In this section, the discrete-time nonlinear networked system presented in Fig. 1 is considered. As shown in Fig. 1, data packet dropouts and quantization are taken into consideration. In the framework of the IT2 T-S fuzzy model, a r-rule fuzzy model is given as follows:

Plant Rule i: IF $f_1(x(k))$ is M_{i1}, and $f_2(x(k))$ is M_{i2} and,

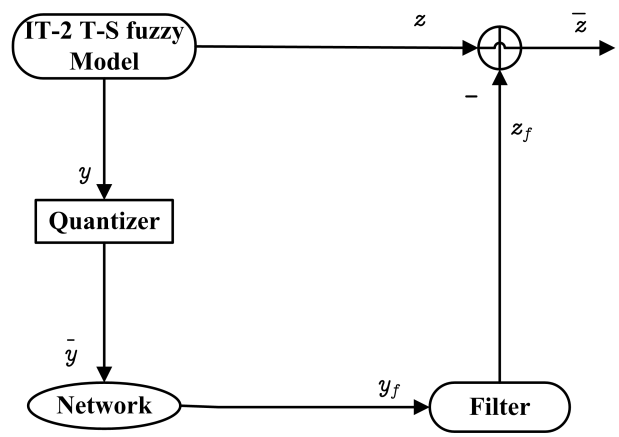

Fig. 1. Structure of networked systems with the quantizer

..., and $f_\theta(x(k))$ is $M_{i\theta}$, THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i w(k), \\ y(k) &= C_i x(k) + D_i w(k), \\ z(k) &= L_i x(k), \end{aligned} \tag{1}$$

where M_{ij} presents the fuzzy set, and $f(x(k)) = [f_1(x(k)), f_2(x(k)), ..., f_\theta(x(k))]$ stands for the premise variable, in which θ is the number of the fuzzy sets. $x(k) \in R^{n_x}$ is the state; $y(k) \in R^{n_y}$ is the measured output; $z(k) \in R^{n_z}$ is the controlled output; $w(k) \in R^{n_w}$ is the disturbance input which belongs to $l_2[0,\infty)$. A_i, B_i, C_i, D_i and L_i are system matrices with appropriate dimensions. $i \in 1,2,...,r$, the scalar r is the number of IF-THEN rules of the system. The following interval sets present the firing strength of the ith rule:

$$W_i(x(k)) = [\underline{m}_i(x(k)), \overline{m}_i(x(k))],$$

where

$$\begin{aligned} \underline{m}_i(x(k)) &= \prod_{p=1}^{\theta} \underline{u}_{M_{ip}}(f_p(x(k))) \geq 0, \\ \overline{m}_i(x(k)) &= \prod_{p=1}^{\theta} \overline{u}_{M_{ip}}(f_p(x(k))) \geq 0, \\ \overline{u}_{M_{ip}}(f_p(x(k))) &\geq \underline{u}_{M_{ip}}(f_p(x(k))) \geq 0, \\ \overline{m}_i(x(k)) &\geq \underline{m}_i(x(k)) \geq 0, \end{aligned}$$

$\underline{u}_{M_{ip}}(f_p(x(k)))$, $\overline{u}_{M_{ip}}(f_p(x(k)))$, $\underline{m}_i(x(k))$ and $\overline{m}_i(x(k))$ denote the lower membership function, upper membership function, lower grade of membership and upper grade of membership, respectively.

The inferred dynamics of the fuzzy system (1) is as follows:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} m_i(x(k)) [A_i x(k) + B_i w(k)], \\ y(k) &= \sum_{i=1}^{r} m_i(x(k)) [C_i x(k) + D_i w(k)], \\ z(k) &= \sum_{i=1}^{r} m_i(x(k)) L_i x(k), \end{aligned} \tag{2}$$

where

$$\begin{aligned} m_i(x(k)) &= \underline{a}_i(x(k))\underline{m}_i(x(k)) + \bar{a}_i(x(k))\overline{m}_i(x(k)), \\ m_i(x(k)) &\geq 0, \\ 0 &\leq \underline{a}_i(x(k)) \leq 1,\ 0 \leq \bar{a}_i(x(k)) \leq 1, \\ 1 &= \sum_{i=1}^{r} m_i(x(k)), \\ 1 &= \underline{a}_i(x(k)) + \bar{a}_i(x(k)) \end{aligned} \tag{3}$$

with $\underline{a}_i(x(k))$ and $\bar{a}_i(x(k))$ being nonlinear weighting functions and $m_i(x(k))$ regarded as the grades of membership.

In this study, the IT2 filter does not share the same membership functions and number of fuzzy rules with the system for enhancing the flexibility of the filter design. The details of an s-rule fuzzy filter are as follows:

Filter Rule j : IF $g_1(x(k))$ is N_{j1}, and $g_2(x(k))$ is N_{j2} and ,..., and $g_{\bar{\Psi}}(x(k))$ is $N_{j\bar{\Psi}}$, THEN

$$\begin{aligned} x_f(k+1) &= A_{fj}x_f(k) + B_{fj}y_f(k), \\ z_f(k) &= L_{fj}x_f(k), \end{aligned} \tag{4}$$

where $x_f(k) \in R^{n_x}$, $z_f(k) \in R^{n_z}$ and $A_{fj},\ B_{fj},\ L_{fj}$ are filter matrices to be determined; s is the number of rules of the filter. The following interval sets describe the firing strength of the jth rule:

$$\Omega_j(x(k)) = [\underline{\omega}_j(x(k)),\ \overline{\omega}_j(x(k))],$$

where

$$\begin{aligned} \underline{\omega}_i(x(k)) &= \prod_{q=1}^{\bar{\Psi}} \underline{u}_{N_{jq}}(g_q(x(k))) \geq 0, \\ \overline{\omega}_i(x(k)) &= \prod_{q=1}^{\bar{\Psi}} \overline{u}_{N_{jq}}(g_q(x(k))) \geq 0, \\ \overline{u}_{N_{jq}}(g_q(x(k))) &\geq \underline{u}_{N_{jq}}(g_q(x(k))) \geq 0, \\ \overline{\omega}_i(x(k)) &\geq \underline{\omega}_i(x(k)) \geq 0, \end{aligned}$$

$\underline{u}_{N_{jq}}(g_p(x(k))),\ \overline{u}_{N_{jq}}(g_p(x(k))),\ \underline{\omega}_i(x(k))$ and $\overline{\omega}_i(x(k))$ denote lower membership function, upper membership function, lower grade of membership and upper grade of membership, respectively.

Then, the overall discrete-time T-S fuzzy filter is as follows:

$$\begin{aligned} x_f(k+1) &= \sum_{j=1}^{s} \omega_j(x(k))\,[A_{fj}x_f(k) + B_{fj}y_f(k)], \\ z_f(k) &= \sum_{j=1}^{s} \omega_j(x(k))\,L_{fj}x_f(k), \end{aligned} \tag{5}$$

where

$$\begin{aligned} \omega_j(x(k)) &= \frac{\tilde{\omega}_j(x(k))}{\sum_{j=1}^{s}\tilde{\omega}_j(x(k))},\ \omega_j(x(k)) \geq 0, \\ \tilde{\omega}_j(x(k)) &= \underline{b}_j(x(k))\underline{\omega}_j(x(k)) + \bar{b}_j(x(k))\overline{\omega}_j(x(k)), \\ 0 &\leq \underline{b}_j(x(k)) \leq 1,\ 0 \leq \underline{b}_j(x(k)) \leq 1, \\ \sum_{j=1}^{s} \omega_j(x(k)) &= 1,\ \underline{b}_j(x(k)) + \bar{b}_j(x(k)) = 1, \end{aligned} \tag{6}$$

$\underline{b}_i(x(k))$ and $\bar{b}_j(x(k))$ are nonlinear functions and $\omega_j(x(k))$ is regarded as the grades of membership. For brevity, $m_i \triangleq m_i(x(k))$, $\omega_j \triangleq \omega_j(x(k))$.

Remark 1: Compared with the existing filter design results, the main advantages of filter (4) designed in this paper lie in two sides. One is that the filter is designed in the framework of the IT2 T-S fuzzy model, which can deal with uncertainties. The other is that the membership functions and the number of fuzzy rules are not the same as those of the system model, which, to some degree, can reduce the implementation and computation complexity.

Remark 2: It can be seen that the expression for $\omega_j(x(k))$ is not the same as that for $m_i(x(k))$. In order to enhance the flexibility of the filter design and obtain less conservative results, this design strategy is adopted.

A. Measurement Quantization

The quantizer used in the system is as in [45] and the model is demoted as

$$\bar{y}(k) = \tilde{U}(y(k)) = [\tilde{U}_1(y_1(k))\ \tilde{U}_2(y_2(k))\ \ldots\ \tilde{U}_{n_y}(y_{n_y}(k))]^T,$$

where $\bar{y}(k) \in R^{n_y}$ is the quantized signal transmitted to the filter via the network. The quantizer $\tilde{U}(y(k))$ is the logarithmic type, whose quantization levels for each $\tilde{U}_c(\cdot)$ $(1 \leq c \leq n_y)$ are described by

$$\begin{aligned} q_c &= \{\pm\chi_l^{(c)}, \chi_l^{(c)} = \kappa_c^l \chi_0^{(c)}, l = 0, \pm1, \pm2, \ldots\} \cup \{0\}, \\ 0 &< \kappa_c < 1,\ \chi_0^{(c)} > 0. \end{aligned}$$

The quantizer maps the whole segments to the quantization level via corresponding each of the quantization level to a segment. Define the logarithmic quantizer $\tilde{U}_c(\cdot)$ as

$$\tilde{U}_c(y_c(k)) = \begin{cases} \chi_l^{(c)}, & \frac{1}{1+\varepsilon_c}\chi_l^{(c)} < y_c(k) \leq \frac{1}{1-\varepsilon_c}\chi_l^{(c)}, \\ 0, & y_c(k) = 0, \\ -\tilde{U}_c(-y_c(k)), & y_c(k) < 0, \end{cases}$$

with $\varepsilon_c = \frac{1-\kappa_c}{1+\kappa_c}$.

According to the sector-bound method provided in [46], the quantization errors can be handled as

$$\bar{y}(k) = (I + \bar{\Delta}(k))y(k), \tag{7}$$

where $\bar{\Delta}(k) = diag\{\bar{\Delta}_1(k), \ldots, \bar{\Delta}_{n_y}(k)\}$, $\left|\bar{\Delta}_c(y_c(k))\right| < \varepsilon_c$, $c = 1, 2, \ldots, n_y$.

B. Communication Links

As shown in Fig. 1, the network is used to exchange information in the system such that the intermittent data missing phenomenon arises. The quantized measurement $\bar{y}(k)$ is hardly transmitted to the filter perfectly (e.g., $\bar{y}(k) \neq y_f(k)$). Therefore, the following model is utilized in this study in order to describe the stochastic measurement missing.

$$y_f(k) = e(k)\,\bar{y}(k), \tag{8}$$

in which $e(k)$ obeys the Bernoulli process and describes the unreliable communication link between the quantizer and the filter. Assume $e(k)$ as follows:

$$\begin{aligned} \text{Prob}\{e(k) = 1\} &= E\{e(k)\} = \bar{e}, \\ \text{Prob}\{e(k) = 0\} &= 1 - \bar{e}. \end{aligned}$$

From (3) and (6), we know

$$\sum_{i=1}^{r} m_i = \sum_{j=1}^{s} \omega_j = \sum_{i=1}^{r}\sum_{j=1}^{s} m_i\omega_j = 1. \tag{9}$$

According to (2), (5), (7) and (8), the filtering error system is represented as

$$\begin{aligned} \bar{x}(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{s} m_i\omega_j[\tilde{A}_{ij}\bar{x}(k) + \tilde{B}_{ij}w(k)], \\ \bar{z}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{s} m_i\omega_j\bar{L}_{ij}\bar{x}(k), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \tilde{A}_{ij} &= A_{1ij} + \tilde{e}(k)A_{2ij}, \;\; \tilde{B}_{ij} = B_{1ij} + \tilde{e}(k)B_{2ij}, \\ A_{1ij} &= \begin{bmatrix} A_i & 0 \\ \bar{e}B_{fj}(I+\bar{\Delta}(k))C_i & A_{fj} \end{bmatrix}, \\ A_{2ij} &= \begin{bmatrix} 0 & 0 \\ B_{fj}(I+\bar{\Delta}(k))C_i & 0 \end{bmatrix}, \\ B_{1ij} &= \begin{bmatrix} B_i \\ \bar{e}B_{fj}(I+\bar{\Delta}(k))D_i \end{bmatrix}, \\ B_{2ij} &= \begin{bmatrix} 0 \\ B_{fj}(I+\bar{\Delta}(k))D_i \end{bmatrix}, \\ \bar{L}_{ij} &= \begin{bmatrix} L_i & -L_{fj} \end{bmatrix}, \;\; \bar{x}(k) = \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix}, \\ \bar{z}(k) &= z(k) - z_f(k), \quad \tilde{e}(k) = e(k) - \bar{e}. \end{aligned}$$

It is obvious that

$$E\{\tilde{e}(k)\} = 0, \;\; E\{\tilde{e}(k)\tilde{e}(k)\} = \bar{e}(1-\bar{e}).$$

To analyze the system (10), we refer to the work in [42]. The state space of interest H is divided into q connected sub-state spaces $H_z(z = 1, 2, ..., q)$ with $H = \cup_{z=1}^{q} H_z$. Then we divide the FOU into $\varsigma + 1$ sub-FOUs. For $l = 1, 2, ..., \varsigma + 1$. Expressions of lower and upper membership functions in the lth sub-FOU are rewritten as follows:

$$\underline{h}_{ijl}(x(k)) = \sum_{z=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}...\sum_{i_{n_x}=1}^{2}\prod_{\tau=1}^{n_x} \upsilon_{\tau i_\tau zl}(x_\tau(k))\underline{\vartheta}, \tag{11}$$

$$\overline{h}_{ijl}(x(k)) = \sum_{z=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}...\sum_{i_{n_x}=1}^{2}\prod_{\tau=1}^{n_x} \upsilon_{\tau i_\tau zl}(x_\tau(k))\overline{\vartheta}, \tag{12}$$

$$\begin{aligned} 0 &\le \underline{h}_{ijl}(x(k)) \le \overline{h}_{ijl}(x(k)) \le 1, \\ \underline{\vartheta} &\le \overline{\vartheta}, \end{aligned}$$

where $\underline{\vartheta}$ and $\overline{\vartheta}$ are constant scalars to be determined and represent $\underline{\vartheta}_{iji_1i_2...i_{n_x}zl}$ and $\overline{\vartheta}_{iji_1i_2...i_{n_x}zl}$ respectively; $0 \le \upsilon_{\tau i_s zl}(x_\tau(k)) \le 1$ with the property $\upsilon_{\tau 1zl}(x_\tau(k)) + \upsilon_{\tau 2zl}(x_\tau(k)) = 1$ for $\tau, s = 1, 2, ..., n_x$; $l = 1, 2, ..., \varsigma + 1$; $i_\tau = 1, 2$; $x(k) \in H_z$; and $\upsilon_{\tau i_s zl}(x_\tau(k)) = 0$ if otherwise. Thus, $\sum\limits_{z=1}^{q}\sum\limits_{i_1=1}^{2}\sum\limits_{i_2=1}^{2}...\sum\limits_{i_{n_x}=1}^{2}\prod\limits_{\tau=1}^{n_x} \upsilon_{\tau i_\tau zl}(x_\tau(k)) = 1$ for all l, which is used to analyze the system stability. For brevity, $\underline{h}_{ijl} \triangleq \underline{h}_{ijl}(x(k))$ and $\overline{h}_{ijl} \triangleq \overline{h}_{ijl}(x(k))$.

Then, system (10) can be rewritten as follows:

$$\begin{aligned} \bar{x}(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{s} h_{ij}(x(k))[\tilde{A}_{ij}\bar{x}(k) + \tilde{B}_{ij}w(k)], \\ \bar{z}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{s} h_{ij}(x(k))\bar{L}_{ij}\bar{x}(k), \end{aligned} \tag{13}$$

where

$$\begin{aligned} h_{ij}(x(k)) &= m_i\omega_j = \sum_{l=1}^{\varsigma+1} \rho_{ijl}(x(k))\Big[\underline{\zeta}_{ijl}(x(k))\underline{h}_{ijl} \\ &\quad + \overline{\zeta}_{ijl}(x(k))\overline{h}_{ijl}\Big], \\ 1 &= \sum_{i=1}^{r}\sum_{j=1}^{s} h_{ij}(x(k)), \\ 0 &\le \underline{\zeta}_{ijl}(x(k)) \le \overline{\zeta}_{ijl}(x(k)) \le 1, \\ \rho_{ijl}(x(k)) &= \begin{cases} 1, & h_{ijl}(x(k)) \in \text{the sub-FOU } l, \\ 0, & \text{else}, \end{cases} \end{aligned}$$

where $\underline{\zeta}_{ijl}(x(k))$ and $\overline{\zeta}_{ijl}(x(k))$ are two functions and unnecessary to be known. $\underline{\zeta}_{ijl}(x(k)) + \overline{\zeta}_{ijl}(x(k)) = 1$ for i, j and l. The following definition is used to obtain the main result in this paper. For brevity, $h_{ij} \triangleq h_{ij}(x(k))$, $\underline{\zeta}_{ijl} \triangleq \underline{\zeta}_{ijl}(x(k))$, $\overline{\zeta}_{ijl} \triangleq \overline{\zeta}_{ijl}(x(k))$ and $\rho_{ijl} \triangleq \rho_{ijl}(x(k))$.

Definition 1: [35] The filtering error system in (13) is stochastically stable in the mean square when $w(k) \equiv 0$ for any initial condition $\bar{x}(0)$ if there is a finite $W > 0$ such that

$$E\left\{\sum_{k=0}^{\infty} |\bar{x}(k)|^2 |_{\bar{x}(0)}\right\} < \bar{x}(0)^T W \bar{x}(0).$$

The purpose of the paper is to design a filter on the basis of IT2 framework such that the following conditions are satisfied simultaneously.

1) The error system in (13) is stochastically stable;

2) Under zero initial condition, with a given positive scalar γ, the error output $\bar{z}(k)$ satisfies

$$E\left\{\sqrt{\sum_{k=0}^{\infty} |\bar{z}(k)|^2}\right\} \le \gamma \|w\|_2.$$

Remark 3: The parameter uncertainties can lead to the state-variable uncertainties, which results in a less accurate model when we model the physical plants. They are fully taken into consideration in the framework of IT2 T-S fuzzy model, in which the membership function is an interval rather than a specific one. The membership functions are determined by lower and upper functions with weighting functions, which makes the parameter uncertainties concluded in the membership functions.

III. Main Results

In this section, for given filter gain matrices A_{fj}, B_{fj}, L_{fj} $(j = 1, 2, ..., s)$, the sufficient condition is provided which ensures error system (13) is stochastically stable with a predefined H_∞ performance.

Theorem 1: With the FOU and state space divided into $\varsigma+1$ sub-FOUs and q connected sub-state spaces, for the given filter gain matrices $A_{fj},\ B_{fj}.\ L_{fj}\ (j=1,2,...,s)$ and positive scalar γ, the system in (13) is stochastically stable and satisfies a given H_∞ performance, if there exist symmetric matrices $P>0,\ W_{ijl}>0\ (i=1,2,...,r,\ j=1,2,...,s,\ l=1,2,...,\varsigma+1)$ and M with appropriate dimensions such that the following inequalities hold:

$$\sum_{i=1}^{r}\sum_{j=1}^{s}\check{\Gamma}-M < 0, \tag{14}$$

$$\bar{Q}_{ijl}-W_{ijl}+M < 0, \tag{15}$$

where

$$\begin{aligned}
\check{\Gamma} &= \underline{\vartheta}\bar{Q}_{ijl}-(\underline{\vartheta}-\overline{\vartheta})W_{ijl}+\underline{\vartheta}M,\\
\check{Q}_{ijl} &= \bar{A}_{ij}^T\bar{P}\bar{A}_{ij}-P,\\
\bar{Q}_{ijl} &= \begin{bmatrix} \check{Q}_{ijl}+\bar{L}_{ij}^T\bar{L}_{ij} & \bar{A}_{ij}^T\bar{P}\bar{B}_{ij} \\ * & -\gamma^2 I+\bar{B}_{ij}^T\bar{P}\bar{B}_{ij}\end{bmatrix},\\
\bar{A}_{ij} &= \begin{bmatrix} A_{1ij}^T & fA_{2ij}^T \end{bmatrix}^T,\ f=\sqrt{\bar{e}\,(1-\bar{e})},\\
\bar{B}_{ij} &= \begin{bmatrix} B_{1ij}^T & fB_{2ij}^T \end{bmatrix}^T, \bar{P}=\mathrm{diag}\{P,P\}.
\end{aligned}$$

Proof: Consider the following Lyapunov function for system (13):

$$V(k)=\bar{x}^T(k)\,P\bar{x}(k),$$

where $P>0$ is the matrix to be determined. Let $\chi^T(k)=\begin{bmatrix}\bar{x}^T(k) & w^T(k)\end{bmatrix}$. Then, the difference is computed as:

$$\begin{aligned}
\Delta V(k) &= E\{V(k+1)\,|\chi(k)\}-V(k)\\
&\le E\left\{\chi^T(k)\sum_{i=1}^{r}\sum_{j=1}^{s}h_{ij}\tilde{F}_{ij}^T P\tilde{F}_{ij}\chi(k)\right\}\\
&\quad -\bar{x}^T(k)\,P\bar{x}(k)\\
&= \chi^T(k)\sum_{i=1}^{r}\sum_{j=1}^{s}h_{ij}Q_{ijl}\chi(k),
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
\tilde{F}_{ij} &= \begin{bmatrix}\tilde{A}_{ij} & \tilde{B}_{ij}\end{bmatrix},\\
Q_{ijl} &= \begin{bmatrix}\check{Q}_{ijl} & \bar{A}_{ij}^T\bar{P}\bar{B}_{ij}\\ * & \bar{B}_{ij}^T\bar{P}\bar{B}_{ij}\end{bmatrix}.
\end{aligned}$$

To achieve the sufficient condition, the following slack matrices are introduced.

$$\left\{\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\left[(1-\bar{\zeta}_{ijl})\underline{h}_{ijl}+\bar{\zeta}_{ijl}\overline{h}_{ijl}\right]-1\right\}M=0, \tag{17}$$

$$-\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}(1-\bar{\zeta}_{ijl})(\underline{h}_{ijl}-\overline{h}_{ijl})W_{ijl}\ge 0. \tag{18}$$

Substituting (17)–(18) into (16), it can be found that

$$\begin{aligned}
\Delta V(k) \le\ & \chi^T(k)\left\{\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\left[\underline{h}_{ijl}Q_{ijl}\right.\right.\\
& \left.\left.-(\underline{h}_{ijl}-\overline{h}_{ijl})W_{ijl}+\underline{h}_{ijl}M\right]-M\right\}\chi(k)\\
& -\chi^T(k)\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\bar{\zeta}_{ijl}(\underline{h}_{ijl}-\overline{h}_{ijl})\\
& \times(Q_{ijl}-W_{ijl}+M)\chi(k).
\end{aligned} \tag{19}$$

When $w(k)=0$, it can be seen from (14) and (15) that $\Xi<0$, where

$$\begin{aligned}
\Xi =\ & \sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\left[\underline{h}_{ijl}\check{Q}_{ijl}-(\underline{h}_{ijl}\right.\\
& \left.-\overline{h}_{ijl})W_{1ijl}+\underline{h}_{ijl}M_1\right]-M_1\\
& -\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\bar{\zeta}_{ijl}(\underline{h}_{ijl}-\overline{h}_{ijl})\\
& \times(\check{Q}_{ijl}-W_{1ijl}+M_1),
\end{aligned}$$

where

$$W_{ijl}=\begin{bmatrix}W_{1ijl} & W_{2ijl}\\ * & W_{3ijl}\end{bmatrix},\ M=\begin{bmatrix}M_1 & M_2\\ * & M_3\end{bmatrix}.$$

Then, the following inequality holds,

$$\begin{aligned}
& E\left\{\bar{x}^T(k+1)\,P\bar{x}(k+1)\right\}-\bar{x}^T(k)\,P\bar{x}(k)\\
\le\ & -\lambda_{\min}(-\Xi)\,\bar{x}^T(k)\,\bar{x}(k)\,.
\end{aligned}$$

Summing the mathematical expectation for both sides of the inequality from $k=0,1,...,d$ with any $d\ge 1$, one can obtain

$$\begin{aligned}
& E\left\{\bar{x}^T(d+1)\,P\bar{x}(d+1)\right\}-\bar{x}^T(0)\,P\bar{x}(0)\\
\le\ & -\lambda_{\min}(-\Xi)\,E\left\{\sum_{k=0}^{d}|\bar{x}(k)|^2\right\},
\end{aligned}$$

which yields

$$E\left\{\sum_{k=0}^{d}|\bar{x}(k)|^2\right\}\le(\lambda_{\min}(-\Xi))^{-1}\,\bar{x}^T(0)\,P\bar{x}(0)\,,$$

with the initial condition $\bar{x}(0)$. When $d=1,2,...,\infty$, considering $E\left\{\bar{x}^T(\infty)\,P\bar{x}(\infty)\right\}\ge 0$, we have

$$E\left\{\sum_{k=0}^{d}|\bar{x}(k)|^2\right\}\le\bar{x}^T(0)\,W\bar{x}(0)\,,$$

where $W\triangleq(\lambda_{\min}(-\Xi))^{-1}P$, which means $W>0$. From Definition 1, the error system in (13) is stochastically stable. Next, consider the H_∞ performance of the error system in (13). Under the zero initial condition, the H_∞ performance index is

$$\begin{aligned}
J \triangleq\ & E\{V(k+1)\,|\chi(k)\}-V(k)\\
& +E\left\{\bar{z}^T(k)\,\bar{z}(k)\,|\chi(k)\right\}-\gamma^2 w^T(k)\,w(k)\,.
\end{aligned}$$

Then, we obtain

$$\begin{aligned} J &= E\left\{\bar{z}^T(k)\bar{z}(k)\right\} - \gamma^2 w^T(k)w(k) + \Delta V(k) \\ &\le \chi^T(k)\left\{\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\left[\underline{h}_{ijl}\bar{Q}_{ijl}\right.\right. \\ &\quad \left.\left. -(\underline{h}_{ijl}-\overline{h}_{ijl})W_{ijl}+\underline{h}_{ijl}M\right]-M\right\}\chi(k) \\ &\quad -\chi^T(k)\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\bar{\zeta}_{ijl}(\underline{h}_{ijl} \\ &\quad -\overline{h}_{ijl})(\bar{Q}_{ijl}-W_{ijl}+M)\chi(k). \end{aligned} \tag{20}$$

Substituting (11) and (12) into (20), we have

$$\begin{aligned} J &\le \chi^T(k)\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl} \\ &\quad \sum_{z=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_{n_x}=1}^{2}\prod_{\tau=1}^{n_x}\upsilon_{\tau i_\tau zl}(x_\tau(k))\breve{\Gamma}\chi(k) \\ &\quad -\chi^T(k)M\chi(k)-\chi^T(k)\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{l=1}^{\varsigma+1}\rho_{ijl}\bar{\zeta}_{ijl} \\ &\quad \times\sum_{z=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_{n_x}=1}^{2}\prod_{\tau=1}^{n_x}\upsilon_{\tau i_\tau zl}(x_\tau(k))(\underline{\vartheta} \\ &\quad -\overline{\vartheta})(\bar{Q}_{ijl}-W_{ijl}+M)\chi(k). \end{aligned} \tag{21}$$

Recalling (11) and (12) and referring to [42], at any time, the equality $\sum_{l=1}^{\varsigma+1}\rho_{ijl}=1$ is ensured by only one $\rho_{ijl}=1$ for each fixed i,j value. According to (14) and (15), one can have

$$E\left\{z^T(k)z(k)\right\}-\gamma^2 w^T(k)w(k)+\Delta V(k)\le 0,$$

which yields $J \le 0$. Then, we can obtain $E\left\{\sqrt{\sum_{k=0}^{\infty}|z(k)|^2}\right\}\le\gamma\|w\|_2$. The proof is completed. ■

Remark 4: Theorem 1 provides a sufficient condition that ensures the error system is stochastically stable and satisfies the guaranteed H_∞ performance. However, due to the existence of quantization effect $\bar{\Delta}(k)$, it has difficulty on checking the effectiveness of Theorem 1. Then, the quantization effect can be eliminated. Next, we will show the new sufficient condition.

Theorem 2: For the given filter gain matrices A_{fj}, B_{fj}, L_{fj}, the quantizer $\tilde{U}_j(\cdot)$ and positive scalar γ, the system in (13) is stochastically stable and satisfies a given H_∞ performance, if there exist a scalar $\varepsilon>0$ and symmetric matrices $P>0$, $\tilde{W}_{1ijl}>0$, $\tilde{W}_{2ijl}$, $\tilde{W}_{3ijl}>0$ $(i=1,2,...,r,\ j=1,2,...,s,\ l=1,2,...,\varsigma+1)$ and $\tilde{M}_1,\ \tilde{M}_2,\ \tilde{M}_3$ with appropriate dimensions such that the following inequalities hold:

$$\begin{bmatrix} \bar{\Sigma}_1 & \bar{\Sigma}_3 & \varepsilon\bar{\Sigma}_2^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} \Sigma_1 & \Sigma_3 & \varepsilon\Sigma_2^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned} \bar{\Sigma}_1 &= \begin{bmatrix} \bar{\Theta}_1 & \bar{\Theta}_2 \\ * & \bar{\Theta}_3 \end{bmatrix},\ \bar{\Sigma}_2=\begin{bmatrix} 0 & 0 & 0 & \bar{C}_i & D_i \end{bmatrix}, \\ \bar{C}_i &= \begin{bmatrix} C_i & 0 \end{bmatrix},\ \bar{\Theta}_1=\mathrm{diag}\{-P^{-1},-P^{-1},-I\}, \\ \bar{\Sigma}_3 &= \begin{bmatrix} \varpi\bar{e}\tilde{B}_{fj} & \varpi f\tilde{B}_{fj} & 0 & 0 & 0 \end{bmatrix}^T,\ \varpi=\sqrt{\underline{\vartheta}}, \\ \bar{\Theta}_2 &= \begin{bmatrix} \varpi\hat{A}_{ij} & \varpi\hat{B}_{ij} \\ \varpi\check{A}_{ij} & \varpi\check{B}_{ij} \\ \varpi\bar{L}_{ij} & 0 \end{bmatrix},\ \hat{A}_{ij}=\begin{bmatrix} A_i & 0 \\ \bar{e}B_{fj}C_i & A_{fj} \end{bmatrix}, \\ \check{A}_{ij} &= \begin{bmatrix} 0 & 0 \\ fB_{fj}C_i & 0 \end{bmatrix},\ \hat{B}_{ij}=\begin{bmatrix} B_i \\ \bar{e}B_{fj}D_i \end{bmatrix}, \\ \check{B}_{ij} &= \begin{bmatrix} 0 \\ fB_{fj}D_i \end{bmatrix},\ \bar{\Theta}_3=\begin{bmatrix} \Phi_{1ijl} & \Phi_{2ijl} \\ * & \Phi_{3ijl} \end{bmatrix}, \\ \tilde{B}_{fj} &= \begin{bmatrix} 0 & B_{fj}^T \end{bmatrix}, \\ \Phi_{1ijl} &= -\underline{\vartheta}P-(\underline{\vartheta}-\overline{\vartheta})\tilde{W}_{1ijl}+(\underline{\vartheta}-\frac{1}{rs})\tilde{M}_1, \\ \Phi_{2ijl} &= -(\underline{\vartheta}-\overline{\vartheta})\tilde{W}_{2ijl}+(\underline{\vartheta}-\frac{1}{rs})\tilde{M}_2, \\ \Phi_{3ijl} &= -\underline{\vartheta}\gamma^2 I-(\underline{\vartheta}-\overline{\vartheta})\tilde{W}_{3ijl}+(\underline{\vartheta}-\frac{1}{rs})\tilde{M}_3, \\ \Sigma_1 &= \begin{bmatrix} \Theta_1 & \Theta_2 \\ * & \Theta_3 \end{bmatrix},\ \Sigma_2=\bar{\Sigma}_2, \\ \Sigma_3 &= \begin{bmatrix} \bar{e}\tilde{B}_{fj} & f\tilde{B}_{fj} & 0 & 0 & 0 \end{bmatrix}^T,\ \Theta_1=\bar{\Theta}_1, \\ \Theta_2 &= \begin{bmatrix} \hat{A}_{ij} & \hat{B}_{ij} \\ \check{A}_{ij} & \check{B}_{ij} \\ \bar{L}_{ij} & 0 \end{bmatrix},\ \Theta_3=\begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix}, \\ \Omega_1 &= -P-\tilde{W}_{1ijl}+\tilde{M}_1,\ \Omega_2=-\tilde{W}_{2ijl}+\tilde{M}_2, \\ \Omega_3 &= -\gamma^2 I-\tilde{W}_{3ijl}+\tilde{M}_3. \end{aligned}$$

Proof: Define

$$W_{ijl}=\begin{bmatrix} \tilde{W}_{1ijl} & \tilde{W}_{2ijl} \\ * & \tilde{W}_{3ijl} \end{bmatrix},\ M=\begin{bmatrix} \tilde{M}_1 & \tilde{M}_2 \\ * & \tilde{M}_4 \end{bmatrix}.$$

For (14) and (15), considering the method handling the quantization error in [25] and under the effect of Schur complement, we can obtain

$$\begin{aligned} \bar{\Sigma}_1+\varepsilon^{-1}\bar{\Sigma}_3\bar{\Sigma}_3^T+\varepsilon\bar{\Sigma}_2^T\bar{\Sigma}_2 &< 0, \\ \Sigma_1+\varepsilon^{-1}\Sigma_3\Sigma_3^T+\varepsilon\Sigma_2^T\Sigma_2 &< 0. \end{aligned}$$

By using Schur complement, (22) and (23) can be obtained. The proof is completed. ■

Based on Theorem 2, the existence condition of the filter can be developed in the following theorem.

Theorem 3: Considering the fuzzy system in (13), for a positive scalar γ and the quantizer $\tilde{U}_j(\cdot)$, the system in (13) is stochastically stable and satisfies a given H_∞ performance, if there exist a scalar $\varepsilon>0$ and symmetric matrices $\bar{P}_1>0$, $\bar{W}>0$, $\bar{W}_{1ijl}>0$, $\bar{W}_{2ijl}$, $\bar{W}_{3ijl}$, $\bar{W}_{4ijl}>0$, $\bar{W}_{5ijl}$, $\bar{W}_{6ijl}>0$, and matrices $\bar{A}_{fj}$, $\bar{B}_{fj}$, $\bar{L}_{fj}$, $\bar{M}_1,\bar{M}_2,\bar{M}_3$, $\bar{M}_4,\bar{M}_5,\bar{M}_6$ satisfying the following conditions:

$$\begin{bmatrix} \hat{P} & \varpi\breve{\Pi}_{ijl} \\ * & \tilde{\Phi} \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} \hat{P} & \breve{\Pi}_{ijl} \\ * & \tilde{\Omega} \end{bmatrix} < 0, \tag{25}$$

where

$$\breve{\Pi}_{ijl} = \begin{bmatrix} \Delta_{1ijl} & \bar{A}_{fj} & \Delta_{3ijl} & \bar{e}\bar{B}_{fj} & 0 \\ \Delta_{2ijl} & \bar{A}_{fj} & \Delta_{4ijl} & \bar{e}\bar{B}_{fj} & 0 \\ f\bar{B}_{fj}C_i & 0 & f\bar{B}_{fj} & f\bar{B}_{fj} & 0 \\ f\bar{B}_{fj}C_i & 0 & f\bar{B}_{fj} & f\bar{B}_{fj} & 0 \\ L_i & -\bar{L}_{fj} & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Phi} = \begin{bmatrix} \bar{\Phi}_{1ijl} & \bar{\Phi}_{2ijl} & \bar{\Phi}_{3ijl} & 0 & \varepsilon C_i^T \\ * & \bar{\Phi}_{4ijl} & \bar{\Phi}_{5ijl} & 0 & 0 \\ * & * & \bar{\Phi}_{6ijl} & 0 & \varepsilon D_i^T \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix},$$

$$\begin{aligned}
\hat{P} &= \text{diag}\{Q, Q, -I\}, \ \Delta_{1ijl} = \bar{P}_1 A_i + \bar{e}\bar{B}_{fj}C_i, \\
\Delta_{2ijl} &= \bar{W}^T A_i + \bar{e}\bar{B}_{fj}C_i, \ Q = \begin{bmatrix} -\bar{P}_1 & -\bar{W} \\ * & -\bar{W} \end{bmatrix}, \\
\Delta_{3ijl} &= \bar{P}_1 B_i + \bar{e}\bar{B}_{fj}D_i, \ \Delta_{4ijl} = \bar{W}^T B_i + \bar{e}\bar{B}_{fj}D_i, \\
\bar{\Phi}_{1ijl} &= -\underline{\vartheta}\bar{P}_1 - (\underline{\vartheta} - \overline{\vartheta})\bar{W}_{1ijl} + (\underline{\vartheta} - \frac{1}{rs})\bar{M}_1, \\
\bar{\Phi}_{2ijl} &= -\underline{\vartheta}\bar{W} - (\underline{\vartheta} - \overline{\vartheta})\bar{W}_{2ijl} + (\underline{\vartheta} - \frac{1}{rs})\bar{M}_2, \\
\bar{\Phi}_{3ijl} &= -(\underline{\vartheta} - \overline{\vartheta})\bar{W}_{3ijl} + (\underline{\vartheta} - \frac{1}{rs})\bar{M}_3, \\
\bar{\Phi}_{4ijl} &= -\underline{\vartheta}\bar{W} - (\underline{\vartheta} - \overline{\vartheta})\bar{W}_{4ijl} + (\underline{\vartheta} - \frac{1}{rs})\bar{M}_4, \\
\bar{\Phi}_{5ijl} &= -(\underline{\vartheta} - \overline{\vartheta})\bar{W}_{5ijl} + (\underline{\vartheta} - \frac{1}{rs})\bar{M}_5, \\
\bar{\Phi}_{6ijl} &= -\underline{\vartheta}\gamma^2 I - (\underline{\vartheta} - \overline{\vartheta})\bar{W}_{6ijl} + (\underline{\vartheta} - \frac{1}{rs})\bar{M}_6,
\end{aligned}$$

$$\tilde{\Omega} = \begin{bmatrix} \bar{\Omega}_1 & \bar{\Omega}_2 & \bar{\Omega}_3 & 0 & \varepsilon C_i^T \\ * & \bar{\Omega}_4 & \bar{\Omega}_5 & 0 & 0 \\ * & * & \bar{\Omega}_6 & 0 & \varepsilon D_i^T \\ * & 0 & * & -\varepsilon I & 0 \\ * & * & 0 & 0 & -\varepsilon I \end{bmatrix},$$

$$\begin{aligned}
\bar{\Omega}_1 &= -\bar{P}_1 - \bar{W}_{1ijl} + \bar{M}_1, \ \bar{\Omega}_2 = -\bar{W} - \bar{W}_{2ijl} + \bar{M}_2, \\
\bar{\Omega}_3 &= -\bar{W}_{3ijl} + \bar{M}_3, \ \bar{\Omega}_4 = -\bar{W} - \bar{W}_{4ijl} + \bar{M}_4, \\
\bar{\Omega}_5 &= -\bar{W}_{5ijl} + \bar{M}_5, \ \bar{\Omega}_6 = -\gamma^2 I - \bar{W}_{6ijl} + \bar{M}_6.
\end{aligned}$$

Furthermore, if the aforementioned conditions hold, the filter gain matrices in the form of (4) can be designed as follows:

$$\begin{bmatrix} A_{fj} & B_{fj} \\ L_{fj} & 0 \end{bmatrix} = \begin{bmatrix} \bar{W}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_{fj} & \bar{B}_{fj} \\ \bar{L}_{fj} & 0 \end{bmatrix}. \quad (26)$$

Proof: Firstly, to facilitate the proof, the matrix P is partitioned as

$$P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix},$$

where $P_3 > 0$; P_2 is non-singular matrix. The matrices $W_{ijl}, \ M$ are repartitioned as

$$\begin{aligned}
W_{ijl} &= \begin{bmatrix} W_{1ijl} & W_{2ijl} & W_{3ijl} \\ * & W_{4ijl} & W_{5ijl} \\ * & * & W_{6ijl} \end{bmatrix}, \\
M &= \begin{bmatrix} M_1 & M_2 & M_3 \\ * & M_4 & M_5 \\ * & * & M_6 \end{bmatrix}.
\end{aligned}$$

Additionally, some matrix variables are defined as follows

$$\begin{aligned}
\bar{P}_1 &= P_1, \ \bar{W} = P_2 P_3^{-T} P_2^T, \ \Gamma = \begin{bmatrix} I & 0 \\ 0 & P_2 P_3^{-1} \end{bmatrix}, \\
\bar{W}_{1ijl} &= W_{1ijl}, \ \bar{W}_{2ijl} = W_{2ijl} P_3^{-T} P_2^T, \ \bar{W}_{3ijl} = W_{3ijl}, \\
\bar{W}_{4ijl} &= P_2 P_3^{-1} W_{4ijl} P_3^{-T} P_2^T, \ \bar{W}_{5ijl} = P_2 P_3^{-1} W_{5ijl}, \\
\bar{W}_{6ijl} &= W_{6ijl}, \ \bar{M}_1 = M_1, \ \bar{M}_2 = M_2 P_3^{-T} P_2^T, \\
\bar{M}_3 &= M_3, \bar{M}_4 = P_2 P_3^{-1} M_4 P_3^{-T} P_2^T, \ \bar{M}_6 = M_6, \\
\bar{M}_5 &= P_2 P_3^{-1} M_5, \ \bar{A}_{fj} = P_2 A_{fj} P_3^{-T} P_2^T, \\
\bar{B}_{fj} &= P_2 B_{fj}, \ \bar{L}_{fj} = L_{fj} P_3^{-T} P_2^T.
\end{aligned}$$

Pre- and post-multiplying (24) by diag$\{\Gamma^{-1}, \Gamma^{-1}, I, \Gamma^{-1}, I, I, I\}$ and its transposition and then performing congruence transformation by diag$\{P^{-1}, P^{-1}, I, P^{-1}, I, I, I\}$ and its transposition, we can obtain the inequality (22). Thus the inequality (24) holds. Similarly, the inequality (25) can be obtained with the same operation. Moreover, with the operation of the similarity transformation to (4), we can get the desired filter matrices form in (26). The proof is completed. ■

Remark 5: From Theorem 3, one can see that the elements considered in the paper (e.g., data loss, data quantization and parameter uncertainties) affects the stability of the system. By solving (24) and (25), the desired filter matrices can be obtained, which guarantees the error system is stochastically stable and preserves the H_∞ performance.

The procedure to apply Theorem 3 to design the IT2 filter is as follows:

- Step 1. Give the IT2 T-S model for the nonlinear NCSs to be considered.
- Step 2. Obtain the IT2 fuzzy model for the system in Step 1 on the basis of the method used in [41] and [42].
- Step 3. Design the IT2 fuzzy filter (5) for the fuzzy system in Step 2.
- Step 4. Give the probability for data transmitted successfully $\bar{\alpha}$, $\bar{\beta}$, the disturbance attenuation γ and the quantizer $\tilde{U}_j(\cdot)$.
- Step 5. Solve the solution to the LMIs (24) and (25) to obtain the filter gains A_{fj}, B_{Fj} and L_{fj}.
- Step 6. If A_{fj}, B_{Fj} and L_{fj} do not exist, return to Step 4 with different γ and $\tilde{U}_j(\cdot)$.

Without considering the data quantization, the following corollary can be obtained, which is used to demonstrate the superiority over the type-1 T-S fuzzy model.

Corollary 1: Considering the fuzzy system in (13) without data quantization, the system in (13) is stochastically stable and satisfies a given H_∞ performance, if there exist symmetric matrices $\bar{P}_1 > 0$, $\bar{W} > 0$, $\bar{W}_{1ijl} > 0$, $\bar{W}_{2ijl}, \bar{W}_{3ijl}, \bar{W}_{4ijl} > 0$, $\bar{W}_{5ijl}, \bar{W}_{6ijl} > 0$, and matrices $\bar{A}_{fj}$, $\bar{B}_{fj}$, $\bar{L}_{fj}$, $\bar{M}_1, \bar{M}_2, \bar{M}_3$, $\bar{M}_4, \bar{M}_5, \bar{M}_6$ satisfying the following conditions:

$$\begin{aligned}
\begin{bmatrix} \hat{P} & \varpi\breve{\Pi}_{1ijl} \\ * & \tilde{\Phi}_1 \end{bmatrix} &< 0, \\
\begin{bmatrix} \hat{P} & \breve{\Pi}_{1ijl} \\ * & \tilde{\Omega}_1 \end{bmatrix} &< 0,
\end{aligned}$$

where

$$\breve{\Pi}_{1ijl}=\begin{bmatrix}\Delta_{1ijl} & \bar{A}_{fj} & \Delta_{3ijl}\\ \Delta_{2ijl} & \bar{A}_{fj} & \Delta_{4ijl}\\ f\bar{B}_{fj}C_i & 0 & f\bar{B}_{fj}\\ f\bar{B}_{fj}C_i & 0 & f\bar{B}_{fj}\\ L_i & -\bar{L}_{fj} & 0\end{bmatrix},$$

$$\tilde{\Phi}_1=\begin{bmatrix}\bar{\Phi}_{1ijl} & \bar{\Phi}_{2ijl} & \bar{\Phi}_{3ijl}\\ * & \bar{\Phi}_{4ijl} & \bar{\Phi}_{5ijl}\\ * & * & \bar{\Phi}_{6ijl}\end{bmatrix}, \tilde{\Omega}_1=\begin{bmatrix}\bar{\Omega}_1 & \bar{\Omega}_2 & \bar{\Omega}_3\\ * & \bar{\Omega}_4 & \bar{\Omega}_5\\ * & * & \bar{\Omega}_6\end{bmatrix},$$

and other matrices are defined the same as in Theorem 3. The desired filter gains are as (26).

Proof: Since the proof is similar to above Theorems, it is omitted here. ∎

IV. Simulation Results

In this section, a practical example is provided to demonstrate the usefulness of the filtering design method proposed in this paper.

Example 1: In the practical example, the tunnel diode circuit [35] shown in Fig. 2 is used to illustrate the effectiveness of the method, whose dynamic equation is

$$i_D(t)=0.002v_D(t)+\partial v_D^3(t),$$

where ∂ is an uncertain parameter and $\partial \in [0.01, 0.03]$.

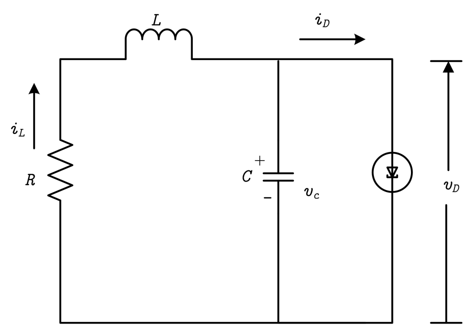

Fig. 2. Tunnel diode circuit

Let $x_1(t)=v_C(t)$ and $x_2(t)=i_L(t)$ be the state variables. Define $\bar{f}=0.002+\partial v_D^2(t)$. The circuit is governed by the following expressions:

$$\begin{aligned}C x_1(t) &= -\bar{f}x_1(t)+x_2(t),\\ L x_2(t) &= -x_1(t)-Rx_2(t)+w(t),\end{aligned}$$

in which $C=20$ mF, $L=1000$ mH and $R=10\ \Omega$.

Assuming $x_1(t)\in[x_{1_{\min}},\ x_{1_{\max}}]=[-3,\ 3]$ and referring to the modeling process in [41], the IT2 T-S fuzzy model is as

$$x(k+1)=\sum_{i=1}^{2} m_i\left(x(k)\right)\left[A_i x(t)+B_i w(t)\right],$$

where

$$A_1=\begin{bmatrix}\frac{-\bar{f}_{\min}}{C} & 50\\ -1 & -10\end{bmatrix}, B_1=\begin{bmatrix}0\\1\end{bmatrix}, \bar{f}_{\min}=0.02,$$

$$A_2=\begin{bmatrix}\frac{-\bar{f}_{\max}}{C} & 50\\ -1 & -10\end{bmatrix}, B_2=\begin{bmatrix}0\\1\end{bmatrix}, \bar{f}_{\max}=0.2720,$$

TABLE I
Lower and upper membership functions of the plant

Lower membership functions	Upper membership functions
$\underline{u}_{M_{11}}(x_1)=\frac{\bar{f}_{\max}-\bar{f}}{\bar{f}_{\max}-\bar{f}_{\min}}$	$\overline{u}_{M_{11}}(x_1)=\frac{\bar{f}_{\max}-\bar{f}}{\bar{f}_{\max}-\bar{f}_{\min}}$
with $\partial=0.03$	with $\partial=0.01$
$\underline{u}_{M_{12}}(x_1)=\frac{\bar{f}-\bar{f}_{\min}}{\bar{f}_{\max}-\bar{f}_{\min}}$	$\overline{u}_{M_{12}}(x_1)=\frac{\bar{f}-\bar{f}_{\min}}{\bar{f}_{\max}-\bar{f}_{\min}}$
with $\partial=0.01$	with $\partial=0.03$

TABLE II
Lower and upper membership functions of the filter

Lower membership functions	Upper membership functions
$\underline{u}_{N_{11}}(x_1)=0.8\times e^{-x_1^2}$	$\overline{u}_{N_{11}}(x_1)=\underline{u}_{N_{11}}(x_1)$
$\underline{u}_{N_{12}}(x_1)=1-\underline{u}_{N_{11}}(x_1)$	$\overline{u}_{N_{12}}(x_1)=\underline{u}_{N_{12}}(x_1)$

and the lower and upper membership function of the system and filter are described in Table I and Table II.

The determination of lower and upper membership functions $\underline{h}_{ijl}(x_1(k))$ and $\overline{h}_{ijl}(x_1(k))$ is related to the state space of interest $x_1(k)$. To this end, $x_1(k)\in[-3,3]$ is divided into 100 regions, equally. Then we can determine the lower and upper bounds of every sub-state, that is, $\underline{x}_{1kl}=3/50(k-51)$, $\overline{x}_{1kl}=3/50(k-50)$. To decrease the computational complexity, we only consider one sub-FOU (i.e., $l=1$). The relative parameters concerning the lower and upper membership functions $\underline{h}_{ijl}(x_1(k))$ and $\overline{h}_{ijl}(x_1(k))$ are listed below:

$$\underline{\vartheta}_{ij1k1}=\underline{m}_i(\underline{x}_{1kl})\underline{\omega}_j(\underline{x}_{1k1}),\ \upsilon_{11k1}(x_1)=1-\frac{x_1-\underline{x}_{1kl}}{\overline{x}_{1kl}-\underline{x}_{1kl}},$$
$$\overline{\vartheta}_{ij1k1}=\overline{m}_i(\underline{x}_{1kl})\overline{\omega}_j(\underline{x}_{1k1}),\ \upsilon_{12k1}(x_1)=1-\upsilon_{11k1}(x_1),$$
$$\underline{\vartheta}_{ij2k1}=\underline{m}_i(\overline{x}_{1kl})\underline{\omega}_j(\overline{x}_{1k1}),\ \overline{\vartheta}_{ij2k1}=\overline{m}_i(\overline{x}_{1kl})\overline{\omega}_j(\overline{x}_{1k1}).$$

Additionally, the membership function h_{ij} is determined by following weighting functions.

$$\begin{aligned}\underline{a}_i(x(k)) &= \sin^2(x(k)),\ \overline{a}_i(x(k))=1-\underline{a}_i(x(k)),\\ \underline{b}_i(x(k)) &= \cos^2(2x(k)),\ \overline{b}_i(x(k))=1-\underline{b}_i(x(k)).\end{aligned}$$

Under sampling period $T=0.5$ s, matrices are as follows:

$$\begin{aligned}A_1 &= \begin{bmatrix}-0.0199 & 0.4586\\ -0.0092 & -0.1107\end{bmatrix}, B_1=\begin{bmatrix}0.9990\\0.0112\end{bmatrix},\\ A_2 &= \begin{bmatrix}-0.0024 & -0.0055\\ 0.0001 & -0.0028\end{bmatrix}, B_2=\begin{bmatrix}0.2699\\0.0733\end{bmatrix},\\ C_1 &= \begin{bmatrix}0.3565 & 0.4256\end{bmatrix}, D_1=0.6998,\\ L_1 &= \begin{bmatrix}0.0007 & -0.0035\end{bmatrix},\\ C_2 &= \begin{bmatrix}1.2016 & 1.4498\end{bmatrix}, D_2=-0.1935,\\ L_2 &= \begin{bmatrix}0.0041 & 0.1262\end{bmatrix}.\end{aligned}$$

Let $\gamma=0.65$, $\bar{e}=0.8$ and the disturbance be

$$w(k)=\begin{cases}-0.2, & 80<k<120,\\ 0.2, & 140<k<200,\\ 0, & \text{else}.\end{cases}$$

Considering the quantization level $\kappa_j=0.9$ and the initial value $\chi_0=0.0001$, Fig. 3 plots the stochastic data missing with the probability $\bar{e}=0.8$ in the error system. Fig. 4 describes the effect of the quantizer. By computing the LMIs

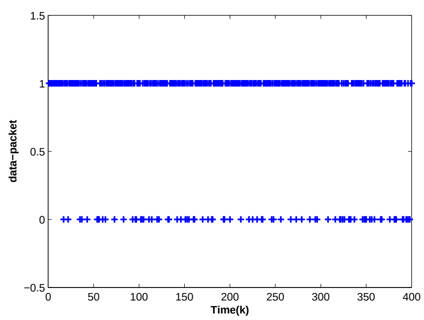

Fig. 3. Data loss with $\bar{e} = 0.8$

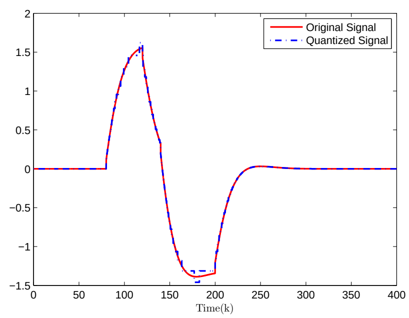

Fig. 4. The effect of quantization

(24) and (25), the desired filter gains are as

$$\begin{aligned} A_{f1} &= \begin{bmatrix} -0.0315 & 0.2788 \\ 0.0003 & -0.0292 \end{bmatrix}, \ B_{f1} = \begin{bmatrix} -0.0844 \\ 0.0010 \end{bmatrix}, \\ L_{f1} &= \begin{bmatrix} -0.0013 & -0.0011 \end{bmatrix}, \\ A_{f2} &= \begin{bmatrix} -0.0312 & 0.2763 \\ -0.0003 & -0.0233 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.0043 \\ 0.0001 \end{bmatrix}, \\ L_{f2} &= \begin{bmatrix} -0.0113 & -0.0223 \end{bmatrix}, \end{aligned}$$

and $\varepsilon = 0.0188$.

Fig. 5 and Fig. 6 present the estimation signal and the estimation error, respectively, which also demonstrates the filter design approach in this paper is effective.

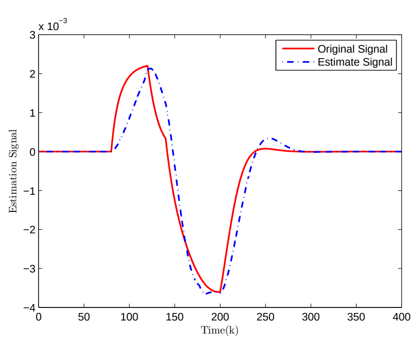

Fig. 5. Estimation signal

Example 2: In this example, the advantages and effectiveness of IT2 T-S fuzzy model are demonstrated. In [35], the tunnel diode circuit model with a fixed value of ∂ (i.e., $\partial = 0.01$) was used to illustrate the effectiveness of the method proposed in their work. However, when ∂ is changeable rather than fixed, the type-1 T-S fuzzy model in [35] does not work. In what follows, the same model as in Example 1, but with different system matrices is applied to Corollary 1. Under sampling period $T = 0.1$ s, we get

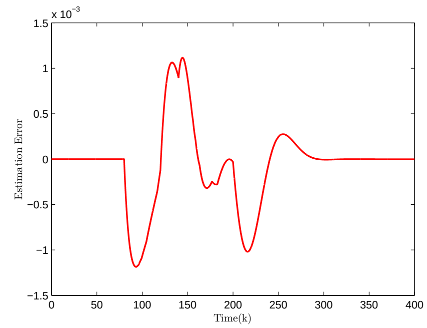

Fig. 6. Estimation error

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.8144 & 2.8909 \\ -0.0578 & 0.2420 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.1763 \\ 0.0582 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0.1871 & 1.4194 \\ -0.0284 & 0.2893 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.1147 \\ 0.0596 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.6248 & 0.9358 \end{bmatrix}, \ D_1 = -1.8275, \\ L_1 &= \begin{bmatrix} 0.0042 & 0.0369 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.8256 & -2.5096 \end{bmatrix}, \ D_2 = -2.8767, \\ L_2 &= \begin{bmatrix} -0.1079 & 0.0328 \end{bmatrix}. \end{aligned}$$

For demonstration, the number of sub-state is defined as 30.

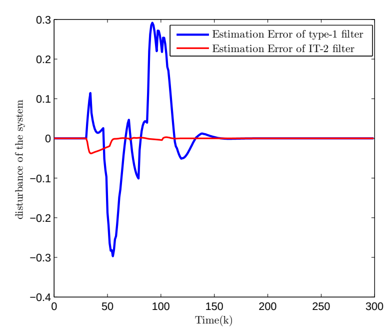

Fig. 7. Comparison of estimation error

Other parameters used to determine the membership functions are the same as those of Example 1. The H_∞ performance index and external disturbance input are defined as those in [35]. Then, Fig. 7 shows the estimation error of IT2 and type-1 T-S fuzzy filter, which demonstrates that the IT2 fuzzy model has less error. Additionally, the minimum $\gamma = 0.0096$ is less than that in [35], which illustrates the merits of IT2 T-S filter.

V. Conclusions

This paper has presented the results on filter design and H_∞ performance for nonlinear networked systems with the measurement missing and the data quantization. By using the

IT2 T-S fuzzy model, the parameter uncertainties have been handled effectively. The filter design conditions have been derived, which can solve the filter gains and guarantee the error system to be asymptotically stable. Finally, some examples are used to validate effectiveness of the method proposed in this paper.

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